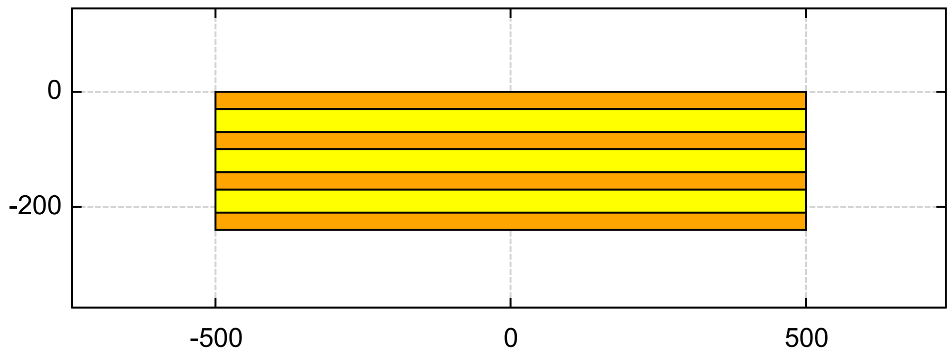
SHEAR FACTOR CALCULATIONS

Section geometry	Width	Height
Layer 1	$b1 := 1000$	$h1 := 30$
Layer 2	$b2 := 1000$	$h2 := 40$
Layer 3	$b3 := 1000$	$h3 := 30$
Layer 4	$b4 := 1000$	$h4 := 40$
Layer 5	$b5 := 1000$	$h5 := 30$
Layer 6	$b6 := 1000$	$h6 := 40$
Layer 7	$b7 := 1000$	$h7 := 30$
Total section height	$h := h1 + h2 + h3 + h4 + h5 + h6 + h7 = 240$	

Section material	Elastic modulus	Shear modulus
Layer 1	$E1 := 12000$	$G1 := 690$
Layer 2	$E2 := 450$	$G2 := 50$
Layer 3	$E3 := 12000$	$G3 := 690$
Layer 4	$E4 := 450$	$G4 := 50$
Layer 5	$E5 := 12000$	$G5 := 690$
Layer 6	$E6 := 450$	$G6 := 50$
Layer 7	$E7 := 12000$	$G7 := 690$

Shear stiffness $GA := G1 \cdot h1 \cdot b1 + G2 \cdot h2 \cdot b2 + G3 \cdot h3 \cdot b3 + G4 \cdot h4 \cdot b4 + G5 \cdot h5 \cdot b5 + G6 \cdot h6 \cdot b6 + G7 \cdot h7 \cdot b7$

Section view



Position of neutral axis from the top edge

$$yy := \frac{b1 \cdot h1 \cdot E1 \cdot \frac{h1}{2} + b2 \cdot h2 \cdot E2 \cdot \left(h1 + \frac{h2}{2} \right) + b3 \cdot h3 \cdot E3 \cdot \left(h1 + h2 + \frac{h3}{2} \right) + b4 \cdot h4 \cdot E4 \cdot \left(h1 + h2 + h3 + \frac{h4}{2} \right) + b5 \cdot \ldots}{b1 \cdot h1 \cdot E1 + b2 \cdot h2 \cdot E2 + b3 \cdot h3 \cdot E\ldots}$$

Position of neutral axis from center

$$yc := yy - \frac{h}{2} = 0$$

Approximation of step function for numerical integration

$$H(y) := \frac{1}{2} \cdot (1 + \mathrm{sign}(y))$$

Elastic modulus expression defined by means of the Heaviside step function

$$E(y) := E1 \cdot H(y + yc + h) - (E1 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1 \right) + (E3 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2 \right) + (E4 - E3) \cdot H\left(y + \ldots \right.$$

Shear modulus expression defined by means of the Heaviside step function

$$G(y) := G1 \cdot H(y + yc + h) - (G1 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1 \right) + (G3 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2 \right) + (G4 - G3) \cdot H\left(y + \ldots \right.$$

