TIMOSHENKO BEAM - Uniformly distribuited load

$$v1(x) := a0 + a1 \cdot x + a2 \cdot x^2 + a3 \cdot x^3 + \frac{q1 \cdot x^4}{24 \cdot EI}$$

$$v1'(x) := \frac{d}{dx} v1(x) = \frac{x^3 \cdot q1 + 6 \cdot EI \cdot \left(a1 + x \cdot \left(2 \cdot (a2 + x \cdot a3) + a3 \cdot x\right)\right)}{6 \cdot EI}$$

$$v1''(x) := \frac{d}{dx} \frac{d}{dx} v1(x) = \frac{x^2 \cdot q1 + 4 \cdot EI \cdot (a2 + 3 \cdot x \cdot a3)}{2 \cdot EI}$$

$$v1'''(x) := \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} v1(x) = \frac{x \cdot q1 + 6 \cdot EI \cdot a3}{EI}$$

$$v1''''(x) := \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} v1(x) = \frac{q1}{EI}$$

$$v2(x) := b0 + b1 \cdot x + b2 \cdot x^2 + b3 \cdot x^3 + \frac{q2 \cdot x^4}{24 \cdot EI}$$

$$v2'(x) := \frac{d}{dx} v2(x) = \frac{x^3 \cdot q2 + 6 \cdot EI \cdot \left(b1 + x \cdot \left(2 \cdot (b2 + x \cdot b3) + b3 \cdot x\right)\right)}{6 \cdot EI}$$

$$v2''(x) := \frac{d}{dx} \frac{d}{dx} v2(x) = \frac{x^2 \cdot q2 + 4 \cdot EI \cdot (b2 + 3 \cdot x \cdot b3)}{2 \cdot EI}$$

$$v2'''(x) := \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} v2(x) = \frac{x \cdot q2 + 6 \cdot EI \cdot b3}{EI}$$

$$v2''''(x) := \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} v2(x) = \frac{q2}{EI}$$

$$\mathrm{Algsys}\left(\begin{bmatrix} v1(0) = 0 \\ v1''(0) = 0 \\ v1(L1) - \frac{EI}{GAc} \cdot v1''(L1) = 0 \\ v2(0) - \frac{EI}{GAc} \cdot v2''(0) = 0 \\ v1'(L1) - v2'(0) = 0 \\ v1''(L1) - v2''(0) = 0 \\ v2(L2) = 0 \\ v2''(L2) = 0 \end{bmatrix} ; \begin{bmatrix} a0 \\ a1 \\ a2 \\ a3 \\ b0 \\ b1 \\ b2 \\ b3 \end{bmatrix}\right) = \begin{cases} a0 = 0 \\ a1 = -\dfrac{\left(GAc \cdot L1^2 - 6 \cdot EI\right) \cdot L2^4 \cdot q2 - L1^3 \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + 2 \cdot L2) \cdots}{48 \cdot EI \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot \cdots} \\ a2 = 0 \\ a3 = \dfrac{GAc \cdot L2^4 \cdot q2 - L1 \cdot \left(L2 \cdot \left(4 \cdot GAc \cdot L1 \cdot L2 + 3 \cdot \left(GAc \cdot L1^2 + 4 \cdot EI\right)\right) + \cdots}{48 \cdot EI \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot EI \cdot L \cdots} \\ b0 = \dfrac{L1 \cdot L2 \cdot \left(L2^3 \cdot q2 + L1^3 \cdot q1\right)}{8 \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot EI \cdot L1)} \\ b1 = \dfrac{\left(GAc \cdot L1^2 + 3 \cdot EI\right) \cdot L2^4 \cdot q2 - L1^4 \cdot \left(GAc \cdot L2^2 + 3 \cdot EI\right) \cdot q1}{24 \cdot EI \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot EI \cdot L1)} \\ b2 = \dfrac{GAc \cdot L1 \cdot L2 \cdot \left(L2^3 \cdot q2 + L1^3 \cdot q1\right)}{16 \cdot EI \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot EI \cdot L1)} \\ b3 = -\dfrac{L2 \cdot \left(L2 \cdot \left(5 \cdot GAc \cdot L1 \cdot L2 + 4 \cdot \left(GAc \cdot L1^2 + 3 \cdot EI\right)\right) + 12 \cdot EI \cdot L1\right) \cdot \cdots}{48 \cdot EI \cdot (L2 \cdot (GAc \cdot L1 \cdot (L1 + L2) + 3 \cdot EI) + 3 \cdot EI \cdots} \end{cases}$$

