Input data:

Input data

Geometry - Concrete

$btop := 590$ $htop := 140$ $ht1 := 80$ $bt1 := 300$ $ht2 := htop - ht1 = 60$

$$ytop := \frac{ht1 \cdot btop \cdot \left(ht2 + \frac{ht1}{2}\right) + \left(\frac{ht2}{2}\right) \cdot (ht2) \cdot bt1}{ht1 \cdot btop + ht2 \cdot bt1} = 80{,}6748$$

$$Itop := \frac{btop \cdot (htop - ytop)^3 - (btop - bt1) \cdot (ht2 - ytop)^3 + bt1 \cdot ytop^3}{3} = 9{,}4424 \cdot 1$$

$Atop := btop \cdot htop - ht2 \cdot (btop - bt1) = 65200$

Material - Concrete - C20/25

$Etop := 24900$ $Gtop := 13000000$ $fck := 20$

$Etopinf := \frac{Etop}{1 + 2{,}5} = 7114{,}2857$ $\gamma c := 25$

Geometry - Timber

$bbot := 590$ $hbot := 60$

$hb1 := 20$ $Abot := bbot \cdot hbot = 35400$

$At := 2 \cdot hb1 \cdot bbot = 23600$

$$Ibot := 2 \cdot \frac{bbot \cdot hb1^3}{12} + 2 \cdot hb1^2 \cdot hb1 \cdot bbot = 1{,}0227 \cdot 10^7$$

$ybot := \frac{hbot}{2} = 30$

$Wt := \frac{2 \cdot Ibot}{hbot} = 3{,}4089 \cdot 10^5$

Material - Timber - C24

$Em0 := 11000$ $Eminf := \frac{11000}{1{,}6} = 6875$

$fmk := 24$ $ft0k := 14$ $ft90k := 0{,}4$ $fc0k := 21$

$ks := 1{,}2$ $fc90k := 2{,}5$ $fvk := 4{,}0$ $\gamma t := 5$

$Ebot := Em0$ $Gbot := 690000$ $Ebotinf := Eminf$

Material - connections

$Econ := 11000$ $Gcon := 690000000$

Geometry - connections

$lef := 100$

Reinforcement

$ds := 8$ $nb := 2$ $As2 := 188 \cdot \frac{btop}{1000} = 110{,}92$

$Asl := nb \cdot \pi \cdot \frac{ds^2}{4} = 100{,}531$ $d1 := htop - \frac{ds}{2} = 136$

$Kser := 850000$ $d2 := 50$

$Ku := 570000$

$$bconser := \sqrt[4]{\frac{12 \cdot Kser \cdot \left((ytop)^3 + (ybot)^3\right)}{3 \cdot Econ}} = 114{,}2929$$

$bconser := bconser$ $Iconser := \frac{bconser^4}{\ldots}$ $Aconser := bconser^2$

$hconser := bconser$ $\quad Iconser := \frac{\ }{12}$ $\quad Aconser := bconser$

$$bconu := \sqrt[4]{\frac{12 \cdot Ku \cdot \left((ytop)^3 + (ybot)^3\right)}{3 \cdot Econ}} = 103{,}4269$$

$hconu := bconu$ $\quad Iconu := \frac{bconu^4}{12}$ $\quad Aconu := bconu^2$

Span length $\quad Lt := 5000$ $\quad nEdof := 6$ $\quad dtb := ytop + ybot = 110{,}6748$

Loads $\quad \gamma g := 1{,}35$ $\quad \gamma q := 1{,}50$

$$egk := \frac{\left(\gamma c \cdot htop - \gamma c \cdot \frac{(btop - bt1)}{btop} \cdot (htop - ht1) + \gamma t \cdot hbot\right)}{10^3} = 3{,}0627$$

$gk := 1{,}0$

$qk := 2{,}8$ $\quad kmod := 0{,}8$ $\quad \gamma m := 1{,}3$ $\quad fvrsk := 1{,}0$

$$qd := (\gamma g \cdot (egk + gk) + \gamma q \cdot qk) \cdot \frac{btop}{1000} = 5{,}714$$

$$qser := ((egk + gk) + qk) \cdot \frac{btop}{1000} = 4{,}049$$

Rod position

$nn := 10$

connection distance $\quad d := \frac{Lt}{nn} = 500$

$nPN := nn + 1$ $\quad nPE := nPN - 1$

Define connector position

$$conns := \begin{bmatrix} 500 \\ 1000 \\ 1500 \\ 3500 \\ 4000 \\ 4500 \end{bmatrix}$$

⊞— Stiffness matrix for a 1D finite element with shear deformation (Timoshenko)

⊞— Stiffness matrix for a 1D finite element with left internal hinge (Timoshenko)

⊞— Stiffness matrix for a 1D finite element with right internal hinge (Timoshenko)

