Code : 100404

B.Tech 4th Semester Exam., 2022

(New Course)

DISCRETE MATHEMATICS

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

*(ii) There are **NINE** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question No. **1** is compulsory.*

1. Choose the correct answer of the following (any *seven*) : 2×7=14

(a) The statement $(\sim P \leftrightarrow Q) \wedge \sim Q$ is true, when

(i) P : True, Q : False

(ii) P : True, Q : True

(iii) P : False, Q : True

(iv) P : False, Q : False

(b) Which of the following statements regarding sets is false?

(i) $A \cap A = A$

(ii) $A \cup A = A$

(iii) $A - (B \cap C) = (A - B) \cup (A - C)$

(iv) $(A \cup B)' = A' \cup B'$

(c) What is the induction hypothesis assumption for the inequality $m! > 2^m$ where $m \geq 4$?

(i) For $m = k$, $k+1! > 2^k$ holds

(ii) For $m = k$, $k! > 2^k$ holds

(iii) For $m = k$, $k! > 3^k$ holds

(iv) For $m = k$, $k! > 2^{k+1}$ holds

(d) A simple graph can have

(i) multiple edges

(ii) self-loops

(iii) parallel edges

(iv) no multiple edges, self-loops and parallel edges

(e) An undirected graph has 8 vertices labelled $1, 2, \cdots, 8$ and 31 edges. Vertices 1, 3, 5, 7 have degree 8 and vertices 2, 4, 6, 8 have degree 7. What is the degree of vertex 8?

(i) 15

(ii) 8

(iii) 5

(iv) 23

(f) A graph which has the same number of edges as its complement must have number of vertices congruent to _____ or _____ modulo 4 (for integral values of number of edges).

(i) $6k$, $6k-1$

(ii) $4k$, $4k+1$

(iii) k, $k+2$

(iv) $2k+1$, k

(g) A graph which consists of disjoint union of trees is called

(i) bipartite graph

(ii) forest

(iii) caterpillar tree

(iv) labelled tree

(h) Which of the following two sets are equal?

(i) $A = \{1, 2\}$ and $B = \{1\}$

(ii) $A = \{1, 2\}$ and $B = \{1, 2, 3\}$

(iii) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

(iv) $A = \{1, 2, 4\}$ and $B = \{1, 2, 3\}$

(i) If C_n is the nth cyclic graph, where $n > 3$ and n is odd, determine the value of $X(C_n)$

(i) 32572

(ii) 16631

(iii) 3

(iv) 310

(j) The number of edges in a regular graph of degree 46 and 8 vertices is

(i) 347

(ii) 230

(iii) 184

(iv) 186

2. *(a)* Let

$D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$

Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false. 6

(i) $\forall x \in D$, if x is odd, then $x > 0$

(ii) $\forall x \in D$, if x is less than 0, then x is even

(iii) $\forall x \in D$, if x is even, then $x \leq 0$

(b) Indicate which of the following statements are true and which are false. Justify your answers as best you can : 8

(i) $\forall x \in \mathbf{Z}^+, \exists y \in \mathbf{Z}^+$ such that $x = y + 1$

(ii) $\forall x \in \mathbf{Z}, \exists y \in \mathbf{Z}$ such that $x = y + 1$

(iii) $\exists x \in \mathbf{R}$ such that $\forall y \in \mathbf{R}, x = y + 1$

(iv) $\forall x \in \mathbf{R}^+, \exists y \in \mathbf{R}^+$ such that $xy = 1$

(v) $\forall x \in \mathbf{R}, \exists y \in \mathbf{R}$ such that $xy = 1$

(vi) $\forall x \in \mathbf{Z}^+$ and $\forall y \in \mathbf{Z}^+, \exists z \in \mathbf{Z}^+$ such that $z = x - y$

(vii) $\forall x \in \mathbf{Z}$ and $\forall y \in \mathbf{Z}, \exists z \in \mathbf{Z}$ such that $z = x - y$

(viii) $\exists u \in \mathbf{R}^+$ such that $\forall v \in \mathbf{R}^+, uv < v$

3. *(a)* Prove the following : 10

(i) There is a real number x such that $x > 1$ and $2^x > x^{10}$.