$L1 := 5000\ \mathrm{mm}$ $\quad L2 := 5000\ \mathrm{mm}$ $\quad h := 100\ \mathrm{mm}$ $\quad b := 100\ \mathrm{mm}$

$$E := 11000\ \frac{\mathrm{N}}{\mathrm{mm}^2} \qquad I := \frac{b \cdot h^3}{12} = \frac{25000000}{3}\ \mathrm{mm}^4 \qquad G := 690\ \frac{\mathrm{N}}{\mathrm{mm}^2}$$

$$EI := E \cdot I = 9{,}1667 \cdot 10^{10}\ \mathrm{N\ mm}^2 \qquad A := b \cdot h = 0{,}01\ \mathrm{m}^2 \qquad q1 := 1\ \frac{\mathrm{kN}}{\mathrm{m}}$$

$$GA := G \cdot A = 6{,}9 \cdot 10^6\ \mathrm{N} \qquad ks := 1{,}2 \qquad q2 := 1\ \frac{\mathrm{kN}}{\mathrm{m}}$$

$$GAc := \frac{G \cdot A}{ks} = 5{,}75 \cdot 10^6\ \mathrm{N}$$

$$a0 := 0$$

$$a1 := -\frac{\left(GAc \cdot L1^{2} - 6 \cdot EI\right) \cdot L2^{4} \cdot q2 - L1^{3} \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + 2 \cdot L2\right) + 12 \cdot EI\right) + 6 \cdot EI \cdot L1\right) \cdot q1}{48 \cdot EI \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$a2 := 0$$

$$a3 := \frac{GAc \cdot L2^{4} \cdot q2 - L1 \cdot \left(L2 \cdot \left(4 \cdot GAc \cdot L1 \cdot L2 + 3 \cdot \left(GAc \cdot L1^{2} + 4 \cdot EI\right)\right) + 12 \cdot EI \cdot L1\right) \cdot q1}{48 \cdot EI \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$b0 := \frac{L1 \cdot L2 \cdot \left(L2^{3} \cdot q2 + L1^{3} \cdot q1\right)}{8 \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$b1 := \frac{\left(GAc \cdot L1^{2} + 3 \cdot EI\right) \cdot L2^{4} \cdot q2 - L1^{4} \cdot \left(GAc \cdot L2^{2} + 3 \cdot EI\right) \cdot q1}{24 \cdot EI \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$b2 := \frac{GAc \cdot L1 \cdot L2 \cdot \left(L2^{3} \cdot q2 + L1^{3} \cdot q1\right)}{16 \cdot EI \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$b3 := -\frac{L2 \cdot \left(L2 \cdot \left(5 \cdot GAc \cdot L1 \cdot L2 + 4 \cdot \left(GAc \cdot L1^{2} + 3 \cdot EI\right)\right) + 12 \cdot EI \cdot L1\right) \cdot q2 + GAc \cdot L1^{4} \cdot q1}{48 \cdot EI \cdot \left(L2 \cdot \left(GAc \cdot L1 \cdot \left(L1 + L2\right) + 3 \cdot EI\right) + 3 \cdot EI \cdot L1\right)}$$

$$u1\left(x\right) := \left(v1\left(x\right) - \frac{EI}{GAc} \cdot v1''\left(x\right)\right) = \frac{x \cdot \left(132503000 \cdot \left(2200\ \mathrm{m} \cdot \left(36717157400\ \mathrm{m}^{2} - 4383752296 \cdot x^{2}\right) + 1285082765120\right.\right.}{374610107578730000000000\ \mathrm{r}}$$

$$u1\left(\frac{L1}{2}\right) = 0{,}0362\ \mathrm{m}$$

$$u2\left(x\right) := v2\left(x\right) - \frac{EI}{GAc} \cdot v2''\left(x\right) = \frac{421436371026071000000000 \cdot \left(2200\ \mathrm{m} \cdot \left(2920642648000\ \mathrm{m} \cdot \left(11\ \mathrm{m}^{2} + 345 \cdot x^{2}\right) - 3363\right.\right.}{}$$

