Input data:

Input data

Geometry - Concrete

$btop := 590$ $htop := 140$ $ht1 := 80$ $bt1 := 300$ $ht2 := htop - ht1 = 60$

$$ytop := \frac{ht1 \cdot btop \cdot \left(ht2 + \frac{ht1}{2} \right) + \left(\frac{ht2}{2} \right) \cdot (ht2) \cdot bt1}{ht1 \cdot btop + ht2 \cdot bt1} = 80,6748$$

$$Itop := \frac{btop \cdot (htop - ytop)^3 - (btop - bt1) \cdot (ht2 - ytop)^3 + bt1 \cdot ytop^3}{3} = 9,4424 \cdot 1$$

$Atop := btop \cdot htop - ht2 \cdot (btop - bt1) = 65200$

Material - Concrete - C20/25

$Etop := 24900$ $Gtop := 13000000$ $fck := 20$

$Etopinf := \frac{Etop}{1 + 2,5} = 7114,2857$ $\gamma c := 25$

Geometry - Timber

$bbot := 590$ $hbot := 60$

$hb1 := 20$ $Abot := bbot \cdot hbot = 35400$

$At := 2 \cdot hb1 \cdot bbot = 23600$

$$Ibot := 2 \cdot \frac{bbot \cdot hb1^3}{12} + 2 \cdot hb1^2 \cdot hb1 \cdot bbot = 1,0227 \cdot 10^7$$

$ybot := \frac{hbot}{2} = 30$

$Wt := \frac{2 \cdot Ibot}{hbot} = 3,4089 \cdot 10^5$

Material - Timber - C24

$Em0 := 11000$ $Eminf := \frac{11000}{1,6} = 6875$

$fmk := 24$ $ft0k := 14$ $ft90k := 0,4$ $fc0k := 21$

$ks := 1,2$ $fc90k := 2,5$ $fvk := 4,0$ $\gamma t := 5$

$Ebot := Em0$ $Gbot := 690000$ $Ebotinf := Eminf$

Material - connections

$Econ := 11000$ $Gcon := 690000000$

Geometry - connections

$lef := 100$

Reinforcement

$ds := 8$ $nb := 2$ $As2 := 188 \cdot \frac{btop}{1000} = 110,92$

$As1 := nb \cdot \pi \cdot \frac{ds^2}{4} = 100,531$ $d1 := htop - \frac{ds}{2} = 136$

$Kser := 850000$ $d2 := 50$

$Ku := 570000$

$$bconser := \sqrt[4]{\frac{12 \cdot Kser \cdot \left((ytop)^3 + (ybot)^3 \right)}{3 \cdot Econ}} = 114,2929$$

$hconser := bconser$ $\quad Iconser := \frac{\quad}{12}$ $\quad Aconser := bconser$

$$bconu := \sqrt[4]{\frac{12 \cdot Ku \cdot \left((ytop)^3 + (ybot)^3 \right)}{3 \cdot Econ}} = 103,4269$$

$hconu := bconu$ $\quad Iconu := \frac{bconu^4}{12}$ $\quad Aconu := bconu^2$

Span length $\quad Lt := 5000$ $\quad nEdof := 6$ $\quad dtb := ytop + ybot = 110,6748$

Loads $\quad \gamma g := 1,35$ $\quad \gamma q := 1,50$

$$egk := \frac{\left(\gamma c \cdot htop - \gamma c \cdot \frac{(btop - bt1)}{btop} \cdot (htop - ht1) + \gamma t \cdot hbot \right)}{10^3} = 3,0627$$

$gk := 1,0$

$qk := 2,8$ $\quad kmod := 0,8$ $\quad \gamma m := 1,3$ $\quad fvrsk := 1,0$

$$qd := \left(\gamma g \cdot (egk + gk) + \gamma q \cdot qk \right) \cdot \frac{btop}{1000} = 5,714$$

$$qser := \left((egk + gk) + qk \right) \cdot \frac{btop}{1000} = 4,049$$

Rod position

$nn := 10$

connection distance $\quad d := \frac{Lt}{nn} = 500$

$nPN := nn + 1$ $\quad nPE := nPN - 1$

Define connector position

$$conns := \begin{bmatrix} 500 \\ 1000 \\ 1500 \\ 3500 \\ 4000 \\ 4500 \end{bmatrix}$$

⊞— Stiffness matrix for a 1D finite element with shear deformation (Timoshenko)

⊞— Stiffness matrix for a 1D finite element with left internal hinge (Timoshenko)

⊞— Stiffness matrix for a 1D finite element with right internal hinge (Timoshenko)

