

**Course Code: ESC106A**

**Course Title: Construction Materials and  
Engineering Mechanics**

**Lecture No. 9:**

**Composition of forces**

**Delivered By: Nimmy Mariam Abraham**



# Lecture Intended Learning Outcomes

**At the end of this lecture, student will be able to:**

- Define composition of forces and resultant force
- Explain and prove parallelogram law of forces
- Apply parallelogram law of forces for specific angles
- Distinguish between resolution and composition of forces



# Contents

- **Engineering Mechanics**

Laws of parallelogram, composition of forces



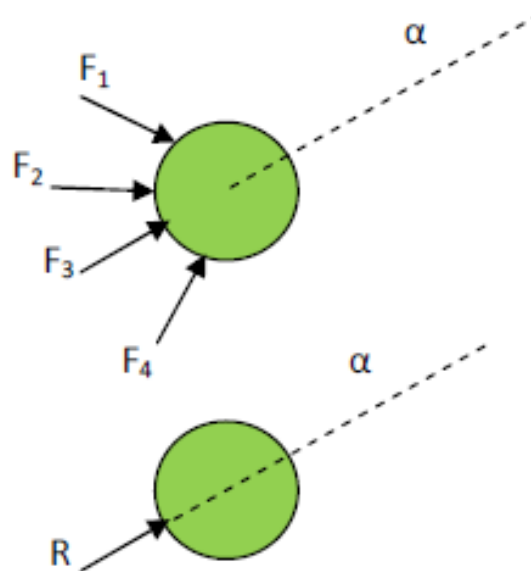
# Resultant Force

- It is possible to find a single force which will have the same effect as that of a number of forces acting on a body.
- Such a single force is called resultant force



# Resultant Force of a System of Forces

- The resultant of a system of forces is a single calculated force which is capable of producing the same effect as that of system of forces on the body
- It is the vector sum of forces of the system



# Composition of Forces

- The reduction of a given system of forces to the simplest system that will be its equivalent is composition of forces

Or

- The technique of finding the resultant of forces is called composition of forces

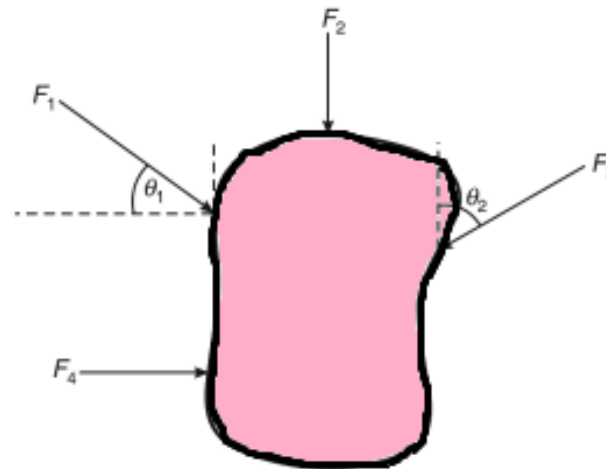


# Composition of Forces

- It is the process of combining a number of forces into a single force such that the net effect produced by the single force is equal to the algebraic sum of the effects produced by the individual forces
- The single force in this case is called the resultant force which produces the same effect on the body as that produced by the individual forces acting together



