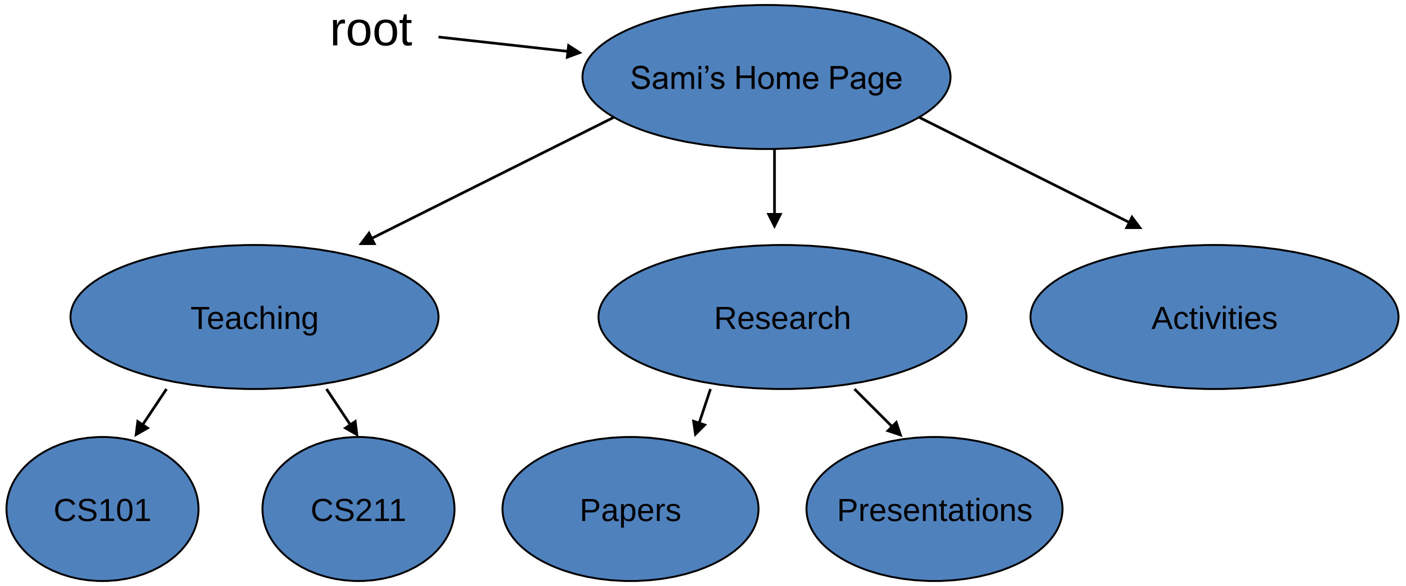
Trees

Example Tree



Definitions

✓root - starting point (top) of the tree

✓parent (ancestor) - the vertex “above” this vertex

✓child (descendent) - the vertices “below” this vertex

Definitions

- any two vertices must have one and only one path between them else its not a tree

- a tree with N nodes has N-1 edges

Definitions

- ✓leaves (terminal nodes) - have no children
- ✓level - the number of edges between this node and the root
- ✓ordered tree - where children's order is significant

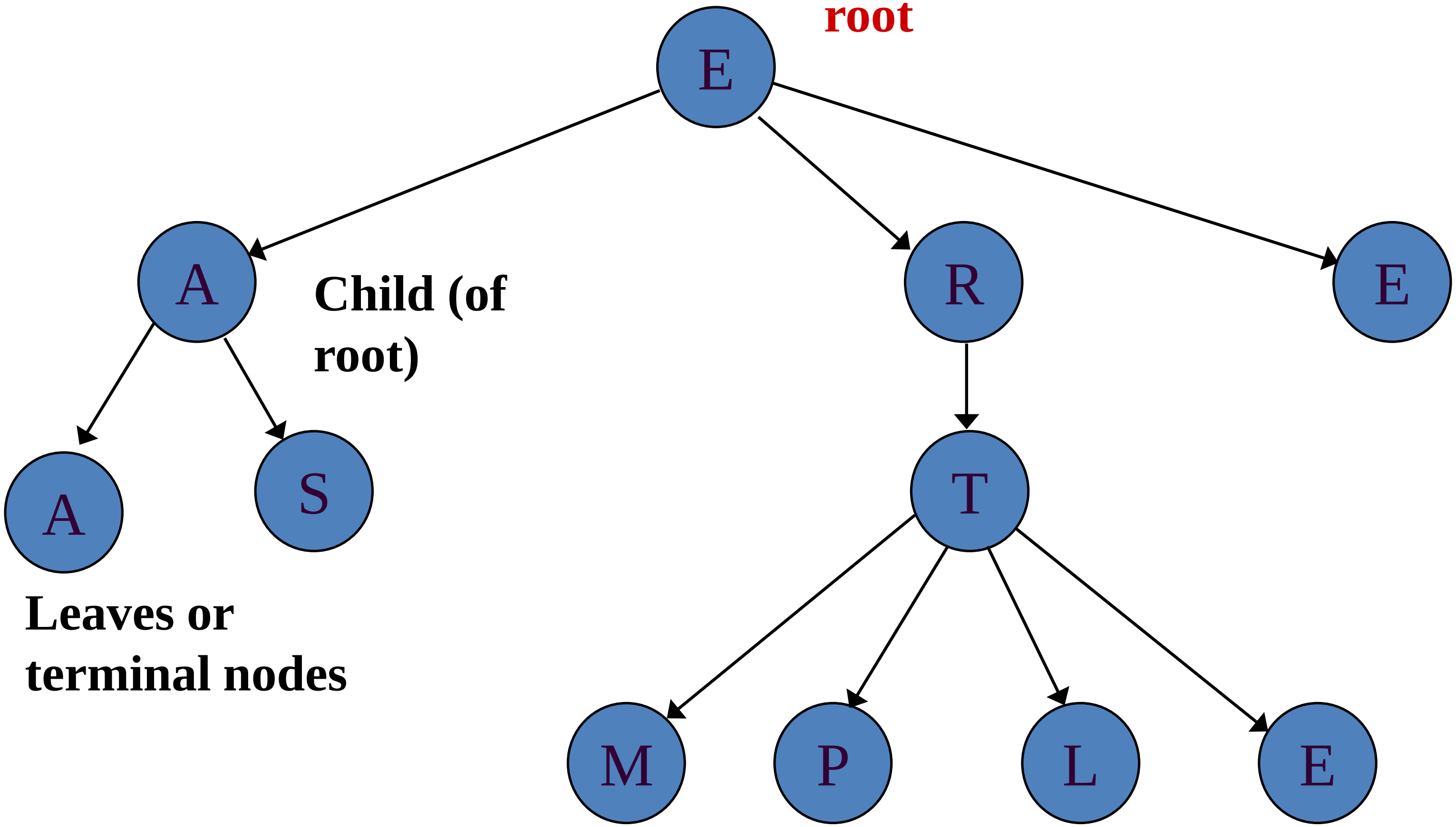
Definitions

- ✓ Depth of a node - the length of the path from the root to that node
  - root: depth 0
- ✓ Height of a node - the length of the longest path from that node to a leaf
  - any leaf: height 0
- ✓ Height of a tree: The length of the longest path from the root to a leaf

Balanced Trees

- the difference between the height of the left sub-tree and the height of the right sub-tree is not more than 1.

Trees - Example



Level

0

1

2

3

root

Child (of root)