⊞— Nodes - basic structure

⊞— Add Nodes - Connectors

t = 0

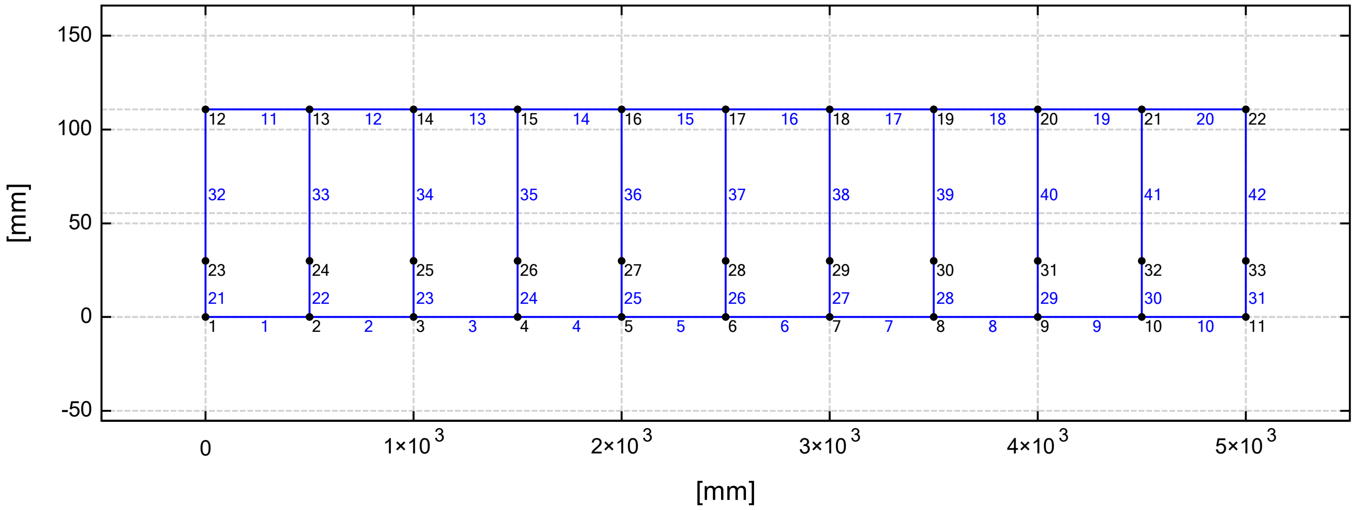
⊞— Elements

⊟— Set connectors

```
for  i ∈ [1..(length(conns))]
  |for  j ∈ [(2·nPE+1)..(2·nPE+2)]
```

```
for j ∈ [(2·nPE+1)..(3·nPE+2)]
  if (Elsle[j 1 = Elsle[j 3) ∧ (Elsle[j 1 = conns[i)
    Elsle[j 9 := "rh"
    Elslu[j 9 := "rh"
for j ∈ [(3·nPE+2)..(4·nPE+2)]
  if (Elsle[j 1 = Elsle[j 3) ∧ (Elsle[j 1 = conns[i)
    Elsle[j 9 := "nh"
    Elslu[j 9 := "nh"
```

System



Stiffness matrix

Boundary conditions (penalty approach)

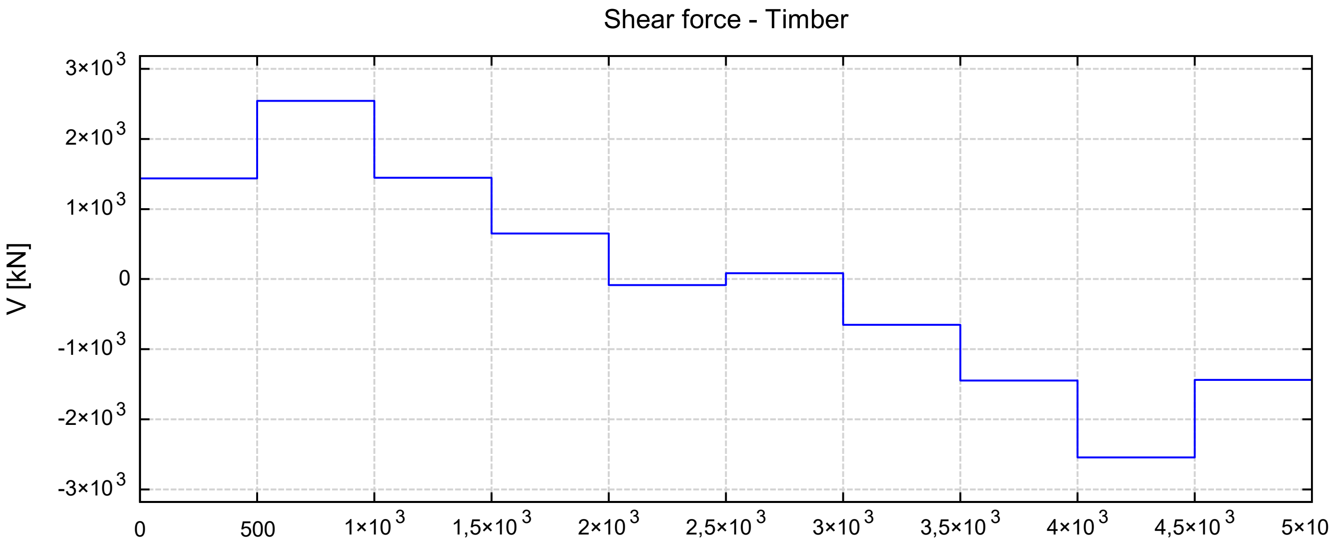
Loads

Displacement (system solution)

