UNIT No 5: Curves and Fractals

Question And Answers

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1) Write short notes on interpolation and approximation. May 23 (4)
2) Explain Blending function for B-spline curves. May 23 (7)
3) What are fractals? Explain Triadic Koch curve in details. May 23 (7)
4) Explain Bezier curve .List it's properties. May 23 (4)
5) Draw and explain Hilbert's curve with an example. May 23 (7)
6) With suitable example write short notes on fractal lines. May 23 (7)
7) What is interpolation? Write short note on interpolating algorithm. April 22 (6)
8) Explain Hilbert curve and give its fractal dimension. April 22 (6)
9) How is coastline measured? What are the methods of measuring length? A.22 (6)

10) Derive blending function of Bezier curve. How blending function is calculated for cubic polynomial curve? April 22 (6)
11) Explain Triadic curve and give its fractal dimension. April 22 (6)
12) How do you find the area under a normal curve? April 22 (6)

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1) Write short notes on interpolation and approximation. (7Q) May 23 (4)

The curve is specified by a set of control points. Position of control points controls the shape of the curve. Fitting a single curve of the higher degree to all the control points is difficult. Normally, the big curve is approximated by joining piecewise polynomials of lower degree, which approximates few control points only.

If the curve passes through the control points, it is called **interpolation**. Interpolation curves are used in animation and specifying camera motion. It is also used in digitizing the coordinates.
If the curve does not pass through the control points and approximate the shape, it is called **approximation or extrapolation.** Such curves are used to estimate the shape of the object surface. Fig. shows the geometry of both methods.

Manipulating spline curves using control point representation is very easy. The designer can adjust the shape of the curve by repositioning the control point. It is highly preferable in interactive graphics design Applying transformation on each point of the curve is equivalent to applying transformation first to control points and redrawing the curve.

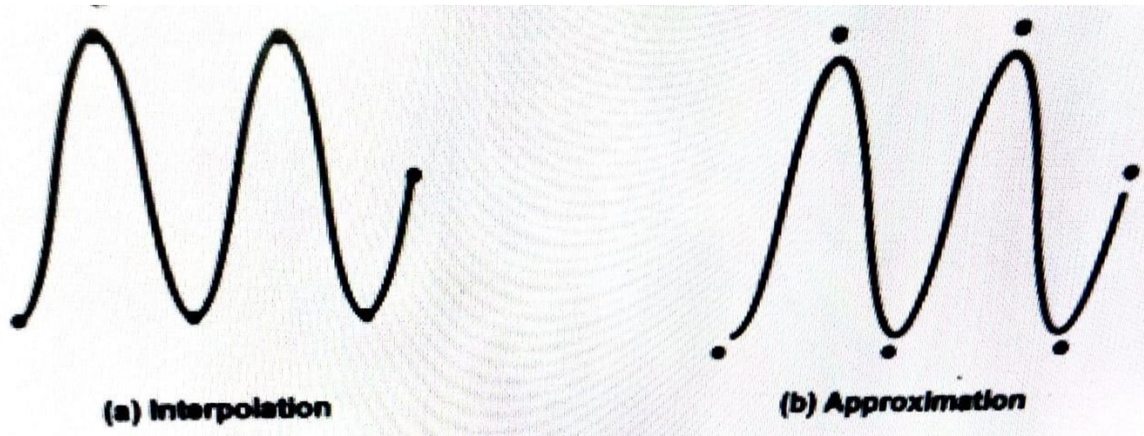

Fig. 10.1.1 : Interpolation v/s approximation

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2) Explain Blending function for B-spline curves. **May 23 (7)**

In the Bezier curve, the degree of the polynomial is always one less than the number of control points. So we cannot have more than four points for cubic Bezier curves. Furthermore, change in the single control point has a global effect on the Bezier curve. B-Spline curves are the most widely used class of curves for approximating the shape due to its following properties:
Degree of a polynomial is independent of a number of control points (which is not true in case of Bezier curve).
They have local control over the curve and surfaces.

However, derivation and generation of B-spline are more complex than Bezier curves.

