

Input data:

☐ — Input data

Geometry - Concrete

$$b_{top} := 590$$

$$h_{top} := 140$$

$$y_{top} := \frac{h_{top}}{2} = 70$$

$$I_{top} := \frac{b_{top} \cdot h_{top}^3}{12} = 1,3491 \cdot 10^8$$

$$W_{top} := \frac{b_{top} \cdot h_{top}^2}{6} = 1,9273 \cdot 10^6$$

$$A_{top} := b_{top} \cdot h_{top} = 82600$$

Material - Concrete - C20/25

$$E_{top} := 24900$$

$$G_{top} := 13000000000$$

$$f_{ck} := 20$$

Geometry - Timber

$$b_{bot} := 590$$

$$h_{bot} := 20$$

$$A_{bot} := b_{bot} \cdot h_{bot} = 11800$$

$$d_l := 40$$

$$I_{bot} := \frac{b_{bot} \cdot h_{bot}^3}{12} = 3,9333 \cdot 10^5$$

$$y_{bot} := \frac{h_{bot}}{2} + d_l = 50$$

Material - Timber - C24

$$E_{m0} := 11000$$

$$E_{bot} := E_{m0}$$

$$G_{bot} := 6900000000$$

$$E_{botinf} := E_{minf}$$

Material - connections

$$E_{con} := 11000$$

$$G_{con} := 6900000000$$

Geometry - connections

$$l_{ef} := 100$$

Reinforcement

$$d_s := 8$$

$$n_b := 6$$

$$A_{s1} := n_b \cdot \pi \cdot \frac{d_s^2}{4} = 301,5929$$

$$d_1 := h_{top} - \frac{d_s}{2} = 136$$

$$K_{ser} := 850000$$

$$d_2 := 50$$

$$K_u := 570000$$

$$b_{conser} := \sqrt[4]{\frac{12 \cdot K_{ser} \cdot \left((y_{top})^3 + (y_{bot})^3 \right)}{3 \cdot E_{con}}} = 109,6688$$

$$h_{conser} := b_{conser}$$

$$I_{conser} := \frac{b_{conser}^4}{12}$$

$$A_{conser} := b_{conser}^2$$

Span length

$$L_t := 5000$$

$$n_{Edof} := 6$$

$$d_{tb} := y_{top} + y_{bot} = 120$$

$$qser := -1$$

$$ks := 1, 2$$

Rod position

$$nn := 5$$

connection distance

$$d := \frac{Lt}{nn} = 1000$$

$$nPN := nn + 1$$

$$nPE := nPN - 1$$

Define connector position

$$conns := \begin{bmatrix} 1000 \\ 2000 \\ 3000 \\ 4000 \end{bmatrix}$$

□ — MOMENT-CURVATURE

Concrete C20/25

$$f_{cm} := 28$$

$$f_{ck} := 20$$

$$\alpha_c := 0,85$$

$$\gamma_c := 1,5$$

$$E_{cm} := 30000$$

$$f_{ctk} := 1,5$$

$$E_c := 24900$$

$$f_{ctm} := 2,2$$

$$\varepsilon_{c1} := 0,002$$

$$\varepsilon_{cu} := 0,0035$$

$$f_{ctd} := \frac{f_{ctk}}{\gamma_c} = 1$$

$$f_{cd} := \alpha_c \cdot \frac{f_{ck}}{\gamma_c} = 11,3333$$

Steel

$$f_{yk} := 450$$

$$\gamma_s := 1,15$$

$$E_s := 200000$$

$$f_{yd} := \frac{f_{yk}}{\gamma_s} = 391,3043$$

$$\varepsilon_e := \frac{f_{yd}}{E_s} = 0,002$$

$$M(\chi; N; y_n) := \begin{cases} x1 := \frac{Asl \cdot f_{yd} + N}{0,8 \cdot b_{top} \cdot f_{cd}} \\ z := d1 - \frac{x1}{3} \\ M1 := \left(Asl \cdot f_{yd} \cdot z + N \cdot \left(y_n - \frac{x1}{3} \right) \right) \\ \chi_e := \frac{\varepsilon_e}{d1 - x1} \\ M_{cr} := f_{ctd} \cdot W_{top} + N \cdot \left(y_n - \frac{h_{top}}{2} \right) \\ \chi_{cr} := \frac{f_{ctd} \cdot W_{top} + N \cdot \left(y_n - \frac{h_{top}}{2} \right)}{E_c \cdot I_{top}} \\ \text{if } ((\chi \geq 0) \wedge (\chi < \chi_{cr})) \end{cases}$$

$$\frac{Mcr \cdot \chi}{\chi cr}$$

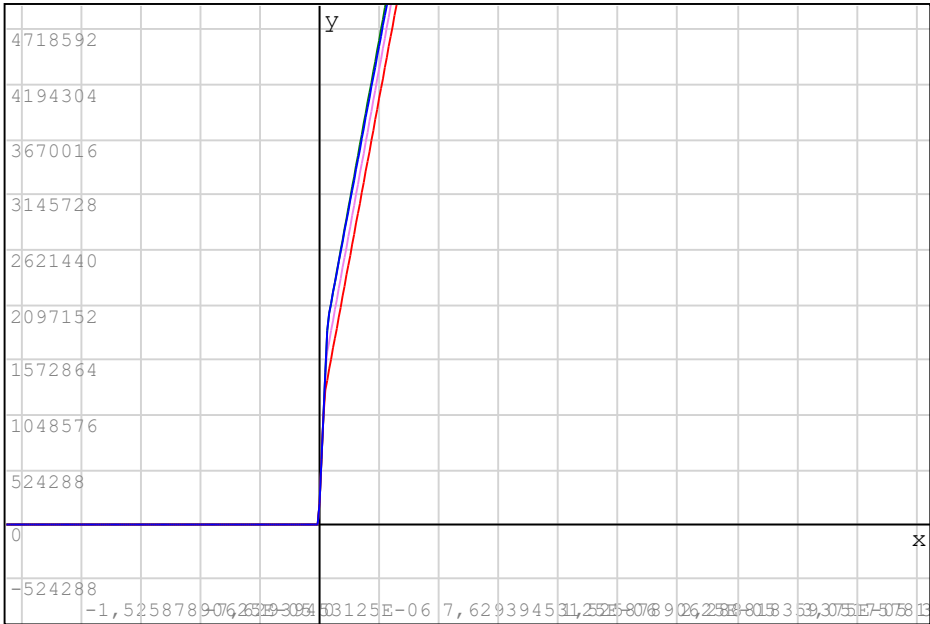
else if $(\chi \geq \chi cr) \wedge (\chi \leq \chi e)$
$$Mcr \cdot \left(1 - \frac{\chi - \chi cr}{\chi e - \chi cr}\right) + Ml \cdot \frac{\chi - \chi cr}{\chi e - \chi cr}$$

else if $\chi > \chi e$
$$Ml$$

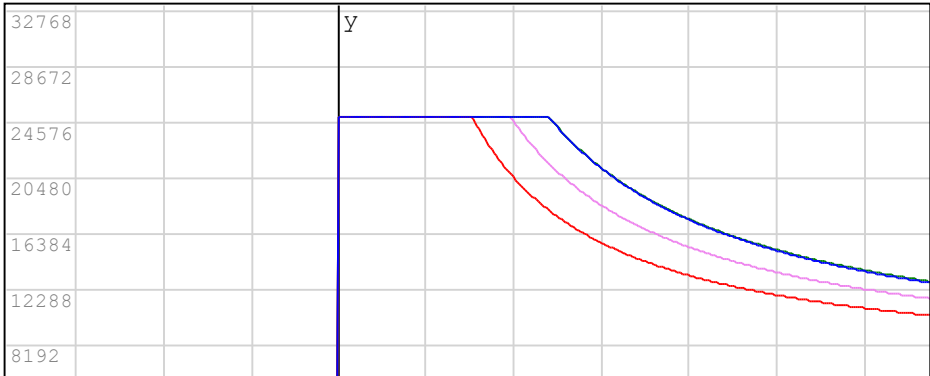
else
0

$$E(\chi; N; y_n) := \frac{M(\chi; N; y_n)}{\chi \cdot I_{top}}$$

$$E\left(\frac{fctd \cdot W_{top}}{Ec \cdot I_{top}}; 0; 0\right) = 24900$$



$$\begin{cases} M(x; 0; 0) \\ M(x; 10000; 0) \\ M(x; 10000; y_{top}) \\ M\left(x; 10000; \frac{y_{top}}{2}\right) \end{cases}$$



| | | | | | | | | | | | |
|-------|--|----------------|--|----------|--|----------------|--|----------------|--|-------|---|
| 4096 | | | | | | | | | | | |
| 0 | | | | | | | | | | | x |
| -4096 | | | | | | | | | | | |
| -8192 | | | | | | | | | | | |
| | | -4,76837158203 | | 0.25E-07 | | 4,768371582031 | | 2,536743164062 | | 5E-07 | |

$$\begin{cases} E(x; 0; 0) \\ E(x; 10000; 0) \\ E(x; 10000; y_{top}) \\ E\left(x; 10000; \frac{y_{top}}{2}\right) \end{cases}$$

☒ — Stiffness matrix for a 1D finite element with shear deformation (Timoshenko) _____

☒ — Stiffness matrix for a 1D finite element with left internal hinge (Timoshenko) _____

☒ — Stiffness matrix for a 1D finite element with right internal hinge (Timoshenko) _____