Reactions

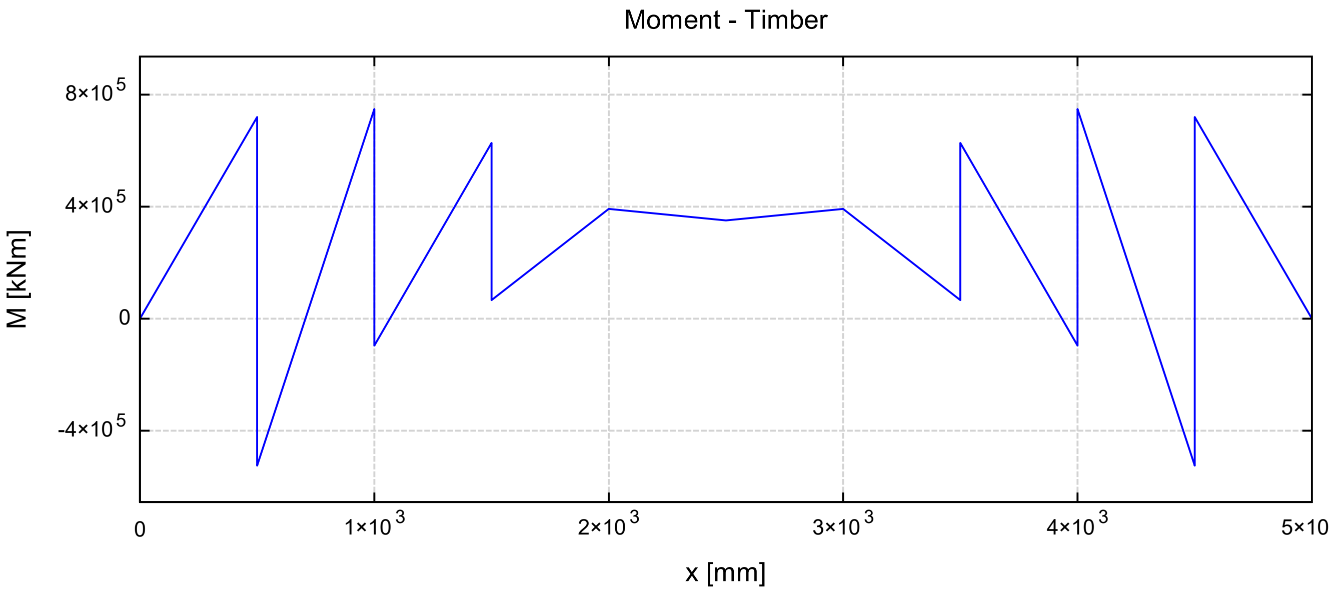
Internal forces

Shear force diagram - Timber

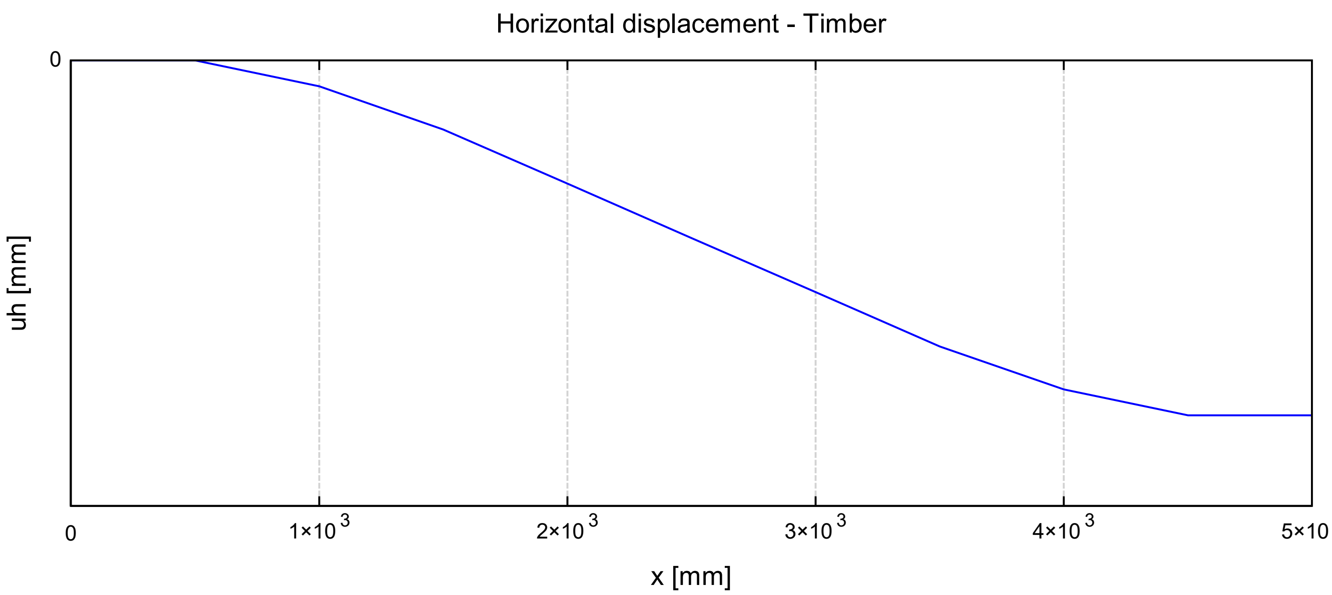


x [mm]