The B-spline curve with (n +1) control points in expressed in the form of blending function as.

$$P(t) = \sum_{i=0}^{n} p_k \cdot B_{i,d}(t), \; t_{min} \leq t \leq t_{max}$$

$$\text{and } 2 \leq d \leq n+1$$

For Bezier curve, the range of t was always between 0 and 1. In the B-spline curve, t depends on types of B spline.

B-Spline bending functions Bi,d are polynomials with degree d- 1. Degree d can take any integer value between 2 and number of control points.

We can achieve local control by defining the blending function on subinterval of the total range of parameter t

Blending functions of Bezier curve are described by Bernstein polynomials, whereas blending functions for B-spline curve are defined using Cox-deBoor recursive formula as shown below:

$$B_{i,1}(t) = \begin{cases} 1, & \text{if } t_i \le t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

$$B_{i,d}(t) = \frac{t - t_i}{t_{i+d-1} - t_i} B_{i,d-1}(t) + \frac{t_{i+d} - t}{t_{i+d} - t_{i+1}} B_{i+1,d-1}(t)$$

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3) What are fractals? Explain Triadic Koch curve in details. May 23 (7)

Fractals is a complex picture created using iteration and a single formula. Sometimes, objects cannot be drawn with a given equation or with a given geometry. Examples: mountains, clouds. Their shape cannot be defined so in this case, we use fractals. So these are nothing but natural objects that can be drawn with the help of fractals. Below is an example of a fractal diagram.

Types of Fractal

There are three types of fractals:

- **Self-similar:** These fractals have parts as a scaled-down version of the entire object. So if we scale up from a smaller part of the fractal, we will scale up to the whole object.
- **Self Affine:** It has parts formed with different scaling parameters in different coordinate directions. For example in the direction, the scaling may be different from that t of y-direction. So the fractal will not be exactly similar in all directions.
- **Invariant:** Formed with nonlinear transformations. The smaller and bigger objects in the fractal are different.

Fractal Dimension

- Fractal is a measure of roughness or fragmentation of an object.
- More fractal dimensions in case of more jagged-looking objects.
- Using some iterative procedure, we can calculate fractal dimension D.

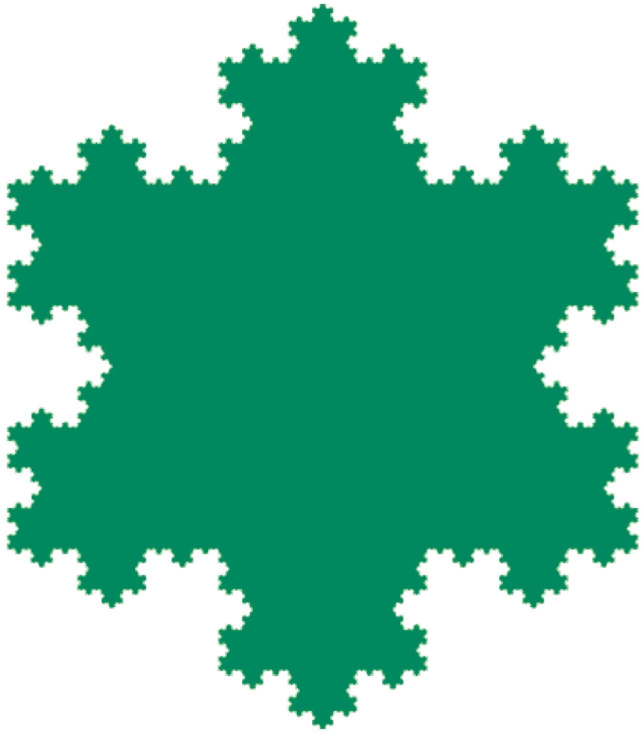
Koch Curve

Next, we will discuss the Koch curve, which is an example of a fractal, which can be drawn with the help of an equation or with the help of a program. In the Koch curve, initially, you have to take a triangle, and in that triangle, each straight line is replaced with four equal-sized lines of scaling factor 1/3. This Koch curve is also known as the snowflake pattern.

Step 1: Below is the diagram explaining how to draw the Koch curve. We have to first divide each side of the triangle into three equal parts and the middle part is again divided to 1/3rd of its original length. This process will be repeated.

