

$$1) \begin{cases} g_1 = f_1 + h_2 * f_2 \\ g_2 = h_1 * f_1 + f_2 \end{cases} \quad \left. \begin{array}{l} \text{Take Fourier transforms on both sides} \\ \text{note } (a * b \rightarrow A \cdot B) \end{array} \right\}$$

$$\therefore \begin{cases} G_1(M) = F_1(M) + H_2(M) \cdot F_2(M) \\ G_2(M) = H_1(M) F_1(M) + F_2(M) \end{cases} \quad \left. \begin{array}{l} \text{Solve for } F_1(M) \\ \text{ \& } F_2(M) \end{array} \right\}$$

$$\Rightarrow \begin{cases} F_1(M) = \frac{G_1(M) - H_2(M) G_2(M)}{1 - H_1(M) H_2(M)} \\ F_2(M) = \frac{G_2(M) - H_1(M) G_1(M)}{1 - H_1(M) H_2(M)} \end{cases} \quad \left. \begin{array}{l} \text{Here } F_1, F_2, G_1, G_2, \\ H_1, H_2 \text{ are the} \\ \text{fourier transforms} \\ \text{of } f_1, f_2, g_1, g_2, h_1, h_2. \end{array} \right\}$$

\therefore Now knowing g_1, g_2, h_1, h_2 , we can find out F_1 & F_2 & then by taking their IFT, we get f_1 & f_2 .

The inherent problem here is that $F_1(M)$ & $F_2(M)$ are undefined when $H_1(M) \cdot H_2(M) = 1$ as the denominator becomes zero.

Also, h_1 & $h_2 \rightarrow$ low pass filter kernels & $\int_{-\infty}^{\infty} h_i(n) dn = 1$ for $i=1,2$.

$$\therefore H_1(0) = \int_{-\infty}^{\infty} h_1(n) e^{-j0 \cdot n} dn = \int_{-\infty}^{\infty} h_1(n) dn = 1$$

$$\text{Similarly } H_2(0) = 1$$

\therefore At $M=0$, $H_1(0) = H_2(0) = 1$ & $F_1(M)$ & $F_2(M)$ become undefined.

\therefore At lower frequencies when $H_1(M) H_2(M) = 1$, reconstruction is not possible, while at higher frequencies ~~is~~ it is possible as $H_1(M)$ & $H_2(M) \downarrow$ on $\uparrow M$ as they are ~~high~~ ^{low} pass filters and so $H_1(M) H_2(M) \neq 1$ any longer & the solution exists.