(ii) There is an integer n such that $2n^2 - 5n + 2$ is prime.

(b) Disprove that for all real numbers a and b, if $a < b$, then $a^2 < b^2$. 4

4. *(a)* Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. This is the distributive law of disjunction over conjunction. 5

(b) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm. 4

(c) If a and b are positive integers, then prove that there exists integers s and t such that $\gcd(a, b) = sa + tb$. 5

5. *(a)* Use mathematical induction to prove this formula for the sum of a finite number of terms of a geometric progression with initial term a and common ratio r

$$\sum_{j=0}^{n} ar^j = a + ar + ar^2 + \cdots + ar^n = \frac{ar^{n+1} - a}{r - 1}$$

when $r \neq 1$

where n is a non-negative integer. 7

(b) Write short notes on the following : 2+3+2=7

(i) Forward proof

(ii) Disjunctive and conjunctive normal form

(iii) Fundamental theorem of arithmetic

6. *(a)* An odd number of people stand in a yard at mutually distinct distances. At the same time each person throws a pie at their nearest neighbour, hitting this person. Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a pie. 7

(b) Prove Bernoulli's inequality that if $h > -1$, then $1 + nh \leq (1 + h)^n$ for all non-negative integers n. 7

7. What is pigeonhole principle? Using it, prove the following : 7+7=14

(a) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

(b) The sequence 8, 11, 9, 1, 4, 6, 12, 10, 5, 7 contains 10 terms. Note that $10 = 3^2 + 1$. There are four strictly increasing subsequences of length four, namely, 1, 4, 6, 12; 1, 4, 6, 7; 1, 4, 6, 10; and 1, 4, 5, 7. There is also a strictly decreasing subsequence of length four, namely, 11, 9, 6, 5.

8. *(a)* In a Round-Robin tournament, the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers beat the Orioles, the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles and the Cardinals beat the Orioles. Model this outcome with a directed graph. 3

(b) Let $G = (V, E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that $u = v$. Show that R is an equivalence relation. 3

(c) Determine whether the following given pair of directed graphs, shown in Fig. 1 and Fig. 2, are isomorphic or not. Exhibit an isomorphism or provide a rigorous argument that none exists. 4+4=8

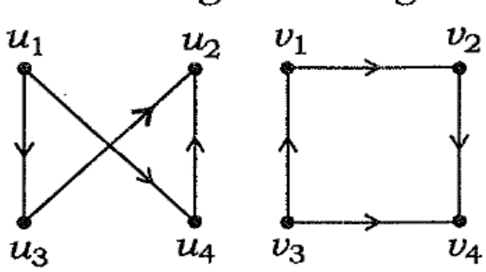

Fig. 1

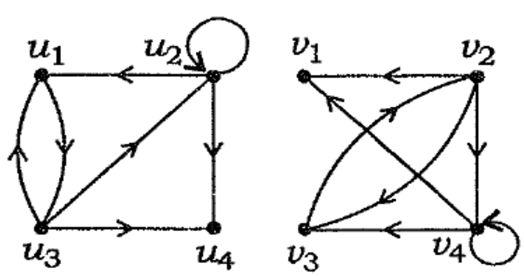

Fig. 2

9. *(a)* Use pseudocode to describe an algorithm for determining the value of a game tree when both players follow a minmax strategy. 4

(b) Suppose that T_1 and T_2 are spanning trees of a simple graph G. Moreover, suppose that e_1 is an edge in T_1 that is not in T_2. Show that there is an edge e_2 in T_2 that is not in T_1 such that T_1 remains a spanning tree if e_1 is removed from it and e_2 is added to it, and T_2 remains a spanning tree if e_2 is removed from it and e_1 is added to it. 5

