Regression Performance

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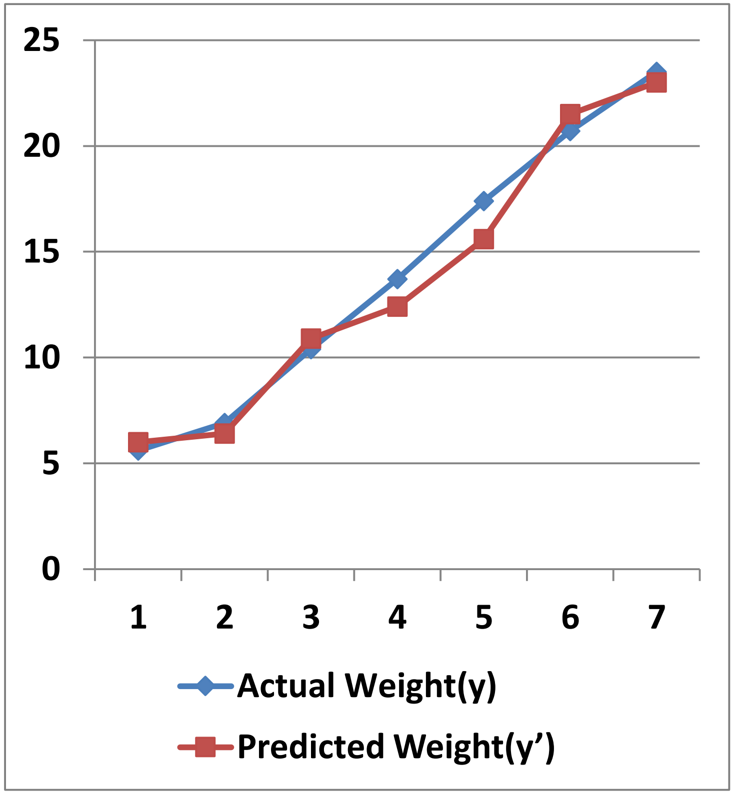
Regression performance

❖ Example: Predict a baby's weight(kg)

Age	Actual Weight(y)	Predicted Weight($\hat{y}$)
1	5.6	6.0
2	6.9	6.4
3	10.4	10.9
4	13.7	12.4
5	17.4	15.6
6	20.7	21.5
7	23.5	23.0



Regression performance

Average error

- Indicate whether the predictions are on average over- or under-predicted.

$$\text{Average error} = \frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})$$
$$= 0.342$$

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Regression performance

❖ Mean absolute error (MAE)

- Gives the magnitude of the average error

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|y-\hat{y}\right| = 0.829$$

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Regression performance

❖ Mean absolute percentage error (MAPE)

- Gives a percentage score of how predictions deviate (on average) from the actual values.

$$\text{MAPE} = 100\% \times \frac{1}{n}\sum_{i=1}^{n}\frac{|y-\hat{y}|}{|y|}$$

$$= 6.43\%$$

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Regression performance

❖ Mean squared error (MSE) and root MSE (RMSE)

- MSE: Standard error of estimate.
- RMSE: Same units as the variable predicted.