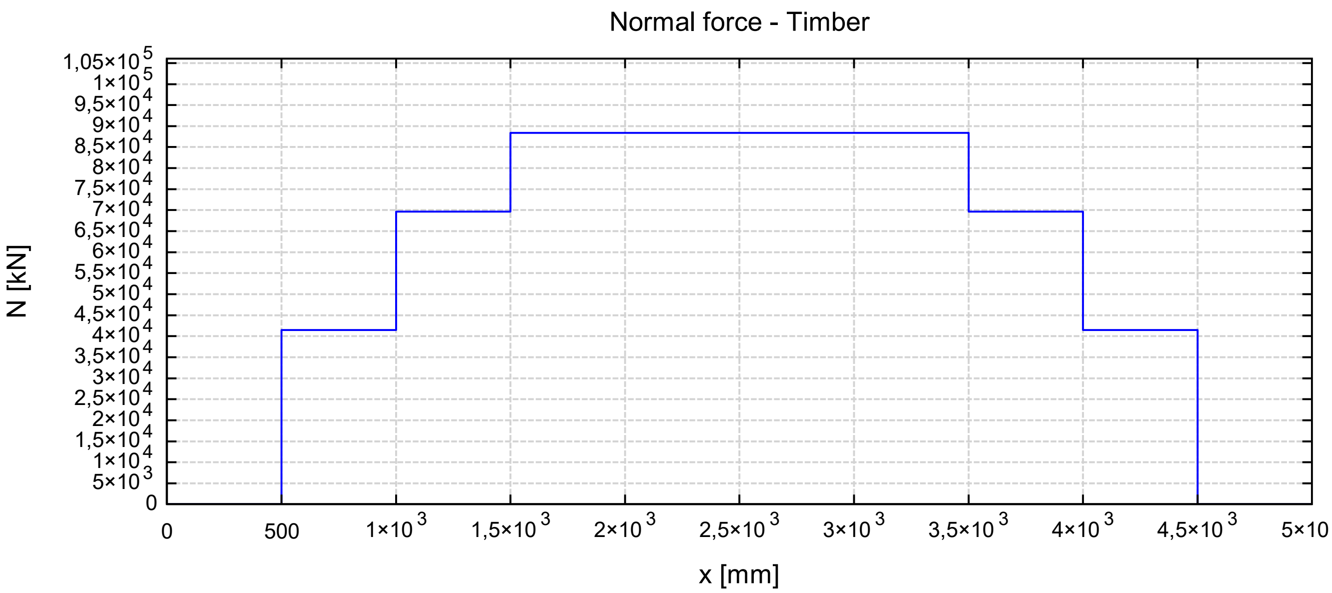
Moment diagram - Timber



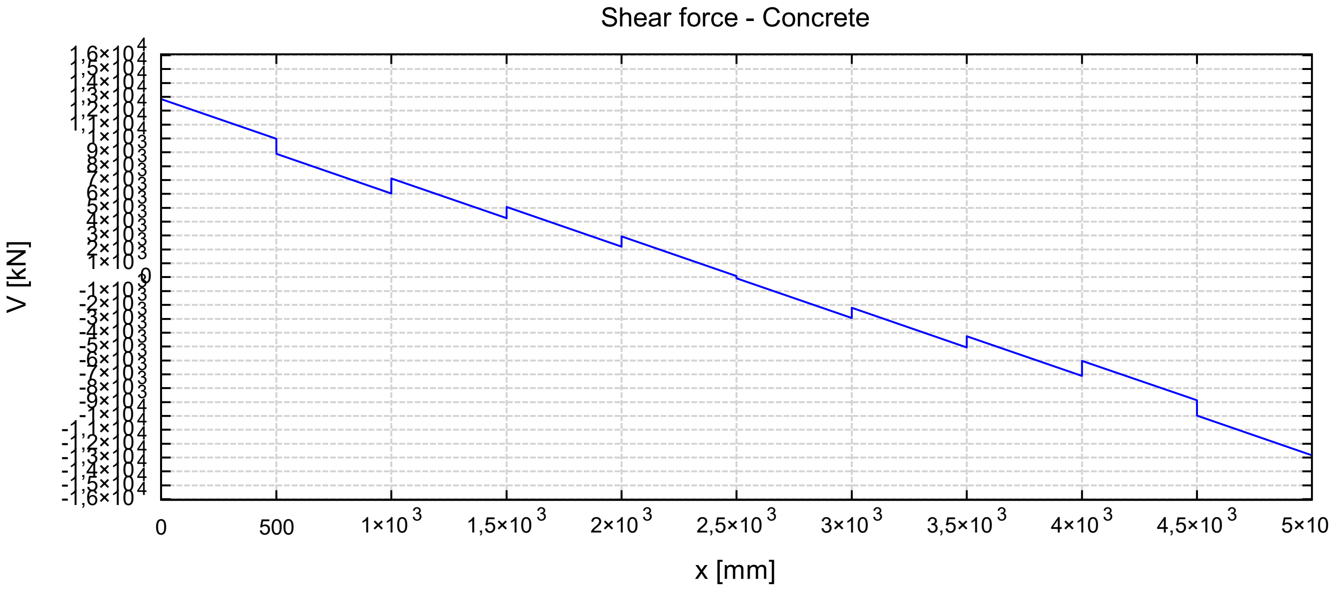
Horizontal displacement - Timber



Normal force diagram - Timber



Shear force diagram - Concrete



Moment diagram - Concrete

$M11 := \mathrm{matrix}(nPE ; 2)$ $\quad$ $M22 := \mathrm{matrix}(nPE ; 2)$

for $i \in [(nPE+1)..(2 \cdot nPE)]$
$\quad M11_{i-nPE\,1} := Nodes_{i+1\,1}$
$\quad M11_{i-nPE\,2} := M1slu_{i}$

for $i \in [(nPE+1)..(2 \cdot nPE)]$
$\quad M22_{i-nPE\,1} := Nodes_{i+2\,1}$
$\quad M22_{i-nPE\,2} := M2slu_{i}$

$M_{max} := \max(\mathrm{col}(M11 ; 2)) = 7{,}9653 \cdot 10^{6}$

$M_{min} := \min(\mathrm{col}(M11 ; 2)) = 2{,}3808 \cdot 10^{5}$

$MM := \mathrm{matrix}(0 ; 2)$

for $i \in [1..(nVal)]$
$\quad MM := \mathrm{stack}\left(MM ; \begin{bmatrix} M11_{i\,1} & M11_{i\,2} \end{bmatrix}\right)$
$\quad MM := \mathrm{stack}\left(MM ; \begin{bmatrix} M22_{i\,1} & M22_{i\,2} \end{bmatrix}\right)$