Koch curve

Step 2: After n iterations, we will get a Koch curve that will look like this.



Koch curve

In the Koch curve, the fractal dimension is 1.2619. This value is calculated using the following procedure:

At each iteration, the scaling factor S is 1/3 because each line segment is divided into 1/3 of its original length, and N is 4 because each line segment is replaced by four smaller line segments. Therefore, we can calculate the fractal dimension of the Koch curve as follows:

$D = \log(4) / \log(1 / 3)$

$D \approx 1.2619$

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4) Explain Bezier curve .List it's properties. **May 23 (4)**

Bezier Curves:-

Bezier curve is an approach for the construction of the Curve.A Bezier curve is determined by a defining polygon. Bezier curves have a number of properties that make them highly useful and convenient for curve and surface design. They are also easy to implement. Therefore Bezier curves are widely available in various CAD systems and in general graphic packages. In this section we will discuss the cubic Bezier curve. The reason for choosing cubic Bezier curve is that they provide reasonable design flexibility and also avoid the large number of calculations.

In general, a Bezier curve section can be fitted to any number of control points. However, as number of control points increases, the degree of the Bezier polynomial also increases. Because in a Bezier curve a degree of a polynomial is one less than the number of control points used. For example. three control points generate a parabola four points generate cubic curve and so on. This is illustrated in Fig. (19)

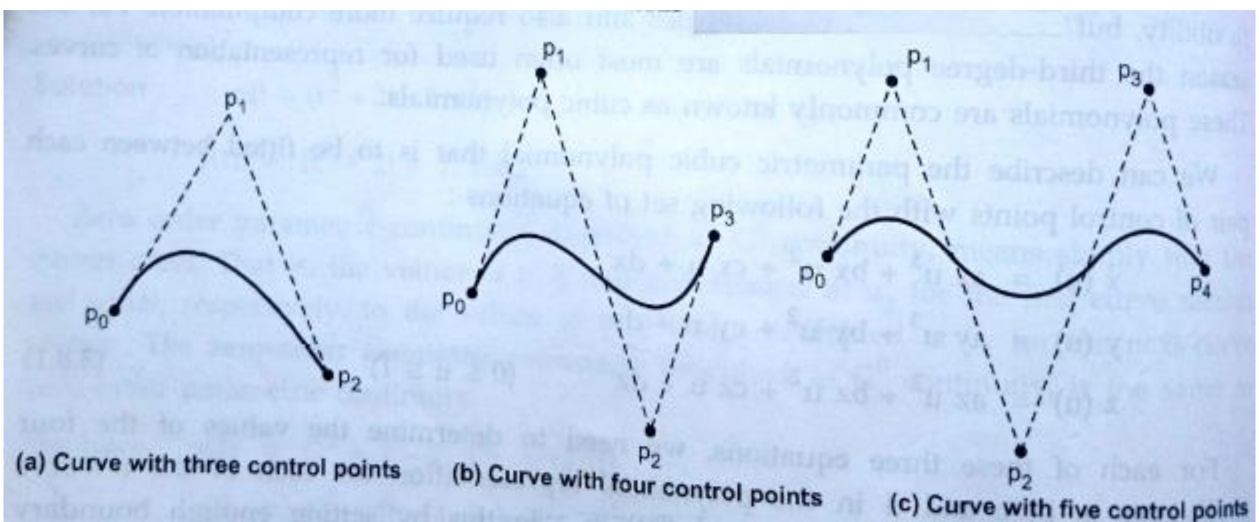


Fig. (19)

