Lecture-40
Equivalent Circuit of 3 - phase Induction Motor

Lecture delivered by:

Topics

- Equivalent Circuit of three phase Induction Motor
- Torque-speed characteristics

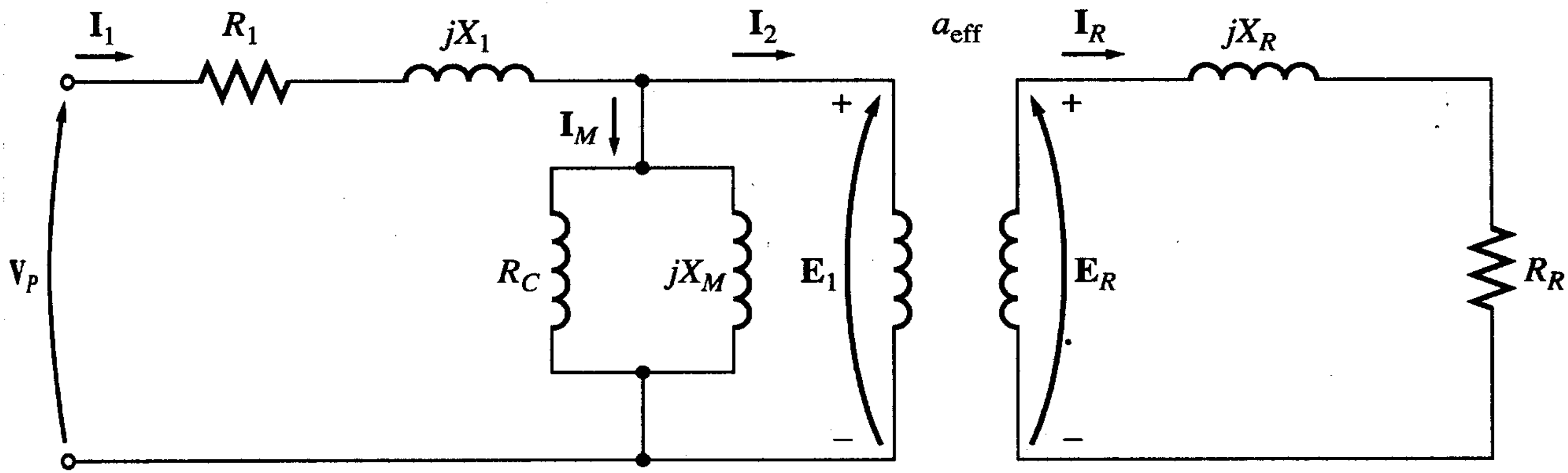
Objectives

At the end of this lecture, student will be able to:

- Develop the equivalent circuit of 3 - phase Induction Motor
- Describe the torque-speed characteristics of 3 - phase Induction Motor
- Derive the Torque equation of 3 - phase Induction Motor
- Describe the power flow of 3 - phase Induction Motor

Equivalent Circuit

- The induction motor is similar to the transformer with the exception that its secondary windings are free to rotate.



Equivalent Circuit

- When the rotor is locked (or blocked), i.e. s =1, the largest voltage and rotor frequency are induced in the rotor, Why?
- On the other side, if the rotor rotates at synchronous speed, i.e. s = 0, the induced voltage and frequency in the rotor will be equal to zero, Why?

$$\boxed{E_R = sE_{R0}}$$

- Where E_{R0} is the largest value of the rotor's induced voltage obtained at s = 1(loacked rotor)

Equivalent Circuit

- The same is true for the frequency, i.e.