$$u2\left(\frac{L2}{2}\right) = 0{,}0362\ \mathrm{m}$$

$$M1\left(x\right) := -\left(v1''\left(x\right) \cdot EI\right) = -\frac{250\ \mathrm{kg} \cdot x \cdot \left(11522 \cdot x - 43235\ \mathrm{m}\right)}{5761\ \mathrm{s}^{2}}$$

$$M1\left(L1\right) = -3119{,}0332\ \mathrm{J}$$

$$M2\left(x\right) := -\left(v2''\left(x\right) \cdot EI\right) = -\frac{3\ \mathrm{kg} \cdot \left(1221554698626290000 \cdot x^{2} + 275\ \mathrm{m} \cdot \left(1267420 \cdot \left(21862995000\ \mathrm{m} - 7298818296 \cdot x\right.\right.\right.}{7329328191757750\ \mathrm{s}^{2}}$$

$$M2\left(0\right) = -3119{,}0332\ \mathrm{J}$$

$$V1\left(x\right) := -\left(v1'''\left(x\right) \cdot EI\right) \qquad V1\left(L1\right) = -3123{,}8066\ \mathrm{N}$$

$$V2\left(x\right) := -\left(v2'''\left(x\right) \cdot EI\right) \qquad V2\left(0\right) = 3123{,}8066\ \mathrm{N}$$

$L := L1 + L2 = 10\ \mathrm{m}$ Beam length

$L_A := 0\ \mathrm{m}$ Distance to the first support

$L_B := L1 = 5\ \mathrm{m}$ Distance to the second support

$L_C := L = 10\ \mathrm{m}$ Distance to the third support

List of the Uniform Loads
(every column - options of a single Load)

$$q := \begin{bmatrix} 2\ \frac{\mathrm{kN}}{\mathrm{m}} \\ 0\ \mathrm{m} \\ L \end{bmatrix}$$

Beam diagram preparing:

$top := 2$

$$base_L(x) := \begin{bmatrix} x & top \\ 1+x & -2+top \\ -1+x & -2+top \\ x & top \end{bmatrix}$$

Support drawing

$$arrow_F(x; sign) := \begin{bmatrix} x & top+0{,}1 \\ -0{,}5+x & 1{,}7 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \\ 0{,}5+x & 1{,}7 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \\ x & 6 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \end{bmatrix}$$

Drawing of the arrow of Point Load

$$arrow_Q(x; sign) := \begin{bmatrix} x & top+0{,}1 \\ -0{,}2+x & 0{,}5 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \\ 0{,}2+x & 0{,}5 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \\ x & 1{,}5 \cdot sign + top + 0{,}1 \\ x & top+0{,}1 \end{bmatrix}$$

Drawing of the arrow of Uniform Load

$$rect_Q(x; width; sign) := \begin{bmatrix} x & top+0{,}1 \\ x & 1{,}5 \cdot sign + top + 0{,}1 \\ x+width & 1{,}5 \cdot sign + top + 0{,}1 \\ x+width & top+0{,}1 \end{bmatrix}$$

Drawing of the area of Uniform Load

$$arr_F := \begin{bmatrix} 0 & top \end{bmatrix}$$

$$\text{for } k := 1; k \le \text{cols}(F); k := k+1$$
$$\quad arr_F := \text{stack}\left(arr_F;\ arrow_F\left(\frac{F_{2\,k}}{\text{dm}};\ \text{sign}\left(F_{1\,k}\right)\right)\right)$$

Drawing of the specified Point Loads

$$arr_Q := \begin{bmatrix} 0 & top \end{bmatrix}$$

$$\text{for } k := 1; k \le \text{cols}(q); k := k+1$$
$$\quad arr_Q := \text{stack}\left(arr_Q;\ rect_Q\left(\frac{q_{2\,k}}{\text{dm}};\ \frac{q_{3\,k}}{\text{dm}};\ \text{sign}\left(q_{1\,k}\right)\right)\right)$$
$$\quad \text{for } j := 1; j \le \frac{q_{3\,k}}{\text{dm}} + 1; j := j+1$$
$$\quad\quad arr_Q := \text{stack}\left(arr_Q;\ arrow_Q\left(\frac{q_{2\,k}}{\text{dm}} + (j-1) \cdot 1{,}0;\ \text{sign}\left(q_{1\,k}\right)\right)\right)$$
$$\quad arr_Q := \text{stack}\left(arr_Q;\ arrow_Q\left(\frac{q_{2\,k}}{\text{dm}} + \frac{q_{3\,k}}{\text{dm}};\ \text{sign}\left(q_{1\,k}\right)\right)\right)$$