Properties of Bezier curve:-

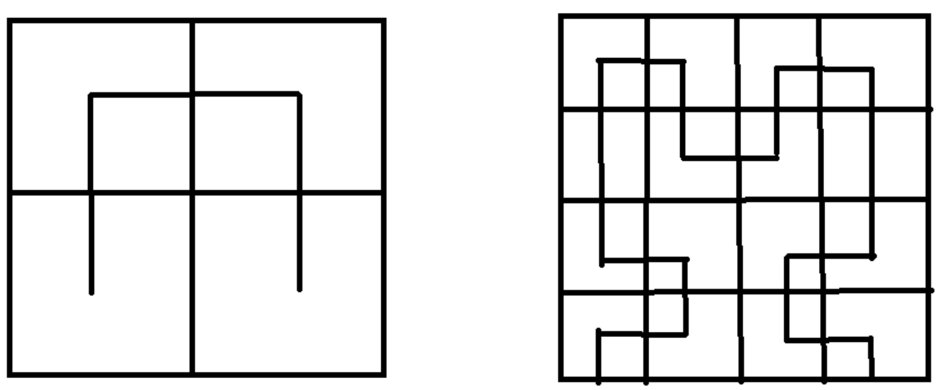
1. The basis functions are real.
2. Bezier curve always passes through the first and last control points i.e. curve has same end points as the guiding Polygon.
3. The degree of the polynomial defining the curve segment is one less than the number of defining polygon point. Therefore, for 4 control points, the degree of the polynomial is three, i.e. cubic polynomial.
4. The curve generally follows the shape of the defining polygon.
5. The direction of the tangent vector at the end points is the same as that of the vector determined by first and last segments.
6. The curve lies entirely within the convex hull formed by four control points.
7. The convex hull property for a Bezier curve ensures that the polynomial smoothly follows the control points.
8. The curve exhibits the variation diminishing property. This means that the curve does not oscillate about any straight line more often than the defining polygon.
9. The curve is invariant under an affine transformation.

5) Draw and explain Hilbert's curve with an example. **May 23 (7)**

The Hilbert curve is a space filling curve that visits every point in a square grid with a size of 2×2, 4×4, 8×8, 16×16, or any other power of 2. It was first described by David Hilbert in 1892. Applications of the Hilbert curve are in image processing: especially image compression and dithering. It has advantages in those operations where the coherence between neighbouring pixels is important. The Hilbert curve is also a special version of a quadtree; any image processing function that benefits from the use of quadtrees may also use a Hilbert curve.

A Hilbert curve' is a particular space-filling curve which, besides possessing aesthetic qualities, seems to have some applications in computer graphics. ' Such a curve is defined by a function which maps a parameter t onto pairs of values (x,y), where t is the length along the curve.

It is also called a peano or space-filling curve. It requires successive approximation. In the first approximation the square is divided into 4 quadrants and draw the curve that connects the center points of each. In the second approximation further, every quadrant is divided which cannot be the center of each.



- here is no limit to the subdivision. Ideally length of the curve is infinite. With every subdivision the length Increase by 4
- The curve is equivalent to line. Its topological dimension is 1.
- Length of the curve changes by 4.

Applications of Hibert Curve:

- It is used for Modelling natural structures like Geographic terrain, mountain, plant structure, clouds, vegetables etc.
- Space research.
- Study of convergence of iterative processes.
- Engineering and architecture.
- Medical science.
- Chemical processes.
- Medical diagnostic images.
- Fluid mechanics.
- Image compression and different Telecommunication purposes.

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6) With suitable example write short notes on fractal lines. **May 23 (7)**

Fractals are infinitely complex patterns that are self similar across different scales. Natural objects are often fractals, for example tree, mountain, clouds, hurricanes etc.

Fractal objects possess the following two properties

-Infinite detail at every point.

-Self-similarity between object parts.

-Natural objects have infinite details, however, we should design a process which produces finite detail because on the computer we cannot display infinite detail.

-After certain levels of zoom in, Euclidean shapes lead to smooth drawing. While in the fractal object, if we keep zoom in, we continue to see the same detail as it appears in the original object. Fig. below describes the discussed concept.