Leaves or terminal nodes

Depth of T: 2

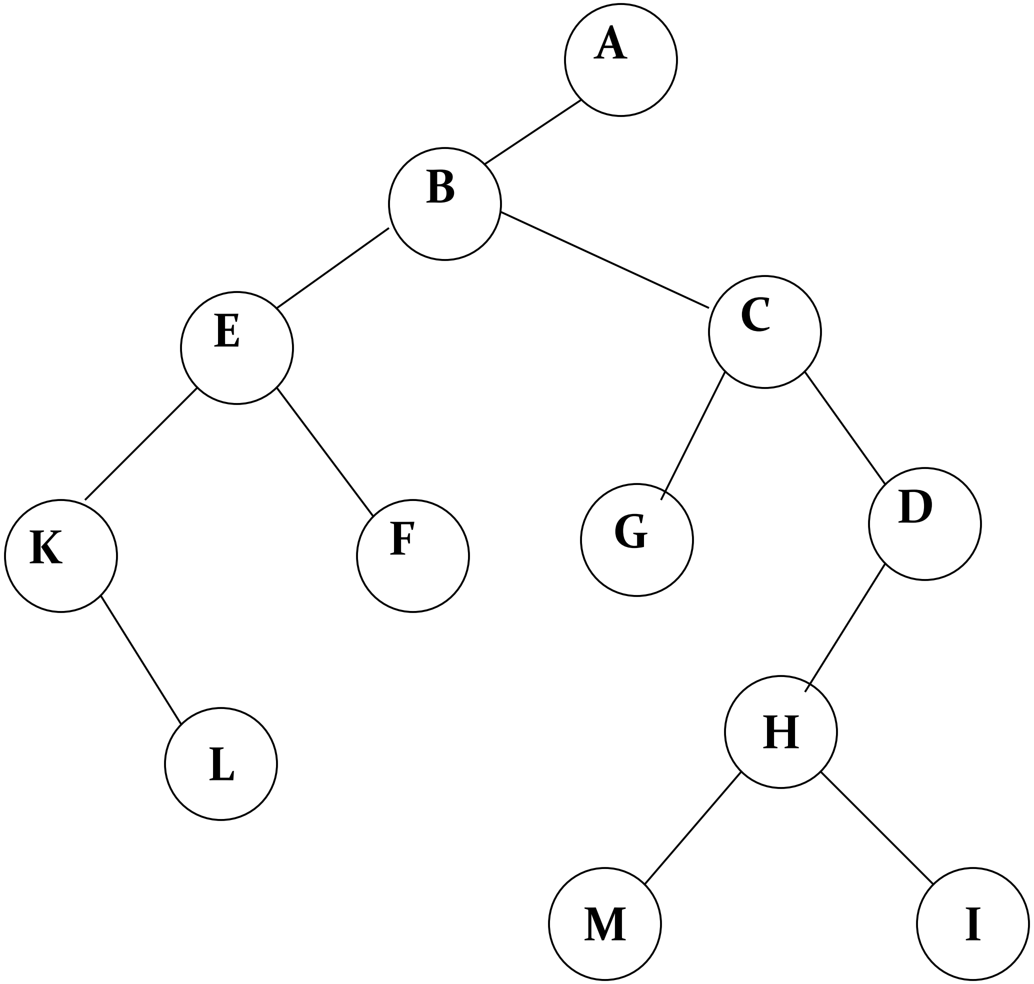
Height of T: 1

Binary Trees

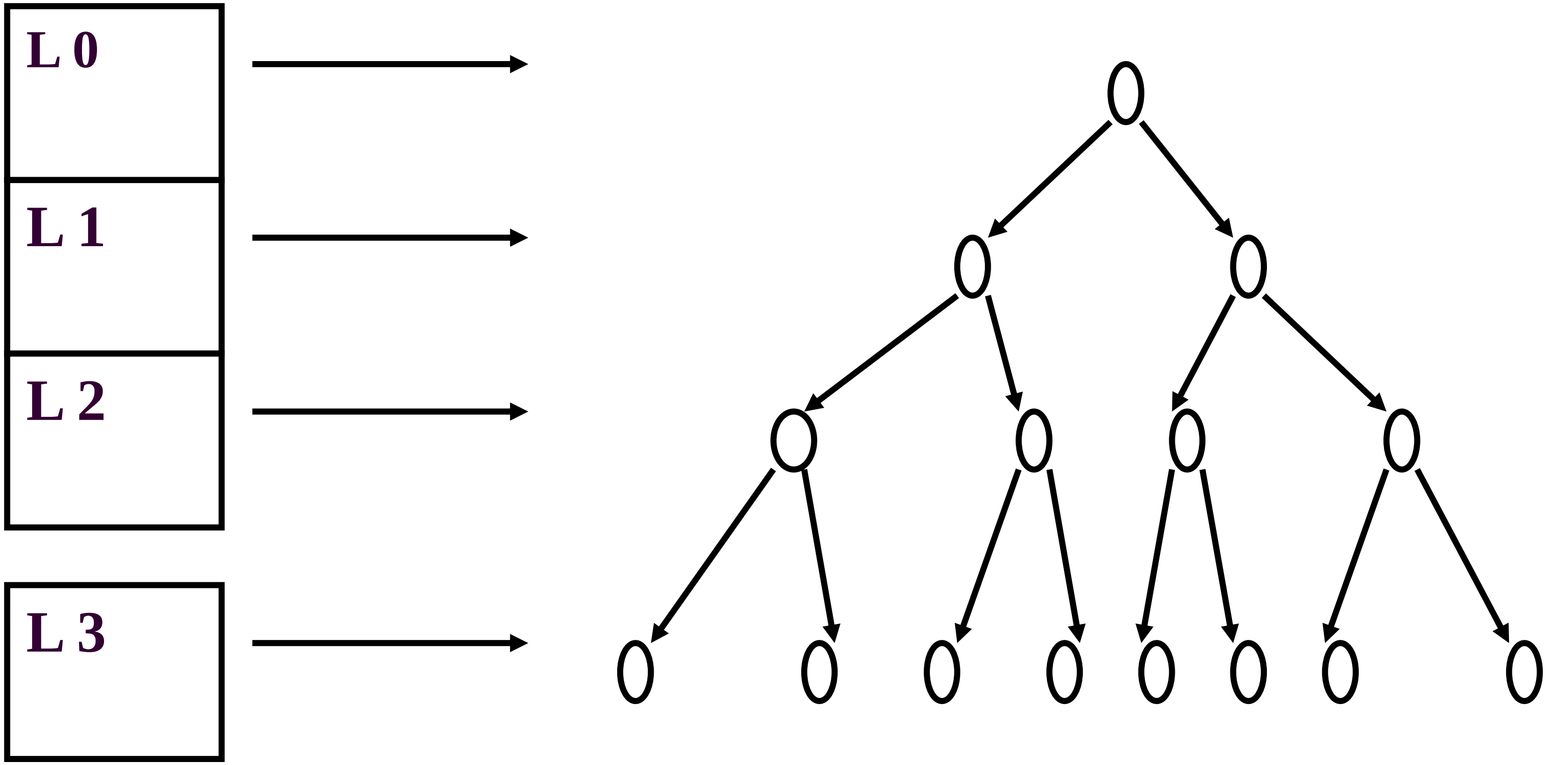
- A special class of trees: max degree for each node is 2

- *Recursive definition*: A binary tree is a finite set of nodes that is either empty or consists of a root and two disjoint binary trees called *the left subtree* and *the right subtree*.

Example



Height of a Complete Binary Tree



L 0

L 1

L 2

L 3

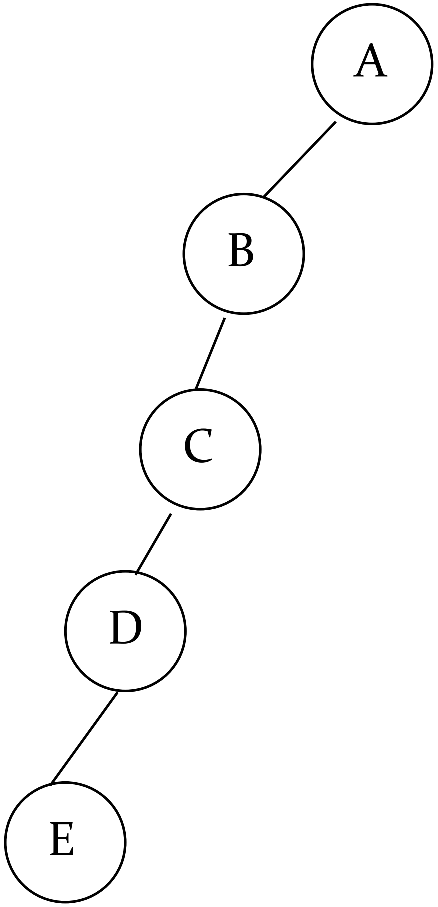
At each level the number of the nodes is doubled.

total number of nodes: $1 + 2 + 2^2 + 2^3 = 2^4 - 1 = 15$

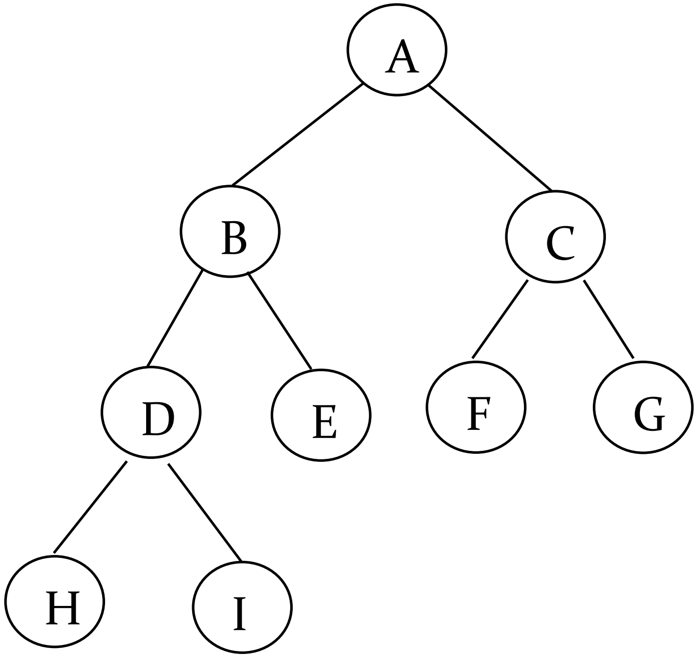
Binary Tree Representations

- If a complete binary tree with *n* nodes is represented sequentially, then for any node with index *i*, 1<=*i*<=*n*, we have:
  - *parent*(*i*) is at *i*/2 if *i*!=1. If *i*=1, *i* is at the root and has no parent.
  - *leftChild*(*i*) is at 2*i* if 2*i*<=*n*. If 2*i*>n, then *i* has no left child.
  - *rightChild*(*i*) is at 2*i*+1 if 2*i* +1 <=*n*. If 2*i* +1 >n, then *i* has no right child.

Sequential / Implicit Array Representation



[1]	A
[2]	B
[3]	--
[4]	C
[5]	--
[6]	--
[7]	--
[8]	D
[9]	--
.	.
[16]	E

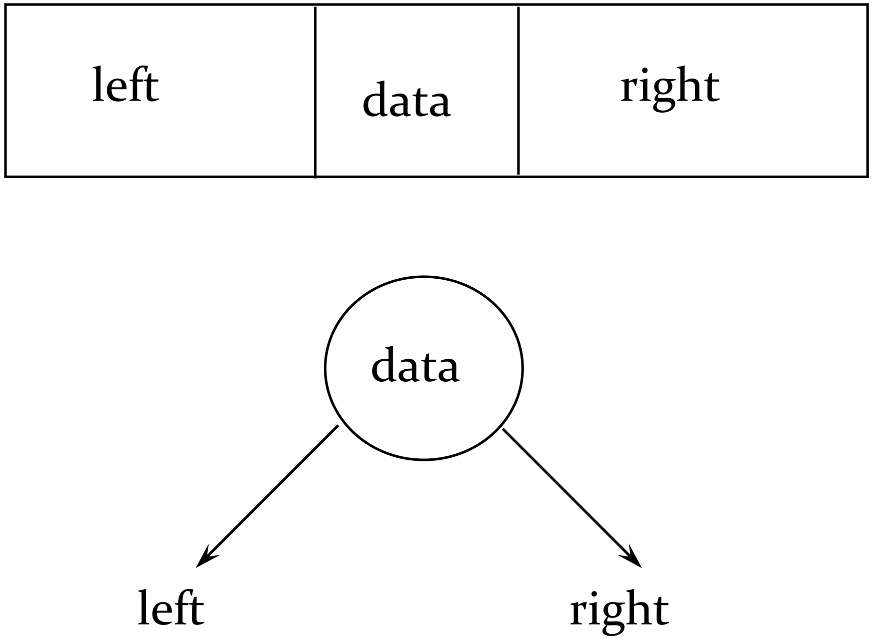


[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

(1) waste space
(2) insertion/deletion
problem

Dynamic/ Linked Representation

```
typedef struct node
{
Int data;
Struct node *left;
Struct node *right;
} bin;
```



Binary Tree Operation

- Creation of binary tree
- Insertion of a node in the tree
- Searching
- Deleting a node

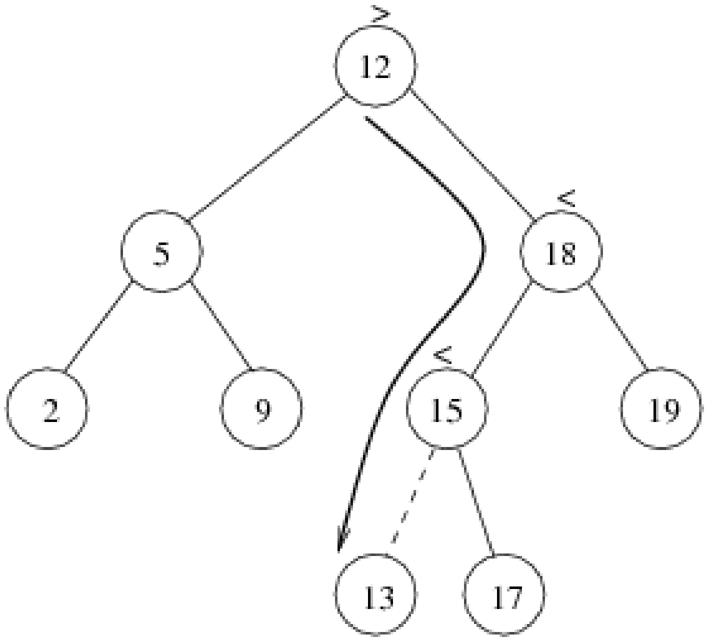
Binary Tree Operations (create)

- The basis of our binary tree node is the following `struct` declaration:

```
struct TreeNode
{
      int value;
      TreeNode *left;
      TreeNode *right;
};
```

Binary Tree Operations (Insert)

- Proceed down the tree as you would with a find
- If X is found, do nothing (or update something)
- Otherwise, insert X at the last spot on the path traversed



Binary Search Tree (BST)

- A binary tree is useful data structure when two way decisions must be made at each point in a process.
- Suppose, we want to find duplicates in a list of numbers.
    - One way is to go on comparing each number with all those that precede it. Number of comparisons increases.
- Comparison can be reduced by using BST.

Binary Tree Operations (Delete a node)

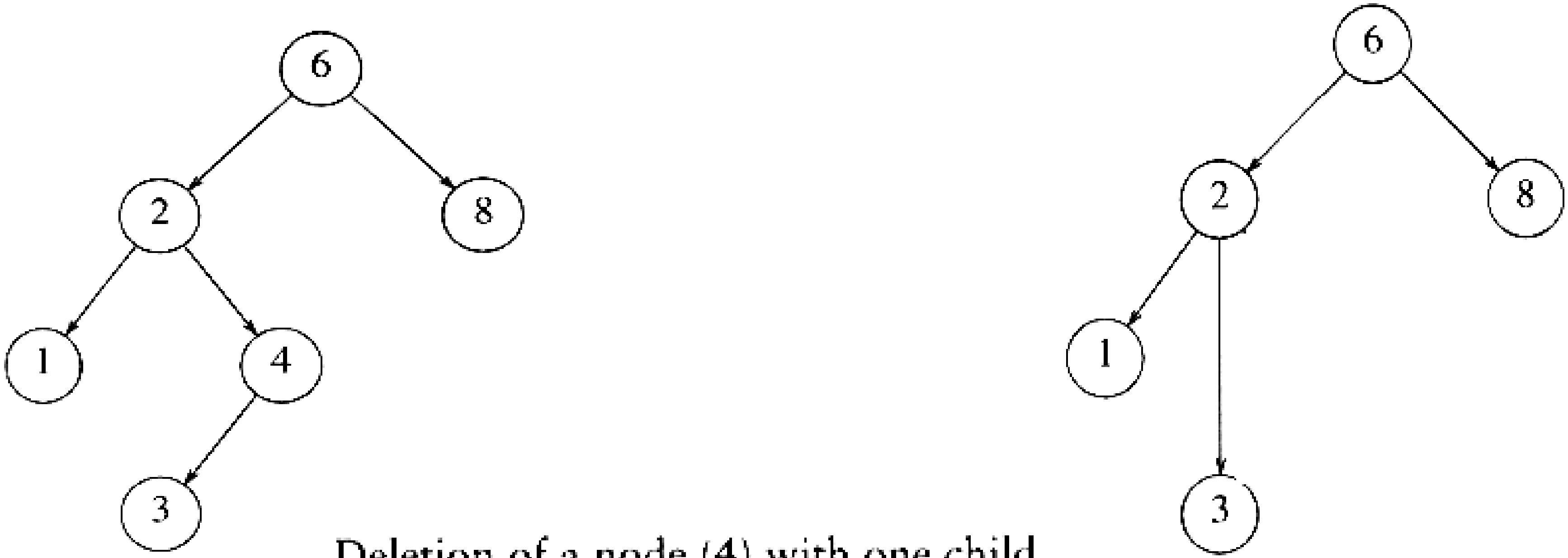
- When we delete a node, we need to consider how we take care of the children of the deleted node.
- This has to be done such that the property of the binary search tree is maintained.
- Three cases:

(1) the node is a leaf

» Delete it immediately

(2) the node has one child

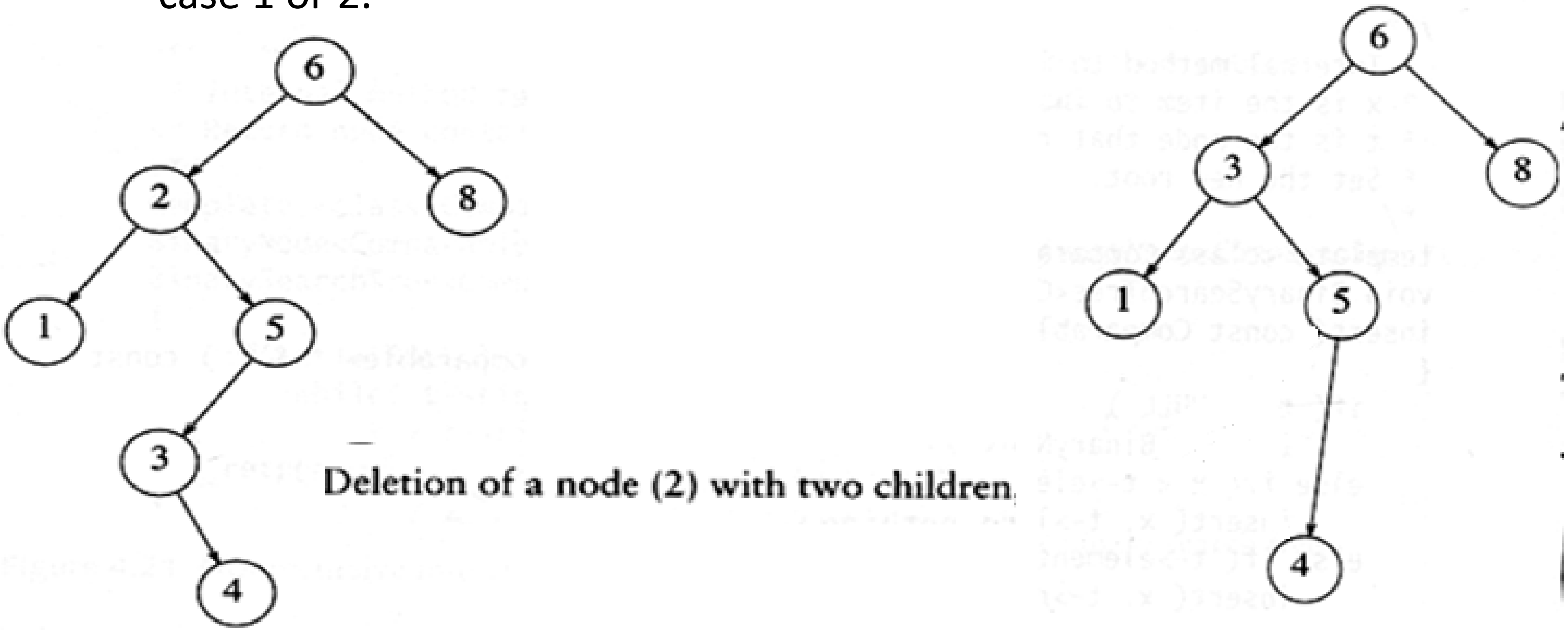
- Adjust a pointer from the parent to bypass that node



Deletion of a node (4) with one child.

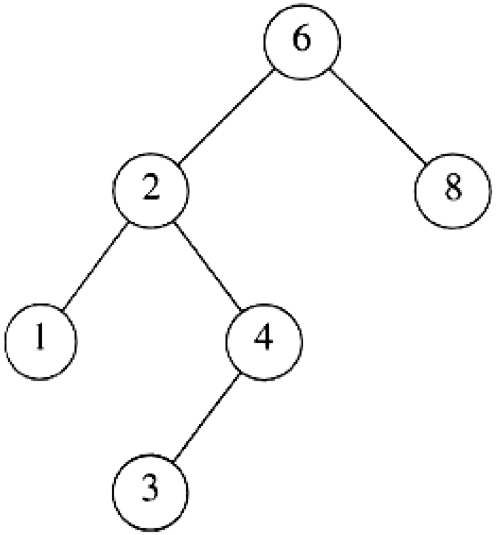
(3) the node has 2 children

- replace the key of that node with the minimum element at the right subtree
- delete the minimum element
  - Has either no child or only right child because if it has a left child, that left child would be smaller and would have been chosen. So invoke case 1 or 2.

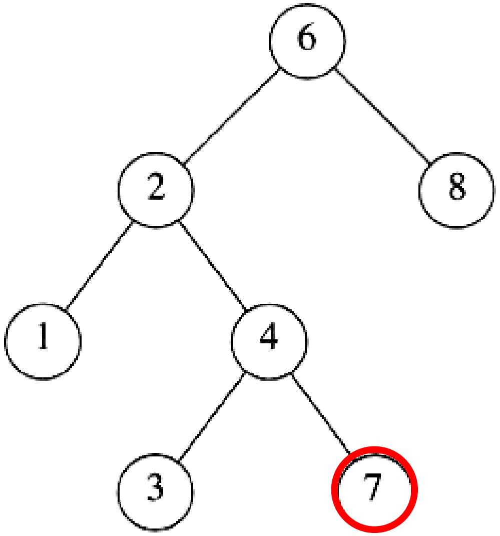


Deletion of a node (2) with two children

Binary Search Trees



A binary search tree



Not a binary search tree

Binary Tree Traversals

- Let L, V, and R stand for moving left, visiting the node, and moving right.
- There are six possible combinations of traversal
  - lVr, lrV, Vlr, Vrl, rVl, rlV
- Adopt convention that we traverse left before right, only 3 traversals remain
  - lVr, lrV, Vlr
  - inorder, postorder, preorder

Binary Tree - Traversal

There are six possible orderings of traversal.

By convention, we always traverse the left subtree before the right one.

That reduces the choices to three:

- **Preorder** - Visit the root node first
- **Inorder** - Visit the left subtree, then the root
- **Postorder** - Visit the root node last

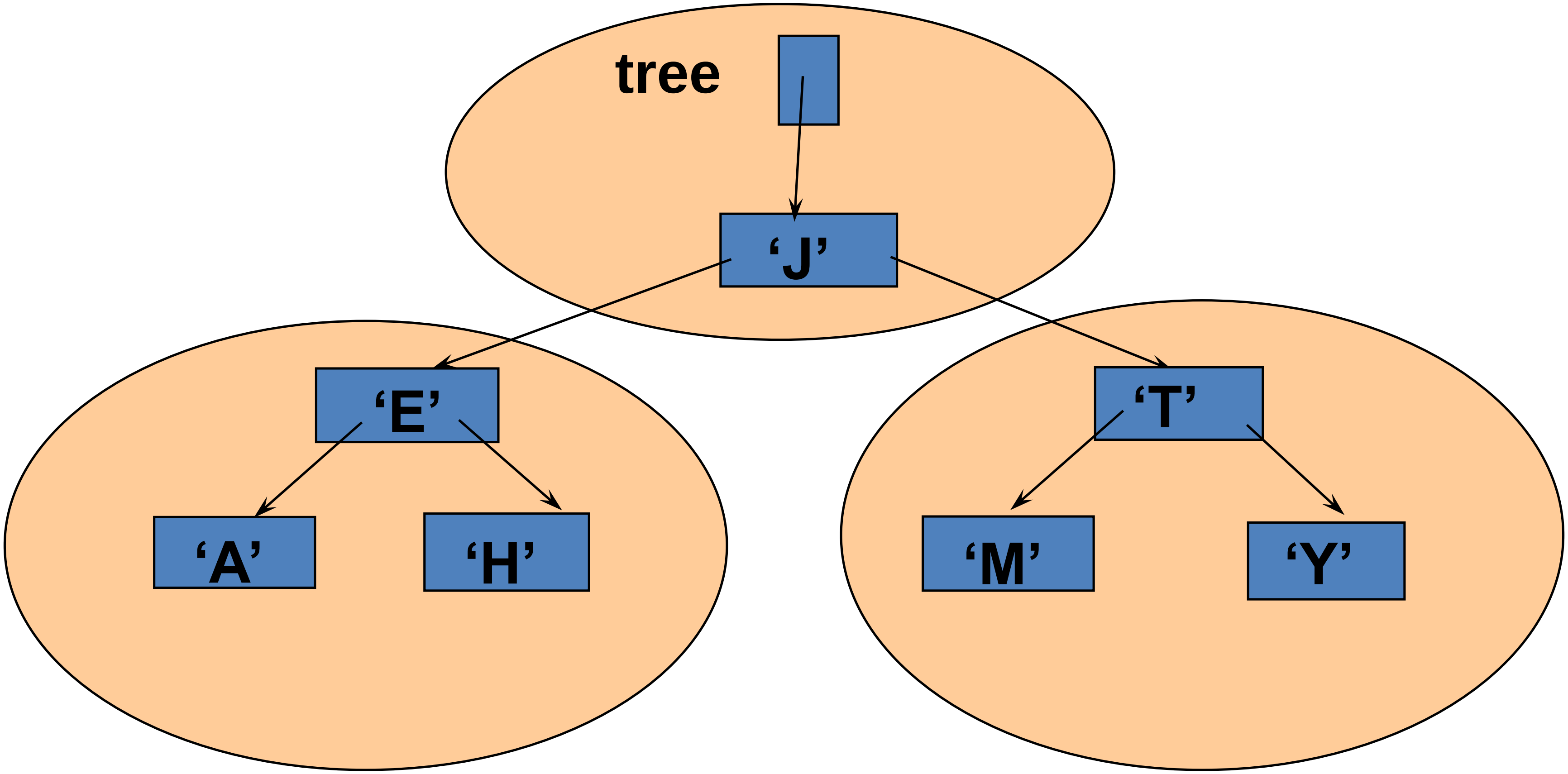
Preorder Traversal

- Visit the root of the tree first,
  then visit the nodes in the left subtree,
  then visit the nodes in the right subtree

```
Preorder(tree)
If tree is not NULL
  Visit Info(tree)
  Preorder(Left(tree))
  Preorder(Right(tree))
```

(Warning: "visit" means that the algorithm does something with the values in the node, e.g., print the value)

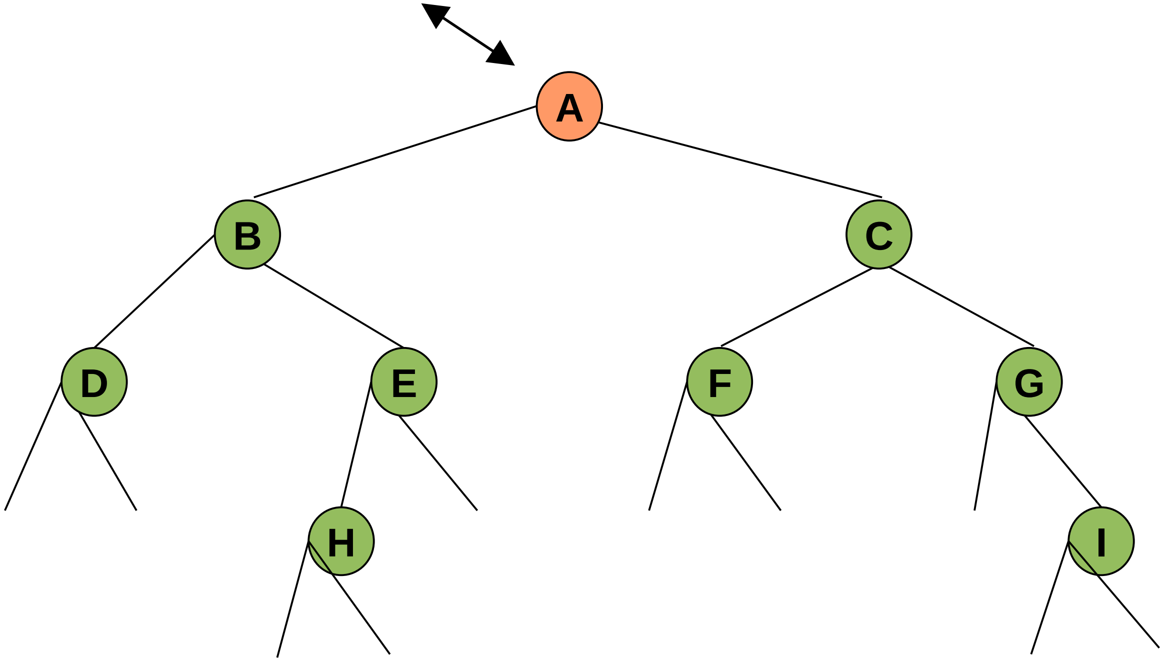
Preorder Traversal:



