

Commutative Algebra

Fall Term

December 13, 2025

To all who find beauty in logic.

Syllabus

We are going to take a brief peek into the field of algebraic number theory in this ongoing seminar. Our ultimate goal is to master some basic tools and techniques, for example, the Dedekind domain and the ramification theory.

In the first part of our seminar, we shall have a review on the rudiments like the Galois theory and some representation theory. We aim to follow what the authors did in *Honors Algebra*.

In the second part, we will briefly discuss some basic concepts in algebraic number theory, like the ring \mathcal{O}_K , the Dedekind domains, primary decomposition and the ramification theory.

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Chapter 1

Rudiments

In this section, we briefly recall some basic concepts in abstract algebra and homological algebra (especially when things happen in $R\text{-Mod}$ category).

1.1 Ring Theory

1.1.1 Radical of Ideals

1.1.2 Zariski Topology

1.2 Homological Algebra

1.2.1 Projective and Injective Objects

1.2.2 Flat Modules

1.2.3 Derived Functors

Chapter 2

Hilbert's Nullstellensatz

In this chapter, we introduce an important theorem in algebraic geometry: Hilbert's Nullstellensatz.