Commutative Algebra

Fall Term

December 13, 2025

To all who find beauty in logic.

Syllabus

We are going to take a brief peek into the field of algebraic number theory in this ongoing seminar. Our ultimate goal is to master some basic tools and techniques, for example, the Dedekind domain and the ramification theory.

In the first part of our seminar, we will have a review on some rudiments from the ring theory and homological algebra. We shall simply follow Atiyah's *An Introduction to Commutative Algebra*.

In the second part, we will briefly discuss some basic concepts in algebraic number theory, like the ring $\mathcal{O}_K$, the Dedekind domains, primary decomposition and the ramification theory.

Contents

Chapter 1

Rudiments

In this section, we briefly recall some basic concepts in abstract algebra and homological algebra (especially when things happen in R-Mod category).

1.1 Ring Theory

1.1.1 Radical of Ideals

1.1.2 Zariski Topology

1.2 Homological Algebra

1.2.1 Projective and Injective Objects

1.2.2 Flat Modules

1.2.3 Derived Functors

Chapter 2

Hilbert's Nullstellensatz

In this chapter, we introduce an important theorem in algebraic geometry: Hilbert's Nullstellensatz.