⊞— Nodes - basic structure

⊞— Add Nodes - Connectors

t = 0

⊞— Elements

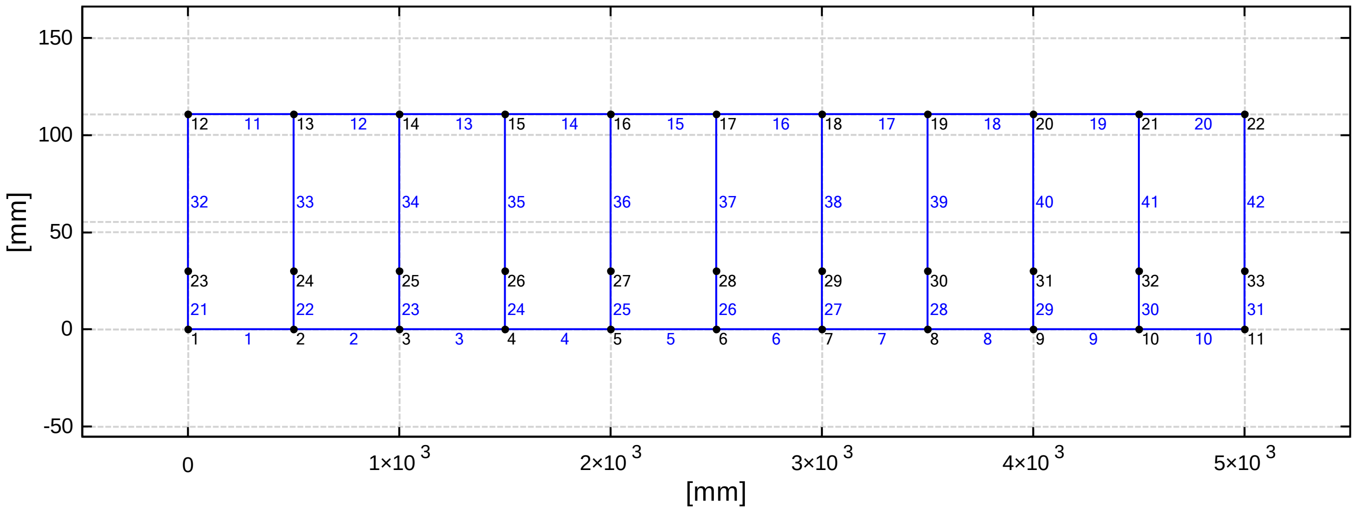
⊟— Set connectors

for $i \in [1..(\text{length}(conns))]$

|for $j \in [(2 \cdot nPE + 1)..(2 \cdot nPE + 2)]$

```
for j ∈ [(2·nPE+1)..(3·nPE+2)]
  if (Elsle_{j 1} = Elsle_{j 3}) ∧ (Elsle_{j 1} = conns_i)
    Elsle_{j 9} := "rh"
    Elslu_{j 9} := "rh"
for j ∈ [(3·nPE+2)..(4·nPE+2)]
  if (Elsle_{j 1} = Elsle_{j 3}) ∧ (Elsle_{j 1} = conns_i)
    Elsle_{j 9} := "nh"
    Elslu_{j 9} := "nh"
```

⊞— System



⊞— Stiffness matrix

⊞— Boundary conditions (penalty approach)

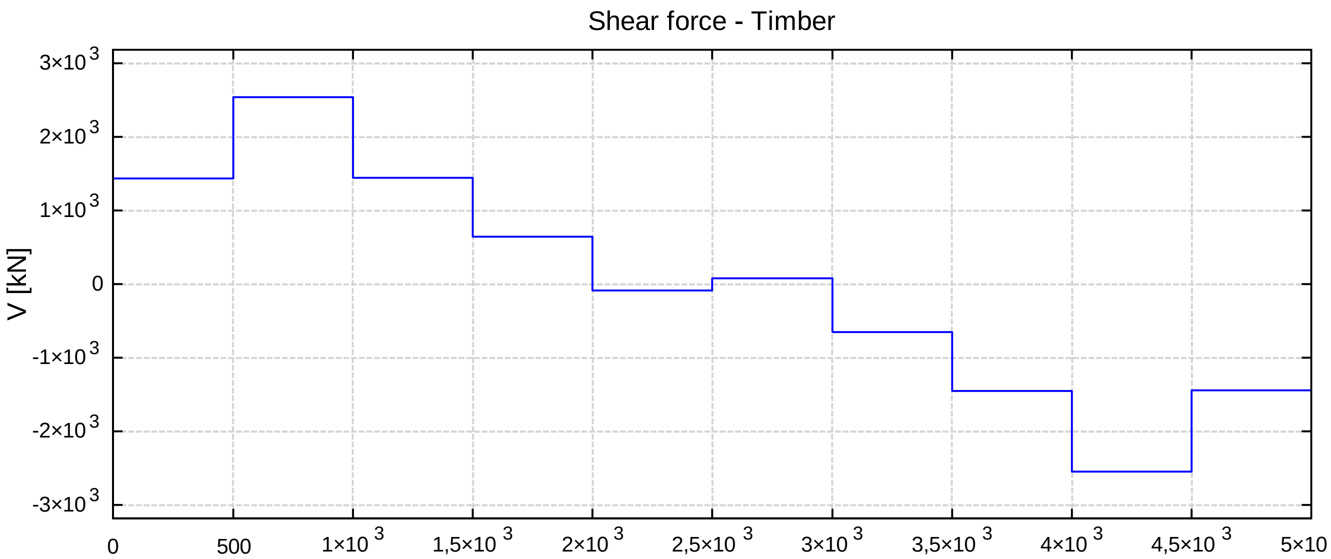
⊞— Loads

⊞— Displacement (system solution)