Drawing of the Specified Uniform Loads

$$op := \text{stack}\left(base_L\left(\frac{L_A}{\text{dm}}\right);\ base_L\left(\frac{L_B}{\text{dm}}\right);\ base_L\left(\frac{L_C}{\text{dm}}\right)\right)$$

$$b_1 := \begin{bmatrix} 0 & top \\ \frac{L}{\text{dm}} & top \\ \frac{L}{\text{dm}} & top + 0{,}1 \\ 0 & top + 0{,}1 \\ 0 & top \end{bmatrix}$$

Beam drawing

$$arr_F := \text{stack}\left(arr_F;\ arrow_F\left(\frac{L_A}{\text{dm}};\ -1\right);\ arrow_F\left(\frac{L_B}{\text{dm}};\ -1\right);\ arrow_F\left(\frac{L_C}{\text{dm}};\ -1\right)\right)$$

$$ground_{op}(x;\ inc) := \begin{bmatrix} x + inc \cdot 0{,}5 - 2{,}7 & top - 2 \\ x + (inc - 1) \cdot 0{,}5 - 2{,}7 & top - 2{,}4 \\ x + inc \cdot 0{,}5 - 2{,}7 & top - 2 \end{bmatrix}$$

$$op := \operatorname{stack}\left(op ; \begin{bmatrix} \frac{L_C}{\mathrm{dm}}+1 & top-2 \end{bmatrix}\right)$$

for $k := 1 ; k \le 10 ; k := k+1$

$$op := \operatorname{stack}\left(op ; ground_{op}\left(\frac{L_C}{\mathrm{dm}} ; k\right)\right)$$

Drawing of the supports

$$op := \operatorname{stack}\left(op ; \begin{bmatrix} \frac{L_C}{\mathrm{dm}}+1 & top-2 \\ \frac{L_C}{\mathrm{dm}} & top \\ \frac{L_B}{\mathrm{dm}} & top \\ \frac{L_B}{\mathrm{dm}}-1 & top-2 \end{bmatrix}\right)$$

for $k := 1 ; k \le 10 ; k := k+1$

$$op := \operatorname{stack}\left(op ; ground_{op}\left(\frac{L_B}{\mathrm{dm}} ; k\right)\right)$$

$$op := \operatorname{stack}\left(op ; \begin{bmatrix} \frac{L_B}{\mathrm{dm}}+1 & top-2 \\ \frac{L_B}{\mathrm{dm}} & top \\ \frac{L_A}{\mathrm{dm}} & top \\ \frac{L_A}{\mathrm{dm}}-1 & top-2 \end{bmatrix}\right)$$

for $k := 1 ; k \le 10 ; k := k+1$

$$op := \operatorname{stack}\left(op ; ground_{op}\left(\frac{L_A}{\mathrm{dm}} ; k\right)\right)$$

$$marks := \begin{bmatrix} \frac{L_A}{\mathrm{dm}} & top-5,5+0,5 & \text{"RA"} & 10 & \text{"Green"} \\ \frac{L_B}{\mathrm{dm}} & top-5,5+0,5 & \text{"RB"} & 10 & \text{"Green"} \\ \frac{L_C}{\mathrm{dm}} & top-5,5+0,5 & \text{"RC"} & 10 & \text{"Green"} \\ \frac{L_A}{\mathrm{dm}}-2,5 & 1 & \text{"A"} & 10 & \text{"Red"} \\ \frac{L_B}{\mathrm{dm}}-2,5 & 1 & \text{"B"} & 10 & \text{"Red"} \\ \frac{L_C}{\mathrm{dm}}-2,5 & 1 & \text{"C"} & 10 & \text{"Red"} \end{bmatrix}$$