(c) Show that a degree-constrained spanning tree of a simple graph in which each vertex has degree not exceeding 2 consists of a single Hamiltonian path in the graph. 5

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Bihar Engineering University, Patna

B.Tech 8th Semester Examination, 2024(S)

Course: B.Tech		***Time: 03 Hours***
Code: 100404	***Subject: Discrete Mathematics***	***Full Marks: 70***

Instructions:-

(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
*(iv) Question No. **1** is compulsory.*

Q.1 Choose the correct option / answer the following (Any seven question only): **[2 x 7 = 14]**

(a) The sequence represented by the function $\frac{1}{1-5x}$ is
(i) $\{3^n\}$ (iii) $\{4^n\}$
(ii) $\{5^n\}$ (iv) $\{5^n + 1\}$

(b) A simple graph have
(i) Multiple edges (iii) Self-loops
(ii) Parallel edges (iv) No multiple edges, self-loops and parallel edges

(c) Which of the following statements regarding sets is false?
(i) $A \cap A = A$ (iii) $A \cup A = A$
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$ (iv) $(A \cup B)' = A' \cup B'$

(d) A graph consisting of simply one circuit, with n ≥ 3 and n being odd, is
(i) 2-chromatic (iii) 3-chromatic
(ii) 4-chromatic (iv) none of these

(e) The number of edges in a bipartite graph with n vertices is at most?
(i) $n^2/2$ (iii) $n^2/4$
(ii) n^2 (iv) $2n$

(f) If p: 'Anil is rich', and q: 'Kanchan is poor', then the symbolic form of the statement 'Either Anil or Kanchan is rich' is
(i) $p \vee q$ (iii) $p \vee \neg q$
(ii) $\neg p \vee q$ (iv) $\neg(p \wedge q)$

(g) The total number of subgroups of group G of prime order is
(i) 1 (iii) 3
(ii) 2 (iv) 4

(h) If a set *A* have *n* elements, then how many relations will be there on set *A*
(i) n^2 (iii) 2^n
(ii) $2n$ (iv) 2^{n^2}

(i) A disjunctive normal form of $P \rightarrow Q$ is
(i) $\neg P \vee Q$ (iii) $P \vee \neg Q$
(ii) $(\neg P \wedge Q) \vee (P \wedge \neg Q)$ (iv) $(P \wedge Q) \vee (P \wedge \neg Q)$

(j) Among 200 people, 150 either swim or jog or both. If 85 swim and 60 swim and jog, how many jog?
(i) 125 (iii) 85
(ii) 225 (iv) 25

Q.2 (a) Obtain the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the statement $(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$. **[7]**

(b) What is mathematical induction? Use mathematical induction to prove that $1.1! + 2.2! + \cdots \ldots + n.n! = (n+1)! - 1$, where n is a positive integer. **[7]**

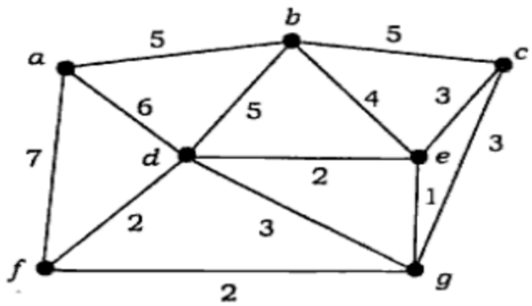
Q.3 (a) If R is a relation on the set of positive integers, such that $(a, b) \epsilon R$ if and only if $a^2 + b$ is even, prove that R is an equivalence relation. **[7]**

(b) If two sets A and B have n elements in commom. Then show that the sets $A \times B$ and $B \times A$ will have 2^nelements in common. **[7]**

Q.4 (a) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. Also, Show that $[(p \wedge q) \rightarrow p] \rightarrow (q \wedge \neg q)$ is a contradiction. **[7]**