⊞— Reactions

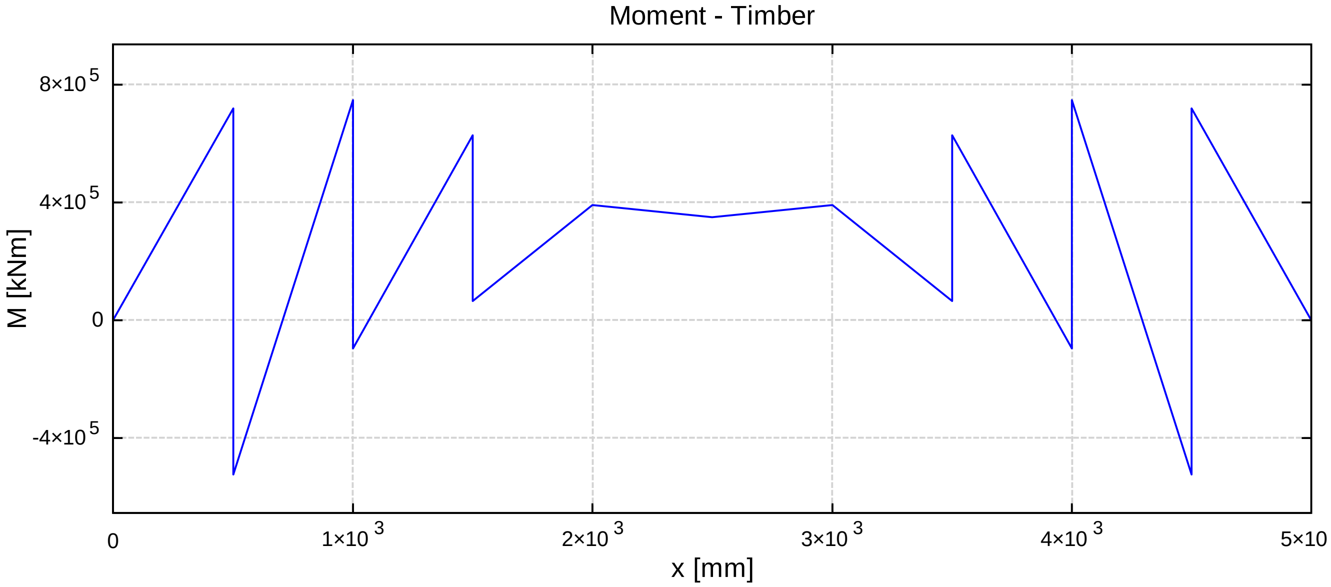
⊞— Internal forces

⊞— Shear force diagram - Timber

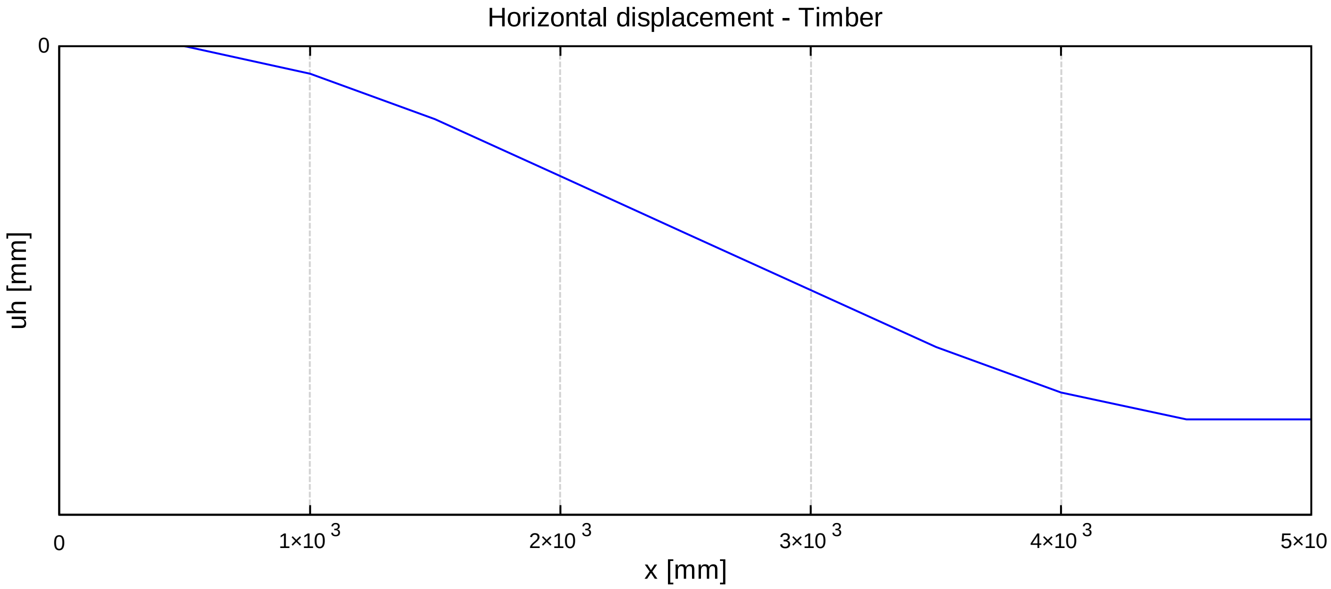


x [mm]