Visit left subtree second

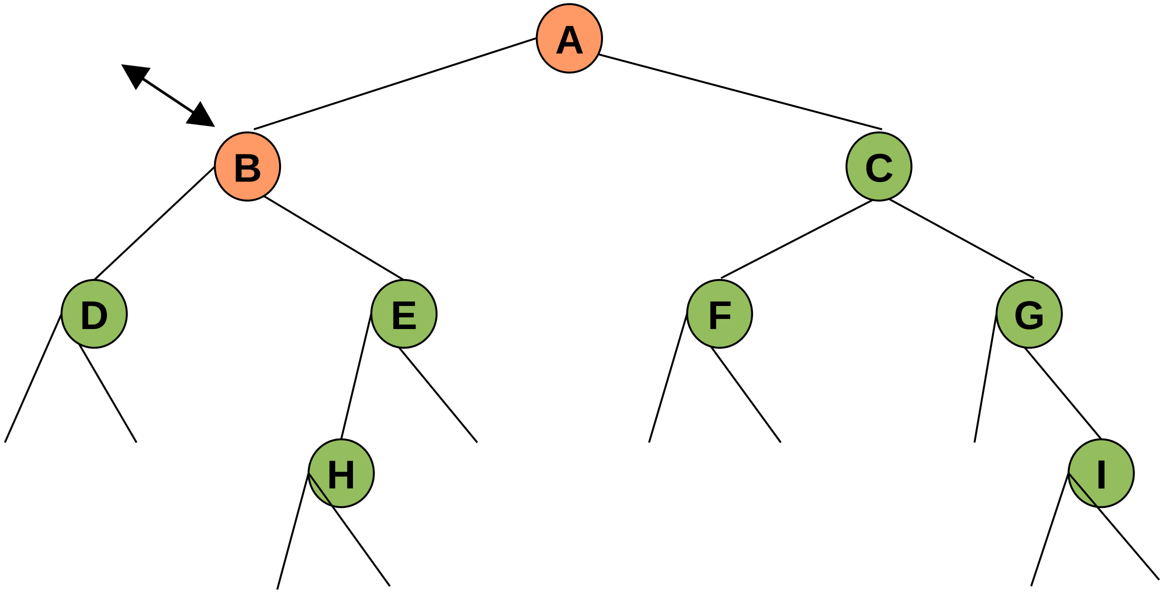
Visit right subtree last

Traversing a Tree Preorder



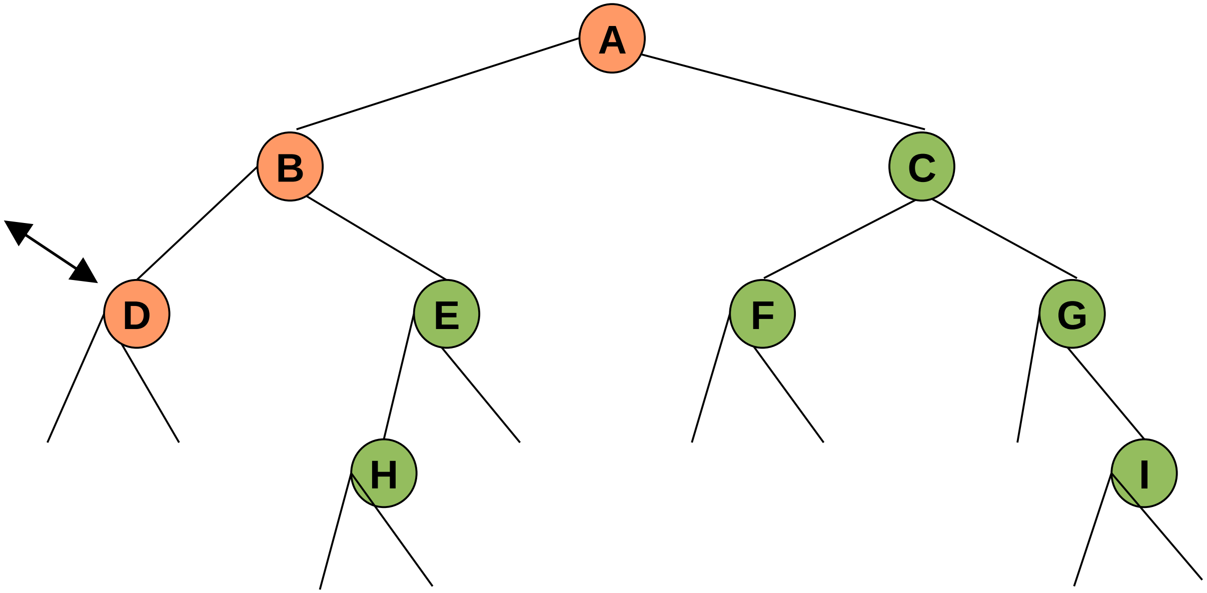
Result: A

Traversing a Tree Preorder



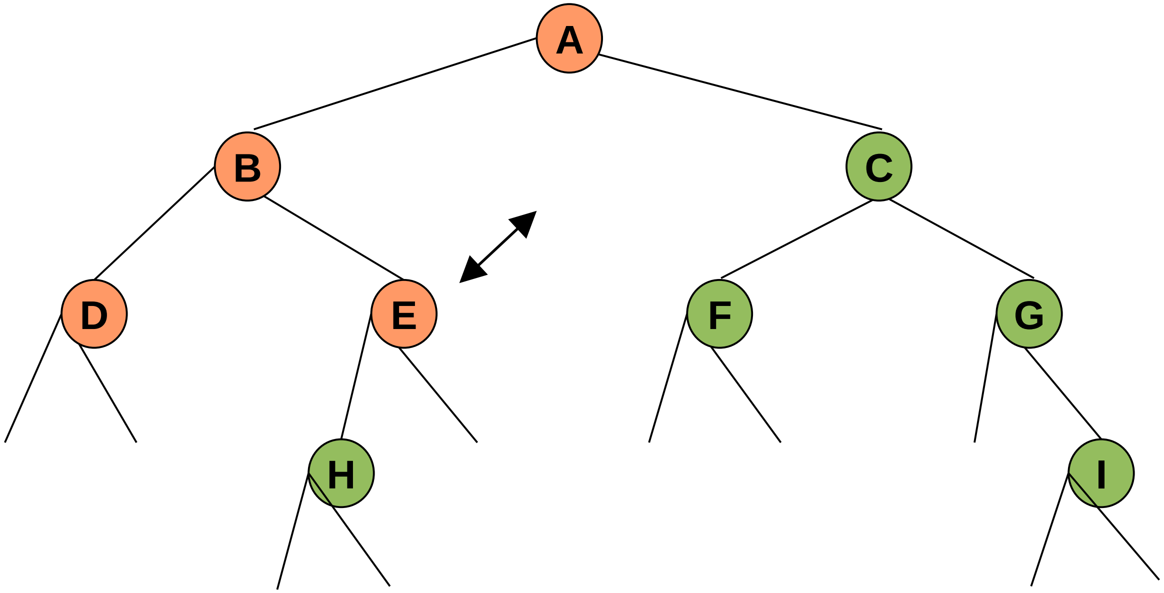
Result: AB

Traversing a Tree Preorder



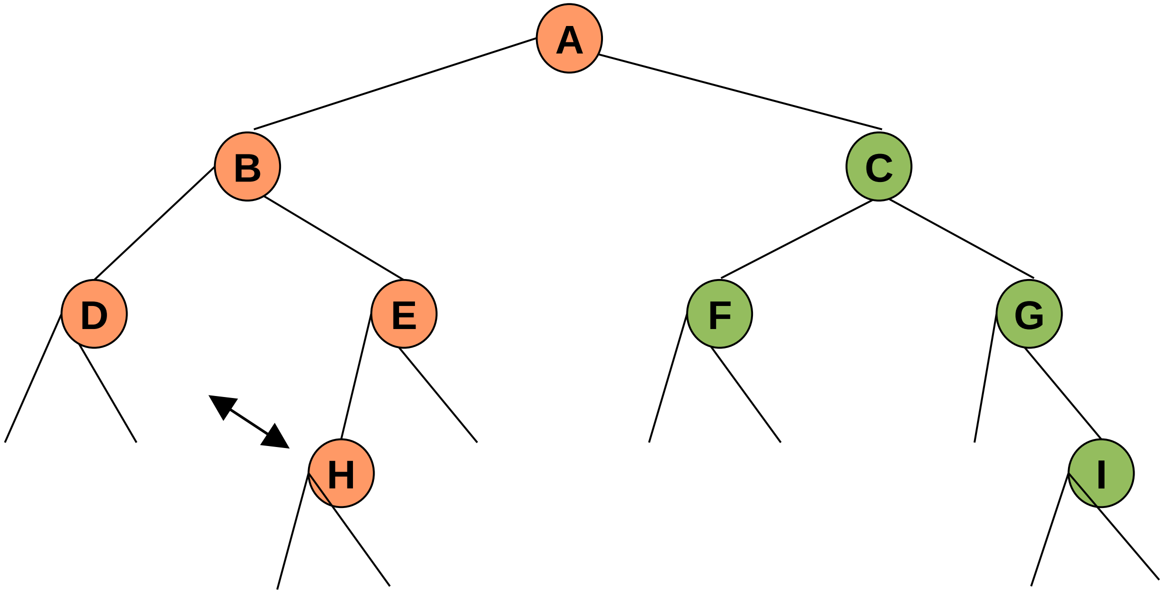
Result: ABD

Traversing a Tree Preorder



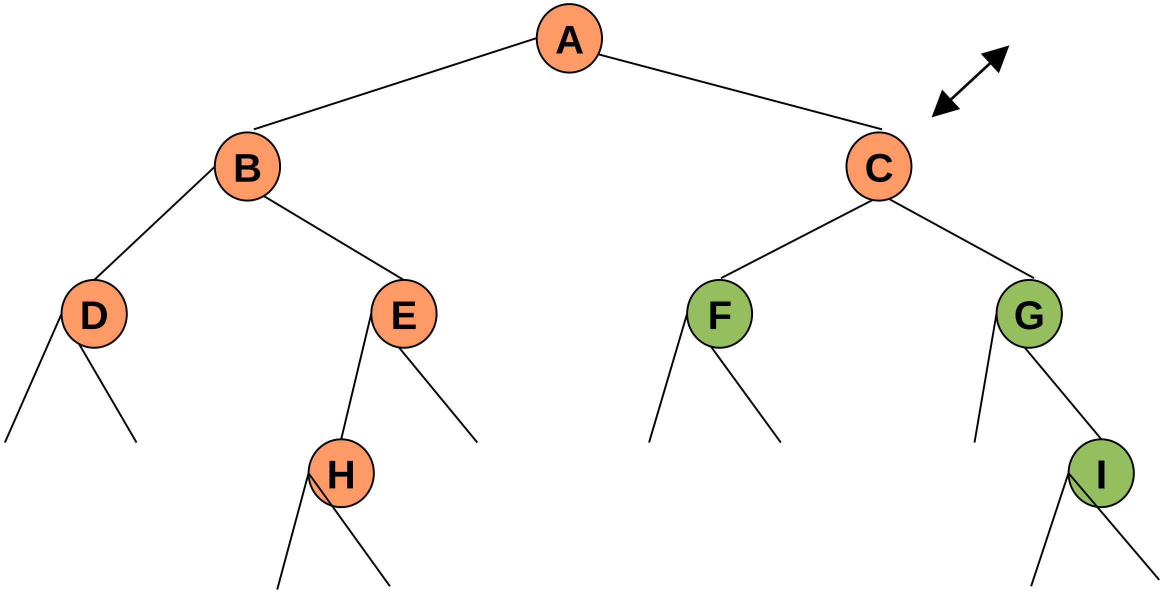
Result: ABDE

Traversing a Tree Preorder



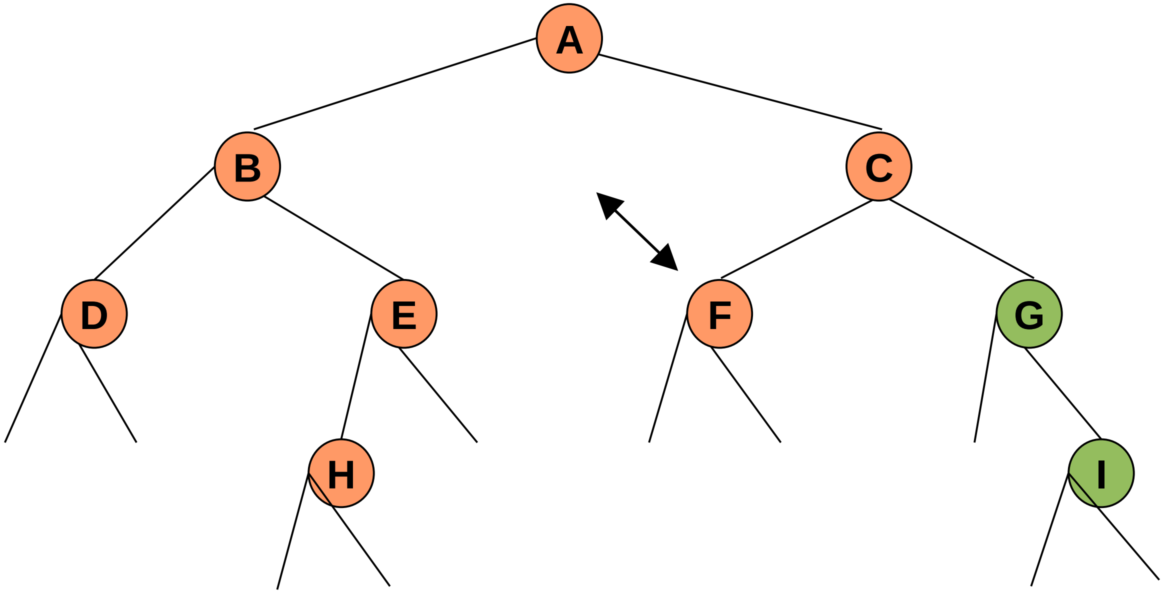
Result: ABDEH

Traversing a Tree Preorder



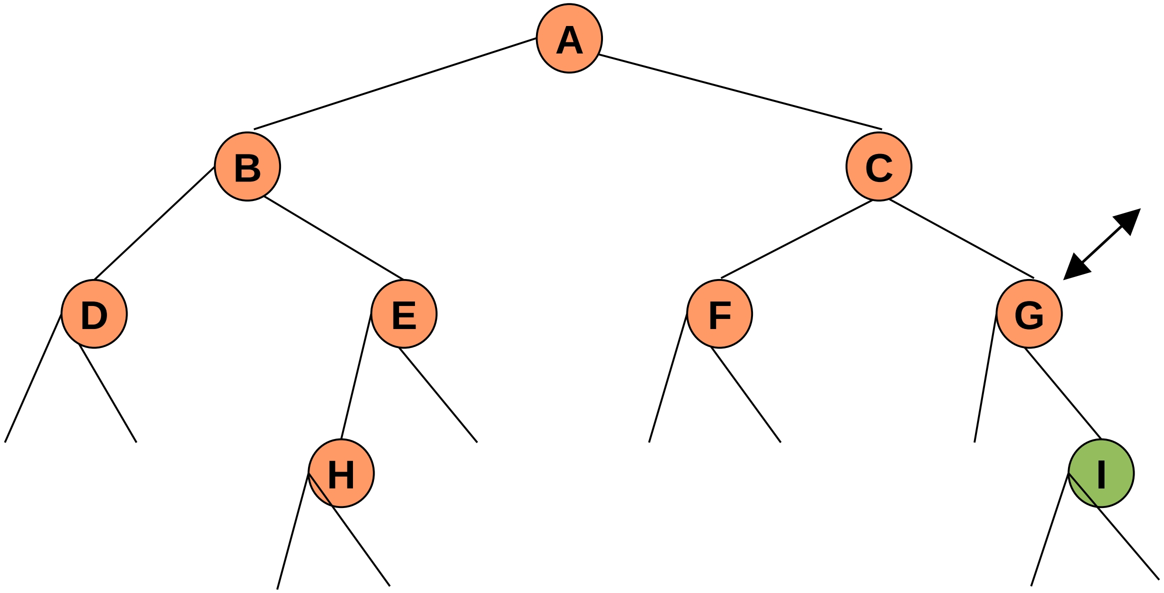
Result: ABDEHC

Traversing a Tree Preorder



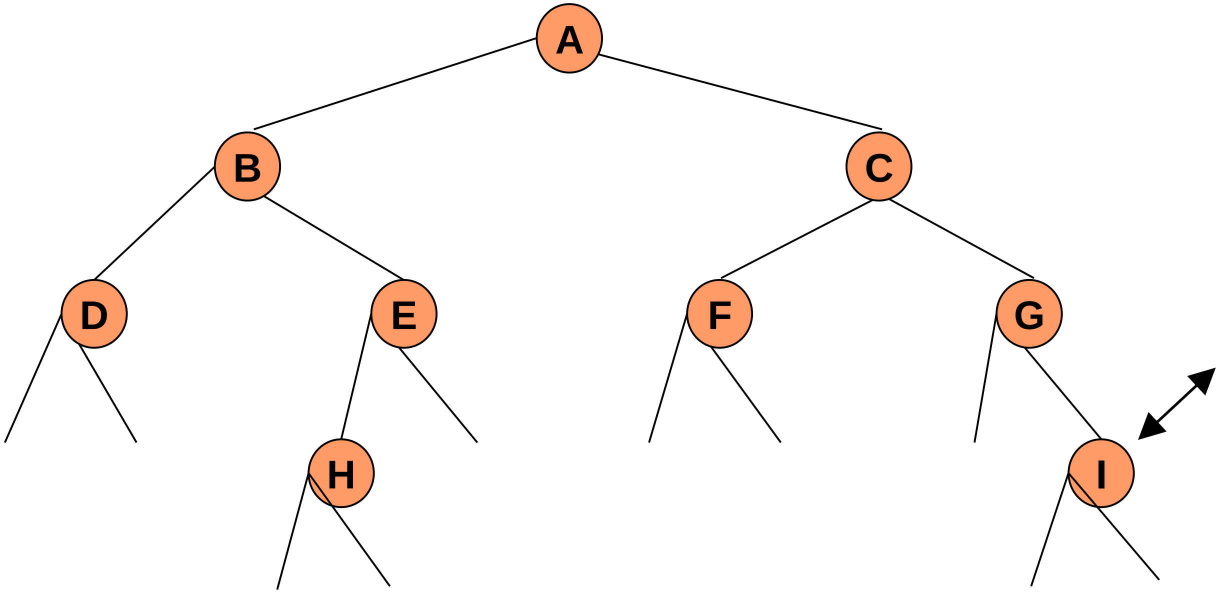
