ASSIGNMENT

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Programme	B. Tech
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Semester/Year	FIRST
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Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

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Contents

Question No. A

Solution to Question No. A:

Overview:

Singular value decomposition (SVD) is a factorization of a real complex matrix. Since we can find rank of matrix using Gaussian elimination to reduce the matrix to row echelon form and then count the number of nonzero rows. This method is applicable/good in all cases but not in the matrix which contain decimal part. In practical, the coefficient matrix A usually involves some errors. So we use method of singular value decomposition. The definition of singular value decomposition: “It is the generalization of the eigen decomposition of a semidefinite normal (for example, a symmetric matrix with positive eigenvalues) to any matrix via an extension of the polar decomposition.

A1 computation of SVD of a matrix with an example

Example: Find the singular values of the matrix A=$\begin{bmatrix} 0 & 1 & 1 \\ \sqrt{2} & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and find the SDV decomposition of A.

We compute AA^T and find AA^T = $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix}$. The characteristic polynomial is:

$\lambda^3 + 10\lambda^2 - 16\lambda = -\lambda(\lambda^2 - 10\lambda + 16)$

$= -\lambda(\lambda - 8)(\lambda - 2)$

So the eigenvalues of AA^T are $\lambda = 8$, $\lambda = 2$, $\lambda = 0$. Thus, the singular values are $\sigma_1 = 2\sqrt{2}$, $\sigma_2 = \sqrt{2}$ (and $\sigma_3 = 0$).

To give the decomposition, we consider the diagonal matrix of singular values Σ = $\begin{bmatrix} 2\sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Next, we find an orthonormal set of eigenvectors for AAT . For $\lambda = 8$, we find an eigenvector (1, 2, 1) - normalizing gives $p_1 = (1/\sqrt{6}, 2/\sqrt{6}, 1/\sqrt{6})$. For $\lambda = 2$ we find $p_2 = (-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$, and finally for $\lambda = 0$ we get $p_3 = (1/\sqrt{2}, 0, -1/\sqrt{2})$.

This gives the matrix P =$\begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$

Finally, we have to find an orthogonal set of eigenvectors for $A^TA = \begin{bmatrix} 2 & 2\sqrt{2} & 0 \\ 2\sqrt{2} & 6 & 2 \\ 0 & 2 & 2 \end{bmatrix}$

This can be done in two ways. We show both ways, starting with orthogonal diagonalization. We already know that the eigenvalues will be λ = 8, λ = 2, λ = 0. This gives eigenvectors

q1 = (1/√6 , 3/√12 ,1/√12), q2 = (1/√3 , 0, − 2/√6) and q3 = (1/√2 ,– 1/ 2 , 1/2). Put these together to get.

$$Q = \begin{bmatrix} 1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 3/\sqrt{12} & 0 & -1/2 \\ 1/\sqrt{12} & -2/\sqrt{6} & 1/2 \end{bmatrix}$$

For a quicker method, we calculate the columns of Q using those of P using the formula

$$p_i = 1/\sigma_i * A^T p_i$$

Thus, we calculate

$p_i = 1/\sigma_i * A^T p_i = 1/\sqrt{8} \begin{bmatrix} 0 & \sqrt{2} & 0 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$ =q1

and similarly, for the other two columns.

Either way we can now we verify that we have $A = P\Sigma Q^T$

A2 comparison of the singular values and singular vector with eigenvalues and eigen vectors of a matrix.

The singular values of a M×N matrix X are the square roots of the eigenvalues of the N × N matrix X^ X (where ^ stands for the transpose-conjugate matrix if it has complex coefficients, or the transpose if it has real coefficients).

The singular value decomposition (SVD) factorizes a linear operator $A : R^n \to R^m$ into three simpler linear operators: 1. Projection z = V T x into an r-dimensional space, where r is the rank of A 2. Element-wise multiplication with r singular values σi , i.e., $z' = Sz$ 3. Transformation $y = Uz'$ to the m-dimensional output space

Combining these statements, A can be re-written as $A = USV^T$

with U an m × r orthonormal matrix spanning A's column space im(A), S an r×r diagonal matrix of singular values, and V an n × r orthonormal matrix spanning A's row space im(A^T)

The eigenvalue decomposition applies to mappings from R n to itself, i.e., a linear operator $A : R^n \to R^n$ described by a square matrix. An eigenvector e of A is a vector that is mapped to a scaled version of itself, i.e., Ae = λe, where λ is the corresponding eigenvalue. For a matrix A of rank r, we can group the r non-zero eigenvalues in an r × r diagonal matrix Λ and their eigenvectors in an n × r matrix E.