$$f_r = S\ f_e$$

- It is known that

$$x = \omega L = 2\ \pi\ f\ L$$

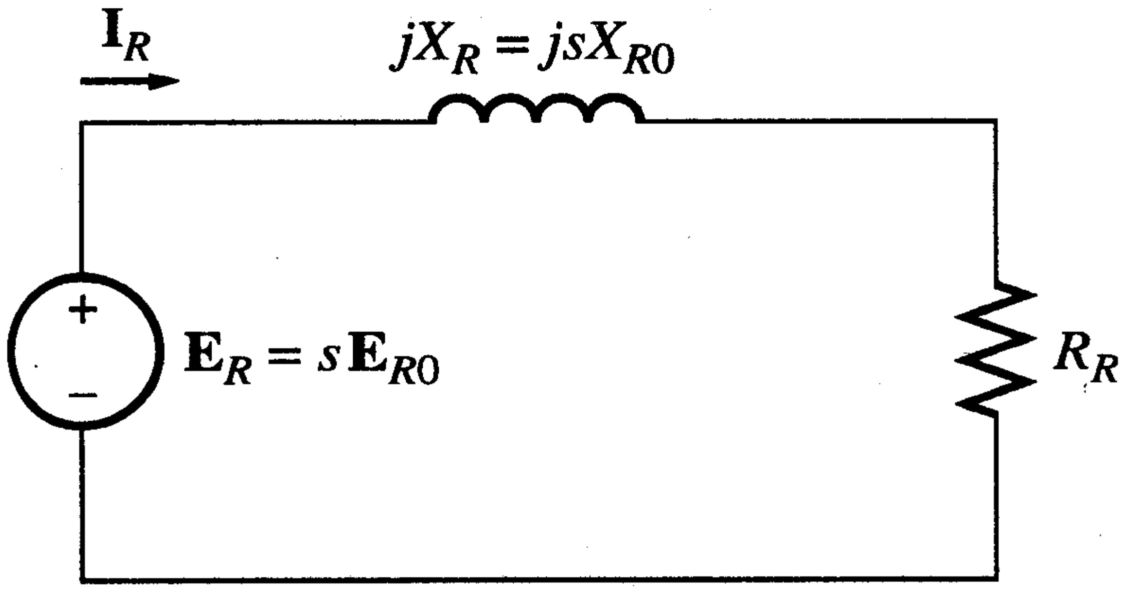
- So, as the frequency of the induced voltage in the rotor changes, the reactance of the rotor circuit also changes

$$\boxed{\begin{aligned} X_r &= \omega_r L_r = 2\pi f_r L_r \\ &= 2\pi s f_e L_r \\ &= sX_{r0} \end{aligned}}$$

Where X_{r0} is the rotor reactance at the supply frequency (at blocked rotor)