☐ — Nodes - basic structure _____

$Nodes := \text{matrix}(3 \cdot nPN; 5)$

```
for i ∈ [1..(nPN)]
  Nodesi 1 := (i - 1) · d
  Nodesi 3 := 3 · i - 2
  Nodesi 4 := 3 · i - 1
  Nodesi 5 := 3 · i
```

```
for i ∈ [(nPN + 1)..(2 · nPN)]
  Nodesi 1 := (i - nPN - 1) · d
  Nodesi 2 := dtb
  Nodesi 3 := 3 · (i) - 2
  Nodesi 4 := 3 · (i) - 1
  Nodesi 5 := 3 · (i)
```

```
for i ∈ [(2 · nPN + 1)..(3 · nPN)]
  Nodesi 1 := (i - 2 · nPN - 1) · d
  Nodesi 2 :=  $\frac{h_{bot}}{2} + dl$ 
  Nodesi 3 := 3 · (i) - 2
```

[+]—Add Nodes - Connectors

□—Elements

$$edof := \text{matrix}(nEl; 6) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & & & & & \end{bmatrix}$$

```
for i ∈ [1..nPE]
```

```

| edofni 1 := i
| edofni 2 := i + 1

```

```

| edofni 1 := i + 1
| edofni 2 := i + 2

```

$$\begin{cases} edofn_{i1} := (i - 2 \cdot nPE) \\ edofn_{i2} := i + 2 \end{cases}$$

```

| edofni 1 := i - nPE + 1
| edofni 2 := i - 2 · nPE

```

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \end{bmatrix}$$

```

edofn =
  7  8
  8  9
  9 10
 10 11
 11 12
  1 13
  2 14
  3 15
  4 16
  5 17
  6 18
 13  7
 14  8
 15  9
 16 10
 17 11
 18 12

```

```

for i ∈ [1..nEl]
  edofi 1 := Nodesi 1 3
  edofi 2 := Nodesi 1 4
  edofi 3 := Nodesi 1 5
  edofi 4 := Nodesi 2 3
  edofi 5 := Nodesi 2 4
  edofi 6 := Nodesi 2 5

```

```

edof =
  1  2  3  4  5  6
  4  5  6  7  8  9
  7  8  9 10 11 12
 10 11 12 13 14 15
 13 14 15 16 17 18
 19 20 21 22 23 24
 22 23 24 25 26 27
 25 26 27 28 29 30
 28 29 30 31 32 33
 31 32 33 34 35 36
  1  2  3 37 38 39
  4  5  6 40 41 42
  7  8  9 43 44 45
 10 11 12 46 47 48
 13 14 15 49 50 51
 16 17 18 52 53 54
 37 38 39 19 20 21
 40 41 42 22 23 24
 43 44 45 25 26 27
 46 47 48 28 29 30
 49 50 51 31 32 33
 52 53 54 34 35 36

```

```
nDof := nNodes · 3 = 54
```

```
Elsle := matrix(nEl; 11)
```

```
for i ∈ [1..nPE]
```

```
  |Elslei 1 := Nodesi 1 3
```

```

      i 1      edofn i 1 1
Elsle i 2 := Nodes edofn i 1 2
Elsle i 3 := Nodes edofn i 2 1
Elsle i 4 := Nodes edofn i 2 2
Elsle i 5 := Abot
Elsle i 6 := Ibot
Elsle i 7 := Ebot
Elsle i 8 := Gbot
Elsle i 9 := "nh"
Elsle i 10 := 0
Elsle i 11 := 0

```

```
for i ∈ [(nPE + 1)..(2 · nPE)]
```

```

Elsle i 1 := Nodes edofn i 1 1
Elsle i 2 := Nodes edofn i 1 2
Elsle i 3 := Nodes edofn i 2 1
Elsle i 4 := Nodes edofn i 2 2
Elsle i 5 := Atop
Elsle i 6 := Itop
Elsle i 7 := Etop
Elsle i 8 := Gtop
Elsle i 9 := "nh"
Elsle i 10 := 0
Elsle i 11 := qser

```

```
for i ∈ [(2 · nPE + 1)..(3 · nPE + 1)]
```

```

Elsle i 1 := Nodes edofn i 1 1
Elsle i 2 := Nodes edofn i 1 2
Elsle i 3 := Nodes edofn i 2 1
Elsle i 4 := Nodes edofn i 2 2
Elsle i 5 := Aconser
Elsle i 6 := Iconser
Elsle i 7 := Econ
Elsle i 8 := Gcon
Elsle i 9 := "lh"
Elsle i 10 := 0
Elsle i 11 := 0

```

I + + +

```

for i ∈ [(3 · nPE + 2) .. (4 · nPE + 2)]
  Elslei 1 := Nodesedofni 1 1
  Elslei 2 := Nodesedofni 1 2
  Elslei 3 := Nodesedofni 2 1
  Elslei 4 := Nodesedofni 2 2
  Elslei 5 := Aconser
  Elslei 6 := Iconser
  Elslei 7 := Econ
  Elslei 8 := Gcon
  Elslei 9 := "rh"
  Elslei 10 := 0
  Elslei 11 := 0

```

□ — Set connectors

```

for i ∈ [1 .. (length(conns))]
  for j ∈ [(2 · nPE + 1) .. (3 · nPE + 2)]
    if (Elslej 1 = Elslej 3) ∧ (Elslej 1 = connsi)
      Elslej 9 := "rh"
  for j ∈ [(3 · nPE + 2) .. (4 · nPE + 2)]
    if (Elslej 1 = Elslej 3) ∧ (Elslej 1 = connsi)
      Elslej 9 := "nh"

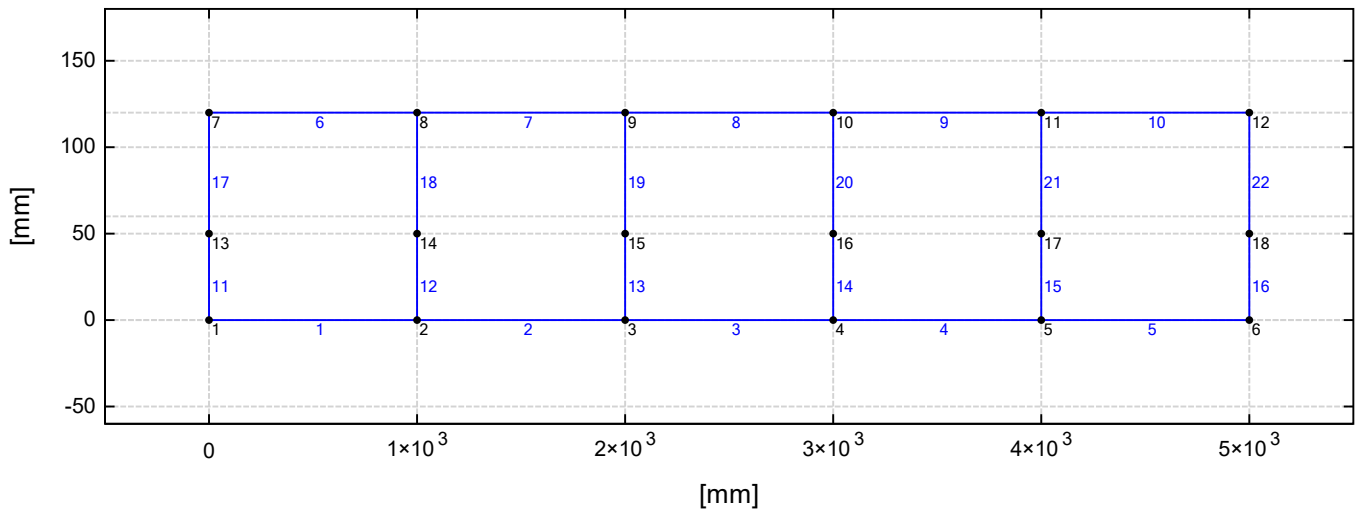
```

Elsle =

| | | | | | | | | | | |
|------|-----|------|-----|------------|---------------------|-------|---------------------|------|---|----|
| 0 | 0 | 1000 | 0 | 11800 | $3,9333 \cdot 10^5$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 1000 | 0 | 2000 | 0 | 11800 | $3,9333 \cdot 10^5$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 2000 | 0 | 3000 | 0 | 11800 | $3,9333 \cdot 10^5$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 3000 | 0 | 4000 | 0 | 11800 | $3,9333 \cdot 10^5$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 4000 | 0 | 5000 | 0 | 11800 | $3,9333 \cdot 10^5$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 0 | 120 | 1000 | 120 | 82600 | $1,3491 \cdot 10^8$ | 24900 | $1,3 \cdot 10^{10}$ | "nh" | 0 | -1 |
| 1000 | 120 | 2000 | 120 | 82600 | $1,3491 \cdot 10^8$ | 24900 | $1,3 \cdot 10^{10}$ | "nh" | 0 | -1 |
| 2000 | 120 | 3000 | 120 | 82600 | $1,3491 \cdot 10^8$ | 24900 | $1,3 \cdot 10^{10}$ | "nh" | 0 | -1 |
| 3000 | 120 | 4000 | 120 | 82600 | $1,3491 \cdot 10^8$ | 24900 | $1,3 \cdot 10^{10}$ | "nh" | 0 | -1 |
| 4000 | 120 | 5000 | 120 | 82600 | $1,3491 \cdot 10^8$ | 24900 | $1,3 \cdot 10^{10}$ | "nh" | 0 | -1 |
| 0 | 0 | 0 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "lh" | 0 | 0 |
| 1000 | 0 | 1000 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |
| 2000 | 0 | 2000 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |
| 3000 | 0 | 3000 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |

| | | | | | | | | | | |
|------|----|------|-----|------------|---------------------|-------|------------------|------|---|---|
| 4000 | 0 | 4000 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |
| 5000 | 0 | 5000 | 50 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "lh" | 0 | 0 |
| 0 | 50 | 0 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |
| 1000 | 50 | 1000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 2000 | 50 | 2000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 3000 | 50 | 3000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 4000 | 50 | 4000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 5000 | 50 | 5000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |

System



Stiffness matrix

```
assemb (KK; K; edofe) := for i ∈ [1..nEdof]
                        for j ∈ [1..nEdof]
                            KKedofei edofej := KKedofei edofej + Ki j
                        KK
```

```
assembf (fl; fle; edofe) := for i ∈ [1..nEdof]
                            fledofei := fledofei + flei
                            fl
```

```
genKsle := K := matrix(nDof; nDof)
           for i ∈ [1..nEl]
               if Elslei 9 = "nh"
                   KKe := Ke(Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5; Elslei 6; Elslei 7; Elslei 8)
               else
```

```

if Elslei 9 = "lh"
    KKe := Kel(Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5; Elslei 6; Elslei 7; Elslei 8)
else
    KKe := Ker(Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5; Elslei 6; Elslei 7; Elslei 8)
edofe := edof [i..i][1..6]
K := assemk(K; KKe; edofe)
K

```

□ — Boundary conditions (penalty approach)

```

dia(Mat) := " Diagonal elements "
for i ∈ [1..nDof]
    diai := Mati i
dia

```

```
km := dia(genKsle)
```

```
c := max(km) · 1000000 = 3,4451 · 1016
```

```
bc := matrix(nDof; 2)
```

```
for i ∈ [1..nDof]
    bci 1 := i

```

```
bc1 2 := 1
```

```
bc2 2 := 1
```

```
bcedofnPE 5 2 := 1
```

```
curv0 := 0
```

```
curv1 := 1
```

```
curv := matrix(nPE; 1)
```

```
for i ∈ [1..nPE]
    curvi 1 := 0,000000001

```

```

for i ∈ [(nPE+1)..(2·nPE)]
    Elslei 1 := Nodesedofni 1 1
    Elslei 2 := Nodesedofni 1 2
    Elslei 3 := Nodesedofni 2 1
    Elslei 4 := Nodesedofni 2 2
    Elslei 5 := Atop
    Elslei 6 := Itop

```

```

Elslei 7 := E ( curvi - nPE 1 ; 0 ; 0 )
Elslei 8 := Gtop
Elslei 9 := "nh"
Elslei 10 := 0
Elslei 11 := -qser

```

```
Kbcsle := genKsle
```

```

for i ∈ [1..nDof]
  if bci 2 = 1
    Kbcslei i := Kbcslei i + c
  else
    continue
Kbc

```

☐ — Loads

```

genFlsle := | fl := matrix ( nDof ; 1 )
              | for i ∈ [1..nEl]
              |   | fle := fle ( Elslei 1 ; Elslei 2 ; Elslei 3 ; Elslei 4 ; Elslei 10 ; Elslei 11 )
              |   | edof := edof [ i .. i ] [ 1 .. 6 ]

```

```

    fl:=assembf(fl; fle; edofe)
  fl

```

```
fsle:=genFlsle
```

▣—Displacement (system solution) —

```
usle:=invert(Kbcsle)·fsle
```

▣—Reactions —

▣—Internal forces —

```
edsle:=matrix(nEl; 6)
```

```

for i∈ [1..nEl]
  for j∈ [1..6]
    edsleij:=usleedofij

```

```

m(ed):=
  b0:=matrix(1; nEl)
  b1:=matrix(1; nEl)
  for i∈ [1..nEl]
    L:=sqrt((Elslei3-Elslei1)2+(Elslei4-Elslei2)2)
    n0i:= $\frac{\text{Elsle}_{i3}-\text{Elsle}_{i1}}{L}$ 
    n1i:= $\frac{\text{Elsle}_{i4}-\text{Elsle}_{i2}}{L}$ 
  m:=matrix(1; nEl)
  for i∈ [1..nEl]
    mi:= $\begin{bmatrix} n0_i & -n1_i & 0 & 0 & 0 & 0 \\ n1_i & n0_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & n0_i & -n1_i & 0 \\ 0 & 0 & 0 & n1_i & n0_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ ·submatrix(ed; i; i; 1; 6)T
  m

```

```
msle:=m(edsle)
```

```

M1:=matrix(nEl; 1)
M2:=matrix(nEl; 1)
V1:=matrix(nEl; 1)
V2:=matrix(nEl; 1)
for i ∈ [1..nEl]
  x1:=Elslei 1
  y1:=Elslei 2
  x2:=Elslei 3
  y2:=Elslei 4
  L:=√((x2-x1)2+(y2-y1)2)
  A:=Elslei 5
  I:=Elslei 6
  E:=Elslei 7
  G:=Elslei 8
  qx:=Elslei 10
  qy:=Elslei 11
  co:= $\frac{12}{L^2} \cdot \frac{ks \cdot E \cdot I}{G \cdot A}$ 
  u1:=msle1 i 1
  u2:=msle1 i 4
  v1:=msle1 i 2
  t1:=msle1 i 3
  v2:=msle1 i 5
  t2:=msle1 i 6
  fle:=fle(x1; y1; x2; y2; qx; qy)
  if Elslei 9 = "nh"
    M1slei :=  $-\left[\left(\frac{1}{1+co}\right) \cdot \left(\frac{6 \cdot E \cdot I}{L^2}\right) \cdot v1 + \left(\frac{E}{1+co} \cdot 4 \cdot \frac{I}{L} \cdot \left(1 + \frac{co}{4}\right)\right) \cdot t1 + \left(-\frac{E}{1+co}\right) \cdot 6 \cdot \frac{I}{L^2} \cdot v2 + \left[\left(\frac{E}{1+co}\right) \cdot 2 \cdot \frac{I}{L} \cdot \left(1 + \frac{co}{4}\right)\right] \cdot t2\right]$ 
    M2slei :=  $\left(\frac{1}{1+co}\right) \cdot \left(\frac{6 \cdot E \cdot I}{L^2}\right) \cdot v1 + \left(\frac{E}{1+co} \cdot 2 \cdot \frac{I}{L} \cdot \left(1 - \frac{co}{2}\right)\right) \cdot t1 + \left(\frac{-E}{1+co}\right) \cdot 6 \cdot \frac{I}{L^2} \cdot v2 + \left[\left(\frac{E}{1+co}\right) \cdot 4 \cdot \frac{I}{L} \cdot \left(1 + \frac{co}{4}\right)\right] \cdot t2$ 
    V1slei :=  $\left(\frac{1}{1+co}\right) \cdot \left(\frac{12 \cdot E \cdot I}{L^3}\right) \cdot v1 + \left(\frac{E}{1+co} \cdot 6 \cdot \frac{I}{L^2}\right) \cdot t1 + \left(-\frac{E}{1+co}\right) \cdot 12 \cdot \frac{I}{L^3} \cdot v2 + \left[\left(\frac{E}{1+co}\right) \cdot 6 \cdot \frac{I}{L^2}\right] \cdot t2 - fle$ 
    V2slei :=  $-\left[\left(\frac{-1}{1+co}\right) \cdot \left(\frac{12 \cdot E \cdot I}{L^3}\right) \cdot v1 + \left(\frac{-E}{1+co} \cdot 6 \cdot \frac{I}{L^2}\right) \cdot t1 + \left(\frac{E}{1+co}\right) \cdot 12 \cdot \frac{I}{L^3} \cdot v2 + \left[\left(\frac{-E}{1+co}\right) \cdot 6 \cdot \frac{I}{L^2}\right] \cdot t2 - fle\right]$ 
  else
    if Elslei 9 = "lh"
      M1slei := fle2
      M2slei :=  $\left(\frac{6 \cdot E \cdot I}{L^2} \cdot \left(\frac{co-2}{co+4}\right) \cdot v1 + \frac{6 \cdot E \cdot I}{L^2} \cdot \left(\frac{co-2}{co+4}\right) \cdot t1 + \left(\frac{E \cdot I}{L^2} \cdot \left(\frac{co+4}{co+2}\right) \cdot v2 + \frac{E \cdot I}{L^2} \cdot \left(\frac{co+4}{co+2}\right) \cdot t2\right)\right]$ 

```

$$M2sle_i := \left(\frac{L^2 \cdot (co+1)}{L^2 \cdot (co+1)} + \frac{L^2 \cdot (co+4) \cdot (co+1)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 - \frac{L^2 \cdot (co+4) \cdot (co+1)}{L^2 \cdot (co+4) \cdot (co+1)} \cdot v2 + \left(\frac{L^2 \cdot (co+1)}{L^2 \cdot (co+1)} - \frac{L^2 \cdot (co+4) \cdot (co+1)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot v1$$

$$V1sle_i := \left(\frac{12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{(-36) \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 + \left(\frac{-12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{36 \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v2 + \left(\frac{6 \cdot E \cdot I}{L^2 \cdot (co+1)} - \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot t1$$

$$V2sle_i := - \left(\left(\frac{-12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{36 \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 + \left(\frac{12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{(-36) \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v2 + \left(\frac{6 \cdot E \cdot I}{L^2 \cdot (co+1)} - \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot t1 \right)$$

else

$$M1sle_i := - \left(\left(\frac{6 \cdot E \cdot I}{L^2 \cdot (co+1)} + \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 - \frac{E \cdot I \cdot (co-2)^2}{L \cdot (co+4) \cdot (co+1)} \cdot t1 - \left(\frac{6 \cdot E \cdot I \cdot (co+4)}{L^2 \cdot (co+1)} + \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot t1 \right)$$

$$M2sle_i := fle_5$$

$$V1sle_i := \left(\frac{12 \cdot E \cdot I}{L^3 \cdot (co+1)} - \frac{36 \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 + \left(\frac{6 \cdot E \cdot I}{L^2 \cdot (co+1)} + \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot t1 + \left(\frac{-12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{36 \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v2$$