A3 Geometrical description of SVD of a matrix.

The linear transformation

$$T : M^n \to M^m$$

$$X \to Kx$$

where σi is the i-th diagonal entry of Σ, and T(Vi) = 0 for i > min(m,n).

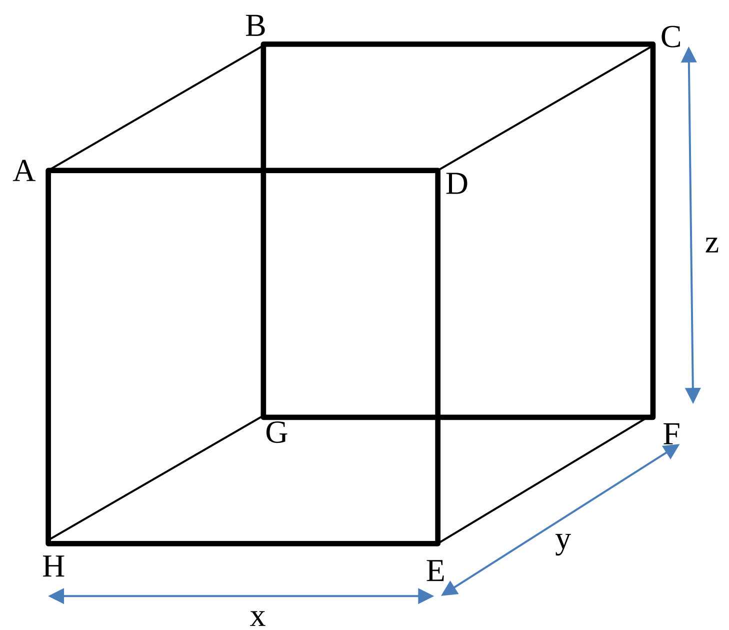
The geometric content of the SVD theorem can thus be summarized as follows: for every linear map T : $M^n \to M^m$one can find orthonormal bases of M^n and M^m and Km such that T maps the i-th basis vector of $M^n$to a non-negative multiple of the i-th basis vector of M^m, and sends the left-over basis vectors to zero. With respect to these bases, the map T is therefore represented by a diagonal matrix with non-negative real diagonal entries.

A4 Application of SVD to solve an engineering problem and conclusion

SVD is used to finding the approximate values of low rank matrices. It make the process simple to convert distance vector to singularity and to find the sensitivity of linear equation that is whether the equation get effected by any change in the value that cause errors.

Solution to Question No. B1:

B1.1 Find the domain of cost as a function of the lengths of the sides with a sketch.



Given, costs

to construct east-west walls= Rs. 10 m^{-2}

to construct North-South walls= Rs. 8 m^{-2}

to construct roof= Rs. 1 m^{-2}

to construct floor= Rs. 5 m^{-2}

let C be the total cost of the construction of all four walls:

therefore:

$$C = 2xz \times 8 + 2yz \times 10 + xy \times 5 + xy \times 1$$

⇨ 16xz + 20yz + 6xy ------------------- (1) where, $x \geq 30$ m, $y \geq 30$ m and $z \geq 4$ m

Given the volume should be 4000 m^2

Therefore, volume = xyz = 4000 m^3 -------------------------- (2)

From the equation (2)

$$z = \frac{4000}{xy}$$

Putting ‘z’ on equation (1)

$$C = 16x \times \frac{4000}{xy} + 20y \times \frac{4000}{xy} + 6xy$$

$$C(x, y) = \frac{64000}{y} + \frac{80000}{x} + 6xy$$

Now, since z≥4 ,

therefore $\frac{4000}{xy} \geq 4$

- ⇨ xy ≤ 1000
- ⇨ $y = \frac{1000}{x}$

So, domain of C = {(x,y):x ≥ 30,30 ≤ y ≤ 1000/x}

Hence, the region bounded below the horizontal

B1.2 Find the dimensions that minimize the cost of construction material.