Equivalent Circuit

- Then, we can draw the rotor equivalent circuit as follows



Where E_R is the induced voltage in the rotor and R_R is the rotor resistance

Equivalent Circuit

- Now we can calculate the rotor current as

$$I_R = \frac{E_R}{(R_R + jX_R)}$$

$$= \frac{sE_{R0}}{(R_R + jsX_{R0})}$$

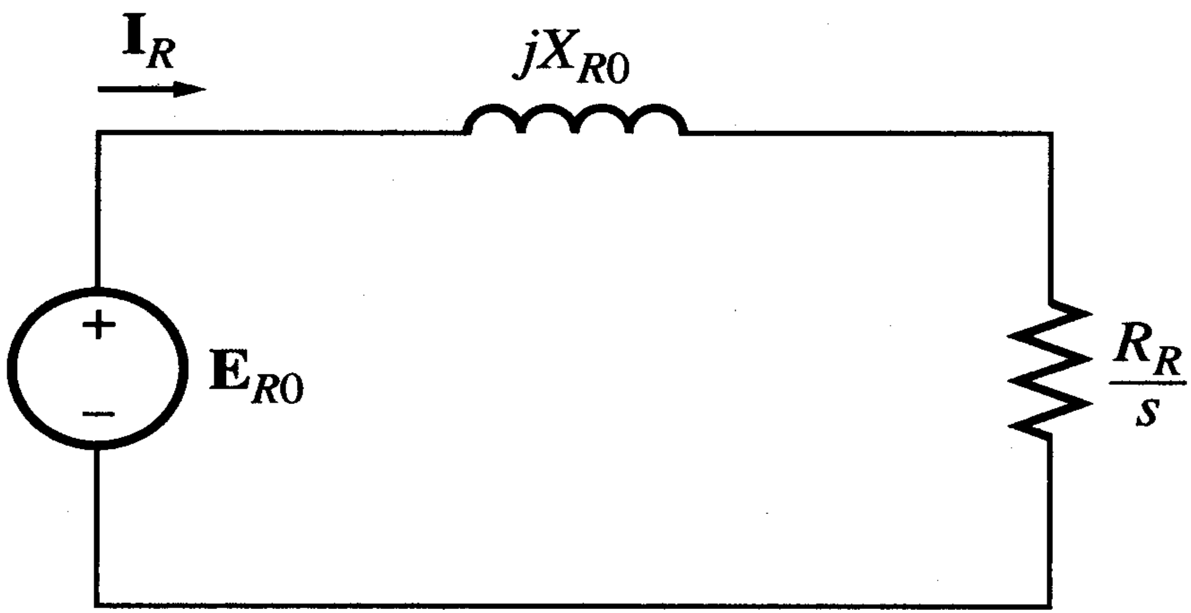
- Dividing both the numerator and denominator by *s* so nothing changes we get

$$I_R = \frac{E_{R0}}{\left(\frac{R_R}{S} + jX_{R0}\right)}$$

Where E_{R0} is the induced voltage and X_{R0} is the rotor reactance at blocked rotor condition (s = 1)

Equivalent Circuit

- Now we can have the rotor equivalent circuit



Equivalent Circuit

- Now as we managed to solve the induced voltage and different frequency problems, we can combine the stator and rotor circuits in one equivalent circuit

