TIMOSHENKO BEAM - Single supported beam with 1 concentrated forces

Beam spans

Span lenght [mm] $L := 5000$

Rectangular cross-section

Section height $h := 200$

Section width $b := 100$

Shear factor $k_s := 1,2$

Moment of inertia $I := \frac{b \cdot h^3}{12}$

Cross - sectional area $A := b \cdot h$

Beam material

Elastic modulus $E := 11000$

Shear modulus $G := 690$

Bending stiffness $EI := E \cdot I$

Shear stiffness $GA := G \cdot A = 1,38 \cdot 10^7$

Corrected shear stiffness $GAc := \frac{G \cdot A}{k_s} = 1,15 \cdot 10^7$

Beam load

Load position $c := \frac{L}{2} = 2500$

Concentrated force [N] $Q := 1000$

$eps := 0,1$

Approximation Dirac delta function $\delta_a(x) := \left(\frac{eps}{\pi} \cdot \frac{1}{x^2 + eps^2} \right)$

Concentrated forces $p(x) := Q \cdot \delta_a(x - c)$

$$I_4(x) := \frac{\text{Int}\left(\text{Int}\left(\text{Int}\left(\text{Int}\left(p(x); x\right); x\right); x\right); x\right)}{EI} \qquad I_3(x) := \frac{\text{Int}\left(\text{Int}\left(\text{Int}\left(p(x); x\right); x\right); x\right)}{EI}$$

$$I_2(x) := \frac{\text{Int}\left(\text{Int}\left(p(x); x\right); x\right)}{EI}$$

$$v(x) := \left(a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3\right) + I_4(x)$$

$$v'(x) := \left(a_1 + 2 \cdot a_2 \cdot x + 3 \cdot a_3 \cdot x^2\right) + I_3(x)$$

$$v''(x) := 2 \cdot a_2 + 6 \cdot a_3 \cdot x + I_2(x)$$

$$\text{Assign}\left(\text{Algsys}\left(\begin{bmatrix} v(0)=0 \\ v''(0)=0 \\ v(L)=0 \\ v''(L)=0 \end{bmatrix}; \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}\right)\right) = \begin{cases} -1,7741 \\ 0,0043 \\ -8,5203\cdot 10^{-7} \\ 0 \end{cases}$$

$$u(x) := \left(v(x) - \frac{EI}{GAc}\cdot v''(x)\right) \qquad M(x) := (-EI)\cdot v''(x)$$

$$\text{eval}\left(u\left(\frac{L}{2}\right)\right) = 3,6596$$

Deflection - midspan

$$w_{max} := \frac{Q\cdot L^3}{48\cdot EI} = 3,5511$$

$$\text{eval}\left(M\left(\frac{L}{2}\right)\right) = 1,2496\cdot 10^6$$

Moment - midspan

$$M_{max} := \frac{Q\cdot L}{4} = 1,25\cdot 10^6$$