Differentiating C partially w.r.t x:-

$$\frac{\partial \mathbf{C}}{\partial \mathbf{x}} = \boldsymbol{C_x} = \frac{\mathbf{-80000}}{\boldsymbol{x^2 + 6y}}$$

Differentiating C partially w.r.t y:

$$\frac{\partial C}{\partial y} = C_y = \frac{-64000}{y^2 + 6x}$$

For the local maxima and local minima $\boldsymbol{C_x}$ and C_y must be 0 so,

$$\boldsymbol{C_x} = \frac{\mathbf{-80000}}{\boldsymbol{x^2 + 6y}} = 0$$

- ⇨ $6yx^2$=80000 -------------------(3)

$$C_y = \frac{-64000}{y^2 + 6x} = 0$$

- ⇨ $6xy^2$=64000 -------------------(4)

Dividing eqn 3 and eqn 4 we get:

$$\frac{6xy^2}{6x^2y} = \frac{64000}{80000}$$

$\Rightarrow \frac{y}{x} = \frac{4}{5}$

$\Rightarrow y = \frac{4x}{5}$

Putting the value of y on eqn (3)

$\Rightarrow \frac{x^3 \times 6 \times 4}{5} = 80000$

$\Rightarrow x^3 = \frac{50000}{3}$

$\Rightarrow x = \sqrt[3]{\frac{50000}{3}} = 25.5436$

For y:

$$y = \frac{4}{5} \times \sqrt[3]{\frac{50000}{3}} = 20.43$$

Critical point for function C are x0=25.5436 and y0 = 20.43

Now, again differentiating C w.r.t x:

$$C_{xx} = \frac{160000}{x^3}$$

$\Rightarrow C_{xx}(x_0, y_0) = \frac{48}{5}$

Similarly,

$$C_{yy} = \frac{128000}{y^3} = \frac{128000}{20.43^3} = 15$$

And, C_{xy}=6

Now, $\Delta(x_0,y_0)=C_{xx} \times C_{yy}- (C_{xy})^2$

$\Rightarrow$ 144-36 = 108

As $\Delta(x_0,y_0)>0$ and $C_{xx}>0$,

Since we know that, (x_0,y_0) is the local minima for the function C

So the cost function C at (x_0,y_0):

$$x_0 = \sqrt[3]{\frac{50000}{3}} = 25.54 \qquad and \quad y_0 = \frac{4}{5} \times \sqrt[3]{\frac{50000}{3}} = 20.43$$

$$C = \frac{640000}{y_0} + \frac{80000}{x_0} + 6x_0y_0$$

$$C(x_0, y_0) = C(25.54, 20.43) = \frac{640000}{20.43} + \frac{80000}{25.54} + 6 \times 25.54 \times 20.43$$

$\Rightarrow$ Rs. 9395.68

according to domain {(x,y):x ≥ 30,30 ≤ y ≤ 1000/x}, The critical points are not present in the function C. Therefore, this point of local minima can't be considered for our requirement

now we are evaluating the boundaries of line $L^{'}$(line passing from (30,30) to (100/3,30)), $L^{''}$ (line passing from (30,100/3) to (100/3,30)) and $L^{'''}$ (line passing from (30,30) to (30,100/3))

On line $L^{'''}$: x=30:

$C(30, y) = \frac{640000}{y} + \frac{80000}{30} + 6 \times 30 \times y$ $\quad\quad\quad\quad$ $where$ (30 ≤ y ≤ 100/3)

C(30,30)=Rs. 10200

C(30,100/3)=Rs. 10586.67

For line $L^{'}$: y=30

$C(x, y) = \frac{640000}{30} + \frac{80000}{x} + 6 \times 30 \times x$ $\quad\quad\quad\quad$ $where$ (30 ≤ x ≤ 100/3)

C(30,30)=Rs. 10200

C(100/3,30)=Rs. 10533.33

On line L_2: y=1000/x

$C(x, 10000/x) = 6000 + 64x + \frac{80000}{x}$ $\quad\quad\quad\quad$ $where$ (30 ≤ x ≤ 100/3)

C(30,100/3)= Rs. 10586.67

C(100/3,30)= Rs. 10533.33

From the value found at $L^{'}$,$L^{''}$ and $L^{'''}$:

Absolute maximum at: (30,100/3)= Rs**.** 10586.67

And , Absolute minimum at : (30,30)=Rs. 10200

Hence, the absolute minima is found at (30,30,z) where z is 4000/(30*30)=4.44 i.e. (30,30,4.44) having cost Rs. 10200.

B1.3 Can you design a building with lesser cost if the restrictions on the lengths of the walls were removed? Justify.

Yes, we can design a building with lesser cost if we can increase or decrease the size of the walls, because of the critical values (25.54,20.43) we got in the eq0 for the cost function C with the least cost value of Rs.

9395.68. So a building of volume 4000m^2 with dimensions x=25.54m, y=20.43m, $z = \frac{4000}{(2554)(20.43)} =$ 7.67 m can be the least building cost

B1.4 Comment and conclude on the result.