Result: ABDEHCF

Traversing a Tree Preorder



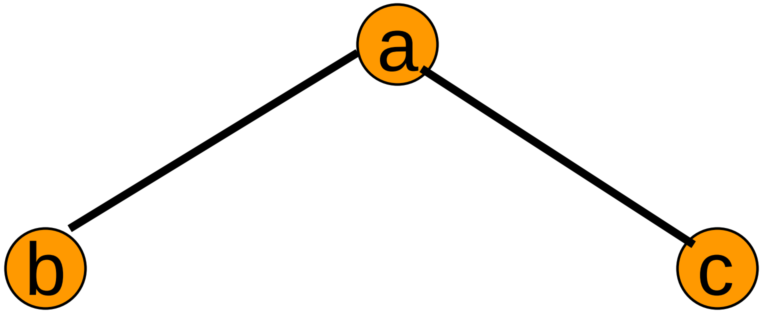
Result: ABDEHCFG

Traversing a Tree Preorder



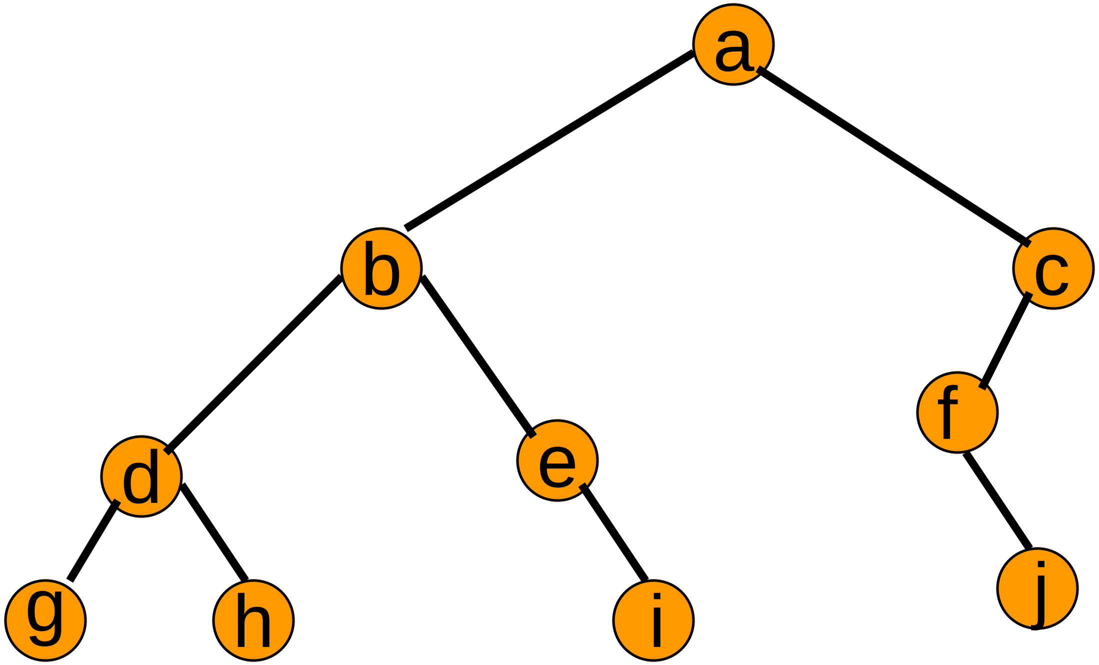
Result: ABDEHCFGI

Preorder Example (Visit = print)



a b c

Preorder Example (Visit = print)



a b d g h e i c f j

Preorder Traversal

```
Void preOrder(BinaryTreeNode
  t)
{
     if (t != null)
     {
        visit(t);
        preOrder(t.leftChild);

  preOrder(t.rightChild);
     }
}
```

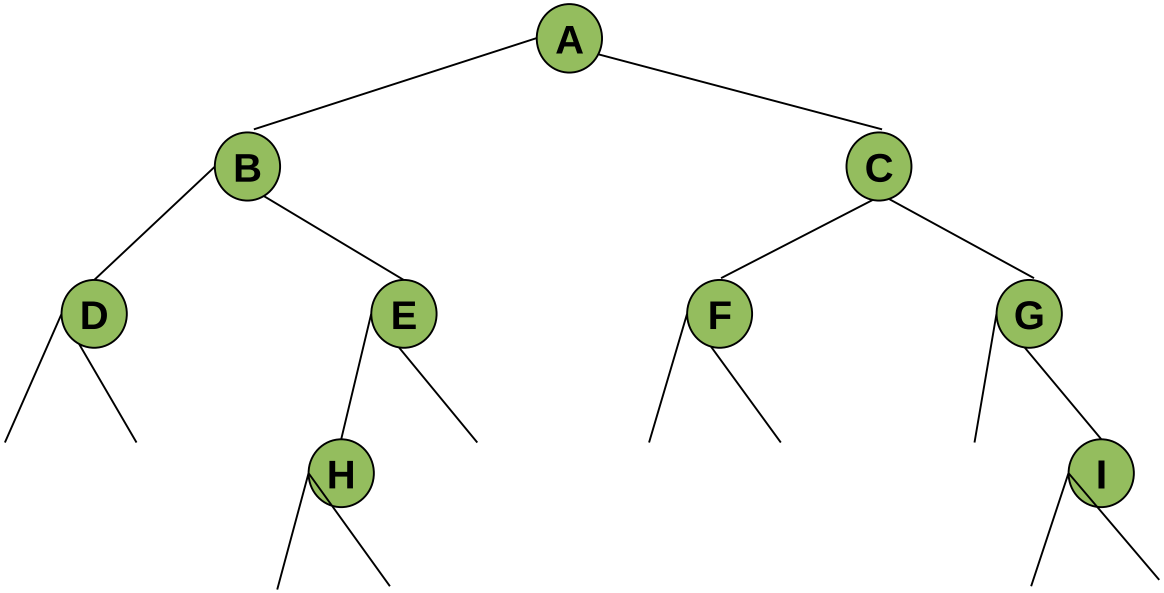
Inorder Traversal

- Visit the nodes in the left subtree,
  then visit the root of the tree,
  then visit the nodes in the right subtree