Cross sectional width defined by means of the Heaviside step function

$$b(y) := b1 \cdot H(y + yc + h) - (b1 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1 \right) + (b3 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2 \right) + (b4 - b3) \cdot H\left(y + \ldots \right.$$

Bending stiffness

$$EI := \mathrm{Int}\left(E(y) \cdot b(y) \cdot y^2 ; y ; -\frac{h}{2} - yc ; \frac{h}{2} - yc \right) = 9{,}1116 \cdot 10^{12}$$

Balance of energy of the beam for linear elasticity

Shear force $V := 1$

Shear flow expression: $T(y) := \frac{-V}{EI} \cdot \mathrm{Int}\left(E(y) \cdot b(y) \cdot y ; y ; -\frac{h}{2} - yc ; y \right)$

Internal energy: $U := \mathrm{eval}\left(\frac{1}{2} \cdot \mathrm{Int}\left(\frac{(T(y))^2}{G(y) \cdot b(y)} ; y ; -\frac{h}{2} - yc ; \frac{h}{2} - yc \right) \right) = 2{,}8561 \cdot 10^{-8}$

External energy: $W(ks) := \frac{1}{2} \cdot \frac{ks}{GA} \cdot V^2 = \frac{ks}{177600000}$

Shear correction factor: $ks := \frac{U \cdot ks}{W(ks)} = 5{,}0723$

Bending stiffness: $\boxed{EI = 9{,}1116 \cdot 10^{12}}$

Shear factor correction: $\boxed{ks = 5{,}0723}$

Corrected shear stiffness: $\boxed{GAc := \frac{GA}{ks} = 1{,}7507 \cdot 10^{7}}$

Simply supported beam: bending deflection vs shear deflection

Uniformly distributed load $q := 10$

Span length $L := 5000$

Cross-section: Cross Laminated Timber KLH 7 layer $h := 240$ $b := 1000$

$$EI := 9{,}1116 \cdot 10^{12}$$

$$GAc := 1{,}7507 \cdot 10^{7}$$

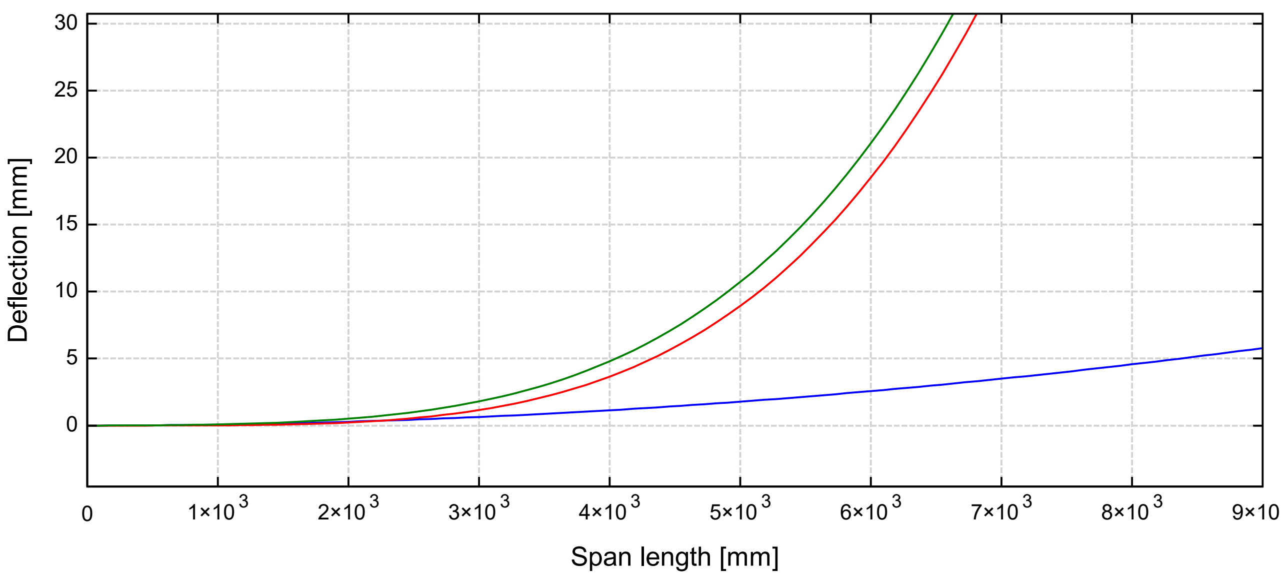
Bending deflection: $fb(L) := \frac{5}{384} \cdot \frac{q \cdot L^4}{EI} = 8{,}9315$