First, we found the cost function in three variables and using the volume, we get the equation in terms of only two variables which are ‘x’ and ‘y’. the we found functions critical point which was present outside the domain, hence we could not consider that point for the calculation of minimum costs. Then we checked for the point on the boundaries of the domain for absolute minima. We get the minima for one of the boundaries.

So, it is not always necessary for the function to have absolute minimum at the critical points, even if the boundaries can produce a minima.

<u>Question No. B2</u>

<u>Solution to Question No. B2:</u>

B2.1 Prove that these families are orthogonal. More precisely, show that if $z0$ = (x0,0) is a point in D which is common to two particular curves $u(x,y)$ = c1 and $v(x,y)$ = c2 and if $f'(z0)$≠ 0, then the lines tangent to those curves at (x0,y0) are perpendicular:

function f(z)=u + iv we have,

$u_x = v_y$ and Using Cauchy Riemann equation $u_y = -v_x$

For u(x,y)=c_1

Partially differentiating u w.r.t x:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \times \frac{\partial u}{\partial x} = 0$$

$$\frac{dy}{dx} = \frac{-u_x}{u_y}$$

$$m1 = \frac{dy}{dx}(x_0, y_0) = \frac{-u_x(x_0,y_0)}{u_y(x_0,y_0)}$$

For v(x,y)=c_2

Partially differentiating v w.r.t x:

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-v_x}{v_y}$$

$$m2 = \frac{dy}{dx}(x_0, y_0) = \frac{-v_x(x_0, y_0)}{v_y(x_0, y_0)}$$

The slope m1 and m2 s are of the tangents at (x_0,y_0).

We also know that,

$m_1 \times m_2=-1$ for straight lines which cut orthogonally.

So,

$$m_1 m_2 = \frac{-u_x \times (-v_x)}{u_y \times v_y}$$

Using Cauchy-Riemann equation $u_x=v_y$ and $u_y = -v_x$

$$\Rightarrow \quad \frac{-u_x \times u_y}{u_y \times u_x} = -1$$

Since $m_1 m_2$ -1, therefore these families of tangent cut orthogonally for the two particular curves u and v.

B2.2 Illustrate the above result by sketching the level curves using MATLAB $(x,y) = c1$ and $v(x,y) = c2$ for $f(z) = z^2$.:

Let u(x,y) and v(x,y) be the real and imaginary parts of z^2 respectively.
Therefore;
$Z^2=(x+iy)^2$
Or, $Z^2=(x^2-y^2+2ixy)$
$u=x^2-y^2=c_1$
$v=2xy=c_2$

```
>> u=@(x,y)x.^2-y.^2;
>> v=@(x,y) 2*x.*y;
>> fcontour(v)
>> hold on
>> fcontour(u)
>>
```

Fig 2.1 code for the plot below

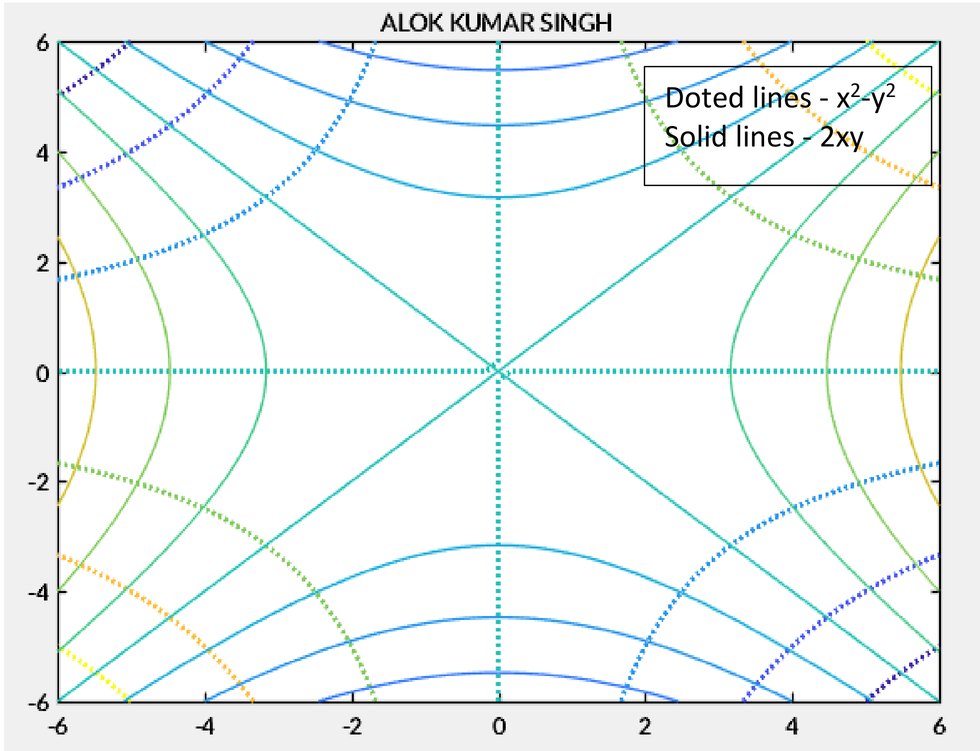


Fig 2.2 plot for f(z)=z^2 with u and v representing real and complex parts of f(z)