$$V2sle_i := - \left(\left(\frac{-12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{36 \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v1 + \left(\frac{12 \cdot E \cdot I}{L^3 \cdot (co+1)} + \frac{(-36) \cdot E \cdot I}{L^3 \cdot (co+4) \cdot (co+1)} \right) \cdot v2 + \left(\frac{6 \cdot E \cdot I}{L^2 \cdot (co+1)} - \frac{6 \cdot E \cdot I \cdot (co-2)}{L^2 \cdot (co+4) \cdot (co+1)} \right) \cdot t1 \right)$$

$$N1sle_i := \frac{E \cdot A}{L} \cdot (u1 - u2)$$

$$N2sle_i := \frac{E \cdot A}{L} \cdot (u1 - u2)$$

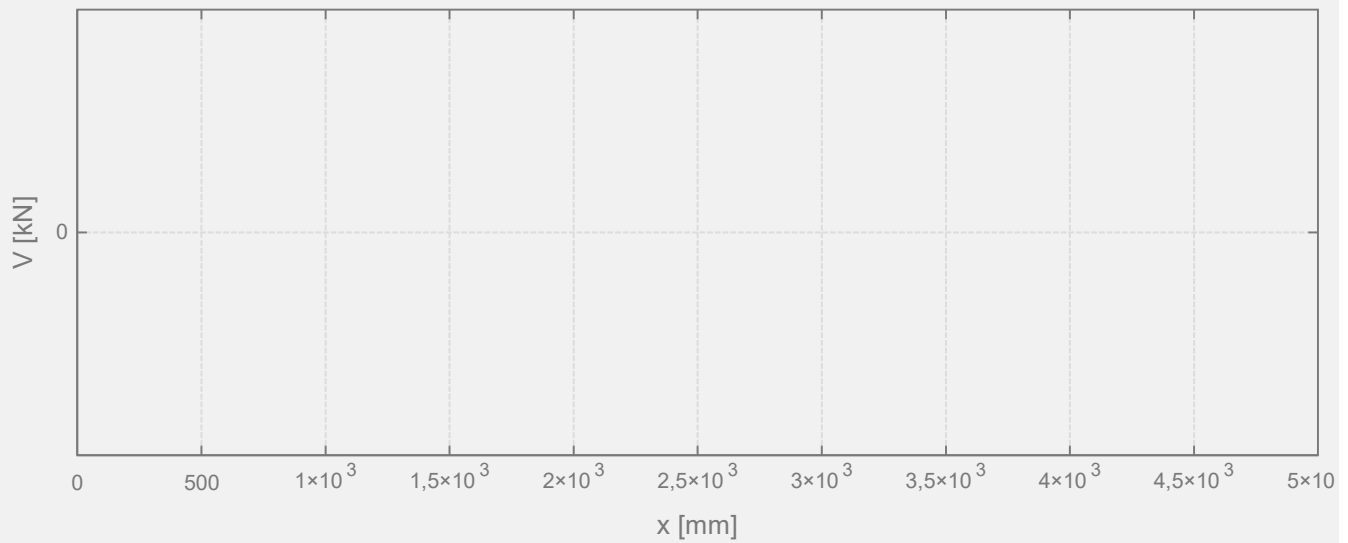
$$M1sle = \begin{bmatrix} -1,3174 \cdot 10^{-10} \\ 43334,0974 \\ 3265,1973 \\ -36934,9241 \\ -31945,4393 \\ -1,6667 \cdot 10^5 \\ -2,0293 \cdot 10^6 \\ -2,8928 \cdot 10^6 \\ -2,9491 \cdot 10^6 \\ -2,1347 \cdot 10^6 \\ 0 \\ -6,5684 \cdot 10^7 \\ -1,1222 \cdot 10^7 \\ 5,0355 \cdot 10^7 \\ 1,0482 \cdot 10^8 \\ 0 \\ 1,1989 \cdot 10^7 \\ -6,9344 \cdot 10^{-8} \\ -4,738 \cdot 10^{-8} \\ 4,7238 \cdot 10^{-8} \\ 1,2365 \cdot 10^{-7} \\ 7,9775 \cdot 10^6 \end{bmatrix}$$

$$M2sle = \begin{bmatrix} -31945,4393 \\ -36934,9241 \\ 3265,1973 \\ 43334,0974 \\ -2,0478 \cdot 10^{-10} \\ -1,9681 \cdot 10^6 \\ -2,7824 \cdot 10^6 \\ -2,7261 \cdot 10^6 \\ -1,8627 \cdot 10^6 \\ -9,1509 \cdot 10^{-8} \\ 2,889 \cdot 10^6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1,0155 \cdot 10^7 \\ 0 \\ 1,0539 \cdot 10^5 \\ 56280,1699 \\ -56280,1699 \\ -1,0539 \cdot 10^5 \\ 0 \end{bmatrix}$$

$$N1sle = \begin{bmatrix} -2,787 \cdot 10^{-9} \\ -1505,5907 \\ -2309,5932 \\ -1505,5907 \\ 2,9209 \cdot 10^{-10} \\ -1,0015 \cdot 10^{-10} \\ 1505,5907 \\ 2309,5932 \\ 1505,5907 \\ 1,4274 \cdot 10^{-10} \\ 2468,0546 \\ -48,3236 \\ 80,269 \\ 80,269 \\ -48,3236 \\ 2468,0546 \\ 2468,0546 \\ -48,3236 \\ 80,269 \\ 80,269 \\ -48,3236 \end{bmatrix}$$