(b) Construct the truth table for $[(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)] \rightarrow R$ and also show that above statement is a tautology by developing a series of logical equivalences. **[7]**

Q.5 (a) Explain Dijkstra's algorithm and apply it to the weighted graph $G = (V, E)$ shown in figure below and determine the shortest distance from vertex a to each of the other vertices in the graph. **[7]**



(b) Show that every group of order 3 is cyclic. **[7]**

Q.6 (a) Solve the recurrence relation: $a_r - 7\,a_{r-1} + 10\,a_{r-2} = 0$ given $a_0 = 0$ and $a_1 = 6$. **[7]**

(b) State Pigeon hole principle. Using the principle, prove that if any five numbers from 1 to 8 are selected, then two of them will add to 9. **[7]**

Q.7 (a) Show that the order of a subgroup of a finite group is a divisor of the order of the group. **[7]**

(b) Prove that for any non-empty binary tree T, if n_0 is the number of leaves and n_2 be the number of nodes having degree 2, then $n_0 = n_2 + 1$. **[7]**

Q.8 (a) Define 'group', 'order of a group', 'cyclic group' and 'Abelian group'. **[7]**

(b) Prove that the maximum number of edges possible in a simple graph of n nodes is $n(n-1)/2$. **[7]**

Q.9 Write short notes on ***any two*** of the following:- **[2 x 7 = 14]**

(a) Principle of Duality

(b) DFS and BFS Traversal

(c) GCD: Euclidean Algorithm

(d) Eulerian and Hamiltonian Walks

Bihar Engineering University, Patna
End Semester Examination – 2023
Semester-IV

Course: B.Tech. | ***Time: 03 Hours***

Code: 100404 | ***Subject: Discrete Mathematics*** | ***Full Marks: 70***

Instructions:-

(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
*(iii) Attempt **FIVE** questions in all.*
*(iv) Question No. **1** is compulsory.*

Q.1 ***Write the answer of the following (Any seven question only):*** **[2 x 7 = 14]**

a) Let A be the set odd positive integers less than 10. Then cardinality of A, $|A|$ is
(i) 5
(ii) 9
(iii)6
(iv) 4

b) If m is the number of objects (pigeons) and n is the number of boxes (pigeonholes), then the function is both one – to – one and onto if
(i) m<n (ii) m=n (iii) m>n (iv) none of these

c) If A x B = B x A, (Where A and B are general matrices) then
(i) A = d (ii) A = B' (iii) B = A, (iv) A' = B

d) A partial ordered relation is transitive, reflexive and
(i) Anti symmetric (ii) bi symmetric (iii) anti reflexive (iv) asymmetric

e) If B is a Boolean Algebra, then which of the following is true
i. B is a finite but complemented lattice
ii. B is a finite, complemented and distributive lattice
iii. B is a finite, Distributive but not complemented lattice
iv. B is not distributive lattice

f) P →q is logically equivalent to
a) ~q →p b) ~P →q
c) ~P ∧ q d) ~p ∨ q

g) if $f(x) = cosx$ and $g(x) = x^3$ then $(f_o g)\ (x)$ is
(i) $(cosx)^3$ (ii) $cos3x$ (iii) $x^{(cosx)^3}$ (iv) $cosx^3$

h) The number of distinguishable permutations of the letters in the word BANANA are
(i) 60 (ii) 36, (iii) 20, (iv) 10

i) Which of the following pair is not congruent modulo 7?
(i) 10, 24
(ii) 25, 56
(iii) -31, -15
(iv) -64, -15

j) Let N = { 1,2,3,,.......} be ordered by divisibility, which of the following subset is totally ordered
(i) (2, 6, 24) (ii) (3, 5, 15) (iii) (2, 9, 16), (iv) (4, 15, 30)

Q2. a.) Let A = B = $\{\frac{x}{-}1 \leq x \leq 1\}$ for each of the following functions state where it is injective, surjective or bijective **[7]**
i) $g(x) = \sin \pi x$
ii) $b(x) = \frac{2x}{3}$