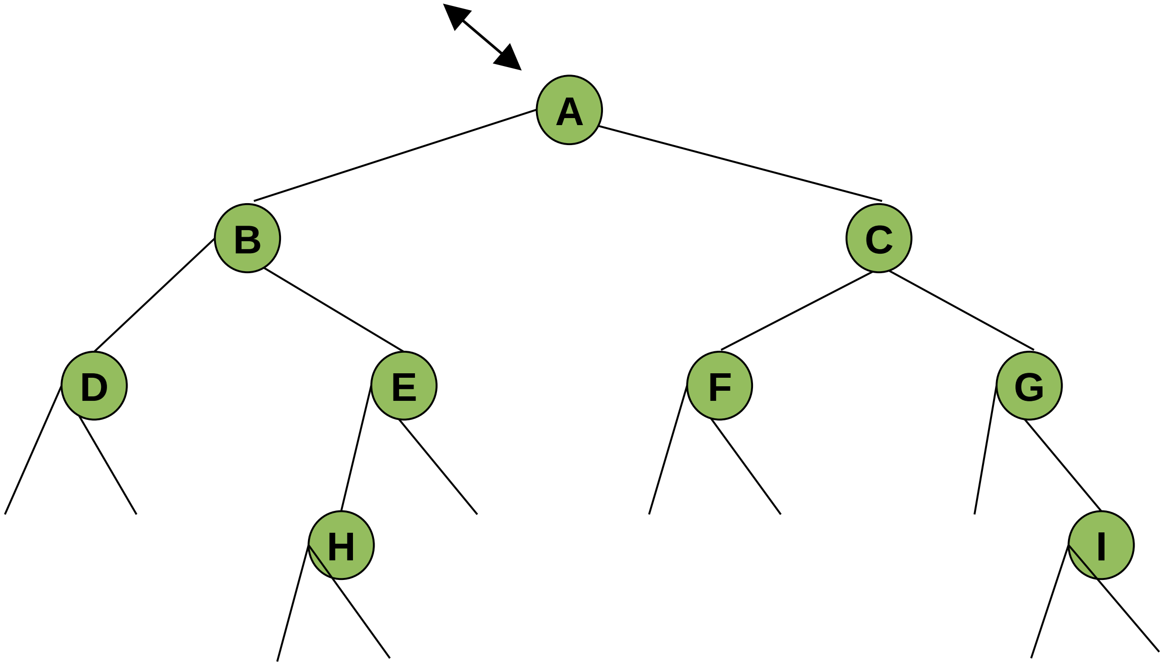
```
Inorder(tree)

If tree is not NULL
  Inorder(Left(tree))
  Visit Info(tree)
  Inorder(Right(tree))
```

(Warning: "visit" means that the algorithm does something with the values in the node, e.g., print the value)

