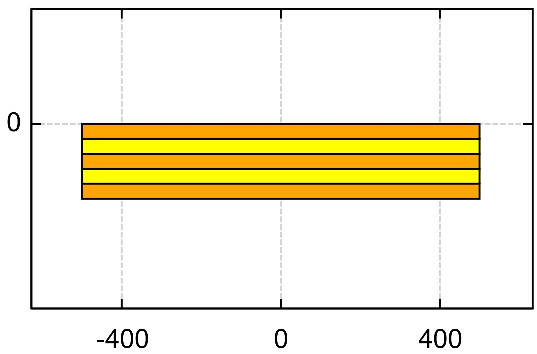
SHEAR FACTOR CALCULATIONS

Section geometry	Width	Height	
Layer 1	$b_1 := 1000$	$h_1 := 20$	[mm]
Layer 2	$b_2 := 1000$	$h_2 := 20$	[mm]
Layer 3	$b_3 := 1000$	$h_3 := 20$	[mm]
Layer 4	$b_4 := 1000$	$h_4 := 20$	[mm]
Layer 5	$b_5 := 1000$	$h_5 := 20$	[mm]
Total section height	$h := h_1 + h_2 + h_3 + h_4 + h_5 = 100$		

Section material	Elastic modulus	Shear modulus	(typical timber properties)
Layer 1	$E_1 := 11000$	$G_1 := 690$	[N/mm²]
Layer 2	$E_2 := 300$	$G_2 := 50$	[N/mm²]
Layer 3	$E_3 := 11000$	$G_3 := 690$	[N/mm²]
Layer 4	$E_4 := 300$	$G_4 := 50$	[N/mm²]
Layer 5	$E_5 := 11000$	$G_5 := 690$	[N/mm²]

Shear stiffness $GA := G_1 \cdot h_1 \cdot b_1 + G_2 \cdot h_2 \cdot b_2 + G_3 \cdot h_3 \cdot b_3 + G_4 \cdot h_4 \cdot b_4 + G_5 \cdot h_5 \cdot b_5 = 4{,}34 \cdot 10^7$ [N]

Section view



Position of neutral axis from the top edge [mm]

$$y_n := \frac{b_2 \cdot h_2 \cdot E_2 \cdot \left(h_1 + \frac{h_2}{2}\right) + b_1 \cdot h_1 \cdot E_1 \cdot \frac{h_1}{2} + b_3 \cdot h_3 \cdot E_3 \cdot \left(h_1 + h_2 + \frac{h_3}{2}\right) + b_4 \cdot h_4 \cdot E_4 \cdot \left(h_1 + h_2 + h_3 + \frac{h_4}{2}\right) + b_5 \cdot h_5 \cdot E_5}{b_1 \cdot h_1 \cdot E_1 + b_2 \cdot h_2 \cdot E_2 + b_3 \cdot h_3 \cdot E_3 + b_4 \cdot h_4 \cdot E_4 + b_5 \cdot h_5 \cdot E_5}$$

Position of neutral axis from center [mm]

$$y_c := y_n - \frac{h}{2} = 0$$

Approximation of step function for numerical integration

$$H(z) := \frac{1}{2} \cdot (1 + \operatorname{sign}(z))$$

Elastic modulus expression for the whole cross-section depending on the distance from the top edge

$$E(z):=E_1\cdot H\left(z+y_c+h\right)-\left(E_1-E_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1\right)+\left(E_3-E_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1-h_2\right)+\left(E_4-E_3\right)\cdot H\left(z+y_c\right.$$

Shear modulus expression for the whole cross-section depending on the distance from the top edge

$$G(z):=G_1\cdot H\left(z+y_c+h\right)-\left(G_1-G_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1\right)+\left(G_3-G_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1-h_2\right)+\left(G_4-G_3\right)\cdot H\left(z+y_c\right.$$

Cross sectional width depending on the distance from the top edge

$$b(z):=b_1\cdot H\left(z+y_c+h\right)-\left(b_1-b_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1\right)+\left(b_3-b_2\right)\cdot H\left(z+y_c+\frac{h}{2}-h_1-h_2\right)+\left(b_4-b_3\right)\cdot H\left(z+y_c\right.$$

Balance of energy of the beam for linear elasticity

Maximal shear force [N] $Q:=1$

Bending stiffness [Nmm²] $EI:=\mathrm{Int}\left(E(z)\cdot b(z)\cdot z^2;\ z;\ -\frac{h}{2};\ \frac{h}{2}\right)=7{,}312\cdot 10^{11}$

Shear flow expression $T(z):=\frac{-Q}{EI}\cdot\mathrm{Int}\left(E(z)\cdot b(z)\cdot z;\ z;\ -\frac{h}{2}-y_c;\ z\right)$

Internal energy $U:=\mathrm{eval}\left(\frac{1}{2}\cdot\mathrm{Int}\left(\frac{(T(z))^2}{G(z)\cdot b(z)};\ z;\ -\frac{h}{2}-y_c;\ \frac{h}{2}-y_c\right)\right)=6{,}2784\cdot 10^{-8}$

External energy $W\left(k_s\right):=\frac{1}{2}\cdot\frac{k_s}{GA}\cdot Q^2=\frac{k_s}{86800000}$

Shear correction factor $k_s:=\frac{U\cdot k_s}{W\left(k_s\right)}=5{,}4497$

Bending stiffness [Nmm²] $\boxed{EI=7{,}312\cdot 10^{11}}$

Shear factor correction [-] $\boxed{k_s=5{,}4497}$

Corrected shear stiffness [N] $\boxed{GAc:=\frac{GA}{k_s}=7{,}9638\cdot 10^{6}}$