Diagram marks drawing

for $k := 1 ; k \le \operatorname{cols}(F) ; k := k+1$

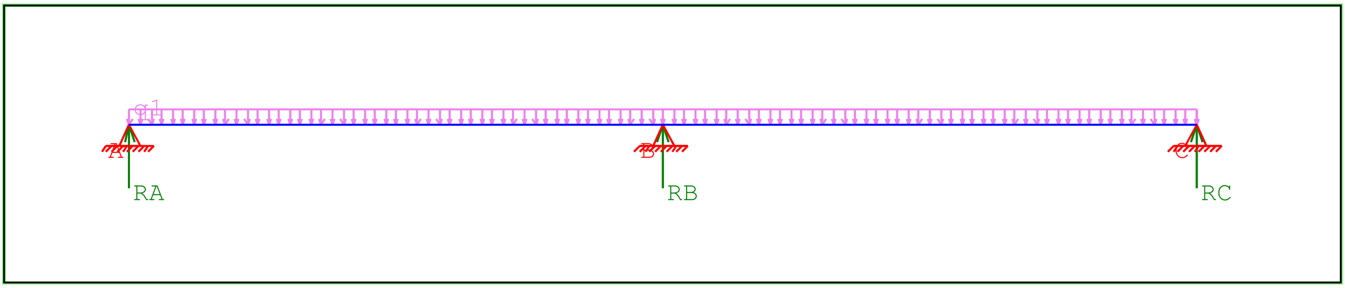
$$marks := \operatorname{stack}\left(marks ; \begin{bmatrix} \frac{F_{2\,k}}{\mathrm{dm}} & top+5,5\cdot\operatorname{sign}\left(F_{1\,k}\right)+0,5 & \operatorname{concat}\left(\text{"F"} ; \operatorname{num2str}(k)\right) & 10 & \text{"Green"} \end{bmatrix}\right)$$

$$\text{for}\ \ k := 1 ; k \le \text{cols}(q) ; k := k+1$$
$$\left| marks := \text{stack}\left(marks ; \left[\frac{q_{2\,k}}{\text{dm}} \quad top + 2{,}5 \cdot \text{sign}\left(q_{1\,k}\right) + 0{,}5 \quad \text{concat}\left(\text{"q"} ; \text{num2str}(k)\right) \quad 10 \quad \text{"Violet"} \right] \right) \right.$$

$$plotter := \begin{cases} b_1 \\ op \\ arr_F \\ arr_Q \\ marks \end{cases}$$

Preparing of the diagram parts to be drawn

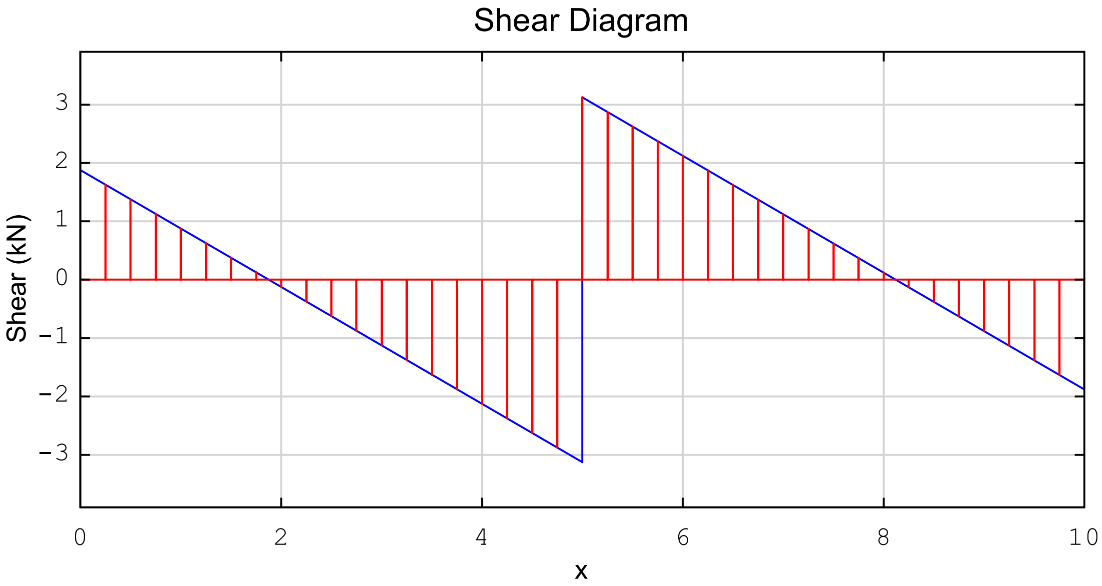
Beam diagram:



$$V(x) := \begin{cases} V1(x) & \text{if}\ \ (x > 0) \wedge (x < L1) \\ V2(x - L1) & \text{if}\ \ x \ge L1 \\ 0 & \text{otherwise} \end{cases}$$

