Class Assignment 1

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Problem . Given that there are two drunk people who start at the origin with an equal probability of taking a step to the left or the right in a one dimensional world, what is the probability of them meeting in N steps? Provide a mathematical model and give computational proof.

Solution We define the following terms:

D, which is the displacement of either person from the origin.

p and q, which are the probability of the walker taking a step to the right or the left respectively

N_{right} and N_{left}, which are the number of steps taken in each direction

We know that:

$$q = 1 - p$$

and

$$p = 1/2$$

At any given point, the displacement from the origin will be

$$D = N_{right} - N_{left} = x - (N - x)$$

if x is the number of steps taken to the right.

$$D = 2x - N \implies x = (N + D)/2 \implies N - x = (N - D)/2$$

The probability of having taken x steps to the right after N steps is a binomial function:

$$P_{N(x)} = \frac{N!}{(x!(N-x)!)} p^x q^{(N-x)} \implies P_{N(x)} = \frac{N!}{\dfrac{(N+D)}{2}!\dfrac{(N-D)}{2}!} p^{\dfrac{(N+D)}{2}} q^{\dfrac{N-D}{2}}$$

Given that $p = q = 1/2$ and $2N$ total steps taken, we find that the probability of them meeting is:

$$P_{2N}(x = N/2) = \frac{2N!}{2^N N!}^{\,2}$$

The code has been attached alongwith this submission for the computational proof.