B2.3 Observe that the curves $(x,y) = 0$ and $v(x,y) = 0$ for $f(z) = z^2$ intersect at the origin and are not orthogonal to each other. Comment.

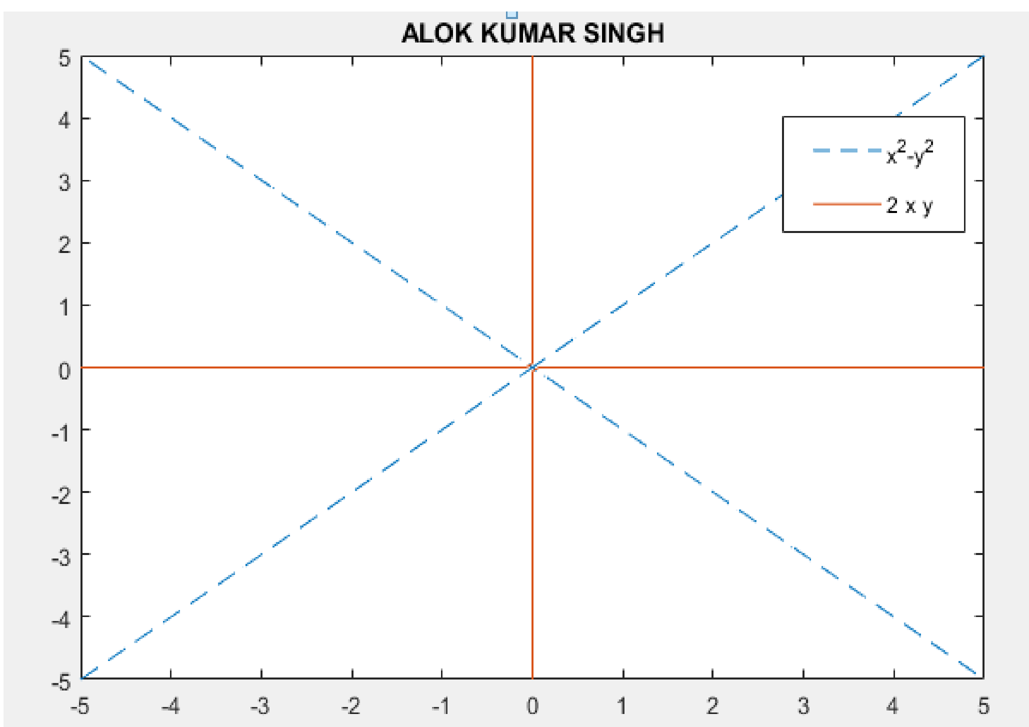


Fig 2.3 plot for the curve u(x,y)=0 and v(x,y)=0 for f(z)=z^2

```
>> u=@(x,y)x.^2-y.^2;
>> v=@(x,y) 2*x.*y;
>> fimplicit(u)
>> hold on
>> fimplicit(v)
```

Fig 2.4 code for plotting u(x,y)=0 and v(x,y)=0 graphs

From the above graph we can say that u(x,y)=0 and v(x,y)=0 are not orthogonal. This is because the two functions don't remain remain analytical function when the u(x,y)=0 and v(x,y)=0 as their derivatives vanish.

<u>Question No. B3</u>

<u>Solution to Question No. B3:</u>

B3.1 Sketch the diagram for the above data and find the transition matrix A for the system:

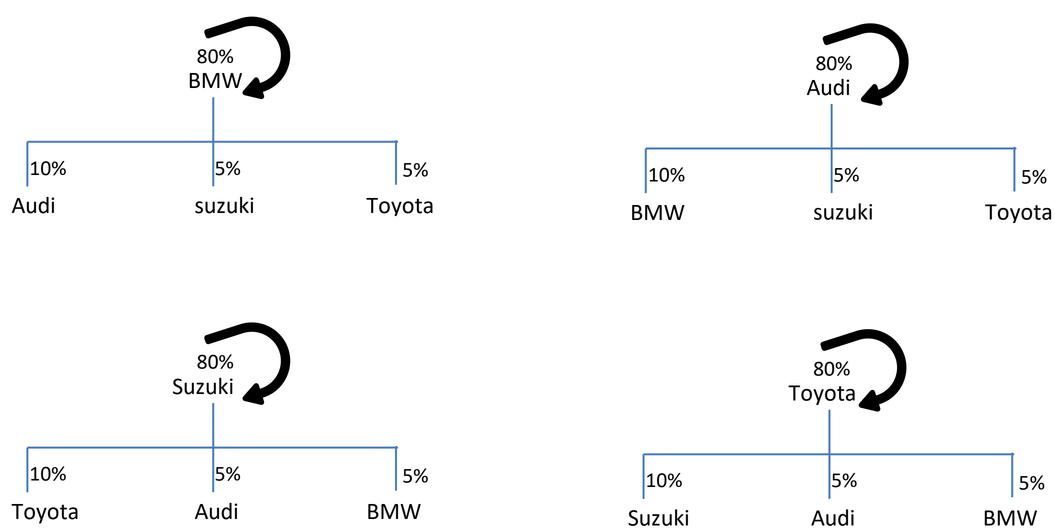


Fig 3.1: Sketch of the relations

From the above relations, we have

$$\begin{array}{cc} & \begin{array}{cccc} \text{Audi} & \text{BMW} & \text{Suzuki} & \text{Toyota} \end{array} \\ A = & \begin{bmatrix} 0.8 & 0.1 & 0.05 & 0.05 \\ 0.1 & 0.8 & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.8 & 0.1 \\ 0.05 & 0.05 & 0.1 & 0.8 \end{bmatrix} \end{array}$$

B3.2 Using MATLAB built-in function find eigenvalue and eigenvectors of the transition matrix A:

```
a =

    0.8000    0.1000    0.0500    0.0500
    0.1000    0.8000    0.0500    0.0500
    0.0500    0.0500    0.8000    0.1000
    0.0500    0.0500    0.1000    0.8000
```

Fig 3.2 transition matrix

```
>> [p,d]=eig(a)

p =

    0.5517    0.4422   -0.5000    0.5000
   -0.5517   -0.4422   -0.5000    0.5000
    0.4422   -0.5517    0.5000    0.5000
   -0.4422    0.5517    0.5000    0.5000


d =

    0.7000         0         0         0
         0    0.7000         0         0
         0         0    0.8000         0
         0         0         0    1.0000

fx >>
```

Fig 3.3 Eigen values (D) and Eigen vectors (P)

Hence,

λ_1=0.7

λ_2=0.7

λ_3=0.8

λ_4=1.0

$$P_1 = \begin{bmatrix} 0.5517 \\ -0.5517 \\ 0.4422 \\ -0.4422 \end{bmatrix} \quad P_2 = \begin{bmatrix} 0.4422 \\ -0.4422 \\ -0.5517 \\ 0.5517 \end{bmatrix} \quad P_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \quad P_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

B3.3 Check whether the matrix A is diagonalizable? If it is diagonalizable then find the diagonalization of A.

A matrix A is said to be diagonalizable if it is similar to its diagonal matrix which contains the Eigen values in its diagonal. Thus a matrix A is diagonalizable if there exists an invertible matrix P such that $P^{-1}DP$.

```
p =

    0.5517    0.4422   -0.5000    0.5000
   -0.5517   -0.4422   -0.5000    0.5000
    0.4422   -0.5517    0.5000    0.5000
   -0.4422    0.5517    0.5000    0.5000
```

Fig 3.4 Eigen vectors (P)

Now,

Inverse of P is:

```
>> inv(p)

ans =

    0.5517   -0.5517    0.4422   -0.4422
    0.4422   -0.4422   -0.5517    0.5517
   -0.5000   -0.5000    0.5000    0.5000
    0.5000    0.5000    0.5000    0.5000

fx >>
```

Fig 3.5 P^{-1} matrix

$P^{-1}AP$:-

```
>> inv(p)*a*p

ans =

    0.7000    0.0000    0.0000   -0.0000
    0.0000    0.7000    0.0000    0.0000
    0.0000    0.0000    0.8000   -0.0000
   -0.0000   -0.0000   -0.0000    1.0000

>>
```

Fig 3.6 $P^{-1}AP$matrix

Therefore, $P^{-1}AP=D$

This proves that the matrix A is diagonalizable and it's diagonalization is nothing but the matrix D which is a diagonal matrix with Eigen values.

B3.4 Suppose, initially there are 200 Audis and 100 each of the other three types leased. Determine the number of people who will lease each type of vehicle after 2, 4 and 6 years.