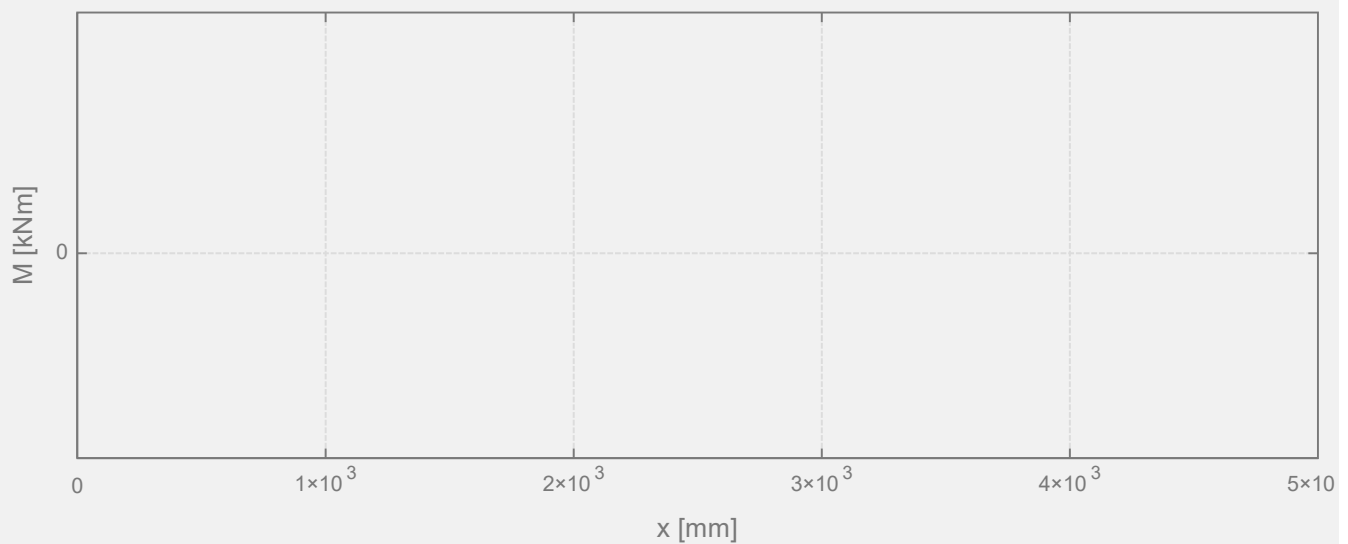
+ Shear force diagram - Timber

Shear force - Timber



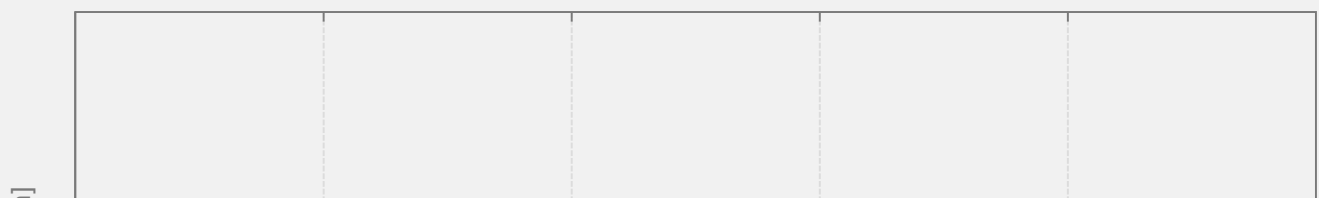
+ Moment diagram - Timber

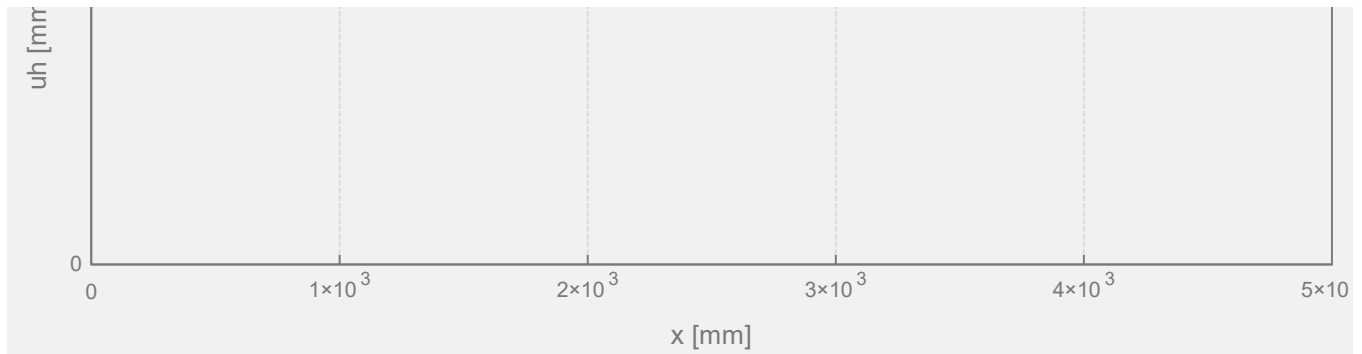
Moment - Timber



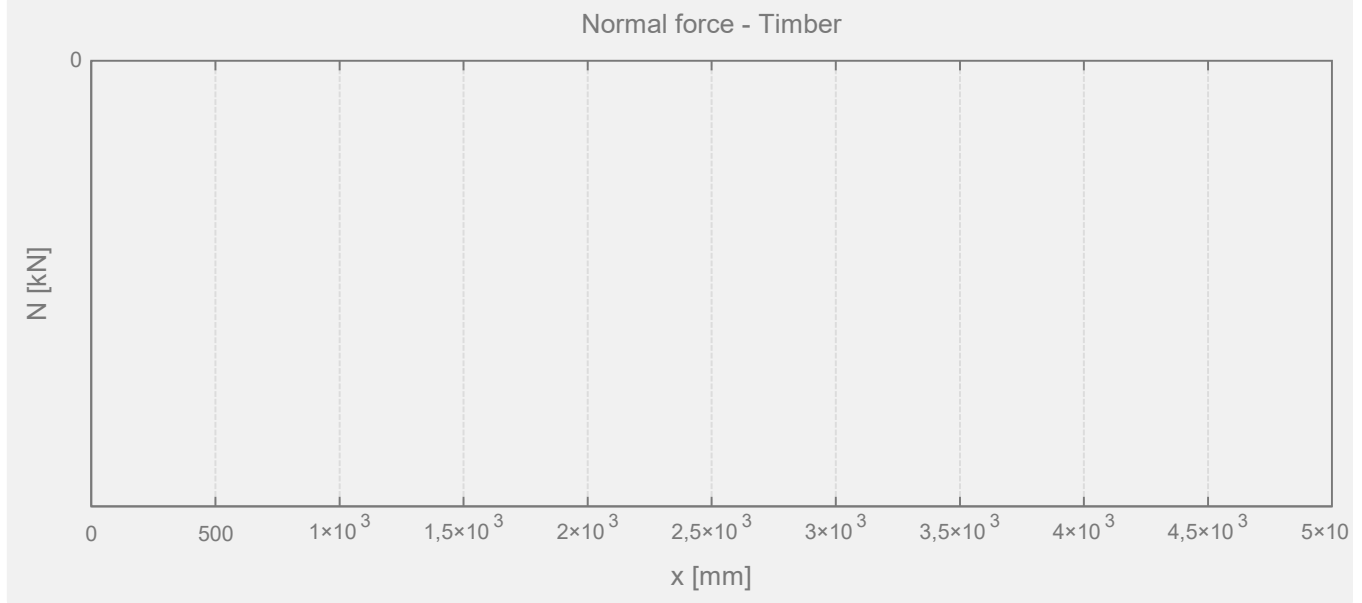
+ Horizontal displacement - Timber

Horizontal displacement - Timber

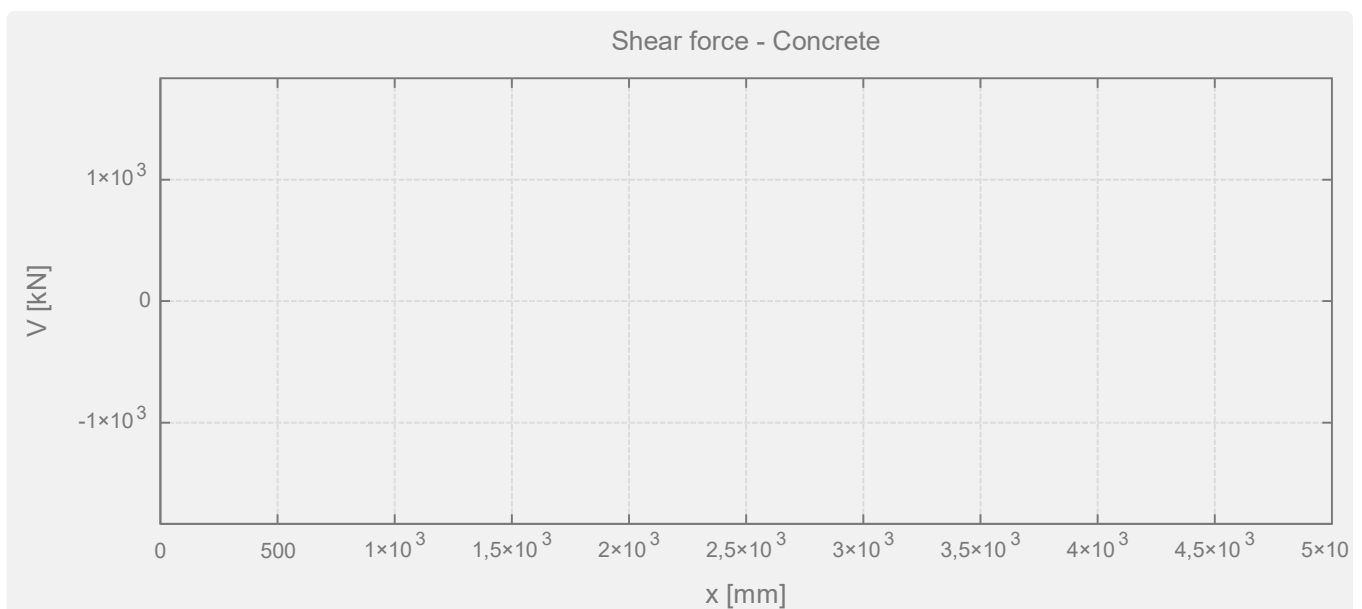




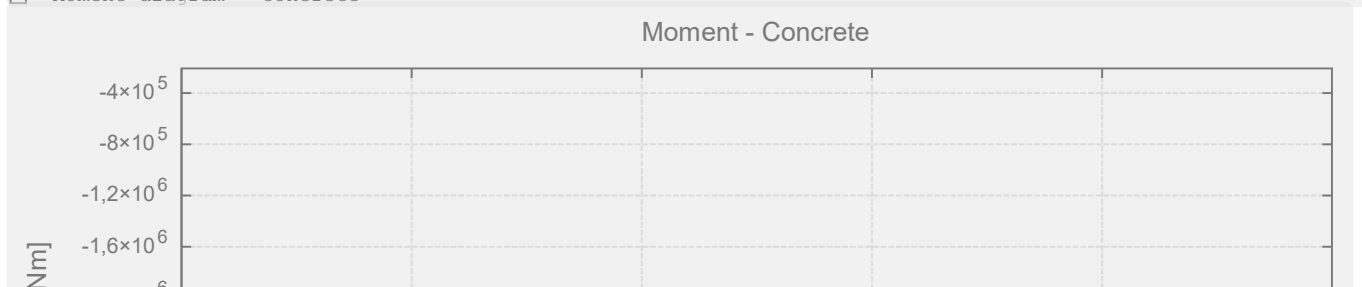
Normal force diagram - Timber

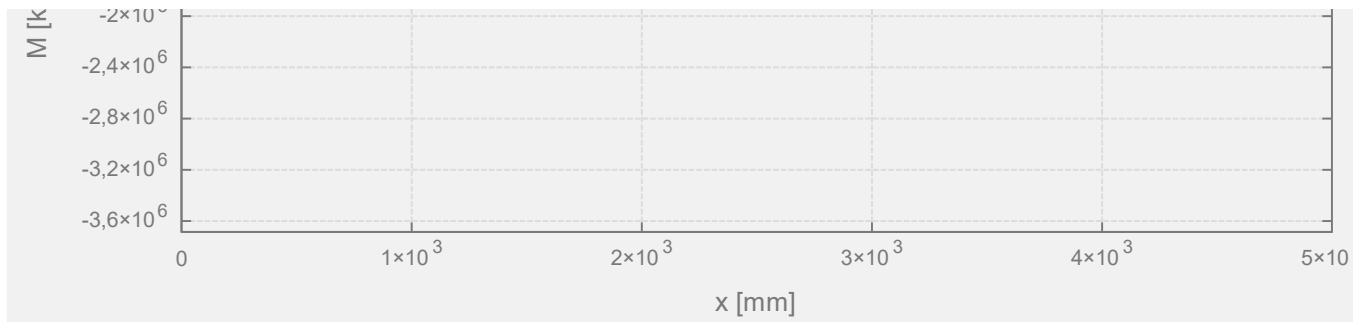


Shear force diagram - Concrete

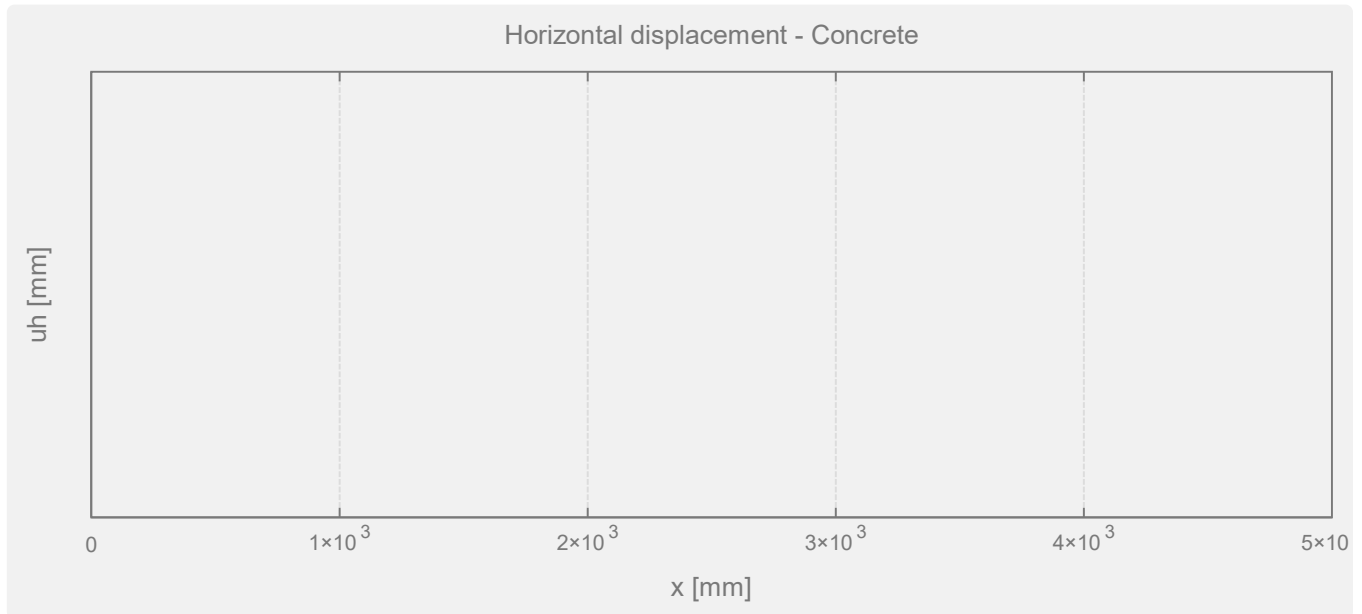


Moment diagram - Concrete

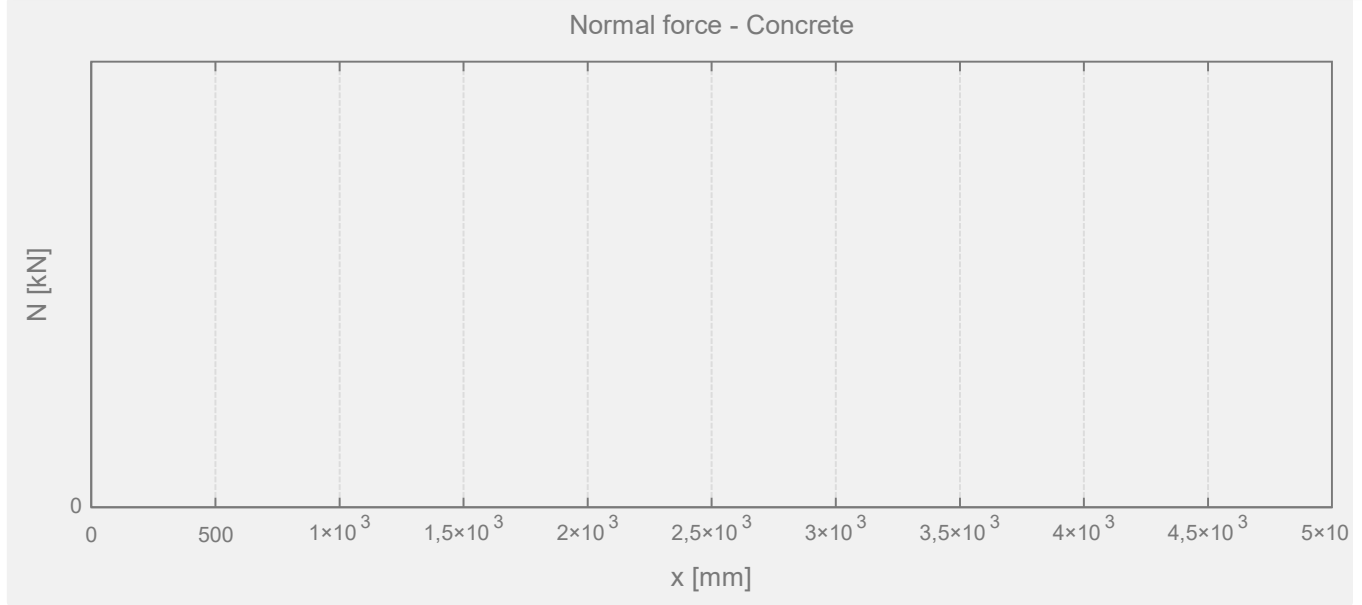




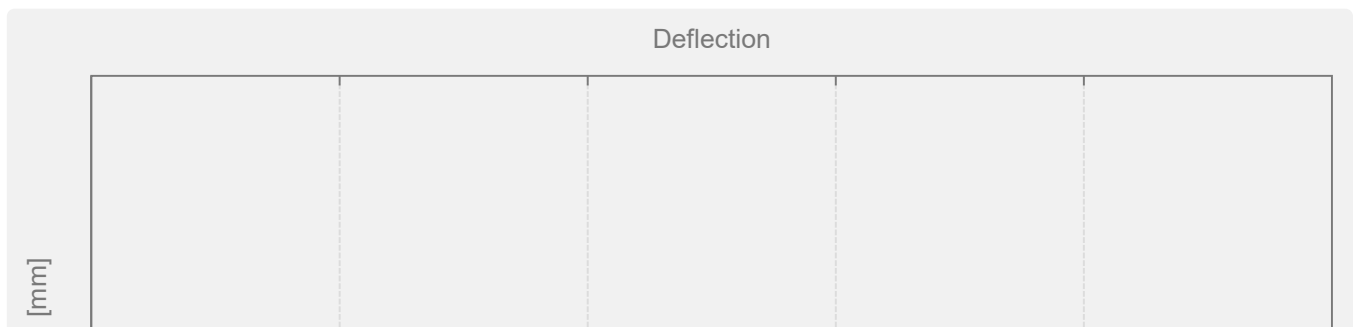
Horizontal displacement - Concrete

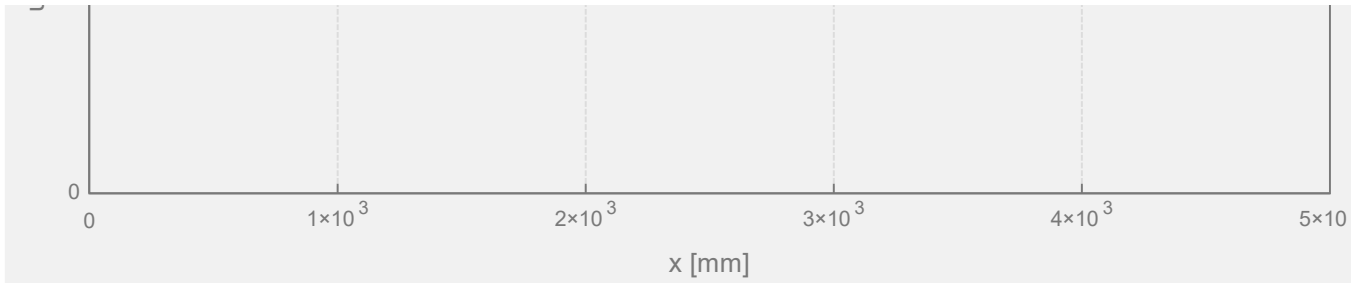


Normal force diagram - Concrete

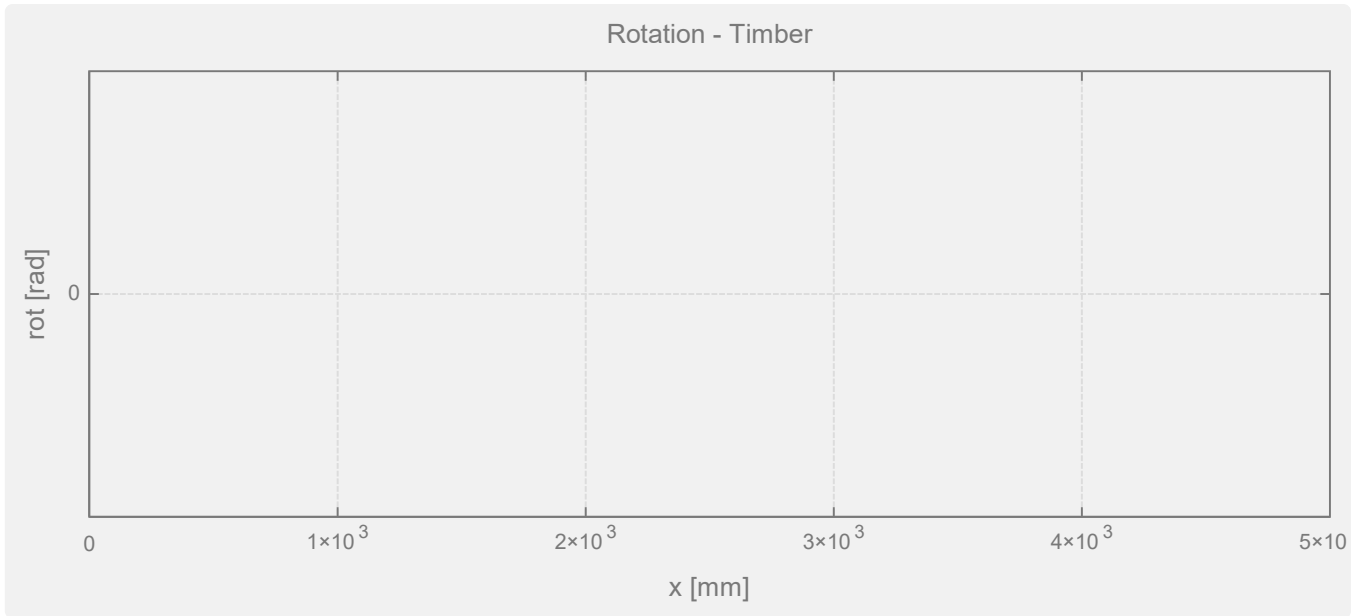


Deflection

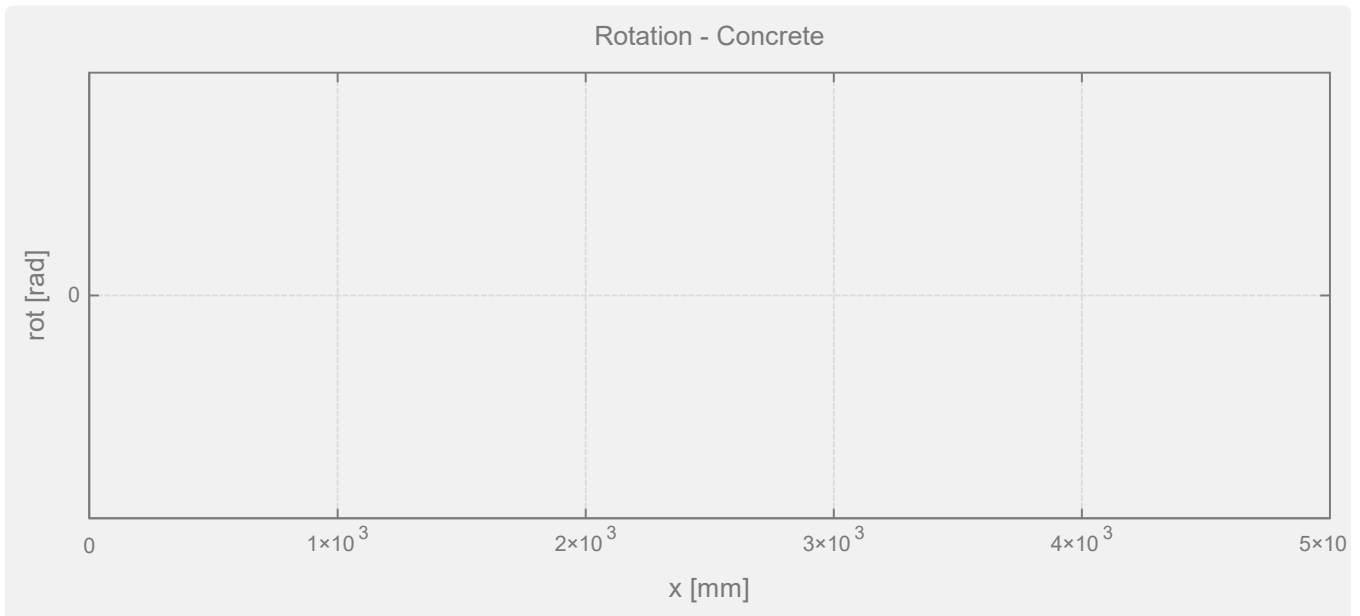




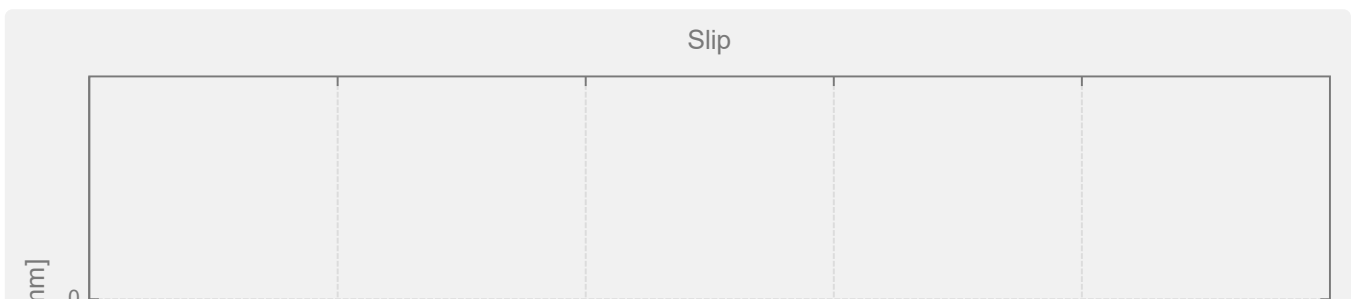
⊞ — Rotation - Timber

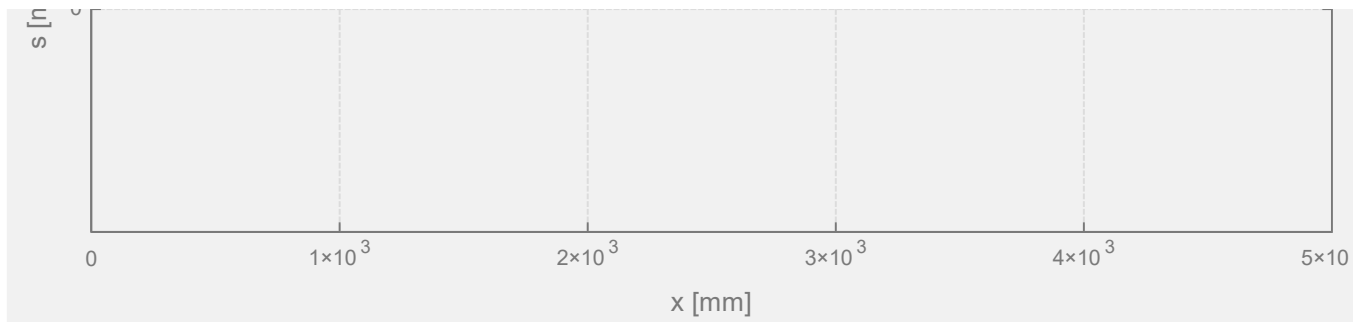


⊞ — Rotation - Concrete



⊞ — Slip





```

for i ∈ [(nPE+1)..(2·nPE)]
  Elsle i 1 := Nodes edofn i 1 1
  Elsle i 2 := Nodes edofn i 1 2
  Elsle i 3 := Nodes edofn i 2 1
  Elsle i 4 := Nodes edofn i 2 2
  Elsle i 5 := Atop
  Elsle i 6 := Itop