$XYPlot_{XLimMin_7} := 0$

$XYPlot_{XLimMax_7} := Lt$

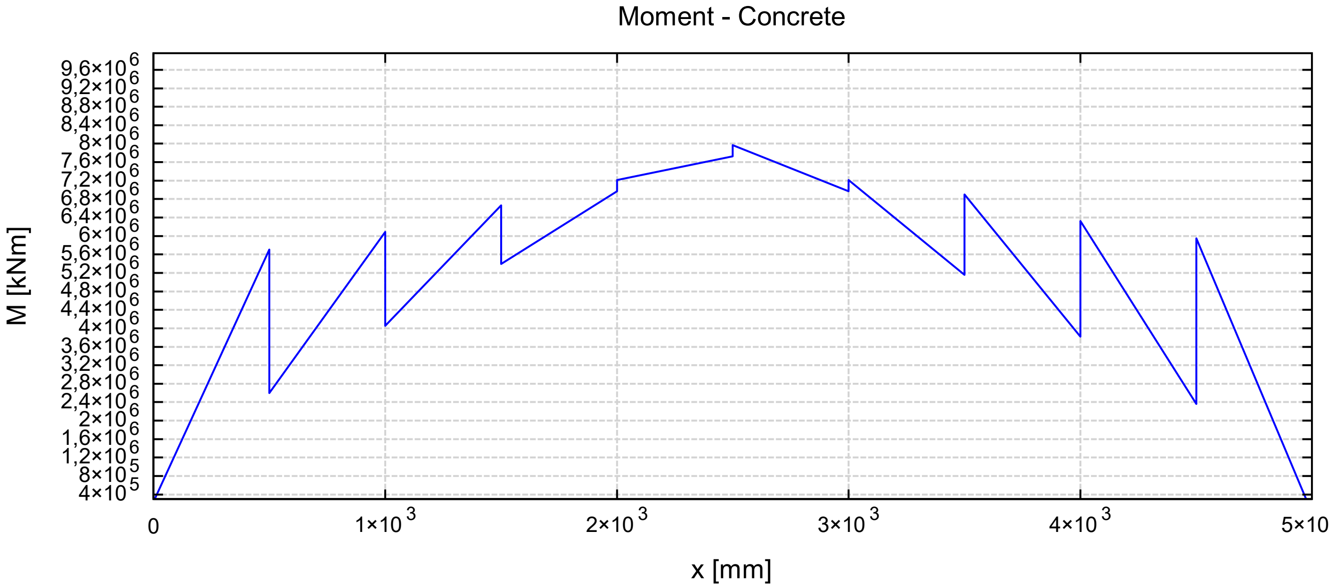
$XYPlot_{YLimMin_7} := \mathrm{round}(1{,}25 \cdot M_{min} ; 2)$ $\quad$ $Mconcmin := M_{min}$

$XYPlot_{YLimMax_7} := \mathrm{round}(1{,}25 \cdot M_{max} ; 2)$ $\quad$ $Mconcmax := M_{max}$

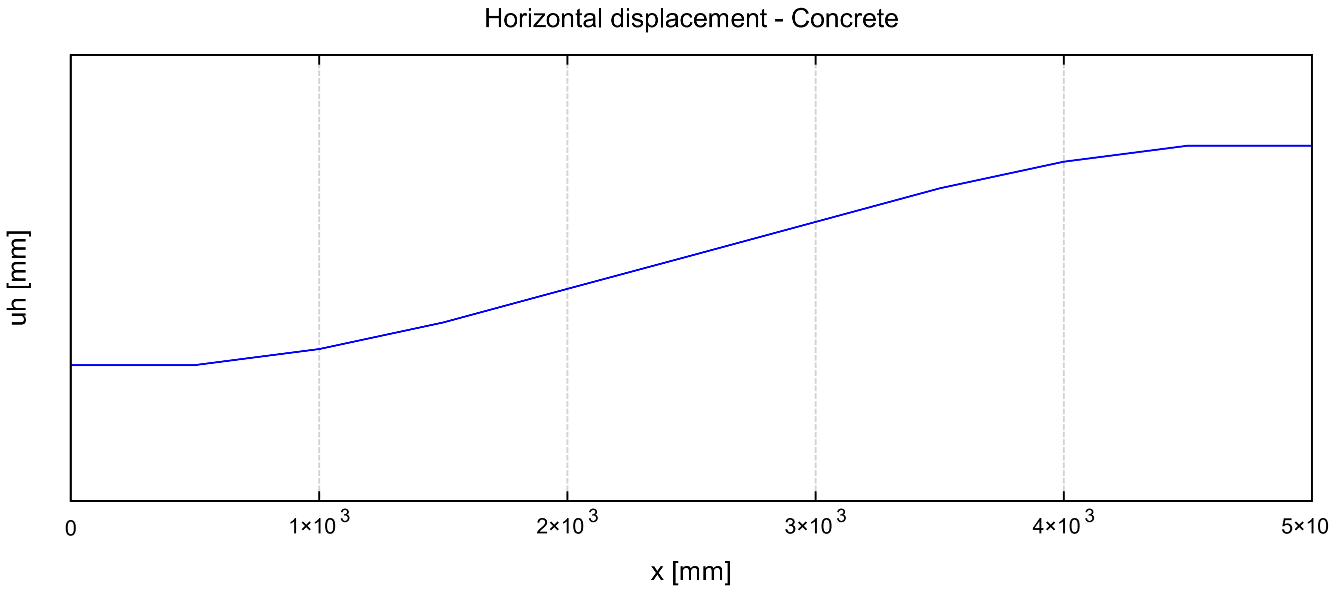
$XYPlot_{XTick_7} := interval\left(\frac{L}{\mathrm{UoM}(L)}\right)$

$range := XYPlot_{YLimMax_7} - XYPlot_{YLimMin_7} = 9{,}659 \cdot 10^{9} \cdot \frac{1}{\mathrm{m}}\ \mathrm{mm}$