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})^2$$

$$= 0.926$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y-\hat{y})^2}$$

$$= 0.962$$

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Regression performance

Train error

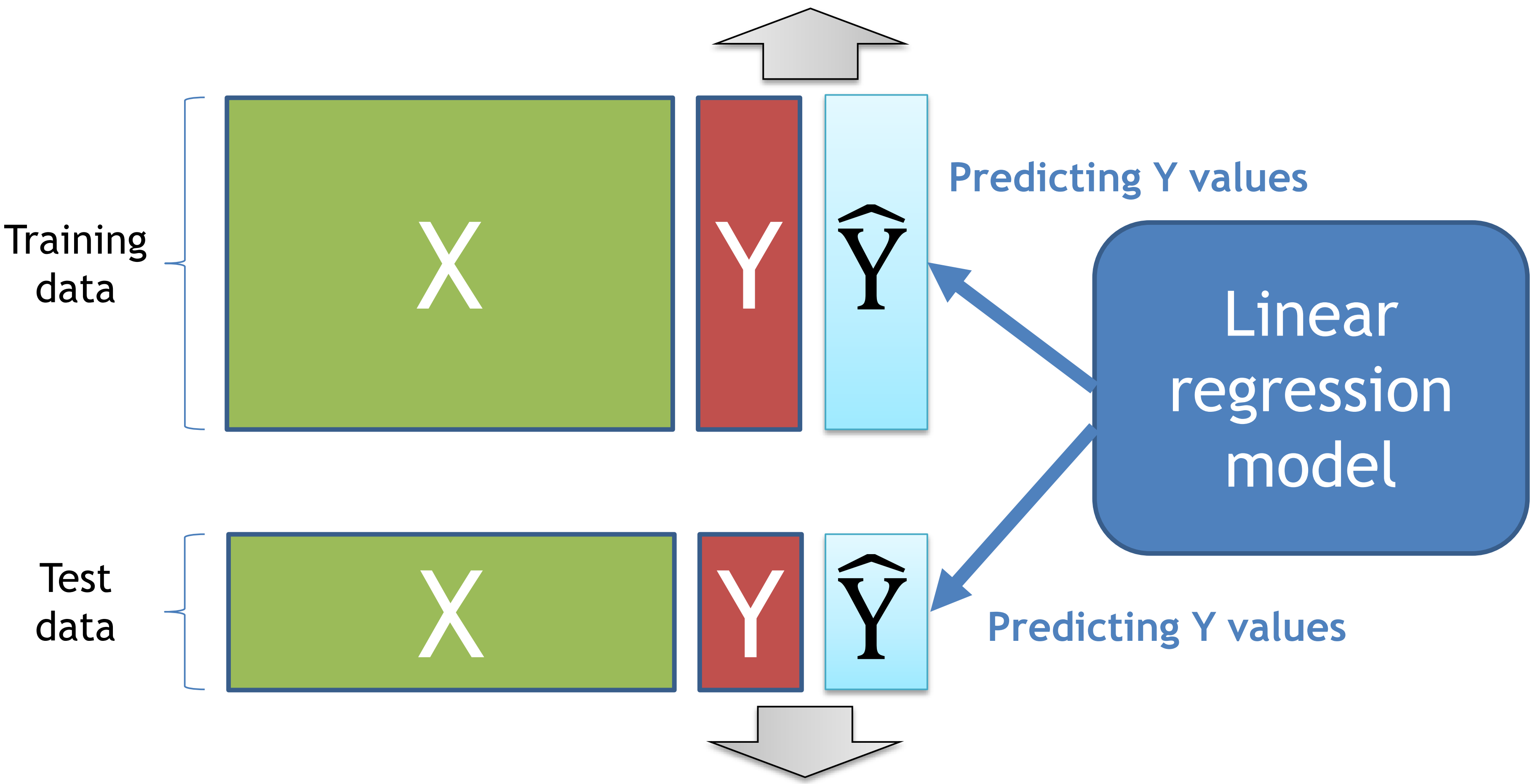
- Regression model이 데이터에 얼마나 적합하였는가?
- Goodness-of-fit

Validation error (or Test error)

- Regression model의 예측 성능이 어느 정도인가?
- Predictive performance

Train error and test error

Calculating “train error” using Y and $\hat{Y}$ of training data



Calculating “test error” using Y and $\hat{Y}$ of test data

Evaluation of regression model in statistics

- Akaike Information Criteria (AIC)
- Bayesian Information Criteria (BIC)
- Adjusted-R^2: 기존의 R^2에 변수의 수를 고려
- Mallow's C_k

$$\mathrm{AIC} = n \cdot \ln\left(\frac{\mathrm{SSE}_k}{n}\right) + 2k$$

$$\mathrm{BIC} = n \cdot \ln\left(\frac{\mathrm{SSE}_k}{n}\right) + k \cdot \ln(n)$$

$$\text{Adjusted-}R^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2)$$

$$C_k = \frac{SSE_k}{s^2} - (n - 2k)$$

n : number of samples

k : number of selected variables

SSE_k : sum of squared error of regression model with k variables

s : sum of squared error of full regression model