# Composition of Forces



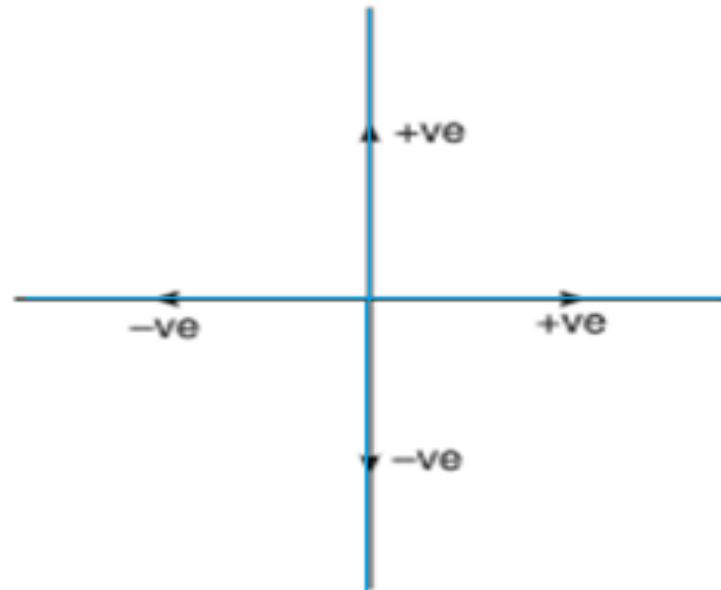
- $\Sigma F_x =$  Algebraic sum of the components of the forces along the x-axis
- $\Sigma F_x = F_4 + F_1 \cos \theta_1 - F_3 \sin \theta_2$
- $\Sigma F_y =$  Algebraic sum of the components of the forces along the y-axis
- $\Sigma F_y = -F_2 - F_1 \sin \theta_1 - F_3 \cos \theta_2$



# Composition of Forces

## Note:

- The positive and negative convention of forces used in the resolution of forces in the previous figure is as that shown in the following figure



## Positive and Negative Convention of Forces



# Composition of Forces

- Therefore the magnitude of the resultant

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

- and the direction of the resultant,

$$\theta = \tan^{-1} \left( \frac{\sum F_y}{\sum F_x} \right)$$



# Two methods of Composition of forces

- 1) Analytical
- 2) Graphical



# Analytical method of Composition of forces

For two forces,

Parallelogram law of forces states that,

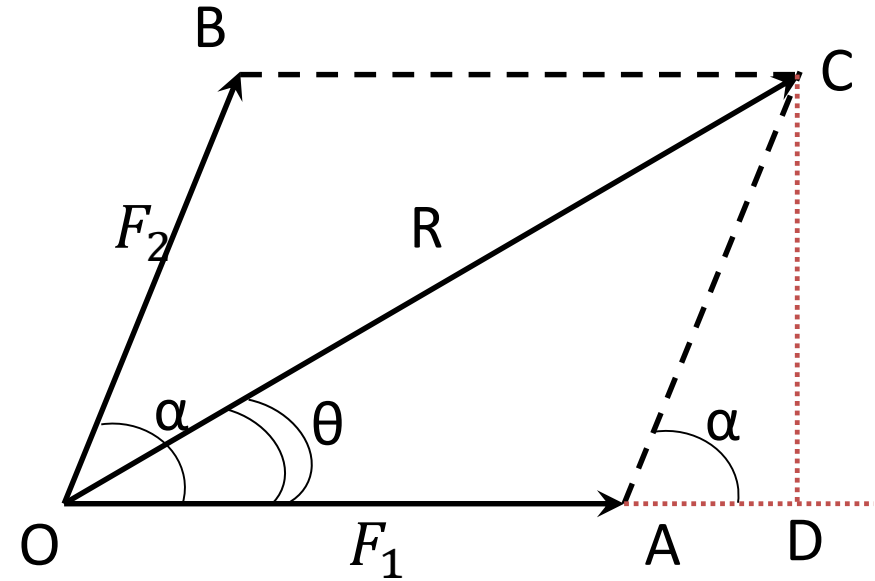
“ If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular points, their resultant is represented by the diagonal of the parallelogram passing through that angular point, in magnitude and direction.”