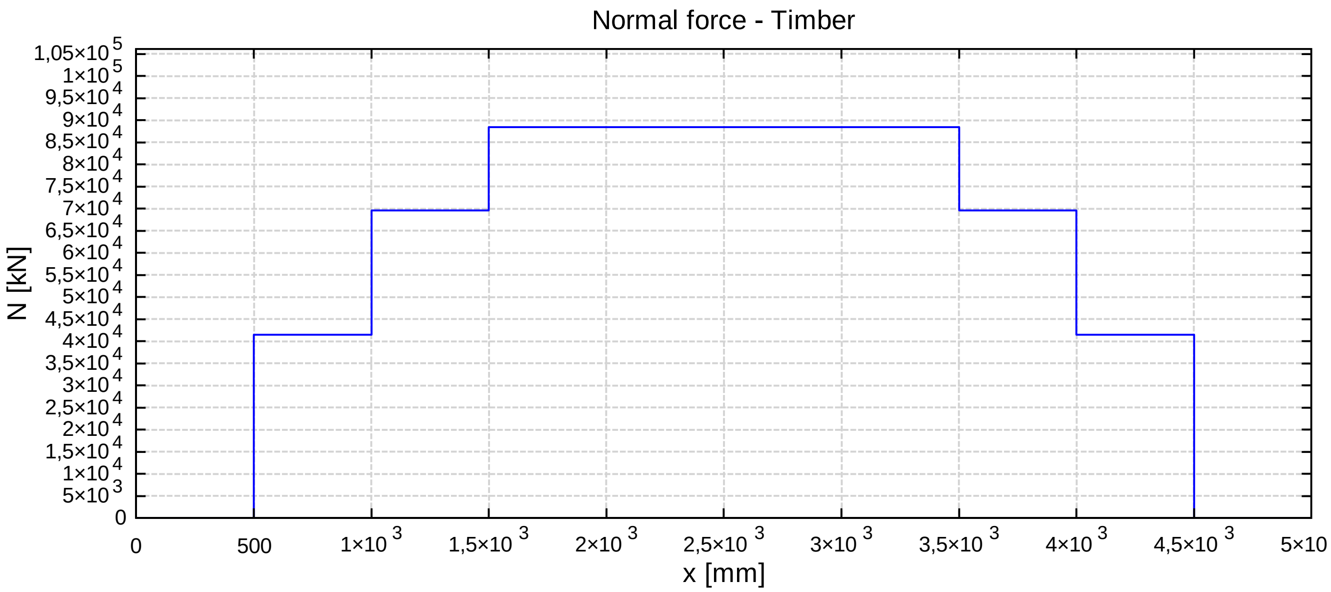
⊞— Moment diagram - Timber



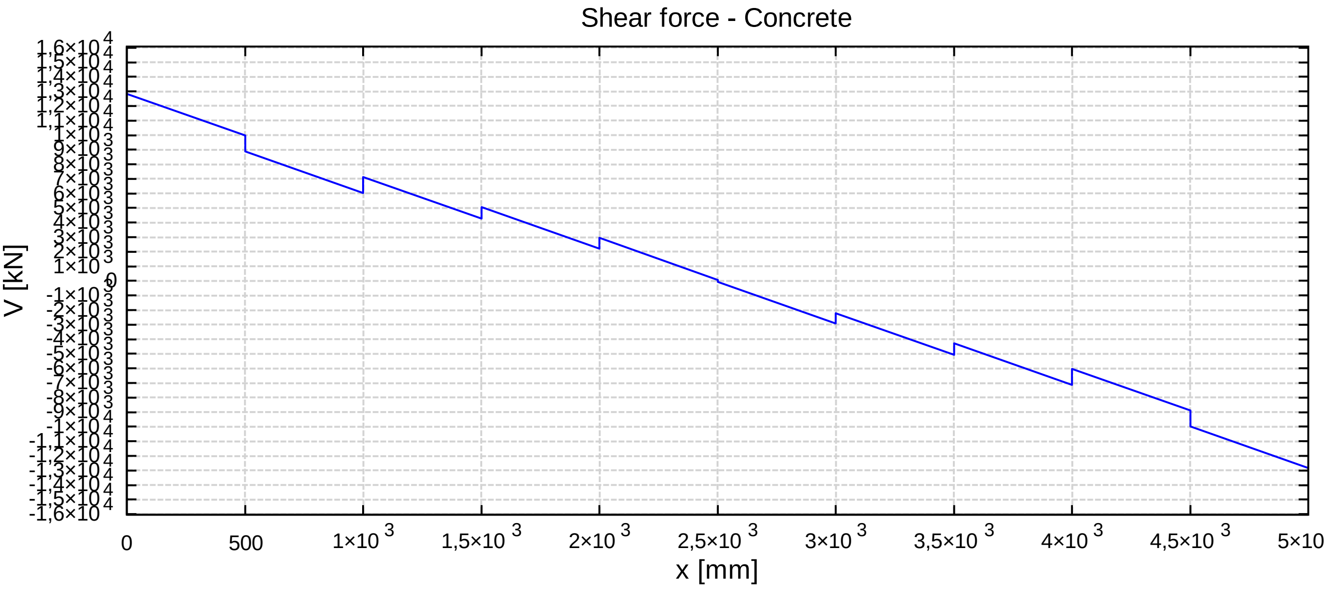
⊞— Horizontal displacement - Timber



⊞— Normal force diagram - Timber



Shear force diagram - Concrete



Moment diagram - Concrete

$M11 := \mathrm{matrix}(nPE; 2)$ $M22 := \mathrm{matrix}(nPE; 2)$

for $i \in [(nPE+1)..(2 \cdot nPE)]$
- $M11_{i-nPE\,1} := Nodes_{i+1\,1}$
- $M11_{i-nPE\,2} := M1slu_{i}$

for $i \in [(nPE+1)..(2 \cdot nPE)]$
- $M22_{i-nPE\,1} := Nodes_{i+2\,1}$
- $M22_{i-nPE\,2} := M2slu_{i}$

$M_{max} := \max(\mathrm{col}(M11; 2)) = 7{,}9653 \cdot 10^{6}$

$M_{min} := \min(\mathrm{col}(M11; 2)) = 2{,}3808 \cdot 10^{5}$

$MM := \mathrm{matrix}(0; 2)$

for $i \in [1..(nVal)]$
- $MM := \mathrm{stack}(MM; [M11_{i\,1}\ M11_{i\,2}])$
- $MM := \mathrm{stack}(MM; [M22_{i\,1}\ M22_{i\,2}])$