  Elsle i 7 := E (curv i - nPE 1; 0; ytop)

```

```

Elslei 8 := Gtop
Elslei 9 := "nh"
Elslei 10 := 0
Elslei 11 := -qser

```

```

dia (Mat) := " Diagonal elements "
for i ∈ [1..nDof]
    diai := Mati i
dia

```

```
curv0 := 0
```

```
curv1 := 1
```

```
curv := matrix(nPE; 1)
```

```

for i ∈ [1..nPE]
    curvi 1 := 0,0000000000000000000001

```

```
times := 0
```

```
NN := matrix(nEl; 1)
```

```

us (curv; Elsle; NN) :=
    while  $\left| \frac{curv_{11} - curv0}{curv_{11}} \right| > 0,001 \wedge (times < 20)$ 
        curv0 := curv11
        times := times + 1
        for i ∈ [(nPE + 1)..(2 · nPE)]
            Elslei 7 := E  $\left( curv_{i - nPE 1}; NN_i; \frac{htop}{2} \right)$ 
        genKsle := K := matrix(nDof; nDof)
        for i ∈ [1..nEl]
            if Elslei 9 = "nh"
                KKe := Ke (Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5; E
            else
                if Elslei 9 = "lh"
                    KKe := Kel (Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5
                else
                    KKe := Ker (Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 5
            edofe := edof

```

```

    [i..i][1..6]
    K := assemk (K; KKe; edofe)
    K