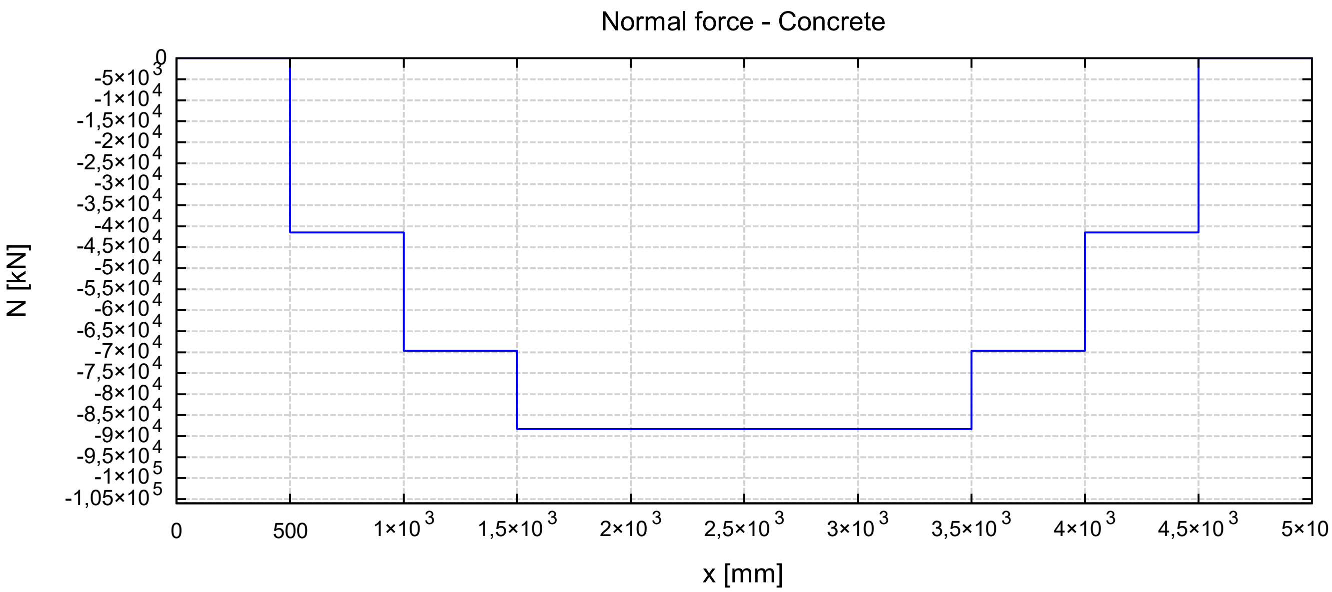
$XYPlot_{YTick_7} := interval\left(\frac{range}{\mathrm{UoM}(range)}\right)$