$XYPlot_{XLimMin\,7} := 0$

$XYPlot_{XLimMax\,7} := Lt$

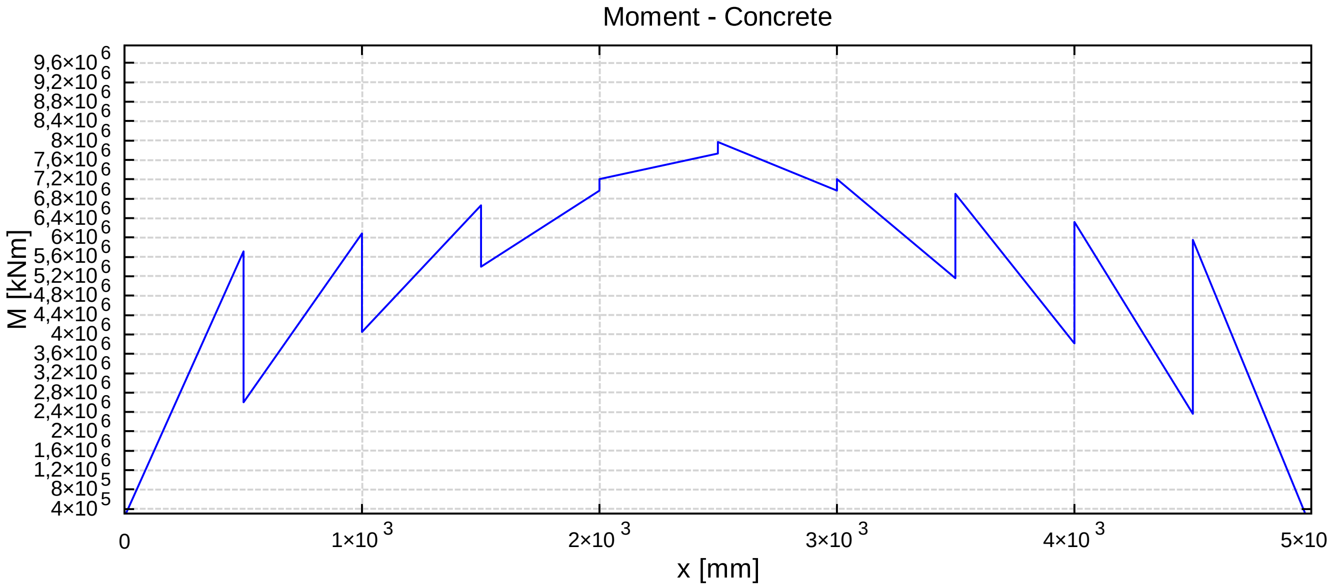
$XYPlot_{YLimMin\,7} := \mathrm{round}(1{,}25 \cdot M_{min}; 2)$ $Mconcmin := M_{min}$

$XYPlot_{YLimMax\,7} := \mathrm{round}(1{,}25 \cdot M_{max}; 2)$ $Mconcmax := M_{max}$

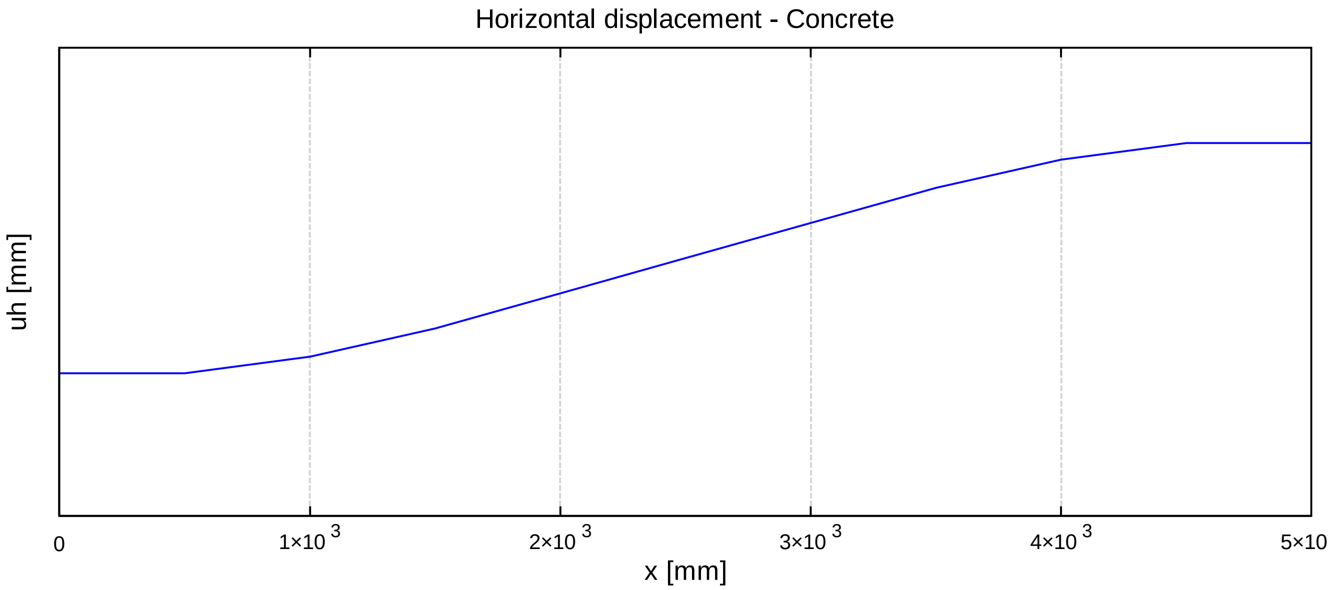
$XYPlot_{XTick\,7} := interval\left(\frac{L}{\mathrm{UoM}(L)}\right)$

$range := XYPlot_{YLimMax\,7} - XYPlot_{YLimMin\,7} = 9{,}659 \cdot 10^{9} \cdot \frac{1}{\mathrm{m}}\ \mathrm{mm}$

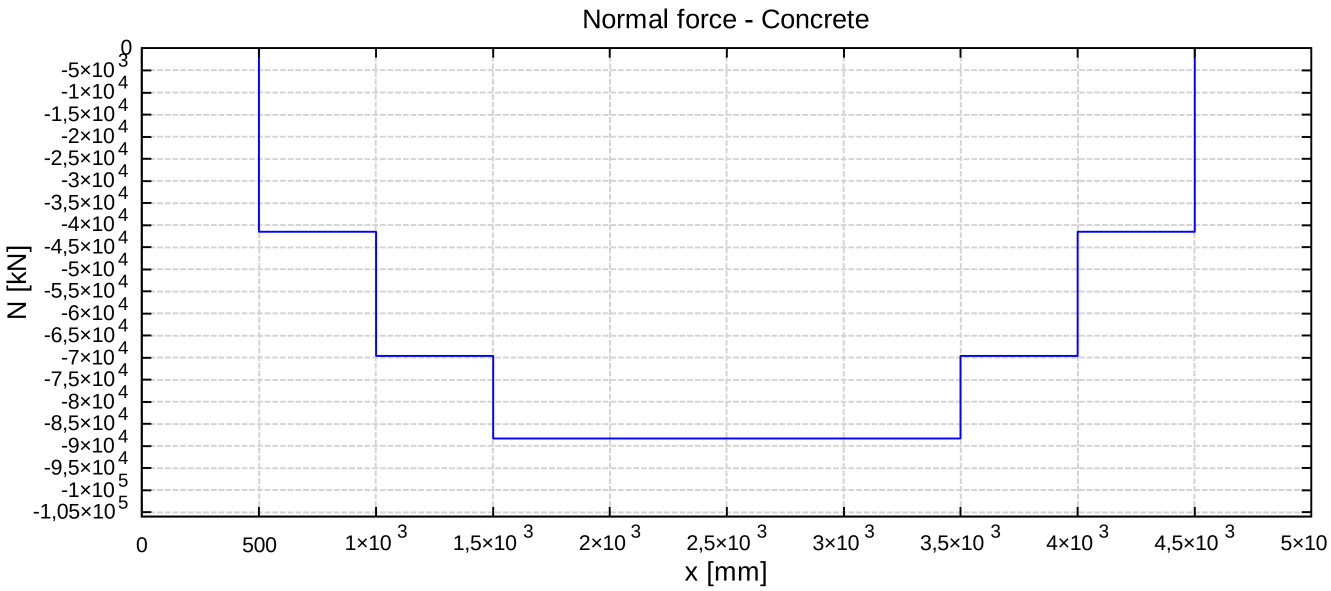
$XYPlot_{YTick\,7} := interval\left(\frac{range}{\mathrm{UoM}(range)}\right)$