Shear deflection: $fs(L) := \frac{q \cdot L^2}{8 \cdot GAc} = 1{,}785$

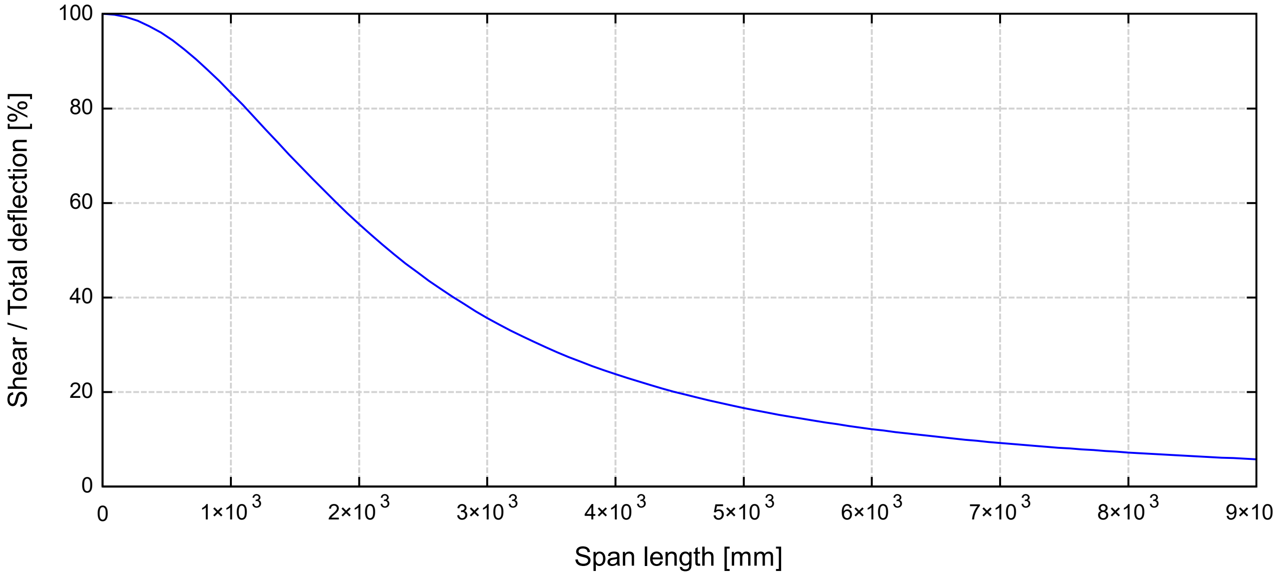
Total deflection: $ft(L) := \frac{5}{384} \cdot \frac{q \cdot L^4}{EI} + \frac{q \cdot L^2}{8 \cdot GAc} = 10{,}7165$