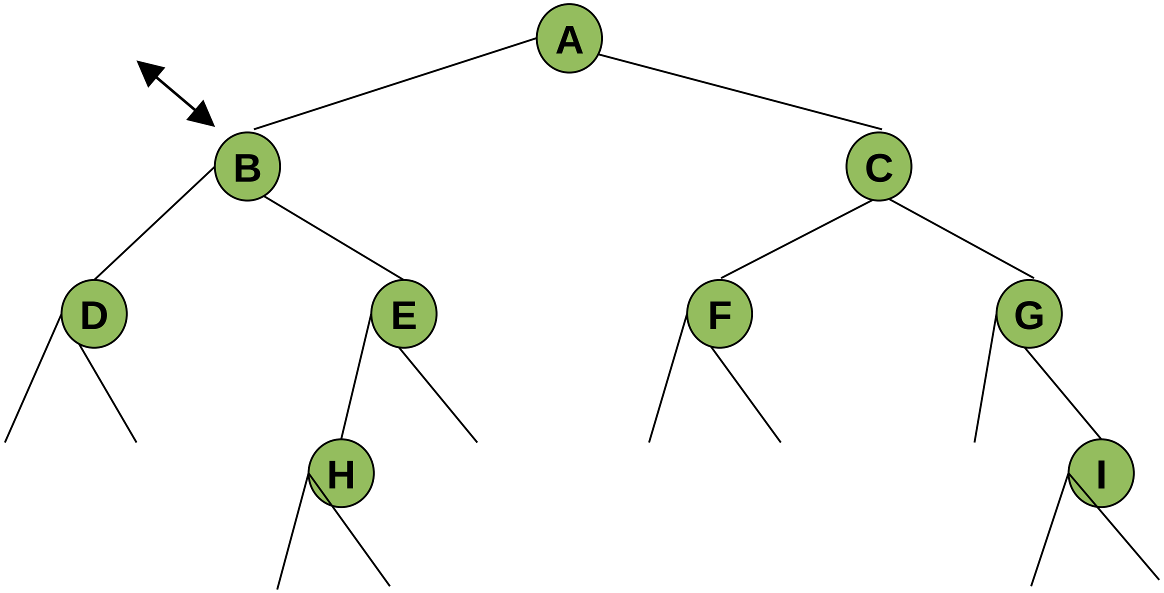
Traversing a Tree Inorder



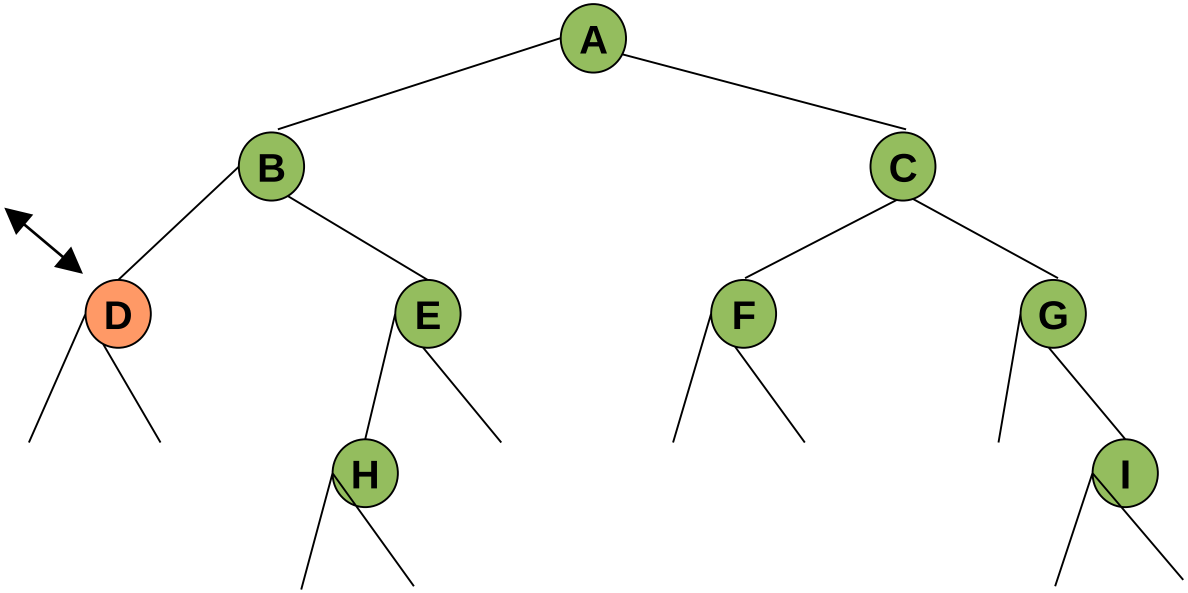
Traversing a Tree Inorder



Traversing a Tree Inorder

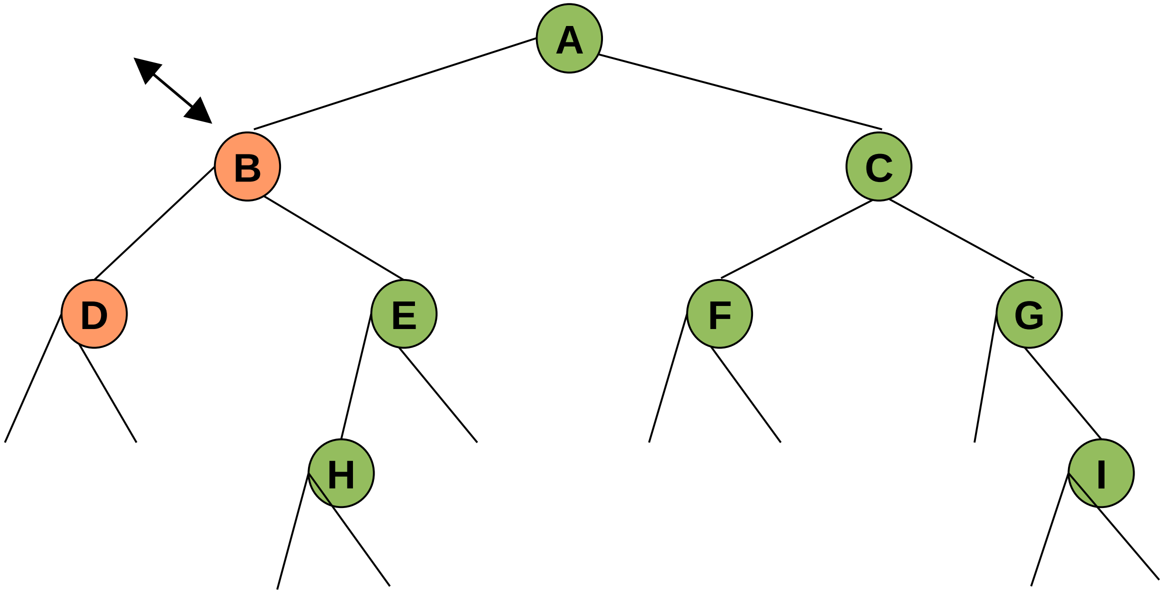


Traversing a Tree Inorder



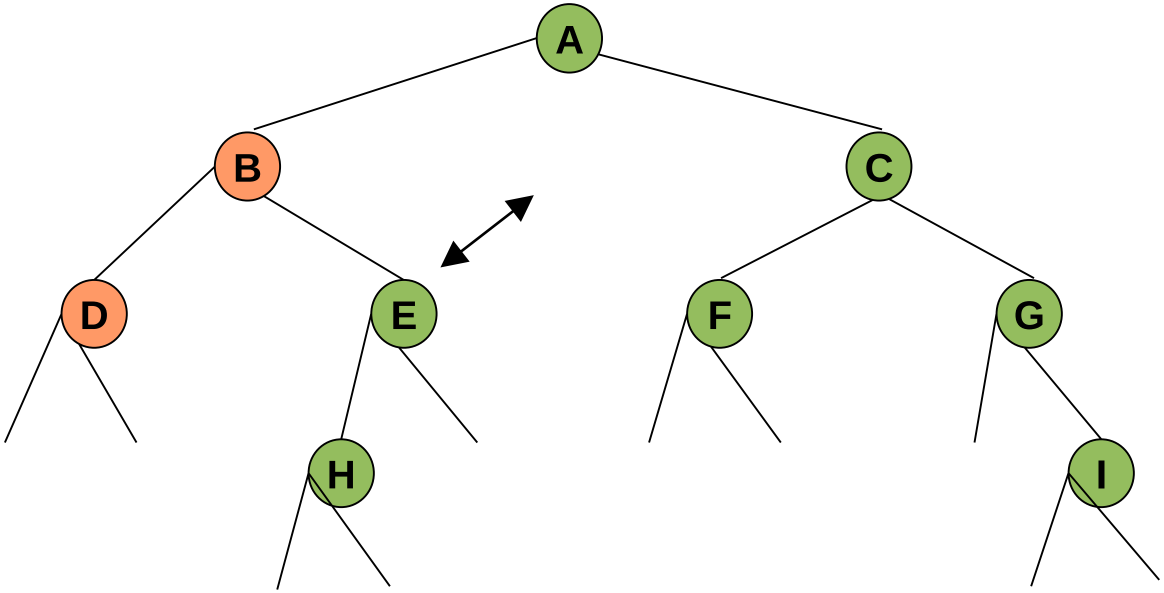
Result: D

Traversing a Tree Inorder



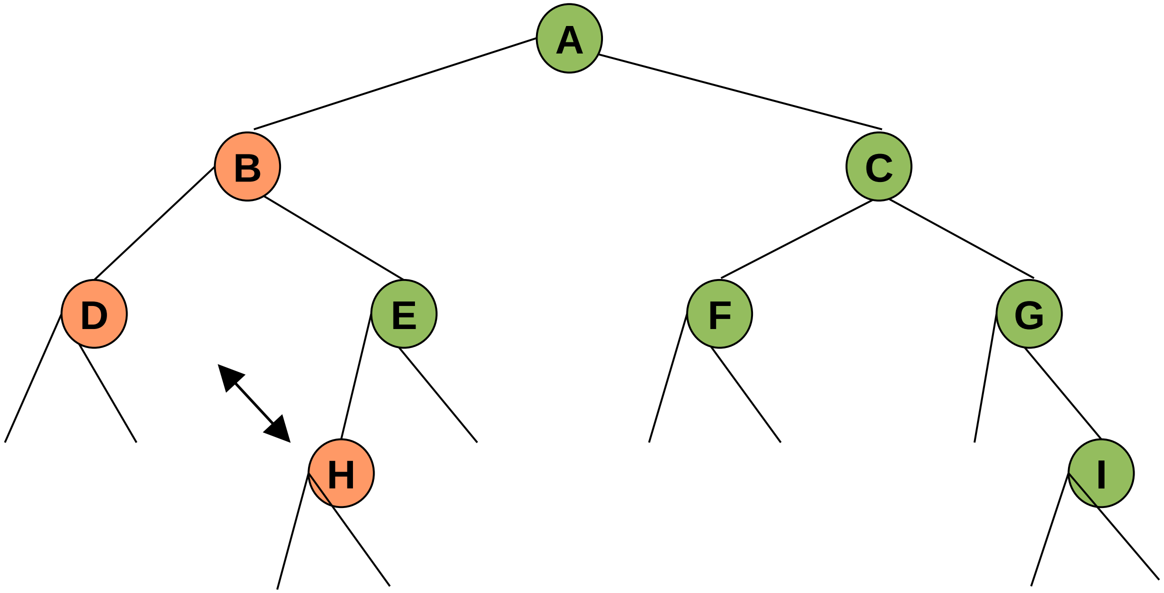
Result: DB

Traversing a Tree Inorder



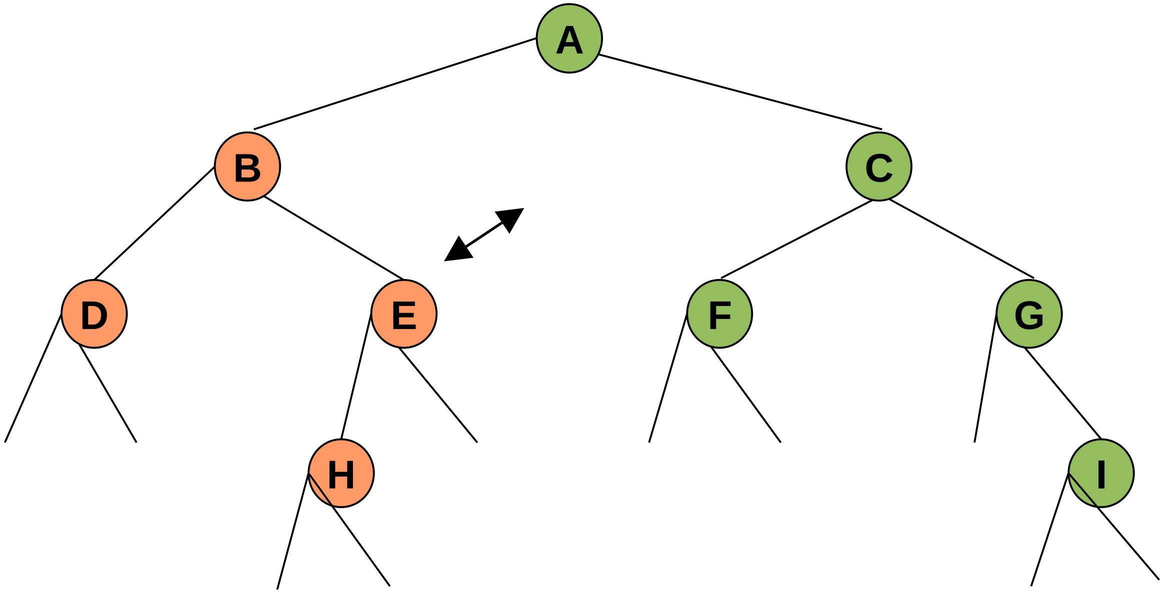
Result: DB

Traversing a Tree Inorder



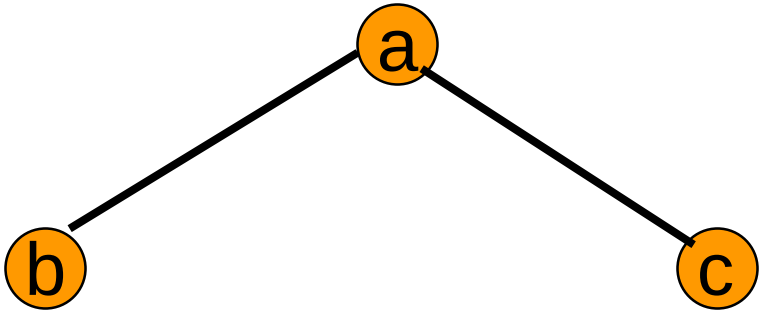
Result: DBH

Traversing a Tree Inorder



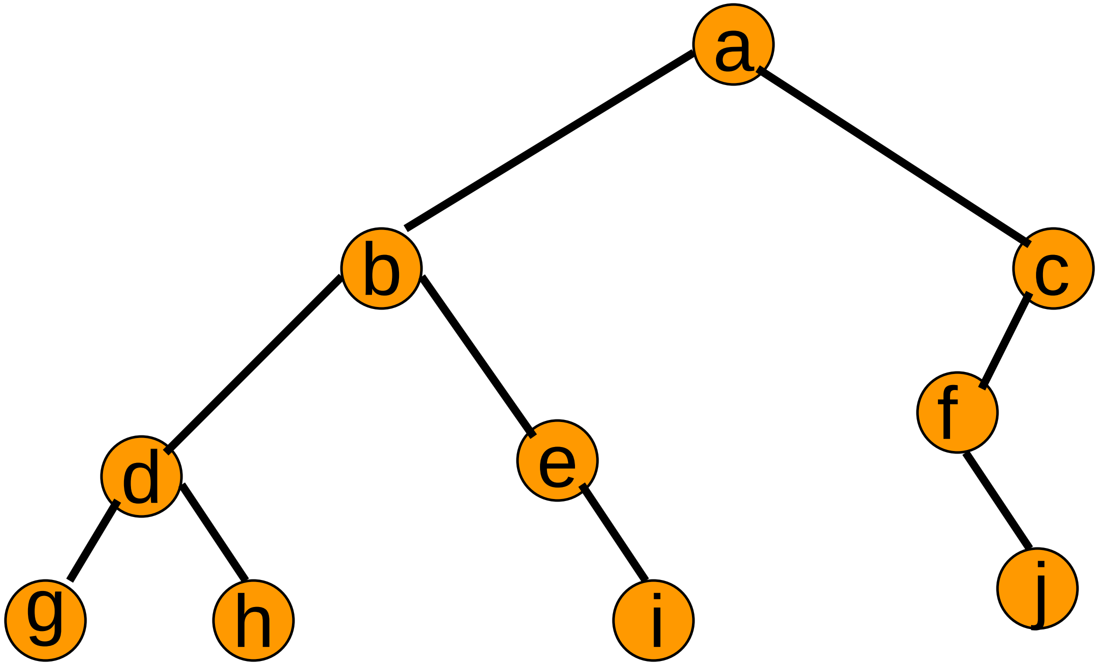
Result: DBHE

Inorder Example (Visit = print)



b a c

Inorder Example (Visit = print)

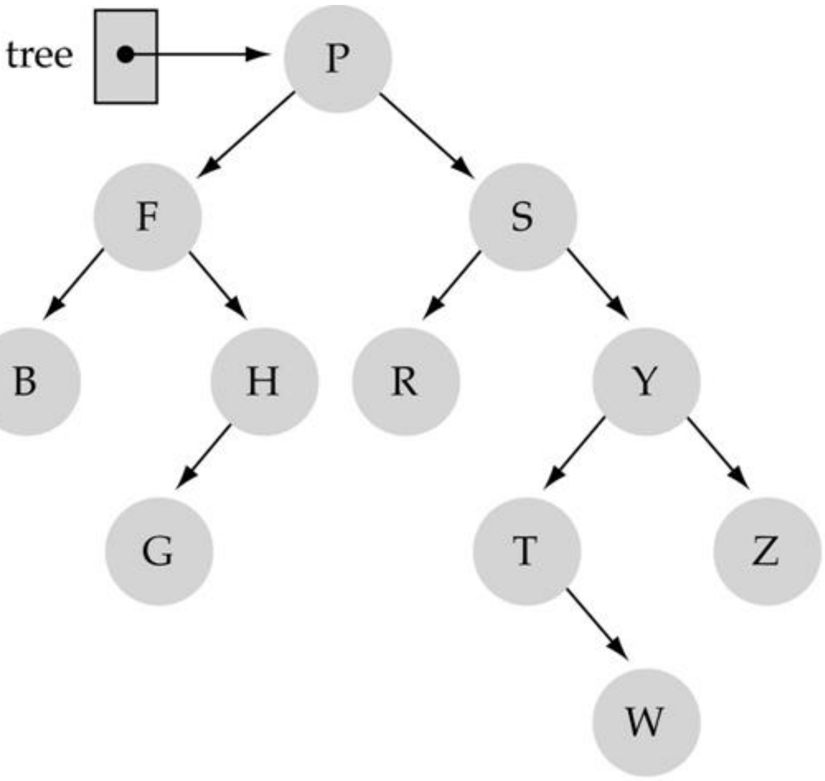


g d h b e i a f j c

Inorder Traversal

```
Void inOrder(BinaryTreeNode t)
{
     if (t != null)
     {
        inOrder(t.leftChild);
        visit(t);
        inOrder(t.rightChild);
     }
}
```

Tree Traversals Example



Inorder: B F G H P R S T W Y Z
Preorder: P F B H G S R Y T W Z
Postorder: B G H F R W T Z Y S P

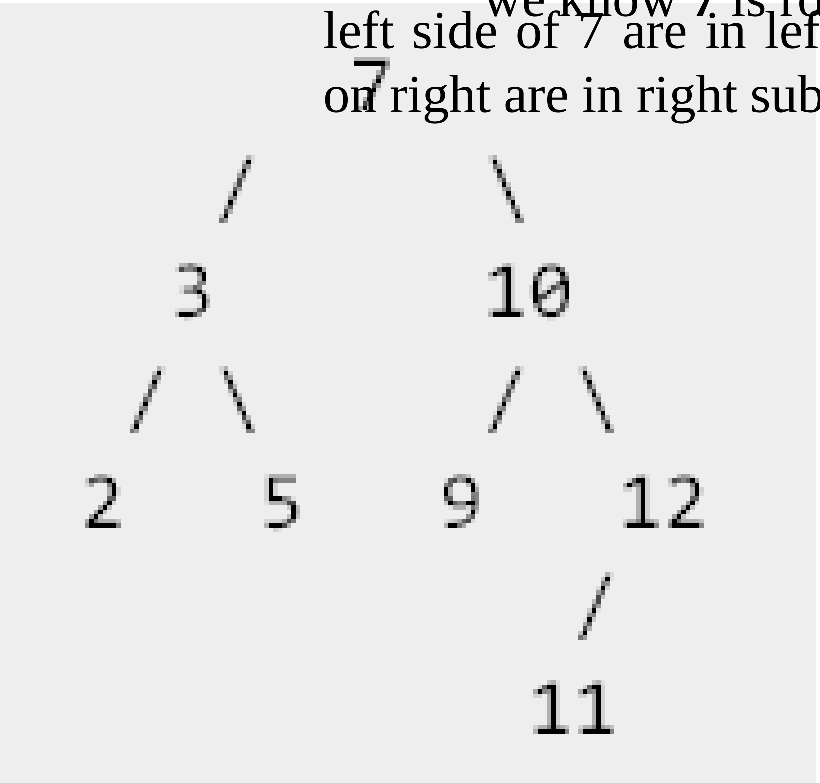
Construct Tree from given Inorder and Postorder traversals

Inorder is: **2 3 5 7 9 10 11 12**

Postorder is: **2 5 3 9 11 12 10 7**

In a Postorder sequence, rightmost element is the root of the tree. So we know **7** is root.

