TIMOSHENKO BEAM - Single supported beam with 2 concentrated forces

Beam spans

Span lenght [mm] $L := 6000$

Rectangular cross-section

Section height $h := 200$

Approximation step-function $H(x) := \dfrac{1}{2} \cdot (1 + \text{sign}(x))$

Section widths $b_1 := 100$ $b_2 := 200$

Width change position $x_b := \dfrac{L}{2}$

Variable section width $b(x) := b_1 + (b_2 - b_1) \cdot H(x - x_b)$

Shear factor $k_s := 1,2$

Moment of inertia $I(x) := \dfrac{b(x) \cdot h^3}{12}$

Cross - sectional area $A(x) := b(x) \cdot h$

Beam material

Elastic modulus $E := 11000$

Shear modulus $G := 690$

Bending stiffness $EI(x) := E \cdot I(x)$

Shear stiffness $GA(x) := G \cdot A(x)$

Corrected shear stiffness $GAc(x) := \dfrac{GA(x)}{k_s}$

Beam load

Load start $x_s := 0$

Load length $x_1 := L = 6000$

Distribuited load [N/mm] $q := 1$

Blocklast $p(x) := q \cdot H(x - x_s) - q \cdot H(x - x_1)$

$$I_4(x) := \frac{\text{Int}\left(\text{Int}\left(\text{Int}\left(\text{Int}\left(p(x); x\right); x\right); x\right); x\right)}{EI(x)} \qquad I_3(x) := \frac{\text{Int}\left(\text{Int}\left(\text{Int}\left(p(x); x\right); x\right); x\right)}{EI(x)}$$

$$I_2(x) := \frac{\text{Int}\left(\text{Int}\left(p(x); x\right); x\right)}{EI(x)}$$

$$v(x) := \left(a_0 + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3\right) + I_4(x)$$

$$v'(x) := \left(a_1 + 2 \cdot a_2 \cdot x + 3 \cdot a_3 \cdot x^2\right) + I_3(x)$$

$$v''(x) := 2 \cdot a_2 + 6 \cdot a_3 \cdot x + I_2(x)$$

$$\text{Assign}\left(\text{Algsys}\left(\begin{bmatrix} v(0) = 0 \\ v''(0) = 0 \\ v(L) = 0 \\ v''(L) = 0 \end{bmatrix}; \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}\right)\right) = \begin{cases} -36,8182 \\ 0,0338 \\ -6,1364 \cdot 10^{-6} \\ 1,7045 \cdot 10^{-10} \end{cases}$$

$$\boxed{u(x) := \left(v(x) - \frac{EI(x)}{GAc(x)} \cdot v''(x)\right)} \qquad \boxed{M(x) := \left(-EI(x)\right) \cdot v''(x)}$$

$$\boxed{\text{eval}\left(u\left(\frac{L}{2}\right)\right) = 17,2011} \qquad \text{Deflection - midspan} \qquad \boxed{w_m := \frac{5}{384} \cdot \frac{q \cdot L^4}{EI\left(\frac{L}{2}\right)} + \frac{q \cdot L^2}{8 \cdot GAc\left(\frac{L}{2}\right)} = 15,6018}$$

$$\boxed{\text{eval}\left(M\left(\frac{L}{2}\right)\right) = 5,625 \cdot 10^6} \qquad \text{Moment - midspan} \qquad \boxed{M_w := \frac{q \cdot L^2}{8} = 4,5 \cdot 10^6}$$