# Parallelogram law of forces

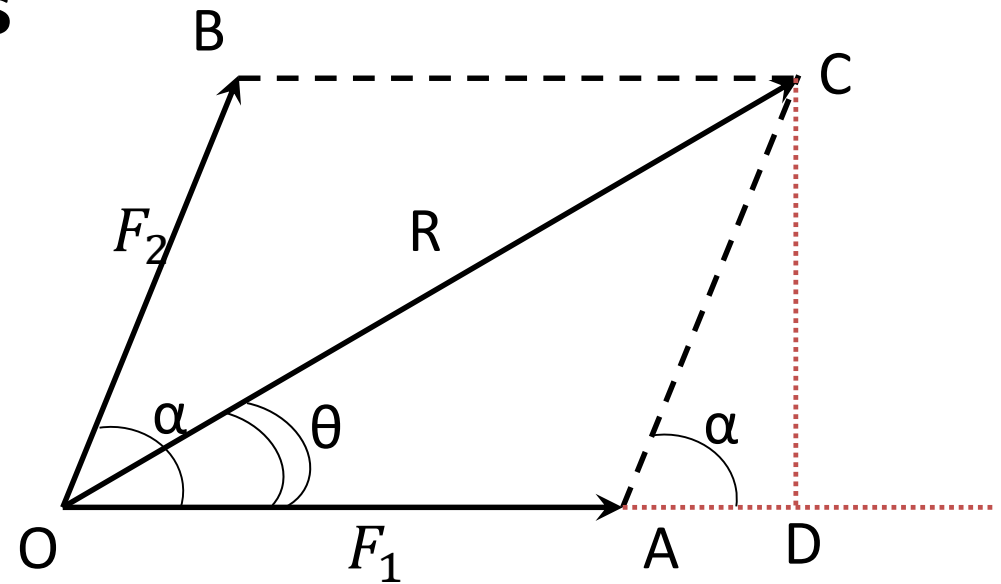
## Proof:

- Consider two forces  $F_1$  and  $F_2$  acting at point O as shown in figure.
- Let ' $\alpha$ ' be the angle between the two forces.
- Complete the parallelogram ACBO.
- Drop perpendicular CD to OA produced.
- Let R be the resultant force of forces
- Let ' $\theta$ ' be the inclination of the resultant force with the line of action of the force.



# Parallelogram law of forces

Proof:



$$OC^2 = OD^2 + CD^2$$

$$OC^2 = (OA + AD)^2 + CD^2$$

$$OA = F_1 ; AD = F_2 \cos \alpha ; CD = F_2 \sin \alpha ; OC = R$$

$$R^2 = (F_1 + F_2 \cos \alpha)^2 + (F_2 \sin \alpha)^2$$

$$R^2 = F_1^2 + 2F_1F_2 \cos \alpha + F_2^2 \cos^2 \alpha + F_2^2 \sin^2 \alpha$$