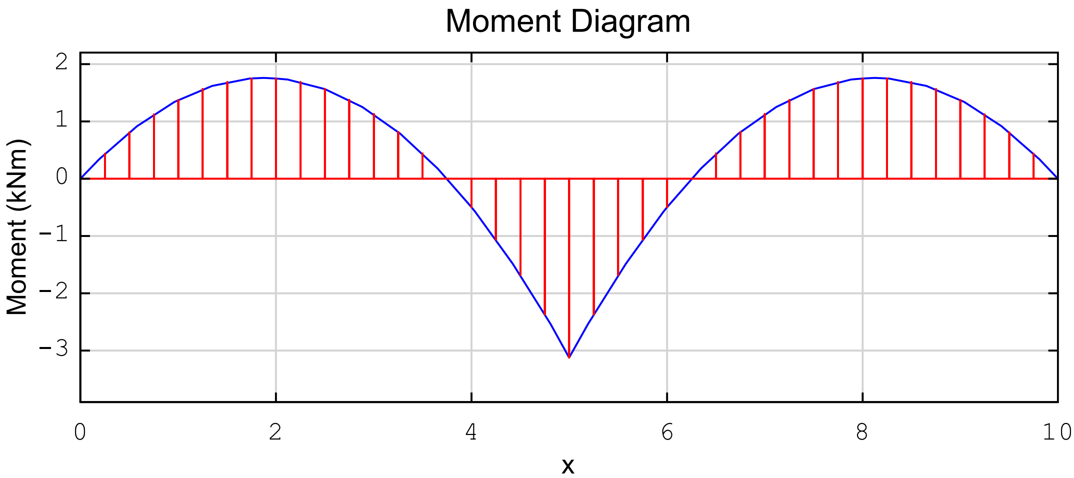
$$M(x) := \begin{cases} M1(x) & \text{if}\ \ (x > 0) \wedge (x < L1) \\ M2(x - L1) & \text{if}\ \ x \ge L1 \\ 0 & \text{otherwise} \end{cases}$$

$$u(x) := \begin{cases} u1(x) & \text{if}\ \ (x > 0) \wedge (x < L1) \\ u2(x - L1) & \text{if}\ \ x \ge L1 \\ 0 & \text{otherwise} \end{cases}$$



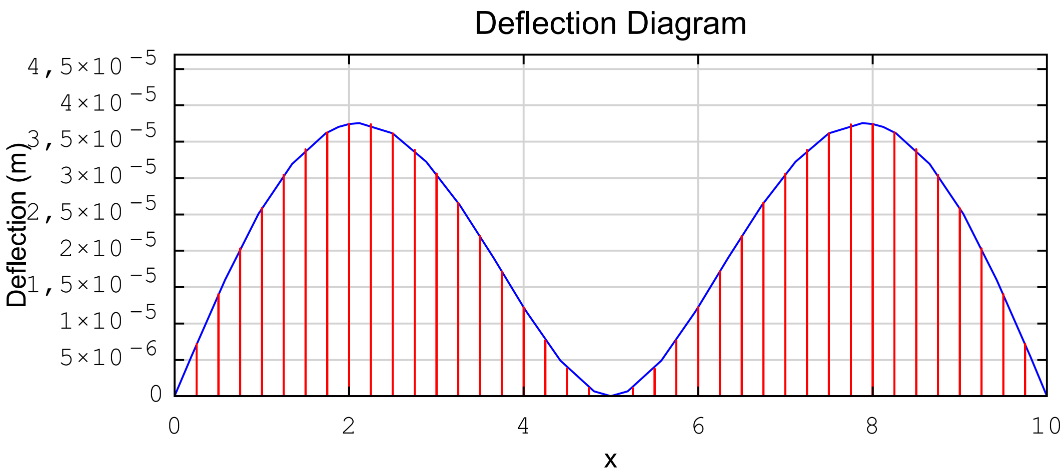
$V_{max} = 3{,}1238$ kN

$V_{min} = -3{,}1238$ kN



$M_{max} = 1{,}7601$ kN m

$M_{min} = -3{,}119$ kN m



$\Delta_{max} = 0{,}0031$ mm

$\Delta_{min} = 0$ mm

Reactions

$R_A := V1\left(0\ \mathrm{m}\right) = 1{,}8762$ kN

$R_B := \left(-V1\left(L1\right) + V2\left(0\ \mathrm{m}\right)\right) = 6{,}2476$ kN

$R_C := -V2\left(L2\right) = 1{,}8762$ kN