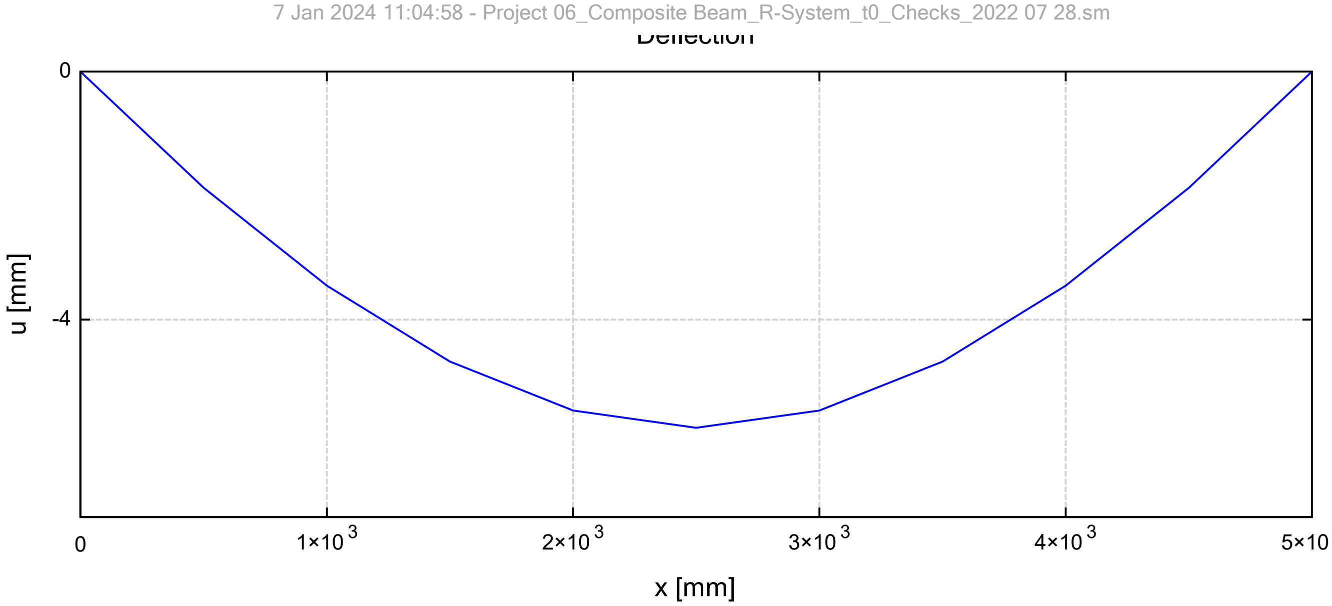
⊞— Horizontal displacement - Concrete



⊞— Normal force diagram - Concrete

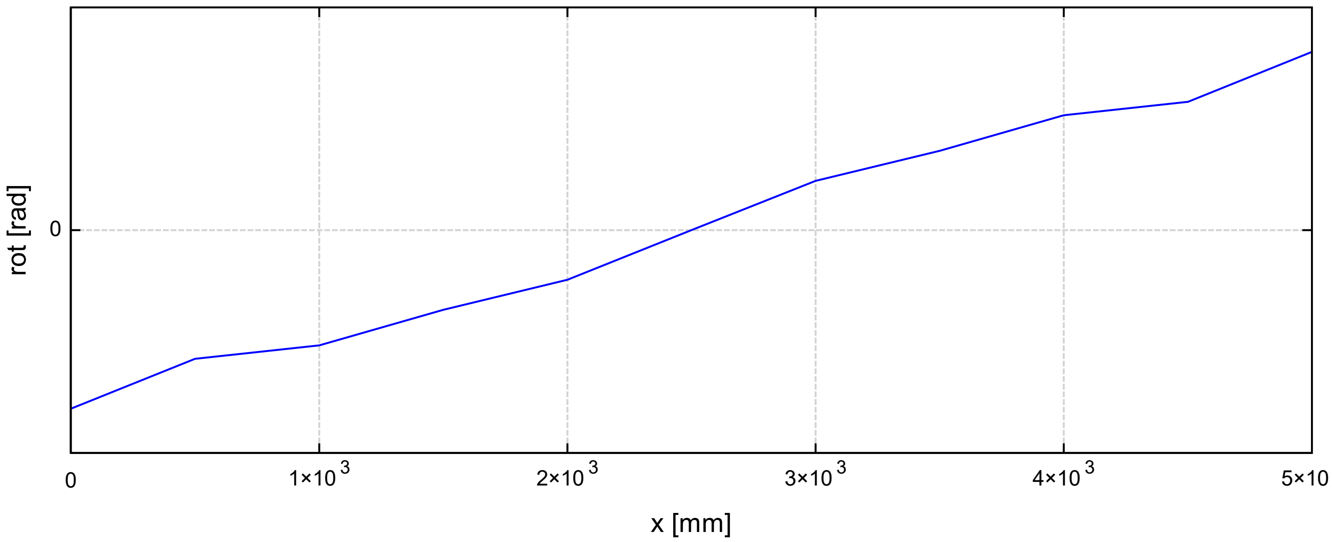


⊞— Deflection



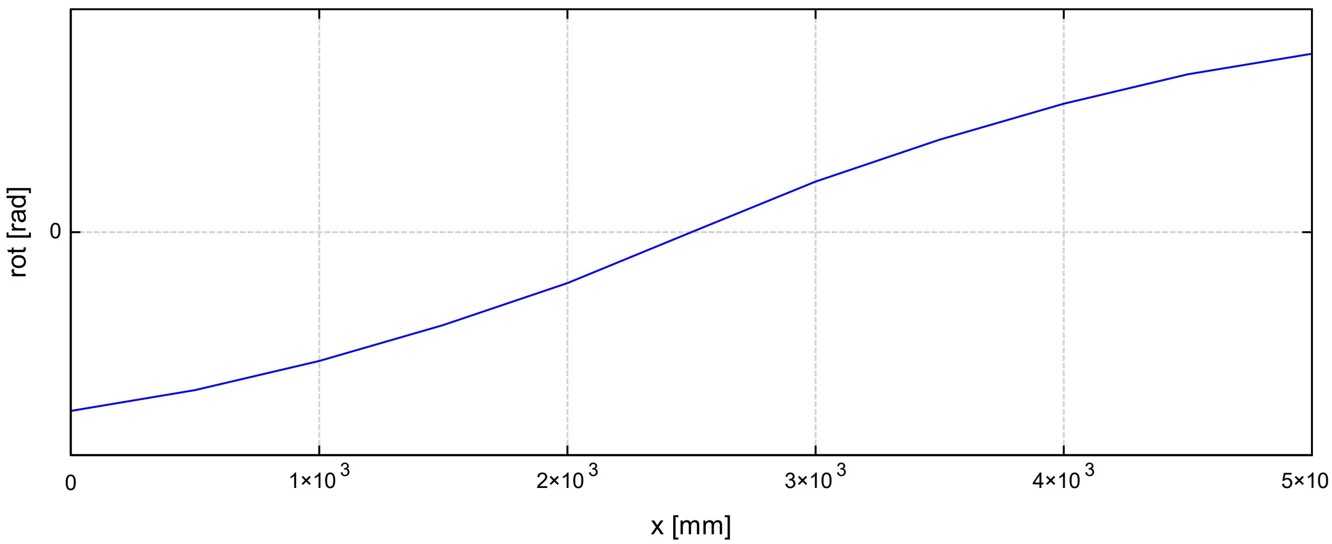
⊞— Rotation - Timber

Rotation - Timber

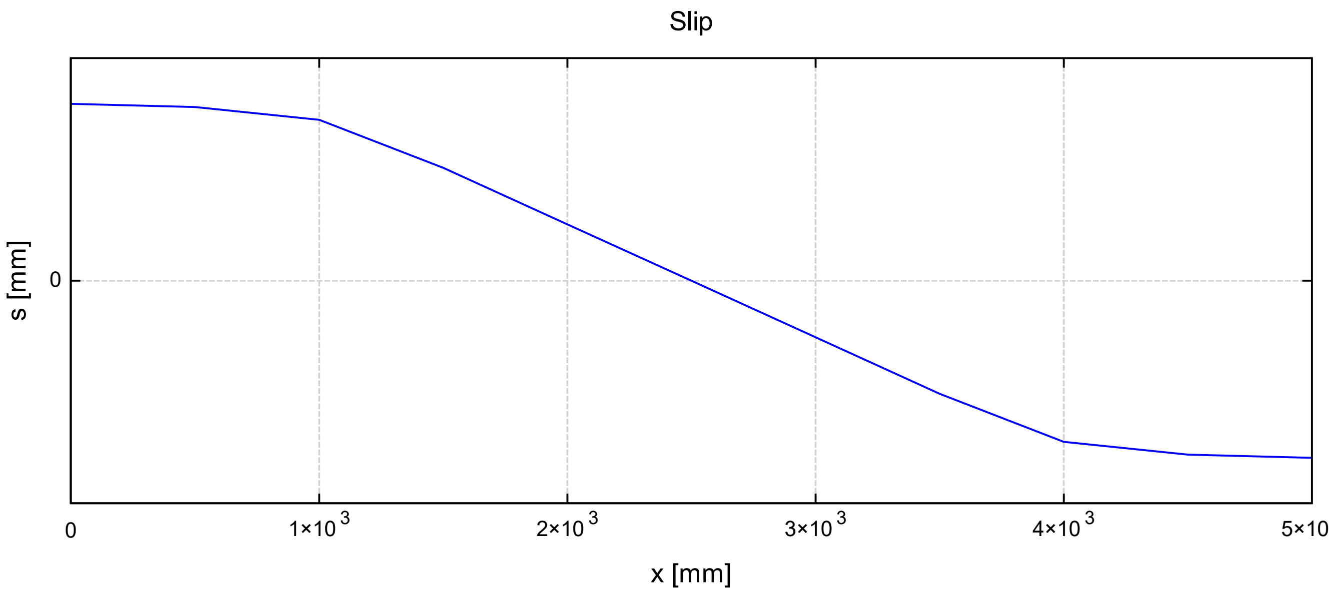


⊞— Rotation - Concrete

Rotation - Concrete



⊞— Slip



⊞— Shear force - connections

⊟— Checks

Shear force - Timber $\quad Vtd := \text{Max}\left(\text{Abs}\left(Vtimmin\right); \text{Abs}\left(Vtimmax\right)\right) = 2546,1431$