$$R^2 = F_1^2 + 2F_1F_2 \cos \alpha + F_2^2$$

$$R = \sqrt{(F_1^2 + 2F_1F_2 \cos \alpha + F_2^2)}$$



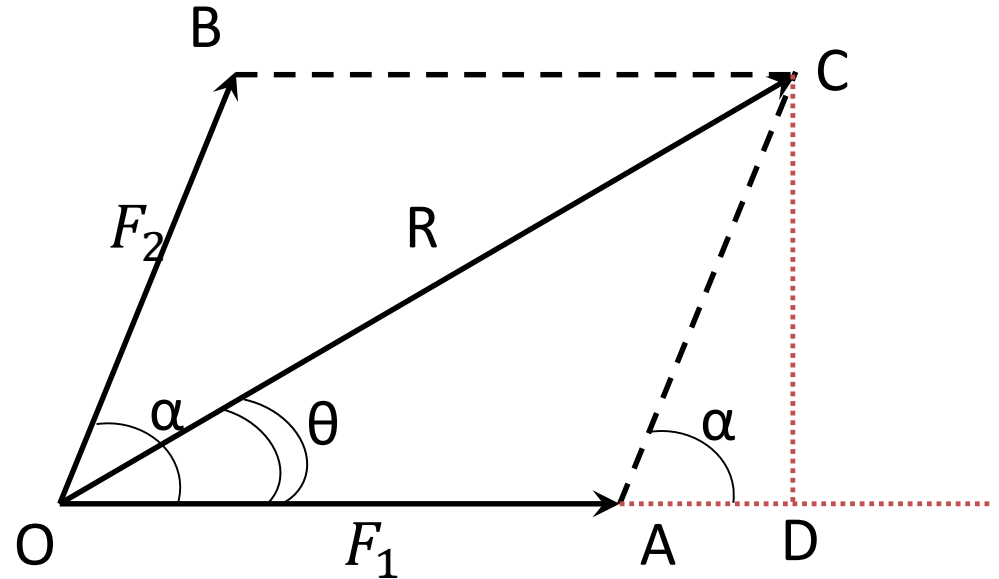
# Parallelogram law of forces

## Proof:

$$\tan \theta = \frac{CD}{OD}$$

$$\tan \theta = \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha}$$

$$\theta = \tan^{-1} \left( \frac{F_2 \sin \alpha}{F_1 + F_2 \cos \alpha} \right)$$



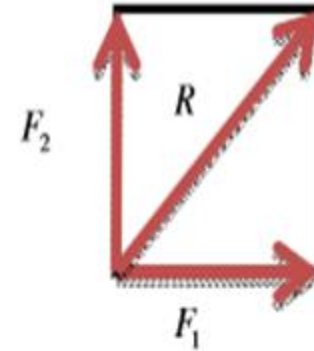
# Parallelogram law of forces

## Proof:

$$\text{If } \alpha = 90^\circ, R = \sqrt{F_1^2 + F_2^2}$$

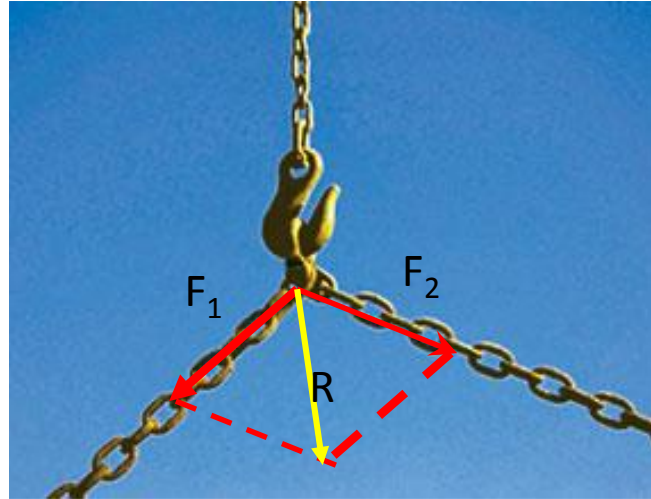
$$\text{If } \alpha = 0^\circ, R = F_1 + F_2$$

$$\text{If } \alpha = 180^\circ, R = F_1 - F_2$$





# Parallelogram law of forces



# Problems

- 1) Two forces of magnitudes 10 N and 8 N are acting at a point. If the angle between the two forces is  $60^\circ$ , determine the direction and magnitude of the resultant force.

$$R=15.62\text{N}, \theta=26.33^\circ$$

- 2) Two equal forces are acting at a point with an angle of  $60^\circ$  between them. If the resultant force is equal to  $20\sqrt{3}$  N find the magnitude of each force.

$$F=20\text{N}, \theta=30^\circ$$



# Summary

- The reduction of a given system of forces to its equivalent simplest system is known as composition of forces
- A single force which will have the same effect as that of a number of forces acting on a body is known as a resultant
- The two methods of composition of forces are analytical and graphical



# Summary

- Application of parallelogram law of forces for specific angles
- Parallelogram law of forces states that:
  - If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram
  - Resultant is represented by the diagonal of the parallelogram passing through that angular point, in magnitude and direction