Percentage of shear deflection in comparison to the total: $per(L) := \frac{fs(L)}{ft(L)}$

$$Defl := \begin{cases} fs(x) \\ fb(x) \\ ft(x) \end{cases}$$



$$per(x) := \frac{\frac{q}{8 \cdot GAc}}{\frac{5}{384} \cdot \frac{q \cdot x^{2}}{EI} + \frac{q}{8 \cdot GAc}}$$



$$H(y) := \frac{1}{2} \cdot (1 + \mathrm{sign}(y))$$

$$E(y) := E1 \cdot H(y + yc + h) - (E1 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (E3 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (E4 - E3) \cdot H\left(y +\right.$$

$$G(y) := G1 \cdot H(y + yc + h) - (G1 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (G3 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (G4 - G3) \cdot H\left(y +\right.$$

$$b(y) := b1 \cdot H(y + yc + h) - (b1 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (b3 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (b4 - b3) \cdot H\left(y +\right.$$

$$nDiv := 100$$

$$T(yy) := \left| \begin{array}{l} step := \frac{h}{nDiv} \\ y := -\frac{h}{2} - yc \\ int := 0 \\ \text{while } \; y < yy \\ \quad \left| \begin{array}{l} y := y + step \\ int := int + step \cdot \left(\frac{E(y) \cdot b(y) \cdot y + E(y - step) \cdot b(y - step) \cdot (y - step)}{2} \right) \end{array} \right. \\ \frac{(-int) \cdot V}{EI} \end{array} \right.$$

$$U := \left| \begin{array}{l} step := \frac{h}{nDiv} \\ y := -\frac{h}{2} - yc \\ int := 0 \\ \text{while } \; y \le \frac{h}{2} - yc \\ \quad \left| \begin{array}{l} y := y + step \\ int := int + step \cdot \left(\frac{\frac{(T(y))^2}{G(y) \cdot b(y)} + \frac{(T(y - step))^2}{G(y - step) \cdot b(y - step)}}{2} \right) \end{array} \right. \\ \frac{int}{2} \end{array} \right.$$

$$U = 2{,}8782 \cdot 10^{-8} \qquad \boxed{2{,}8561 \cdot 10^{-8}}$$

$$W(kk) := \frac{1}{2} \cdot \frac{kk}{GA} \cdot V^2 = \frac{kk}{177600000}$$

$$kk := \frac{U \cdot kk}{W(kk)} = 5{,}1117 \qquad \boxed{5{,}0723}$$

$$H(y) := \frac{1}{2} \cdot (1 + \mathrm{sign}(y))$$

$$E(y) := E1 \cdot H(y + yc + h) - (E1 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (E3 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (E4 - E3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$$G(y) := G1 \cdot H(y + yc + h) - (G1 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (G3 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (G4 - G3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$$b(y) := b1 \cdot H(y + yc + h) - (b1 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (b3 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (b4 - b3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$$T(yy) := \left| \begin{array}{l} step := \frac{h}{200} \\ y := -\frac{h}{2} - yc \\ int := 0 \\ \text{while } y \le yy \\ \quad \left| \begin{array}{l} y := y + step \\ int := int + \frac{step}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - step}{2}\right) \cdot b\left(\frac{2 \cdot y - step}{2}\right) \cdot \left(\frac{2 \cdot y - step}{2}\right) + E(y - step) \cdot b(y - \right. \end{array} \right. \\ \left(\frac{-V}{EI}\right) \cdot int \end{array} \right.$$

$$U := \left| \begin{array}{l} step := \frac{h}{200} \\ y := -\frac{h}{2} - yc \\ int := 0 \\ \text{while } y \le \frac{h}{2} - yc \\ \quad \left| \begin{array}{l} y := y + step \\ int := int + \frac{step}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - step}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - step}{2}\right) \cdot b\left(\frac{2 \cdot y - step}{2}\right)} + \frac{(T(y - step))^2}{G(y - step) \cdot b(y - step)} \right) \end{array} \right. \\ \frac{int}{2} \end{array} \right.$$