km := dia (genKsle)
c := max (km) * 1000000
bc := matrix (nDof; 2)
for i ∈ [1..nDof]
    bci 1 := i
    bc1 2 := 1
    bc2 2 := 1
    bcedofnPE 5 2 := 1
Kbcsle := genKsle
for i ∈ [1..nDof]
    if bci 2 = 1
        Kbcslei i := Kbcslei i + c
    else
        continue
genFlsle := fl := matrix (nDof; 1)
for i ∈ [1..nEl]
    fle := fle (Elslei 1; Elslei 2; Elslei 3; Elslei 4; Elslei 10;
    edofe := edof [i..i][1..6]
    fl := assemf (fl; fle; edofe)
    fl
fsle := genFlsle
usle := invert (Kbcsle) * fsle
edsle := matrix (nEl; 6)
for i ∈ [1..nEl]
    for j ∈ [1..6]
        edslei j := usleedofi j
m (ed) := b0 := matrix (1; nEl)
        b1 := matrix (1; nEl)
        for i ∈ [1..nEl]
            L :=  $\sqrt{(Elsle_{i 3} - Elsle_{i 1})^2 + (Elsle_{i 4} - Elsle_{i 2})^2}$ 
            n0i :=  $\frac{Elsle_{i 3} - Elsle_{i 1}}{L}$ 
            n1i :=  $\frac{Elsle_{i 4} - Elsle_{i 2}}{L}$ 
        m := matrix (1; nEl)
        for i ∈ [1..nEl]
            mi :=  $\begin{bmatrix} n0_i & -n1_i & 0 & 0 & 0 & 0 \\ n1_i & n0_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & n0_i & -n1_i & 0 \\ 0 & 0 & 0 & n1_i & n0_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \text{submatrix} (ed; i; i; 1; 6)^T$ 
        m
msle := m (edsle)