(b) Let f(x) = x+2 , g(x) = x-2 , h (x) = 3x find (i) fog (ii) fogoh **[7]**

Q3. a.) find the power set of each of these sets **[7]**
i) { a, b} ii) { φ, {φ}}

b.) Use Cantor's diagonal argument to prove that set F of all functions f: (0,1) → R has larger Cardinality than $|R|$ **[7]**

Q4. Determine if the sets are countable or uncountable **[14]**
a.) the set A of all function $g: Z_+ \to Z_+$
b.) The set B of all functions $f: Z_+ \to \{0,1\}$

Q5. Prove the following by using the principle of mathematical induction for all n € N: **[14]**

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

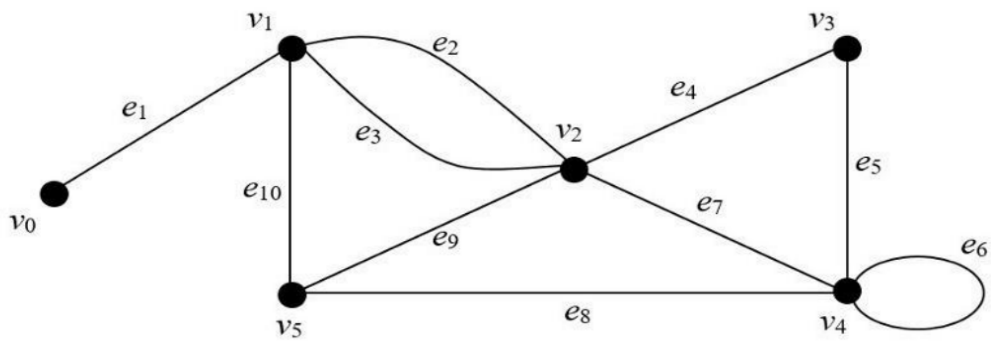
Q6. State and prove Division algorithm theorem well-ordering principle. **[14]**

Q7. (a) Check the validity of the following argument all integers are rational numbers. Some integers are powers of 5. Therefore, some rational numbers are powers of 5 **[7]**

(b) A grocery store employee is stocking apples. Each apple is a different color. There are 10 apples left in the box and the employee pulls out 2 of them at random. What is the probability that the employee pulls out one pink apple and yellow apple? **[7]**

Q8. Let $\Psi : G \rightarrow H$ be a homomorphism of groups. Show that if $a \in G$ has order n, then $\Psi(a) \in H$ has order dividing n. **[14]**

Q9. Consider the following graph



(a) Does a Hamiltonian path exist? If so describe it. If not say why not. **[7]**

(b) Does an Eulerian path exist? If so describe it. If not say why not. **[7]**

Bihar Engineering University, Patna

B. Tech 4th Semester Examination, 2024

Course: B.Tech | ***Time: 03 Hours***
Code: 100404 | ***Subject: Discrete Mathematics*** | ***Full Marks: 70***

Instructions:-

(i) The marks are indicated in the right-hand margin.
*(ii) There are **NINE** questions in this paper.*
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*(iv) Question No. **1** is compulsory.*

Q.1 Choose the correct option / answer the following (Any seven question only):- **[2 x 7 = 14]**

(a) The function $f: R \to R$ defined by $f(x) = x^3 + 5$
(i) One-One but not onto (iii) Both One-One and Onto
(ii) Onto but not One-One (iv) None of these

(b) Consider the following relation R on a set $A = \{1, 2, 3, 4\}$:
$R = \{(1,1), (1,3), (3,2), (2,2), (3,3), (3,1), (2,3), (1,4), (4, 1)\}$ Then
(i) Reflexive (iii) Transitive
(ii) Symmetric (iv) Equivalence

(c) Out of the following which of these integers is not prime?
(i) 23 (iii) 21
(ii) 59 (iv) 71

