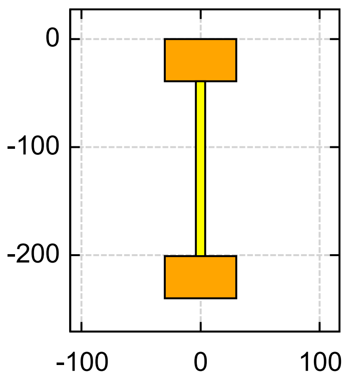
SHEAR FACTOR CALCULATIONS

Section geometry	Width	Height	
Top subsection	$b_1 := 60$	$h_1 := 39$	[mm]
Middle subsection	$b_2 := 8$	$h_2 := 162$	[mm]
Bottom subsection	$b_3 := 60$	$h_3 := 39$	[mm]
Total section height	$h := h_1 + h_2 + h_3 = 240$		
Material properties	Elastic modulus	Shear modulus	(typical timber properties)
Top subsection	$E_1 := 14500$	$G_1 := 600$	[N/mm²]
Middle subsection	$E_2 := 5300$	$G_2 := 2100$	[N/mm²]
Bottom subsection	$E_3 := 14500$	$G_3 := 600$	[N]
Shear stiffness	$GA := G_1 \cdot h_1 \cdot b_1 + G_2 \cdot h_2 \cdot b_2 + G_3 \cdot h_3 \cdot b_3 = 5{,}5296 \cdot 10^{6}$		

Section view



$$y_n := \frac{b_2 \cdot h_2 \cdot E_2 \cdot \left(h_1 + \frac{h_2}{2}\right) + b_1 \cdot h_1 \cdot E_1 \cdot \frac{h_1}{2} + b_3 \cdot h_3 \cdot E_3 \cdot \left(h_1 + h_2 + \frac{h_3}{2}\right)}{b_1 \cdot h_1 \cdot E_1 + b_2 \cdot h_2 \cdot E_2 + b_3 \cdot h_3 \cdot E_3}$$

Position of neutral axis from the top edge [mm] $y_n = 120$

$$EI := \frac{{h_1}^3 \cdot b_1 \cdot E_1}{12} + \frac{{h_2}^3 \cdot b_2 \cdot E_2}{12} + \frac{{h_3}^3 \cdot b_3 \cdot E_3}{12} + b_1 \cdot h_1 \cdot E_1 \cdot \left(y_n - \frac{h_1}{2}\right)^2 + b_2 \cdot h_2 \cdot E_2 \cdot \left(h_1 + \frac{h_2}{2} - y_n\right)^2 + b_3 \cdot h_3 \cdot E_3 \cdot \left(h \ldots\right.$$

Bending stiffness [Nmm²] $EI = 7{,}0903 \cdot 10^{11}$

Position of neutral axis from center [mm] $y_c := y_n - \frac{h}{2} = 0$

General expressions

Approximation for step function $H(z) := \frac{1}{2} \cdot (1 + \mathrm{sign}(z))$

Elastic modulus expression for the whole cross-section depending on the distance from the top edge

$$E(z) := E_1 \cdot H(z + y_c + h) - (E_1 - E_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1\right) + (E_3 - E_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1 - h_2\right)$$

Shear modulus expression for the whole cross-section depending on the distance from the top edge

$$G(z) := G_1 \cdot H(z + y_c + h) - (G_1 - G_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1\right) + (G_3 - G_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1 - h_2\right)$$

Cross sectional width depending on the distance from the top edge

$$b(z) := b_1 \cdot H(z + y_c + h) - (b_1 - b_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1\right) + (b_3 - b_2) \cdot H\left(z + y_c + \frac{h}{2} - h_1 - h_2\right)$$

Balance of energy of the beam for linear elasticity

Properties for the simply supported beam (useful for the further calculations):

- Linear distributed load [kN/m] $q := 1$
- Span length [m] $L := 1$

Bending stiffness $EI := \mathrm{Int}\left(E(z) \cdot b(z) \cdot z^2 ; z ; -\frac{h}{2} - y_c ; \frac{h}{2} - y_c\right) = 7{,}0903 \cdot 10^{11}$

Shear flow expression $T(z) := (-q) \cdot \frac{L}{2 \cdot EI} \cdot \mathrm{Int}\left(E(z) \cdot b(z) \cdot z ; z ; -\frac{h}{2} - y_c ; z\right)$

Internal energy [Nmm] $U := \mathrm{eval}\left(\frac{1}{2} \cdot \mathrm{Int}\left(\frac{(T(z))^2}{G(z) \cdot b(z)} ; z ; -\frac{h}{2} - y_c ; \frac{h}{2} - y_c\right)\right) = 3{,}1719 \cdot 10^{-8}$

External energy [Nmm] $W(k_s) := \frac{1}{2} \cdot \frac{k_s}{GA} \cdot \left(\frac{q \cdot L}{2}\right)^2 = \frac{k_s}{44236800}$

Shear correction factor $k_s := \mathrm{solve}\left(U - W(k_s) ; k_s\right) = 1{,}4032$

Bending stiffness [Nmm²] $\boxed{EI = 7{,}0903 \cdot 10^{11}}$

Shear correction factor [-] $\boxed{k_s = 1{,}4032}$

Corrected shear stiffness [N] $\boxed{GAc := \frac{GA}{k_s} = 3{,}9408 \cdot 10^{6}}$