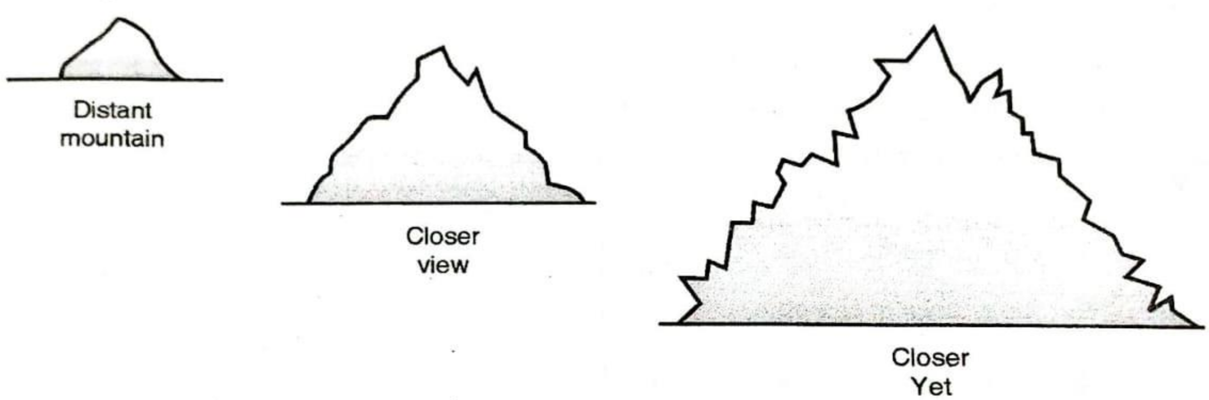


Fig. 10.2.1 : View of the mountain at different zoom level

Zoom in effect on the monitor is achieved by selecting a smaller and smaller window with the same viewport. Less and less detail is mapped to the same size as the viewport.

11 Q ans-> continue (Explain Triadic curve and give its fractal dimension.)

Fractal Dimension

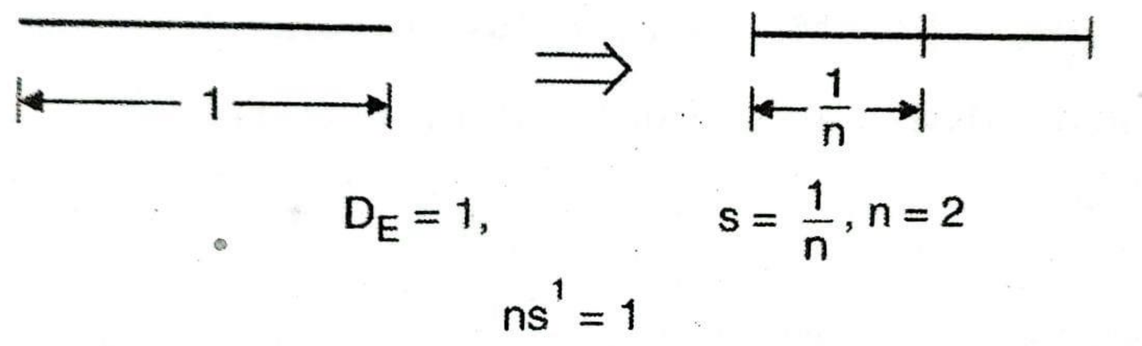
Amount of variation in object detail is described by a number called fractal dimension. Fractal dimension can be any real number.

Detail variation in a fractal is controlled by the fractal dimension number D. We can consider D as a measure of the roughness of the object.

Objects with jaggy boundary have larger D value. We can write iterative procedure controlling detail by specifying D. However, it is difficult to estimate the value of D for real objects.

Self-similar fractals are obtained by recursively applying the same scaling factor to Euclidean shapes.

Fig. below shows the relationship between scaling factor and a number of parts for unit length line, unit square and unit cube with s = 1/2. With s = ½, the line is divided into two equal segments, the square is divided into four equal part of equal area and cube is divided into eight subcubes of equal volume.



(a)

