2) $g = h * f$, $g \to$ gradient image, $h \to$ kernel (1D)

$\therefore$ We can have 'h' as $[-1 \;\; 1]$ for 'g' to be gradient of f.

$\therefore \; g = f(x+1) - f(x), \quad x \in [1, N]$

$f(x) \xrightarrow{DFT} F(u)$, then $f(x+1) \xrightarrow{DFT} F(u).e^{j\frac{2\pi u}{N}}$

$\therefore \; G(u) = F(u).\left(e^{j\frac{2\pi u}{N}} - 1\right) \Rightarrow \boxed{F(u) = \dfrac{G(u)}{\left(e^{j\frac{2\pi u}{N}} - 1\right)}}$

$\Rightarrow$ Again, at $u = 0$, denominator $= 0$ & $F(u)$ is undefined.

$\therefore$ We can't get back the DC value of the image's pixels and we need to assume ~~the~~ it.

This is like assuming boundary conditions for the gradients, ie. if f has N elements, $f(N+1) = 0$ we assume.

For a 2D image, gradient in direction :-

$x \Rightarrow \hat{F}_x(u,v) = \left(e^{j\frac{2\pi u}{N}} - 1\right) F(u,v)$

$y \Rightarrow \hat{F}_y(u,v) = \left(e^{j\frac{2\pi v}{N}} - 1\right) F(u,v)$

$\therefore$ If we know $\hat{F}_x(u,v)$ only, $F(u,v)$ can be found at all points except at $u = 0$.

Similarly with $\hat{F}_y(u,v)$, $F(u,v)$ can be found except at $v = 0$

If we know both, $\hat{F}_x(u,v)$ & $\hat{F}_y(u,v)$, we still have a problem to recover $F(u,v)$ at $u = v = 0$ which is the DC component of the 2D image.

Hence we again need to make assumptions for the DC component.

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