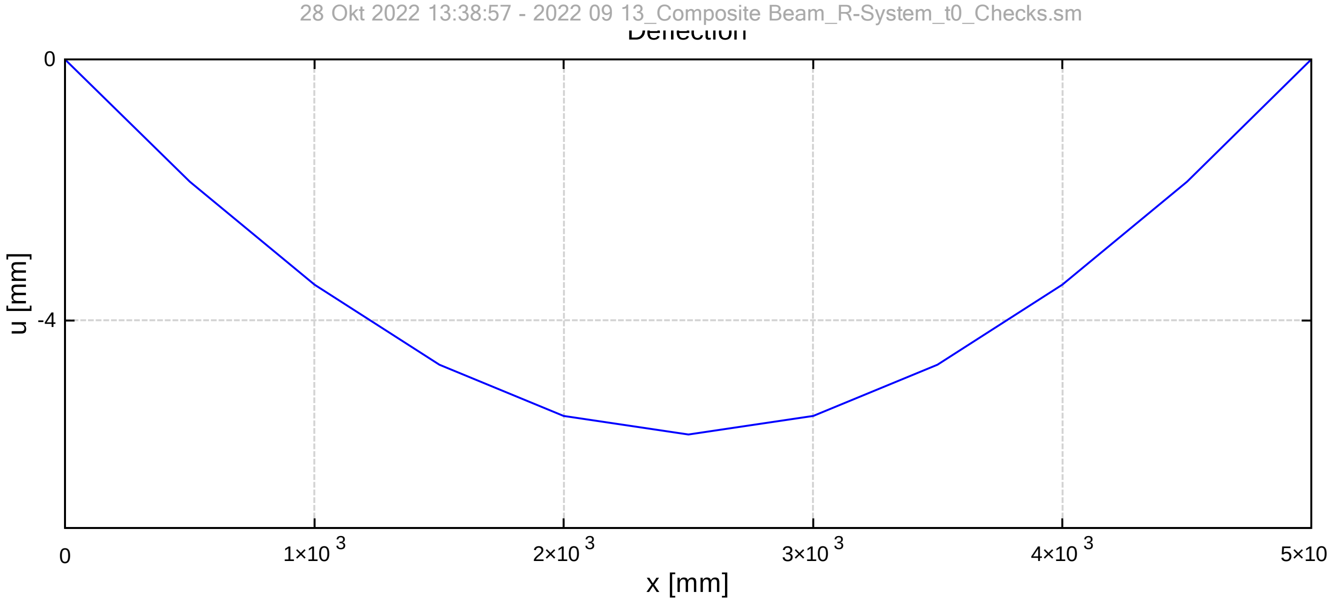
⊞— Horizontal displacement - Concrete



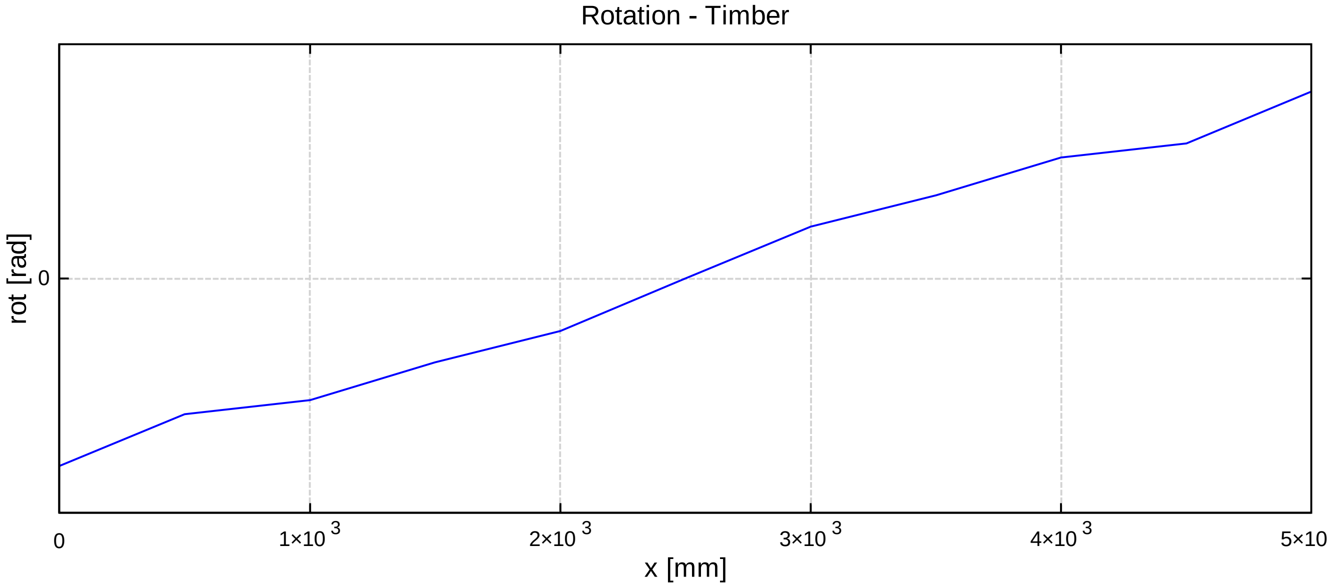
⊞— Normal force diagram - Concrete



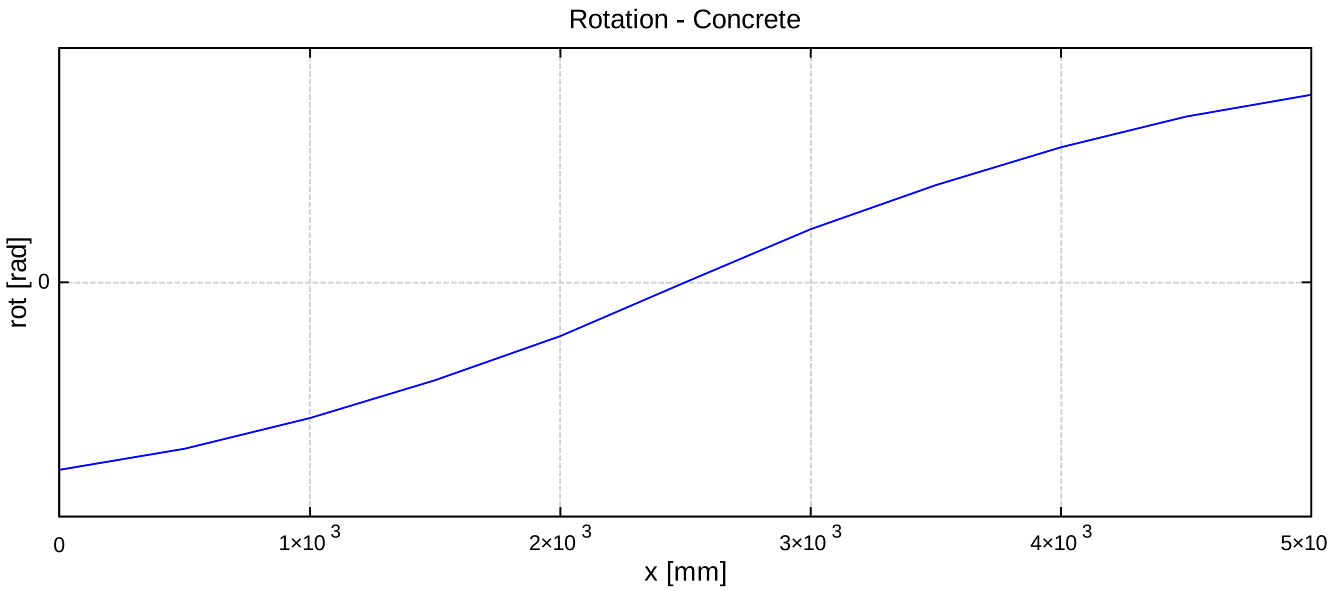
⊞— Deflection



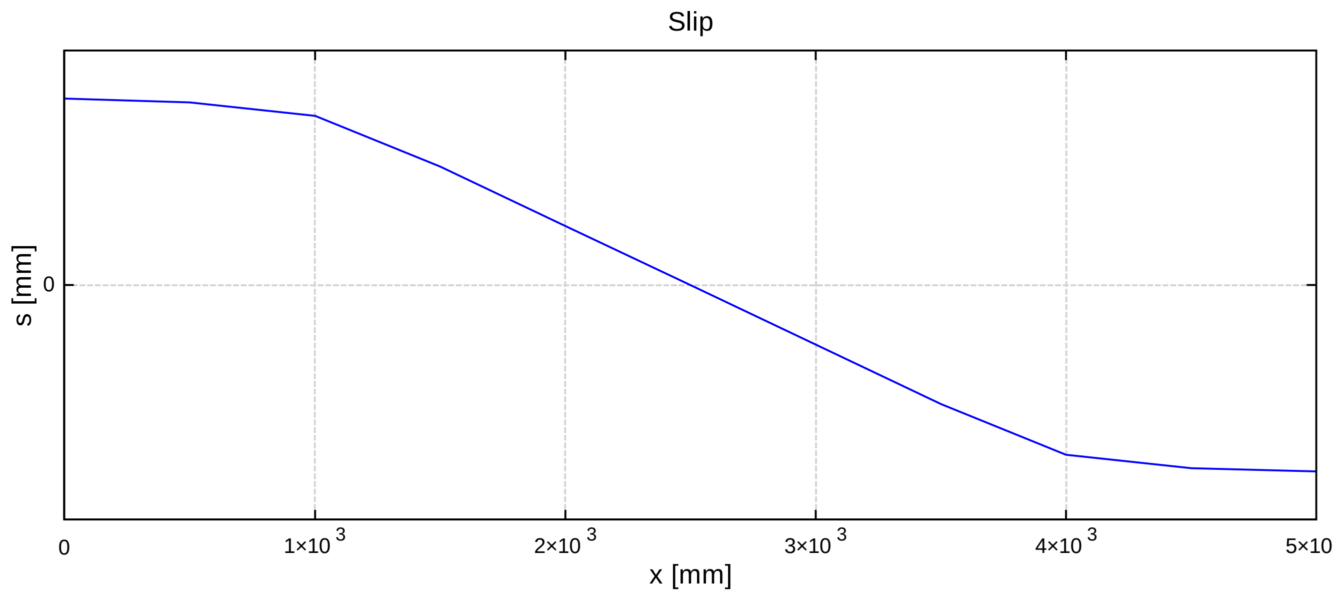
⊞— Rotation - Timber



⊞— Rotation - Concrete



⊞— Slip



⊞— Shear force - connections

⊟— Checks

Shear force - Timber $Vtd := \text{Max}\left(\text{Abs}\left(Vtimmin\right); \text{Abs}\left(Vtimmax\right)\right) = 2546{,}1431$