$U = ▮$

$\boxed{2{,}8561 \cdot 10^{-8}}$

$$W(kk) := \frac{1}{2} \cdot \frac{kk}{GA} \cdot V^2 = \frac{kk}{177600000}$$

$$kk := \frac{U \cdot kk}{W(kk)} = 5{,}0788$$

$\boxed{5{,}0723}$

$$H(y) := \frac{1}{2} \cdot (1 + \mathrm{sign}(y))$$

$$E(y) := E1 \cdot H(y + yc + h) - (E1 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (E3 - E2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (E4 - E3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$$G(y) := G1 \cdot H(y + yc + h) - (G1 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (G3 - G2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (G4 - G3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$$b(y) := b1 \cdot H(y + yc + h) - (b1 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1\right) + (b3 - b2) \cdot H\left(y + yc + \frac{h}{2} - h1 - h2\right) + (b4 - b3) \cdot H\left(y + yc + \frac{h}{2} -\right.$$

$T(yy) :=$

$s1 := \frac{h1}{10}$

$s2 := \frac{h2}{10}$

$s3 := \frac{h3}{10}$

$s4 := \frac{h4}{10}$

$s5 := \frac{h5}{10}$

$s6 := \frac{h6}{10}$

$s7 := \frac{h7}{10}$

$y := -\frac{h}{2} - yc$

$int := 0$

$\text{while } \left((y \le yy) \wedge \left(y \le h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y \ge -\frac{h}{2} - yc\right)$

$\quad y := y + s1$

$\quad int := int + \frac{s1}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s1}{2}\right) \cdot b\left(\frac{2 \cdot y - s1}{2}\right) \cdot \left(\frac{2 \cdot y - s1}{2}\right) + E(y - s1) \cdot b(y - s1) \cdot (y - s1\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h2 + h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s2$

$\quad int := int + \frac{s2}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s2}{2}\right) \cdot b\left(\frac{2 \cdot y - s2}{2}\right) \cdot \left(\frac{2 \cdot y - s2}{2}\right) + E(y - s2) \cdot b(y - s2) \cdot (y - s2\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h3 + h2 + h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h2 + h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s3$

$\quad int := int + \frac{s3}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s3}{2}\right) \cdot b\left(\frac{2 \cdot y - s3}{2}\right) \cdot \left(\frac{2 \cdot y - s3}{2}\right) + E(y - s3) \cdot b(y - s3) \cdot (y - s3\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s4$

$\quad int := int + \frac{s4}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s4}{2}\right) \cdot b\left(\frac{2 \cdot y - s4}{2}\right) \cdot \left(\frac{2 \cdot y - s4}{2}\right) + E(y - s4) \cdot b(y - s4) \cdot (y - s4\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s5$

$\quad int := int + \frac{s5}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s5}{2}\right) \cdot b\left(\frac{2 \cdot y - s5}{2}\right) \cdot \left(\frac{2 \cdot y - s5}{2}\right) + E(y - s5) \cdot b(y - s5) \cdot (y - s5\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h6 + h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s6$

$\quad int := int + \frac{s6}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s6}{2}\right) \cdot b\left(\frac{2 \cdot y - s6}{2}\right) \cdot \left(\frac{2 \cdot y - s6}{2}\right) + E(y - s6) \cdot b(y - s6) \cdot (y - s6\right.$

$\text{while } \left((y \le yy) \wedge \left(y \le h7 + h6 + h5 + h4 + h3 + h2 - \frac{h}{2} - yc\right)\right) \wedge \left(y > h6 + h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$\quad y := y + s7$

$\quad int := int + \frac{s7}{6} \cdot \left(E(y) \cdot b(y) \cdot y + 4 \cdot E\left(\frac{2 \cdot y - s7}{2}\right) \cdot b\left(\frac{2 \cdot y - s7}{2}\right) \cdot \left(\frac{2 \cdot y - s7}{2}\right) + E(y - s7) \cdot b(y - s7) \cdot (y - s7\right.$