(d) The Greatest common divisor of 0 and 11
(i) 0 (iii) 11
(ii) 1 (iv) Does not exist

(e) Suppose G is a group with a binary operation '*' defined by $a * b = a + b + 1, \forall\, a, b \in G$ then
(i) Identity element = -1 (iii) Identity element = 1
(ii) Identity element = -2 (iv) Identity element = 2

(f) The field which contains at least _______ element/elements
(i) 0 (iii) 2
(ii) 1 (iv) 3

(g) The set of integers with respect to addition is
(i) Abelian group (iii) Both (i) and (ii)
(ii) Cyclic group (iv) None of these

(h) The total number of edges in a complete graph of n vertices is
(i) n (iii) $n^2 - 1$
(ii) $\frac{n}{2}$ (iv) $\frac{n(n-1)}{2}$

(i) In any undirected graph, the sum of degrees of all vertices
(i) must be even (iii) must be odd
(ii) is a twice the number of edges (iv) Both (i) and (ii)

(j) $(p \to q) \wedge (r \to q)$ is equivalent to
(i) $(p \vee r) \to q$ (iii) $p \vee (r \to q)$
(ii) $p \vee (r \to p)$ (iv) $p \to (q \to r)$

Q.2	(a)	Given A = {1, 2, 3}, B = {7, 8} and R = { (1, 7), (2, 7), (1, 8), (3, 8) }, find R^{-1} (inverse of R) and R' complement of R.	**[7]**
	(b)	Consider $A = B = C = R$ set of real numbers and let $f: A \to B, g: B \to C$ defined by $f(x) = x + 9$ and $g(y) = y^2 + 3$, find the following composition functions: $(fog)(a), (gof)(a), (gof)(3), (fog)(-3)$.	**[7]**
Q.3	(a)	Describe the Fundamental Theorem of Arithmetic. Factorize the number '324' and represent it as a product of primes.	**[7]**
	(b)	Use the Euclidean algorithm to find the greatest common divisor of each pair of integers, (i) 60, 90 (ii) 414, 662	**[7]**
Q.4	(a)	What is the negation of each of the following propositions? (i) Today is Tuesday (ii) A cow is an animal (iii) No one wants to buy my house	**[7]**
	(b)	Determine the truth value of each of the following statements (i) 4+3=7 and 6+2=8 (ii) 2+3=4 and 3+1=2.	**[7]**
Q.5	(a)	Show that the set of rational numbers Q forms an abelian group under the composition defined by $a * b = a + b - ab$.	**[7]**
	(b)	Define a ring, give an example of a commutative ring without unity, and a non-commutative ring with identity.	**[7]**
Q.6	(a)	Describe a graph model that can be used to represent all forms of electronic communication between two people in a single graph. What kind of graph is needed?	**[7]**
	(b)	Explain the complete graph and prove that k_5 (complete graph) is a nonplanar graph.	**[7]**
Q.7	(a)	Explain the congruence relation. Let n be a positive integer then: (i) If $a \equiv b \ (mod\ n)$, then $b \equiv a \ (mod\ n)$ (ii) If $a \equiv b \ (mod\ n)$ and $b \equiv c \ (mod\ n)$, then $a \equiv c \ (mod\ n)$.	**[7]**
	(b)	Show that $p \Leftrightarrow q \equiv (p \Longrightarrow q) \wedge (q \Longrightarrow p)$.	**[7]**
Q.8	(a)	What is the field? Show that $(Q, +, \times)$ is a field.	**[7]**
	(b)	If in a ring R with unity, $(xy)^2 = x^2y^2$ for all $x, y \in R$, then R is commutative.	**[7]**
Q.9	(a)	Write short notes of the following: (i) Eulerian Graph (ii) Graph Coloring	**[7]**
	(b)	Write short notes of the following: (i) Chromatic number (ii) Perfect Graph	**[7]**