```

```

M1:=matrix(nEl;1)
M2:=matrix(nEl;1)
V1:=matrix(nEl;1)
V2:=matrix(nEl;1)
for i∈[1..nEl]
  x1:=Elsle_i 1
  y1:=Elsle_i 2
  x2:=Elsle_i 3
  y2:=Elsle_i 4
  L:=sqrt((x2-x1)^2+(y2-y1)^2)
  A:=Elsle_i 5
  I:=Elsle_i 6
  E:=Elsle_i 7
  G:=Elsle_i 8
  qx:=Elsle_i 10
  qy:=Elsle_i 11
  co:=frac(12,2)*frac(ks*E*I,G*A)
  u1:=msle_1 i 1
  u2:=msle_1 i 4
  v1:=msle_1 i 2
  t1:=msle_1 i 3
  v2:=msle_1 i 5
  t2:=msle_1 i 6
  fle:=fle(x1;y1;x2;y2;qx;qy)
  if Elsle_i 9=="nh"
    M1sle_i:=left[ -left[ left[frac(1,1+co)]*left[frac(6*E*I,L^2)]*v1+left[frac(E,1+co)*4*frac(I,L]*left[1+frac(co,4)]right]*t1+left[-frac(E,1+co)
    M2sle_i:=left[frac(1,1+co)]*left[frac(6*E*I,L^2)]*v1+left[frac(E,1+co)*2*frac(I,L]*left[1-frac(co,2)]right]*t1+left[frac(-E,1+co)]*6*frac(I,L
    V1sle_i:=left[frac(1,1+co)]*left[frac(12*E*I,L^3)]*v1+left[frac(E,1+co)*6*frac(I,L^2)]*t1+left[-frac(E,1+co)]*12*frac(I,L^3)
    V2sle_i:=left[-left[frac(-1,1+co)]*left[frac(12*E*I,L^3)]*v1+left[frac(-E,1+co)*6*frac(I,L^2)]*t1+left[frac(E,1+co)]*12*frac(I,L^3)
  else
    if Elsle_i 9=="lh"
      M1sle_i:=fle_2
      M2sle_i:=left[frac(6*E*I,L^2*(co+1))+frac(6*E*I*(co-2),L^2*(co+4)*(co+1))right]*v1-frac(6*E*I*(co-2),L^2*(co+4)*(co+1))
      V1sle_i:=left[frac(12*E*I,3)+frac((-36)*E*I,3)right]*v1+left[frac(-12*E*I,3)+frac(12*E*I,3)right]*t1

```

```

    ( L3 · (co + 1) - L3 · (co + 4) · (co + 1) ) ( L3 · (co + 1) - L3 · (co + 4) · (co + 1) )
    V2slei := - ( ( -12 · E · I / L3 · (co + 1) + 36 · E · I / L3 · (co + 4) · (co + 1) ) · v1 + ( 12 · E · I / L3 · (co + 1) + 36 · E · I / L3 · (co + 4) · (co + 1) ) · v2 )
else
    M1slei := - ( ( 6 · E · I / L2 · (co + 1) + 6 · E · I · (co - 2) / L2 · (co + 4) · (co + 1) ) · v1 - E · I · (co - 2)2 / L · (co + 4) · (co + 1) )
    M2slei := fle5
    V1slei := ( 12 · E · I / L3 · (co + 1) - 36 · E · I / L3 · (co + 4) · (co + 1) ) · v1 + ( 6 · E · I / L2 · (co + 1) + 6 · E · I · (co - 2) / L2 · (co + 4) · (co + 1) ) · v2
    V2slei := - ( ( -12 · E · I / L3 · (co + 1) + 36 · E · I / L3 · (co + 4) · (co + 1) ) · v1 + ( 12 · E · I / L3 · (co + 1) + 36 · E · I / L3 · (co + 4) · (co + 1) ) · v2 )
    N1slei := E · A / L · (u1 - u2)
    N2slei := E · A / L · (u1 - u2)
    NNi := (N1slei + N2slei) / 2
for i ∈ [(nPE + 1) .. (2 · nPE)]
    curvi - nPE 1 := (msle1 i 6 - msle1 i 3) / (Elslei 3 - Elslei 1)
usle

```

```

    1,9912 · 10-26
    -8,2564 · 10-14
    -0,0037
    4,3754 · 10-15
    -1,9197
    0,0017
    -0,0174
    -3,1621
    0,0007
    -0,0447
    -3,1621
    -0,0007
    -0,0621
    -1,9197
    -0,0017
    -0,0621
    -8,2564 · 10-14
    0,0037
    -0,0342
    -0,0022

```

$$us(curv; Elsie; NN) =$$

$$\begin{bmatrix} -0,002 \\ -0,0342 \\ -1,9196 \\ -0,0017 \\ -0,0326 \\ -3,1622 \\ -0,0007 \\ -0,0295 \\ -3,1622 \\ 0,0007 \\ -0,0279 \\ -1,9196 \\ 0,0017 \\ -0,0279 \\ -0,0022 \\ 0,002 \\ -0,0142 \\ -0,0009 \\ -0,0003 \\ 0,0836 \\ -1,9196 \\ -0,0017 \\ 0,0161 \\ -3,1621 \\ -0,0007 \\ -0,0782 \\ -3,1621 \\ 0,0007 \\ -0,1457 \\ -1,9196 \\ 0,0017 \\ -0,0479 \\ -0,0009 \\ 0,0003 \end{bmatrix}$$

$$curv = \begin{bmatrix} 3,1557 \cdot 10^{-7} \\ 9,9994 \cdot 10^{-7} \\ 1,4223 \cdot 10^{-6} \\ 9,9994 \cdot 10^{-7} \\ 3,1557 \cdot 10^{-7} \end{bmatrix}$$

$$NN = \begin{bmatrix} -5,6793 \cdot 10^{-10} \\ 2262,5285 \\ 3538,4062 \\ 2262,5285 \\ 6,7931 \cdot 10^{-10} \\ 1,9318 \cdot 10^{-10} \\ -2262,5285 \\ -3538,4062 \\ -2262,5285 \\ -4,4112 \cdot 10^{-10} \\ -2453,5577 \\ 78,3058 \\ -124,7481 \\ -124,7481 \\ 78,3058 \\ -2453,5577 \\ -2453,5577 \end{bmatrix}$$

$$\begin{bmatrix} 78,3058 \\ -124,7481 \\ -124,7481 \\ 78,3058 \\ \vdots \end{bmatrix}$$

$$\begin{aligned}
 \text{Elsle} = & \begin{bmatrix}
 0 & 0 & 1000 & 0 & 11800 & 3,9333 \cdot 10^5 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 1000 & 0 & 2000 & 0 & 11800 & 3,9333 \cdot 10^5 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 2000 & 0 & 3000 & 0 & 11800 & 3,9333 \cdot 10^5 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 3000 & 0 & 4000 & 0 & 11800 & 3,9333 \cdot 10^5 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 4000 & 0 & 5000 & 0 & 11800 & 3,9333 \cdot 10^5 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 0 & 120 & 1000 & 120 & 82600 & 1,3491 \cdot 10^8 & 24900 & 1,3 \cdot 10^{10} & \text{"nh"} & 0 & -1 \\
 1000 & 120 & 2000 & 120 & 82600 & 1,3491 \cdot 10^8 & 17168,7007 & 1,3 \cdot 10^{10} & \text{"nh"} & 0 & -1 \\
 2000 & 120 & 3000 & 120 & 82600 & 1,3491 \cdot 10^8 & 13885,0578 & 1,3 \cdot 10^{10} & \text{"nh"} & 0 & -1 \\
 3000 & 120 & 4000 & 120 & 82600 & 1,3491 \cdot 10^8 & 17168,7007 & 1,3 \cdot 10^{10} & \text{"nh"} & 0 & -1 \\
 4000 & 120 & 5000 & 120 & 82600 & 1,3491 \cdot 10^8 & 24900 & 1,3 \cdot 10^{10} & \text{"nh"} & 0 & -1 \\
 0 & 0 & 0 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"lh"} & 0 & 0 \\
 1000 & 0 & 1000 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"rh"} & 0 & 0 \\
 2000 & 0 & 2000 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"rh"} & 0 & 0 \\
 3000 & 0 & 3000 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"rh"} & 0 & 0 \\
 4000 & 0 & 4000 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"rh"} & 0 & 0 \\
 5000 & 0 & 5000 & 50 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"lh"} & 0 & 0 \\
 0 & 50 & 0 & 120 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"rh"} & 0 & 0 \\
 1000 & 50 & 1000 & 120 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 2000 & 50 & 2000 & 120 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0 \\
 3000 & 50 & 3000 & 120 & 12027,2418 & 1,2055 \cdot 10^7 & 11000 & 6,9 \cdot 10^8 & \text{"nh"} & 0 & 0
 \end{bmatrix}
 \end{aligned}$$

NN =

$$\begin{bmatrix}
 -5,679 \\
 2262 \\
 3538 \\
 2262 \\
 6,793 \\
 1,931 \\
 -2262 \\
 -3538 \\
 -2262 \\
 -4,41 \\
 -245 \\
 78 \\
 -12 \\
 -12 \\
 78 \\
 -245 \\
 -245 \\
 78 \\
 -12 \\
 -12 \\
 78 \\
 -245
 \end{bmatrix}$$

| | | | | | | | | | | |
|------|----|------|-----|------------|---------------------|-------|------------------|------|---|---|
| 4000 | 50 | 4000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "nh" | 0 | 0 |
| 5000 | 50 | 5000 | 120 | 12027,2418 | $1,2055 \cdot 10^7$ | 11000 | $6,9 \cdot 10^8$ | "rh" | 0 | 0 |