Given:- n=2 years

So, n for 2, 4 and 6 years would be 1, 2 and 3 respectively

Since, $A=PDP^{-1}$ (By the property of a diagonalizable matrix, $D=P^{-1}AP$)

Now, $A^2=A.A$

- ⇨ $(PDP^{1}).(PDP^{-1})$
- ⇨ $A^2 = PD^2P^{-1}$
- ⇨ $A^2 = PD^nP^{-1}$

```
>> P*D*inv(P)*[200;100;100;100]

ans =

  180.0000
  110.0000
  105.0000
  105.0000
```

Fig 3.7 For n=1: Number vehicles in 2years

```
>> P*D^2*inv(P)*[200;100;100;100]

ans =

  165.5000
  116.5000
  109.0000
  109.0000
```

Fig 3.8 For n=2: Number vehicles in 4years

```
>> P*D^3*inv(P)*[200;100;100;100]

ans =

  154.9500
  120.6500
  112.2000
  112.2000
```

Fig 3.9 For n=3: Number vehicles in 6years

In 2 years:

No. of people leases Audi : 180

No. of people leases BMW = 110

No. of people leases Suzuki = 105

No. of people leases Toyota = 105

In 6 years :

No. of people leases Audi = 154.95≈155
No. of people leases BMW = 120.65≈121
No. of people leases Suzuki = 112.2≈112
No. of people leases Toyota =112.2 ≈112

In 4 years :

No. of people leases Audi = 165.5 ≈ 166
No. of people leases BMW = 116.5 ≈117
No. of people leases Suzuki = 109
No. of people leases Toyota =109

B3.5 Find the steady state vector as lim $n \to \infty$ x_n and comment on the result.

```
>> B=[200;100;100;100];
>> A=[0.8 0.1 0.05 0.05;0.1 0.8 0.05 0.05;0.05 0.05 0.8 0.1;0.05 0.05 0.1 0.8];
```

```
>> P*D^20*inv(P)*B

ans =

  125.3281
  125.2483
  124.7118
  124.7118
```

```
>> P*D^53*inv(P)*B

ans =

  125.0002
  125.0002
  124.9998
  124.9998
```

```
P*D^10*inv(P)*B

s =

129.0967
126.2720
122.3156
122.3156
```

```
>> P*D*inv(P)*B

ans =

  180.0000
  110.0000
  105.0000
  105.0000
```

```
>> P*D^5*inv(P)*B

ans =

  141.5955
  124.7885
  116.8080
  116.8080
```

```
>> P*D^30*inv(P)*B

ans =

  125.0321
  125.0298
  124.9691
  124.9691
```

```
>> P*D^40*inv(P)*B

ans =

  125.0034
  125.0033
  124.9967
  124.9967
```

```
>> P*D^48*inv(P)*B

ans =

  125.0006
  125.0006
  124.9994
  124.9994
```

```
>> P*D^25*inv(P)*B

ans =

  125.1012
  125.0877
  124.9056
  124.9056
```

```
>> P*D^57*inv(P)*B

ans =

  125.0001
  125.0001
  124.9999
  124.9999
```

```
>> P*D^59*inv(P)*B

ans =

  125.0000
  125.0000
  125.0000
  125.0000
```

```
>> P*D^60*inv(P)*B

ans =

  125.0000
  125.0000
  125.0000
  125.0000
```

Fig 3.10 evaluating the steady state of the transition probability of matrix A (part 1)

```
>> P*D^70*inv(P)*B

ans =

  125.0000
  125.0000
  125.0000
  125.0000
```

```
>> P*D^80*inv(P)*B

ans =

  125.0000
  125.0000
  125.0000
  125.0000
```

```
>> P*D^100*inv(P)*B

ans =

  125.0000
  125.0000
  125.0000
  125.0000
```

Fig 3.11 evaluating the steady state of the transition probability of matrix A (part 2)

We can observe that, taking power after 59 the matrix just stayed stable and didn't vary with more power. . After which the matrix just stayed stable and didn't vary with more powers. c: $\begin{bmatrix}125\\125\\125\\125\end{bmatrix}$

Hence, the steady state vector is $\begin{bmatrix}125\\125\\125\\125\end{bmatrix}$

Question No. B4

Solution to Question No. B4:

We are using:

$Ar=M[1-(1+r)^{-n}]$
$fr=40r+(1+r)^{-60}-1$
$dfr=40-60(1+r)^{-61}$

we have:

Ar=600000r

M=15000

N==60 and $r_0=x_0=0.015$

4.1 Explain the steps involved in the algorithm:

```
function [ a ] = newton_raphson( f,df,x0 )
a(1)=x0;
MAXIT=10;
TOL=10^(-6);  %decimal value after which error is not shown
for i=1:MAXIT  %for loop from value of i to MAXIT i.e. upto limit
  a(i+1)=a(i)-f(a(i))/df(a(i));    % formula for approximation of root
  err(i+1)=abs((a(i+1)-a(i))/a(i+1))*100;             %error generated from each test case
  if err(i+1)<TOL     %condition for the approximation
    fprintf("The required root at %d iteration is %f",i,a(i+1));
    break;
  end
end
end
```

B4.2 Assuming initial guess $r0$ = 0.015, write MATLAB function to implement Newton-Raphson method

```
function [ a ] = newton_raphson( f,df,x0 )
a(1)=x0;
MAXIT=10;
TOL=10^(-6);
for i=1:MAXIT
    a(i+1)=a(i)-f(a(i))/df(a(i));
    err(i+1)=abs((a(i+1)-a(i))/a(i+1))*100;
    if err(i+1)<TOL
        fprintf("The required root at %d iteration is %f",i,a(i+1));
        break;
    end
end
end
```

fig 4.1 MATLAB function for Newton-Raphson method

```
>> dfr=@(r) 40-60*(1+r).^-61;
>> fr=@(r) 40*r +(1+r).^-60 -1;
>> x0=0.015;
>> [x]=newton_raphson( fr,dfr,x0 )
The required root at 4 iteration is 0.014395
x =

    0.0150    0.0144    0.0144    0.0144    0.0144

>>
```

fig 4.2 Approximate places of the roots by using above Newton-Raphson function/method

B4.3 Plot the given function and indicate the root in the same graph.

```
The required root at 4 iteration is 0.014395
x =

    0.0150    0.0144    0.0144    0.0144    0.0144

>> t=-0.1:0.001:0.1;
>> y=fr(t);
>> plot(t,y)
>> hold on
>> plot (x(end),0,'ro')
>>
```

fig 4.3 MATALAB algorithm to plot the indicating root of function

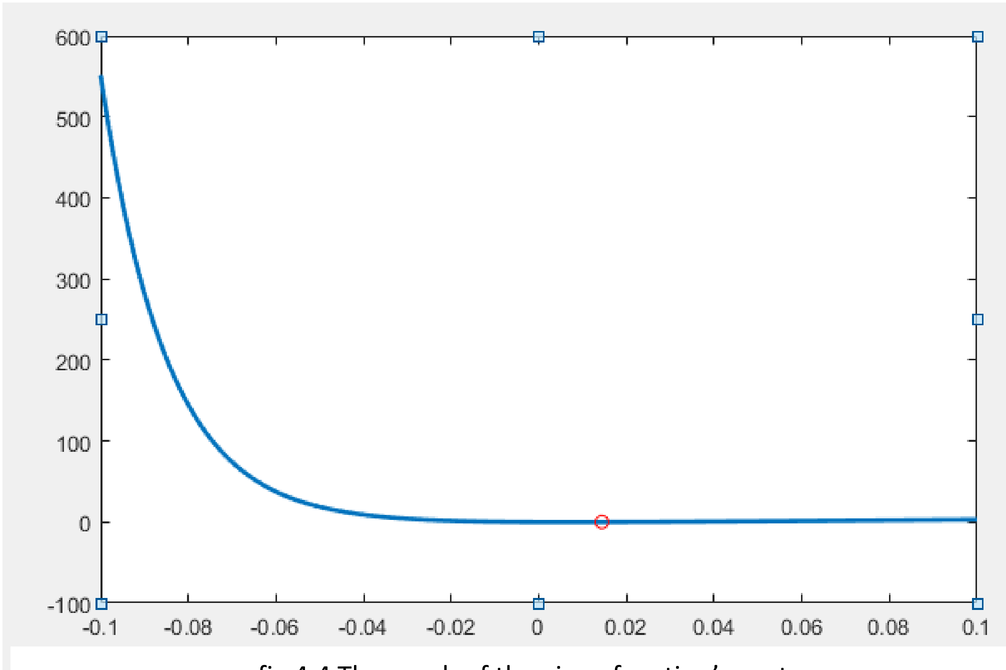


fig 4.4 The graph of the given function’s root

B4.4 Comment on the result obtained

from the graph we can see that the error in the root calculation keeps on decreasing and reaches to the tolerance which is the point where we get error about zero. So the roots keep on converging to the root till the last error and it reaches till the error approaches 0;