Moment - Timber $\quad Mtd := \text{Max}\left(\text{Abs}\left(Mtimmin\right); \text{Abs}\left(Mtimmax\right)\right) = 7,484 \cdot 10^{5}$

Normal force - Timber $\quad Ntd := \text{Max}\left(\text{Abs}\left(Ntimmin\right); \text{Abs}\left(Ntimmax\right)\right) = 88353,6767$

Shear force - Concrete $\quad Vcd := \text{Max}\left(\text{Abs}\left(Vconcmin\right); \text{Abs}\left(Vconcmax\right)\right) = 12845,1328$

Moment - Concrete $\quad Mcd := \text{Max}\left(\text{Abs}\left(Mconcmin\right); \text{Abs}\left(Mconcmax\right)\right) = 7,9653 \cdot 10^{6}$

Normal force - Concrete $\quad Ncd := -\text{Max}\left(\text{Abs}\left(Nconcmin\right); \text{Abs}\left(Nconcmax\right)\right) = -88353,6767$

Deflection $\quad uu := \text{Max}\left(\text{Abs}\left(uu_{min}\right); \text{Abs}\left(uu_{max}\right)\right) = 5,7484$

Shear force - connection $\quad Fd := \text{Max}\left(\text{Abs}\left(Vconnmin\right); \text{Abs}\left(Vconnmax\right)\right) = 41484,7005$

$\gamma c := 1,5$

$Crdc := \frac{0,15}{\gamma c} = 0,1 \qquad bw := btop - bt1 = 290 \qquad k := \text{Min}\left(1 + \sqrt{\frac{200}{ds}}; 2\right) = 2$

$Asl = 100,531$

$\rho 1 := \text{Min}\left(\frac{Asl}{bw \cdot d1}; 0,02\right) = 0,0025 \qquad \sigma cp := 0$

$vmin := \left(\frac{0,0525}{\gamma c} \cdot \sqrt{k^{3} \cdot fck}\right) = 0,4427$

$Vrdcmin := \left(vmin + 0,12 \cdot \sigma cp\right) \cdot bw \cdot d1 = 17460,8323$

$Vrd1c := \left(Crdc \cdot k \cdot \sqrt[3]{100 \cdot \rho 1 \cdot fck} + 0,12 \cdot \sigma cp\right) \cdot bw \cdot d1 = 13575,7724$

$\boxed{Vrdc := \text{Max}\left(Vrd1c; Vrdcmin\right) = 17460,8323}$

Check - shear - concrete: $\quad \eta sfc := \frac{Vcd}{Vrdc} = 0,7357$

$Vrdt := \frac{Abot}{1,5} \cdot \frac{kmod}{\gamma m} \cdot fvrsk = 14523,0769$

Check - shear - timber (rolling shear):

$$\boxed{\eta sft := \frac{Vtd}{Vrdt} = 0,1753}$$

Check - tension / bending - timber:

$ft0d := \frac{kmod}{\gamma m} \cdot ft0k = 8,6154$ $\quad fmd := \frac{kmod}{\gamma m} \cdot fmk = 14,7692$

$\sigma t0d := \frac{Ntd}{At} = 3,7438$ $\quad \sigma md := \frac{Mtd}{Wt} = 2,1954$

$$\boxed{\eta mt := \frac{\sigma t0d}{ft0d} + \frac{\sigma md}{fmd} = 0,5832}$$

Check - compression / bending - concrete at support: $\quad \alpha cc := 0,85$

$fyd := \frac{500}{1,15} = 434,7826$ $\quad fcd := \frac{\alpha cc}{\gamma c} \cdot fck = 11,3333$ $\quad beff := btop$

Verifica in mezzeria e all'appoggio

Axial load position: $\quad ya := 0$

$yc := \text{solve}\left(0,8 \cdot beff \cdot yc \cdot fcd - As2 \cdot fyd - As1 \cdot fyd = \text{Abs}\left(Ncd\right); yc; 0; htop\right) = 33,7031$

$Mrd := \left(0,8 \cdot beff \cdot yc \cdot fcd \cdot \left(htop - ytop - 0,4 \cdot yc\right) - As2 \cdot fyd \cdot \left(htop - ytop - d2\right) + As1 \cdot fyd \cdot \left(ytop - \frac{ds}{2}\right)\right) + ya \cdot A$

Check - notch:

- shear in concrete

$\beta := 0,25$ $\quad bn := 300$ $\quad ln := 300$ $\quad \nu := 0,516$

$Fcsk := \beta \cdot 0,5 \cdot bn \cdot ln \cdot \nu \cdot fck = 1,161 \cdot 10^{5}$ $\quad Fcsd := \frac{\alpha cc}{1,5} \cdot Fcsk = 65790$

- compression in concrete (crush)

$hn := 15$ $\quad An := bn \cdot hn = 4500$

$Fcck := An \cdot fck = 90000$ $\quad Fccd := \frac{\alpha cc}{1,5} \cdot Fcck = 51000$

- compression parallel to the grain in timber (crush)

$Ftck := An \cdot fc0k = 94500$ $\quad Ftcd := \frac{0,8}{1,3} \cdot Ftck = 58153,8462$

- longitudinal shear in timber

$Ftsk := bn \cdot ln \cdot 0,5 \cdot fvk = 1,8 \cdot 10^{5}$ $\quad Ftsd := \frac{0,8}{1,3} \cdot Ftsk = 1,1077 \cdot 10^{5}$

$Fcrd := \text{Min}\left(Fcsd; Fccd; Ftcd; Ftsd\right) = 51000$