$\left(\frac{-V}{EI}\right) \cdot int$

$U :=$

$s1 := \frac{h1}{10}$

$s2 := \frac{h2}{10}$

$s3 := \frac{h3}{10}$

$s4 := \frac{h4}{10}$

$s5 := \frac{h5}{10}$

$s6 := \frac{h6}{10}$

$s7 := \frac{h7}{10}$

$y := -\frac{h}{2} - yc$

$int := 0$

while $\left(y \le h1 - \frac{h}{2} - yc\right) \wedge \left(y \ge -\frac{h}{2} - yc\right)$

$y := y + s1$

$$int := int + \frac{s1}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s1}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s1}{2}\right) \cdot b\left(\frac{2 \cdot y - s1}{2}\right)} + \frac{(T(y - s1))^2}{G(y - s1) \cdot b(y - s1)} \right)$$

while $\left(y \le h2 + h1 - \frac{h}{2} - yc\right) \wedge \left(y > h1 - \frac{h}{2} - yc\right)$

$y := y + s2$

$$int := int + \frac{s2}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s2}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s2}{2}\right) \cdot b\left(\frac{2 \cdot y - s2}{2}\right)} + \frac{(T(y - s2))^2}{G(y - s2) \cdot b(y - s2)} \right)$$

while $\left(y \le h3 + h2 + h1 - \frac{h}{2} - yc\right) \wedge \left(y > h2 + h1 - \frac{h}{2} - yc\right)$

$y := y + s3$

$$int := int + \frac{s3}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s3}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s3}{2}\right) \cdot b\left(\frac{2 \cdot y - s3}{2}\right)} + \frac{(T(y - s3))^2}{G(y - s3) \cdot b(y - s3)} \right)$$

while $\left(y \le h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right) \wedge \left(y > h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$y := y + s4$

$$int := int + \frac{s4}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s4}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s4}{2}\right) \cdot b\left(\frac{2 \cdot y - s4}{2}\right)} + \frac{(T(y - s4))^2}{G(y - s4) \cdot b(y - s4)} \right)$$

while $\left(y \le h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right) \wedge \left(y > h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$y := y + s5$

$$int := int + \frac{s5}{6} \cdot \left(\frac{(T(y))^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s5}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s5}{2}\right) \cdot b\left(\frac{2 \cdot y - s5}{2}\right)} + \frac{(T(y - s5))^2}{G(y - s5) \cdot b(y - s5)} \right)$$

while $\left(y \le h6 + h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right) \wedge \left(y > h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc\right)$

$y := y + s6$

$$\mathrm{int} := \mathrm{int} + \frac{s6}{6} \cdot \left(\frac{\left(T(y)\right)^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s6}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s6}{2}\right) \cdot b\left(\frac{2 \cdot y - s6}{2}\right)} + \frac{\left(T(y - s6)\right)^2}{G(y - s6) \cdot b(y - s6)} \right)$$

$$\text{while } \left(y \le h7 + h6 + h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc \right) \wedge \left(y > h6 + h5 + h4 + h3 + h2 + h1 - \frac{h}{2} - yc \right)$$

$$y := y + s7$$

$$\mathrm{int} := \mathrm{int} + \frac{s7}{6} \cdot \left(\frac{\left(T(y)\right)^2}{G(y) \cdot b(y)} + 4 \cdot \frac{\left(T\left(\frac{2 \cdot y - s7}{2}\right)\right)^2}{G\left(\frac{2 \cdot y - s7}{2}\right) \cdot b\left(\frac{2 \cdot y - s7}{2}\right)} + \frac{\left(T(y - s7)\right)^2}{G(y - s7) \cdot b(y - s7)} \right)$$

$$\frac{\mathrm{int}}{2}$$

$U = \blacksquare$ $\qquad \boxed{2{,}8561 \cdot 10^{-8}}$

$$W(kk) := \frac{1}{2} \cdot \frac{kk}{GA} \cdot V^2 = \frac{kk}{177600000}$$

$kk := \frac{U \cdot kk}{W(kk)} = \blacksquare$ $\qquad \boxed{5{,}0723}$