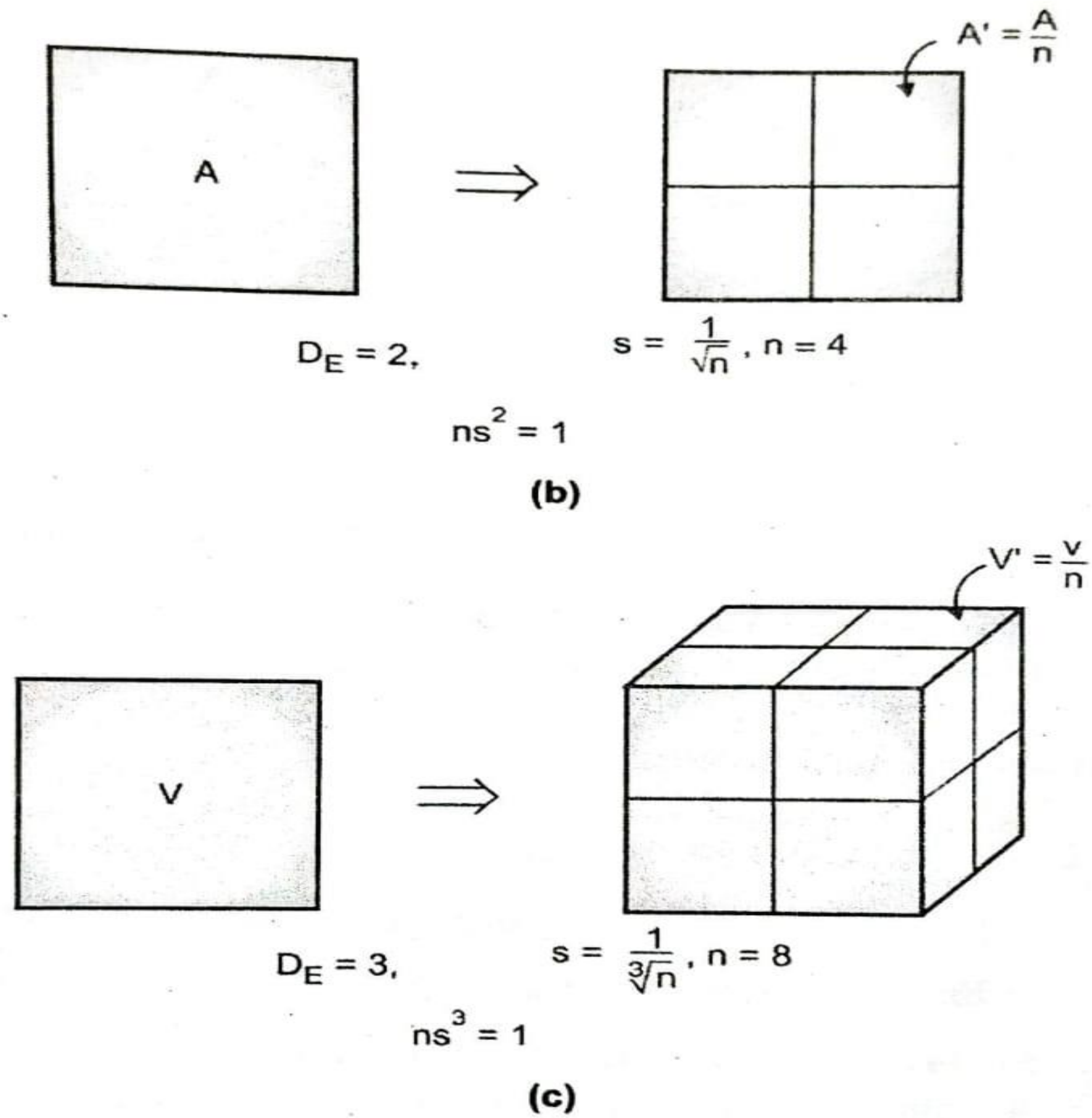


Fig. 10.2.3 : Subdivision of object with scaling factor $s = ½$ and Euclidean dimensions ((a) $D_E = 1$, (b) $D_E = 2$ and (c) $D_E = 3$)

For these objects, the relationship between a number of subparts and the scaling factor is $n \cdot s^{D_E} = 1$. For self-similar objects,

$$n \cdot s^D = 1$$

9) What is coastline measured? What are the methods of measuring length? 22 (6)

Measuring the length of a coastline is not as straightforward as it might seem, and different methods can yield different results. The coastline paradox, discovered by the mathematician Lewis Fry Richardson in 1961, highlights the idea that the measured length of a coastline depends on the length of the measuring stick or unit used. As the measuring stick gets smaller, more details along the coastline are captured, leading to a longer measured length.

There are several methods used to measure the length of coastlines:

Cartographic Measurement:

Traditional maps provide a simple way to measure coastlines using rulers or measuring devices. However, the accuracy of this method depends on the scale of the map. Smaller scales may not capture intricate details of the coastline.