Search for **7** in Inorder sequence. Once we know position of 7 (or index of 7) in Inorder sequence, we also know that all elements on left side of 7 are in left subtree and elements on right are in right subtree.



Construct Tree from given Preorder and Postorder traversals

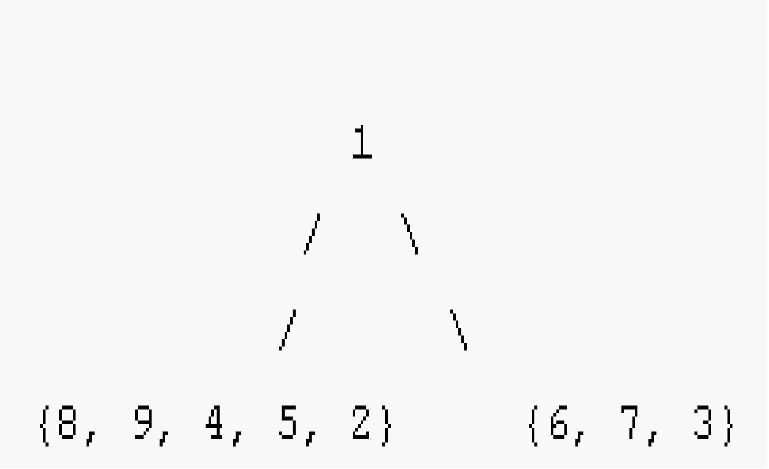
preorder = {1, 2, 4, 8, 9, 5, 3, 6, 7}
postorder = {8, 9, 4, 5, 2, 6, 7, 3, 1};

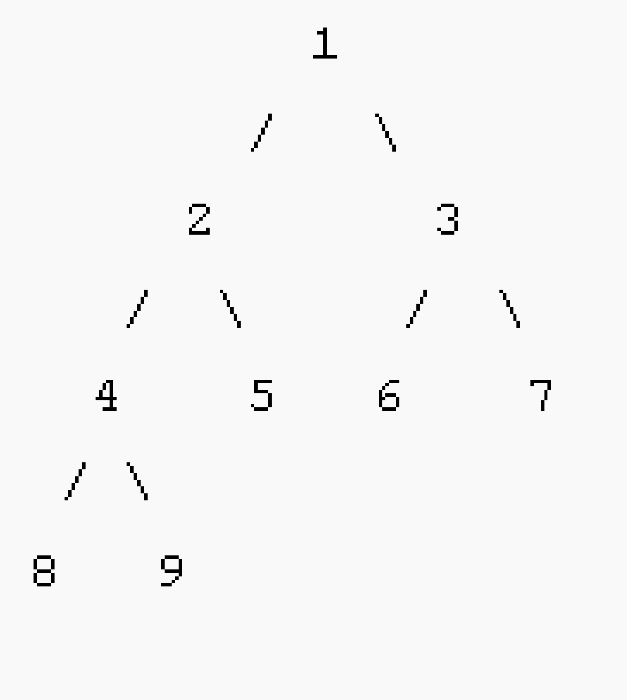
In pre[], the leftmost element is root of tree. Since the tree is full and array size is more than 1. The value next to 1 in pre[], must be left child of root.

So we know 1 is root and 2 is left child.

All nodes before 2 in post[] must be in left subtree

Now we know 1 is root, elements {8, 9, 4, 5, 2} are in left subtree, and the elements {6, 7, 3} are in right subtree.

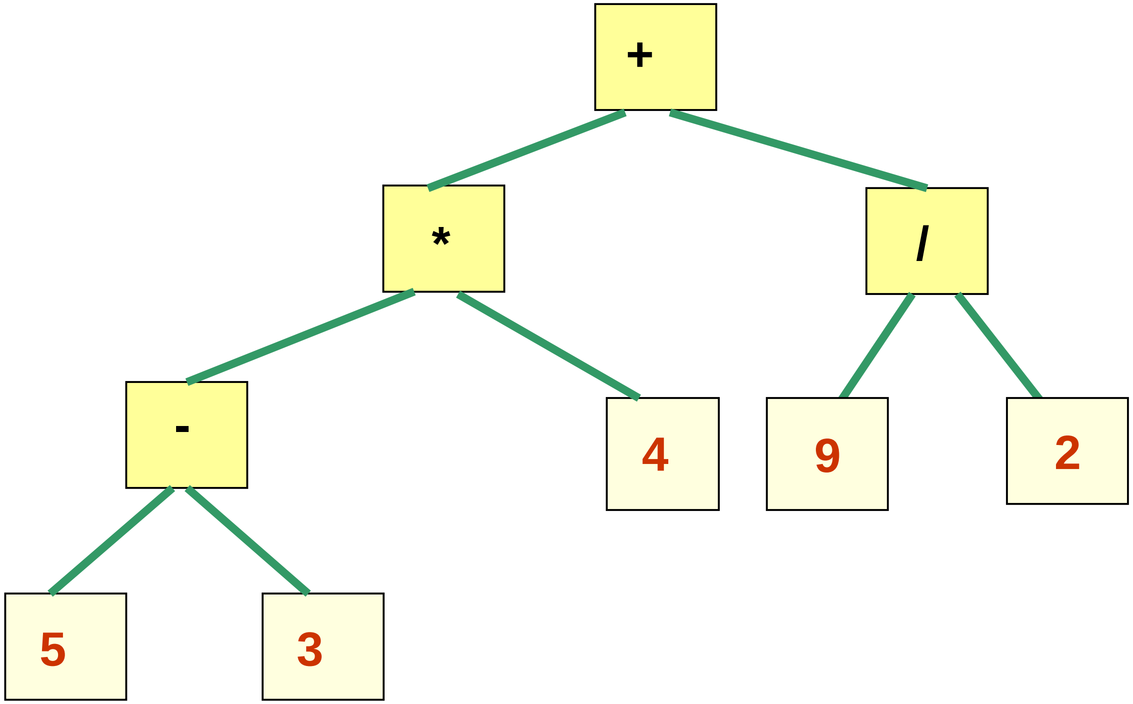




Expression tree

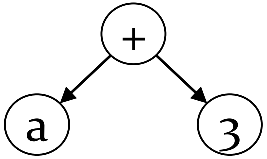
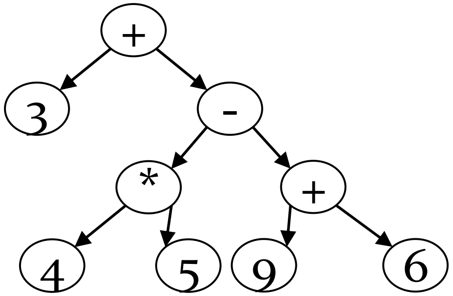
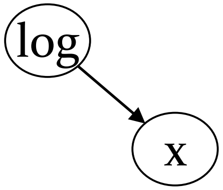
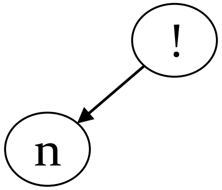
- An expression tree for an arithmetic, relational, or logical expression is a binary tree in which:
  - The parentheses in the expression do not appear.
  - The leaves are the variables or constants in the expression.
  - The non-leaf nodes are the operators in the expression:
    - A node for a binary operator has two non-empty subtrees.
    - A node for a unary operator has one non-empty subtree.
- The operators, constants, and variables are arranged in such a way that an inorder traversal of the tree produces the original expression without parentheses.

Example: An Expression Tree



(5 – 3) * 4 + 9 / 2

Expression Tree Examples

Expression	Expression Tree	Inorder Traversal Result
(a+3)		a + 3
3+(4*5-(9+6))		3+4*5-9+6
log(x)		log x
n!		n !

Why Expression trees?

- Expression trees are used to remove ambiguity in expressions.
- Consider the algebraic expression 2 - 3 * 4 + 5.
- Without the use of precedence rules or parentheses, different orders of evaluation are possible:

((2-3)*(4+5)) = -9

((2-(3*4))+5) = -5

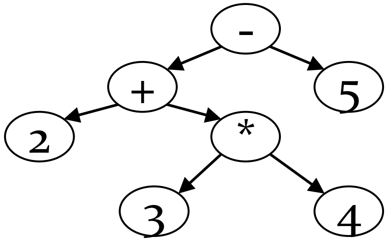
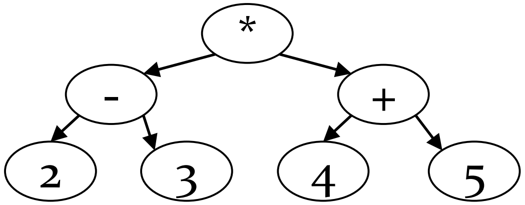
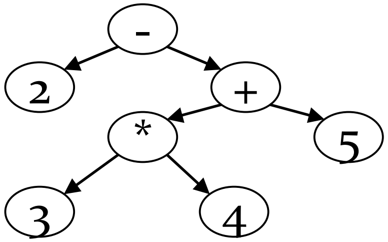
(2-((3*4)+5)) = -15

(((2-3)*4)+5) = 1

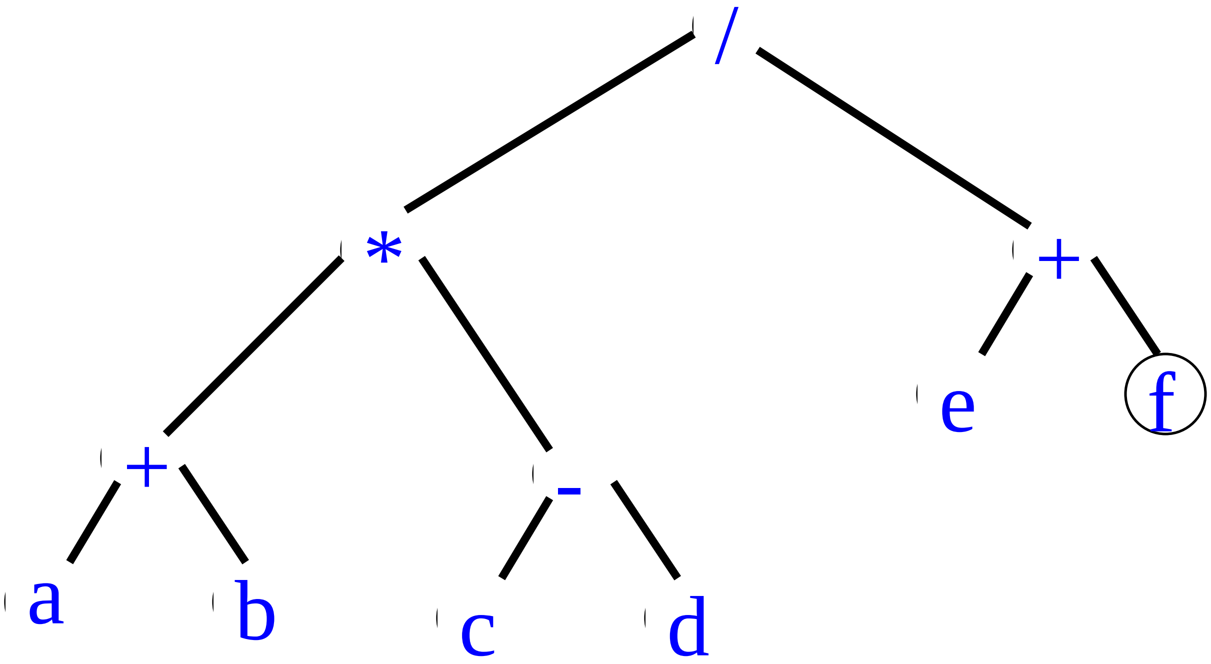
(2-(3*(4+5))) = -25

- The expression is ambiguous because it uses infix notation: each operator is placed between its operands.

- Expression trees can be very useful for:
    - Evaluation of the expression.
    - Generating correct compiler code to actually compute the expression's value at execution time.
    - Performing symbolic mathematical operations (such as differentiation) on the expression.

Expression	Expression Tree	Infix form
2 - 3 * 4 + 5		2 - 3 * 4 + 5
(2 - 3) * (4 + 5)		2 - 3 * 4 + 5
2 - (3 * 4 + 5)		2 - 3 * 4 + 5

Constructing Expression Tree Using Postfix Expression



a b + c d - * e f +   /