Moment - Timber $Mtd := \text{Max}\left(\text{Abs}\left(Mtimmin\right); \text{Abs}\left(Mtimmax\right)\right) = 7{,}484 \cdot 10^{5}$

Normal force - Timber $Ntd := \text{Max}\left(\text{Abs}\left(Ntimmin\right); \text{Abs}\left(Ntimmax\right)\right) = 88353{,}6767$

Shear force - Concrete $Vcd := \text{Max}\left(\text{Abs}\left(Vconcmin\right); \text{Abs}\left(Vconcmax\right)\right) = 12845{,}1328$

Moment - Concrete $Mcd := \text{Max}\left(\text{Abs}\left(Mconcmin\right); \text{Abs}\left(Mconcmax\right)\right) = 7{,}9653 \cdot 10^{6}$

Normal force - Concrete $Ncd := -\text{Max}\left(\text{Abs}\left(Nconcmin\right); \text{Abs}\left(Nconcmax\right)\right) = -88353{,}6767$

Deflection $uu := \text{Max}\left(\text{Abs}\left(uu_{min}\right); \text{Abs}\left(uu_{max}\right)\right) = 5{,}7484$

Shear force - connection $Fd := \text{Max}\left(\text{Abs}\left(Vconnmin\right); \text{Abs}\left(Vconnmax\right)\right) = 41484{,}7005$

$\gamma c := 1{,}5$

$Crdc := \frac{0{,}15}{\gamma c} = 0{,}1$ $bw := btop - bt1 = 290$ $k := \text{Min}\left(1 + \sqrt{\frac{200}{ds}}; 2\right) = 2$

$Asl = 100{,}531$

$\rho 1 := \text{Min}\left(\frac{Asl}{bw \cdot d1}; 0{,}02\right) = 0{,}0025$ $\sigma cp := 0$

$\nu min := \left(\frac{0{,}0525}{\gamma c} \cdot \sqrt{k^{3} \cdot fck}\right) = 0{,}4427$

$Vrdcmin := \left(\nu min + 0{,}12 \cdot \sigma cp\right) \cdot bw \cdot d1 = 17460{,}8323$

$Vrd1c := \left(Crdc \cdot k \cdot \sqrt[3]{100 \cdot \rho 1 \cdot fck} + 0{,}12 \cdot \sigma cp\right) \cdot bw \cdot d1 = 13575{,}7724$

$\boxed{Vrdc := \text{Max}\left(Vrd1c; Vrdcmin\right) = 17460{,}8323}$

Check - shear - concrete: $\eta sfc := \frac{Vcd}{Vrdc} = 0{,}7357$

$Vrdc$

Check - shear - timber (rolling shear):

$$Vrdt := \frac{Abot}{1,5} \cdot \frac{kmod}{\gamma m} \cdot fvrsk = 14523,0769$$

$$\boxed{\eta sft := \frac{Vtd}{Vrdt} = 0,1753}$$

Check - tension / bending - timber:

$$ft0d := \frac{kmod}{\gamma m} \cdot ft0k = 8,6154 \qquad fmd := \frac{kmod}{\gamma m} \cdot fmk = 14,7692$$

$$\sigma t0d := \frac{Ntd}{At} = 3,7438 \qquad \sigma md := \frac{Mtd}{Wt} = 2,1954$$

$$\boxed{\eta mt := \frac{\sigma t0d}{ft0d} + \frac{\sigma md}{fmd} = 0,5832}$$

Check - compression / bending - concrete at support: $\alpha cc := 0,85$

$$fyd := \frac{500}{1,15} = 434,7826 \qquad fcd := \frac{\alpha cc}{\gamma c} \cdot fck = 11,3333 \qquad beff := btop$$

Axial load position: $ya := 0$

$$yc := \mathrm{solve}\left(0,8 \cdot beff \cdot yc \cdot fcd - As2 \cdot fyd - Asl \cdot fyd = \mathrm{Abs}\left(Ncd\right); yc; 0; htop\right) = 33,7031$$

$$Mrd := \left(0,8 \cdot beff \cdot yc \cdot fcd \cdot \left(htop - ytop - 0,4 \cdot yc\right) - As2 \cdot fyd \cdot \left(htop - ytop - d2\right) + Asl \cdot fyd \cdot \left(ytop - \frac{ds}{2}\right)\right) + ya \cdot A$$

Check - notch:

- shear in concrete

$$\beta := 0,25 \qquad bn := 300 \qquad ln := 300 \qquad \nu := 0,516$$

$$Fcsk := \beta \cdot 0,5 \cdot bn \cdot ln \cdot \nu \cdot fck = 1,161 \cdot 10^{5} \qquad Fcsd := \frac{\alpha cc}{1,5} \cdot Fcsk = 65790$$

- compression in concrete (crush)

$$hn := 15 \qquad An := bn \cdot hn = 4500$$

$$Fcck := An \cdot fck = 90000 \qquad Fccd := \frac{\alpha cc}{1,5} \cdot Fcck = 51000$$

- compression parallel to the grain in timber (crush)

$$Ftck := An \cdot fc0k = 94500 \qquad Ftcd := \frac{0,8}{1,3} \cdot Ftck = 58153,8462$$

- longitudinal shear in timber

$$Ftsk := bn \cdot ln \cdot 0,5 \cdot fvk = 1,8 \cdot 10^{5} \qquad Ftsd := \frac{0,8}{1,3} \cdot Ftsk = 1,1077 \cdot 10^{5}$$

$$Fcrd := \mathrm{Min}\left(Fcsd; Fccd; Ftcd; Ftsd\right) = 51000$$