Aerial and Satellite Imagery:

Aerial and satellite imagery have become powerful tools for measuring coastlines. High-resolution images allow for detailed measurements, and the use of Geographic Information System (GIS) software enables accurate calculations of coastline length.

GIS (Geographic Information System):

GIS technology integrates various data sources, including satellite imagery, aerial photographs, and topographic maps, to create accurate digital representations of coastlines. GIS software can then calculate the length of the coastline using algorithms designed for such measurements.

LIDAR (Light Detection and Ranging):

LIDAR is a remote sensing technology that uses laser light to measure distances. It can be employed to create highly detailed and accurate digital elevation models, which include coastal features. LIDAR data can be processed to measure the length of coastlines.

Fractal Geometry:

Recognizing that coastlines often exhibit fractal-like patterns, fractal geometry provides a mathematical approach to estimating coastline length. By applying fractal dimensions, one can account for the intricate details at various scales.

Perimeter-Area Ratio Method:

This method involves measuring the perimeter of a region at different levels of detail and analyzing the relationship between perimeter and enclosed area. The coastline length is then estimated based on this ratio.

10) **Derive blending function of Bezier curve. How blending function is calculated for cubic polynomial curve? 2022 6 Marks**

The blending function for a cubic polynomial curve is typically associated with the Bézier curve. Bézier curves are widely used in computer graphics and computer-aided design (CAD) to represent smooth curves. The blending functions for a cubic Bézier curve describe how control points influence the shape of the curve.

A cubic Bézier curve is defined by four control points, P0, P1, P2, and P3. The curve is parametrically defined by the parameter (t) in the range [0, 1]. The formula for the cubic Bézier curve is given by:

[B(t) = (1-t)^3 cdot P0 + 3 cdot (1-t)^2 cdot t cdot P1 + 3 cdot (1-t) cdot t^2 cdot P2 + t^3 cdot P3]

The blending functions for the four control points are the coefficients in front of each control point in the formula. These blending functions are calculated as follows:

1. (1-t) 3) corresponds to the influence of the first control point (P0).
2. 3 . (1-t) 2 .t) corresponds to the influence of the second control point (P1).
3. 3 cdot (1-t) . t2) corresponds to the influence of the third control point (P2).
4. (t^3) corresponds to the influence of the fourth control point (P3).

The sum of these blending functions is always equal to 1, ensuring that the curve starts at (P0) when (t = 0) and ends at (P3) when (t = 1).

These blending functions are derived from the binomial expansion of the polynomial expressions that describe the Bézier curve. Understanding the algebraic properties of the Bézier curve helps in implementing it computationally, such as in graphics programming or design software.

12) **How do you find the area under a normal curve   April 22 (6)**

To find the area under a normal curve (a bell-shaped curve also known as a Gaussian distribution), you typically use a statistical table or a calculator. The standard normal distribution, with a mean of 0 and a standard deviation of 1, is commonly used for these calculations.

Here are the general steps:

1. **Standardize the Variable:**

If you're working with a normal distribution with a mean ((mu)) and standard deviation ((sigma)) different from 0 and 1, respectively, you need to standardize the variable. This involves converting the original variable ((X)) to a standard normal variable ((Z)) using the formula:

[Z = frac{(X - mu)}{sigma}]

This transforms the variable into a standard normal distribution.

2. Use a Z-Table or Calculator:
Once you have the standardized variable (Z), you can use a standard normal distribution table (Z-table) or a calculator to find the area under the curve. The Z-table provides the cumulative probability up to a certain Z-value.

If using a calculator, you can use built-in functions or statistical software to find the cumulative probability.

For example, if you want to find the probability that (Z) is less than a specific value (z), you look up (z) in the Z-table or use the calculator to find (P(Z < z)).

If you're looking for the probability in a specific range, you may need to find two Z-values and subtract the corresponding probabilities.

3. **Interpret the Result:**
The result from the Z-table or calculator gives you the cumulative probability. This represents the area under the normal curve up to the specified Z-value.

Remember that the total area under the normal curve is 1, so probabilities are expressed as proportions of the total area.

Keep in mind that many statistical software packages also provide functions to directly calculate the probabilities without the need for standardization or manual lookup in a table.