Hybrid Storage Model

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1 Introduction

This model describes the energy flows in a two- storage system. It will be formulated as a dynamic programming problem.

2 Model

2.1 Definitions

- Indices
  i: discrete time step index
  j: storage device number (1: battery, 2: supercapacitor)

- Parameters
  α^C: charging efficiency
  α^D: discharging efficiency
  β: storage efficiency factor (constant)
  N: number of steps (DP horizon)
  K: cost weighting factor for rate (relative to cost of energy loss)

- Variables
  L: load energy demand (Random Variable!!)
  E: energy state of storage device
  D: energy released by discharging (AFTER loss)
  C: energy consumed by charging (BEFORE loss)
  J: value function

NOTE: C_1 does not exist because not possible to charge the battery while driving. (Assuming no regenerative braking at the moment.)

2.2 Constraints

- Supply-demand balance:

$$[D_1(i)] + [D_2(i) - C_2(i)] = L(i) \tag{1}$$

- Bounds on stored energy:

$$E_j^{min} \leq E_j(i) \leq E_j^{max} \tag{2}$$

- Bounds on charging:

$$0 \leq C_2(i) \leq C_2^{max} \tag{3}$$

- Bounds on discharging:

$$0 \leq D_j(i) \leq D_j^{max} \tag{4}$$

2.3 Recursive State Equations

The state of the system is the energy in a storage device ($E_j(i)$). This evolves according to the charging and discharging of the storage device, which is the control.

The following recursive equations describe the changes in the state, including due to constant leakage loss:

$$E_1(i+1) = \beta_1 E_1(i) + \left[-\frac{1}{\alpha_1^D} D_1(i) \right] \tag{5}$$

$$E_2(i+1) = \beta_2 E_2(i) + \left[\alpha_2^C C_2(i) - \frac{1}{\alpha_2^D} D_2(i) \right] \tag{6}$$

Substituting (1) into (6), one obtains:

$$E_2(i+1) = \beta_2 E_2(i) + \left[\alpha_2^C [D_1(i) + D_2(i) - L(i)] - \frac{1}{\alpha_2^D} D_2(i) \right] \tag{7}$$

2.4 Objective Function

- Minimize charge and discharge rates for the first storage device (battery):

$$J_{rate} = min \left[\sum_{i=0}^{N} K \left[D_1(i) \right]^2 \right] \tag{8}$$

- Minimize power loss due to energy transfers

$$J_{loss} = min \left[\sum_{i=0}^{N} (1-\alpha_1^D) D_1(i) + (1-\alpha_2^C) C_2(i) + (1-\alpha_2^D) D_2(i) \right] \tag{9}$$

Hence, the cost functions are as follow:

- Battery storage:

$$J_i[E_1(i)] = \mathop{\mathbb{E}}_{L(i)}\{\min_{D_1,D_2}(1-\alpha_1^D)D_1(i) + K[D_1(i)]^2 + (1-\alpha_2^D)[D_2(i)]$$
$$+(1-\alpha_2^C)[D_1(i)+D_2(i)-L(i)] + \mathop{\mathbb{E}}_{L(i+1)}\{J_{i+1}[f_1(E_1(i), D_1(i)), L(i+1)]\}\} \tag{10}$$

where $f_1(\cdot)$ is (5), the state equation for the battery.

- Supercapacitor storage:

$$J_i[E_2(i)] = \mathop{\mathbb{E}}_{L(i)}\{\min_{D_1,D_2}(1-\alpha_1^D)D_1(i) + K[D_1(i)]^2 + (1-\alpha_2^D)[D_2(i)]$$
$$+(1-\alpha_2^C)[D_1(i)+D_2(i)-L(i)] + \mathop{\mathbb{E}}_{L(i+1)}\{J_{i+1}[f_2(E_2(i), D_2(i)), L(i+1)]\}\} \tag{11}$$

where $f_2(\cdot)$ is (7), the state equation for the supercapacitor.

References