Where

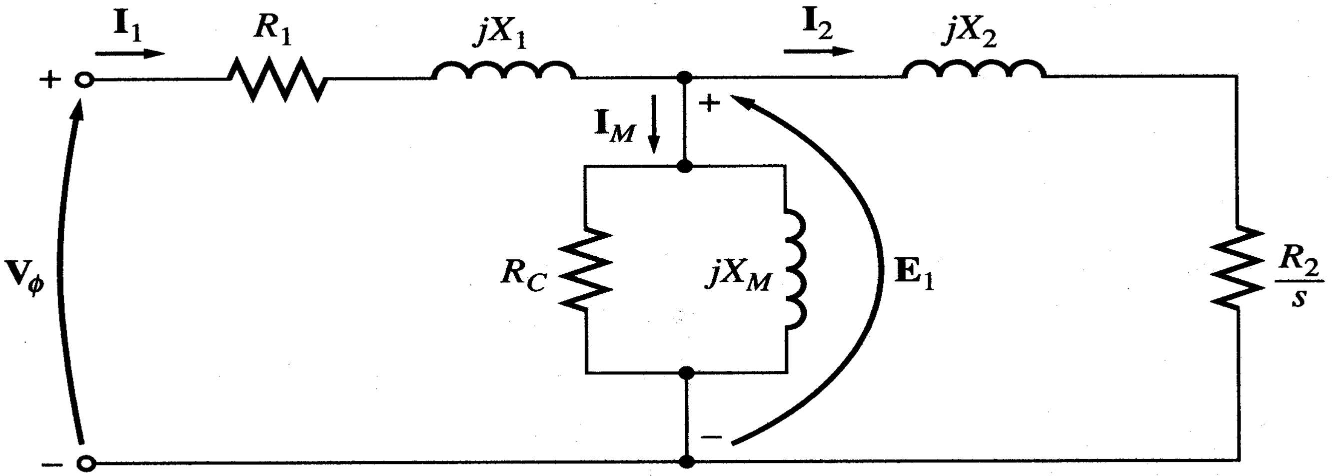
$$X_2 = a_{eff}^2 X_{R0}$$

$$R_2 = a_{eff}^2 R_R$$

$$I_2 = \frac{I_R}{a_{eff}}$$

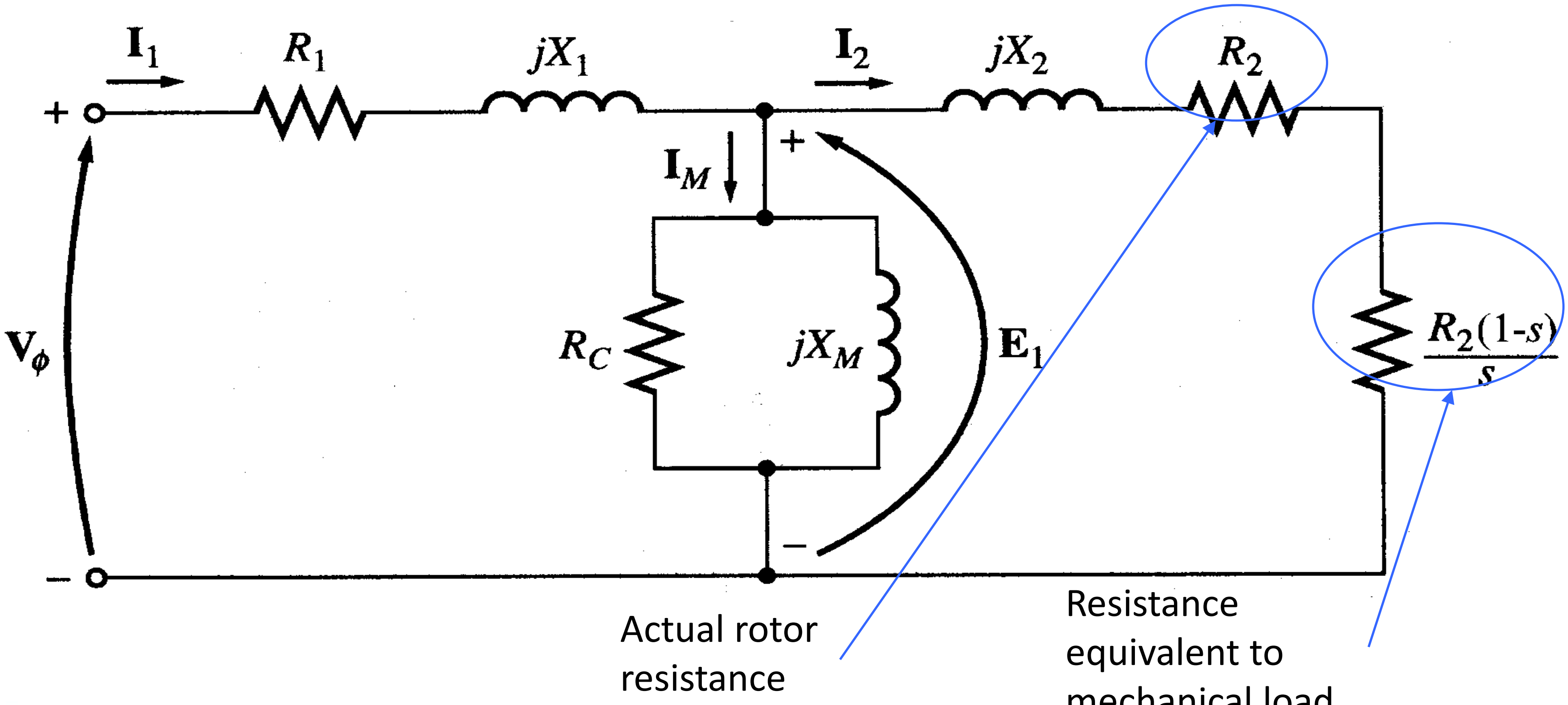
$$E_1 = a_{eff} E_{R0}$$

$$a_{eff} = \frac{N_S}{N_R}$$

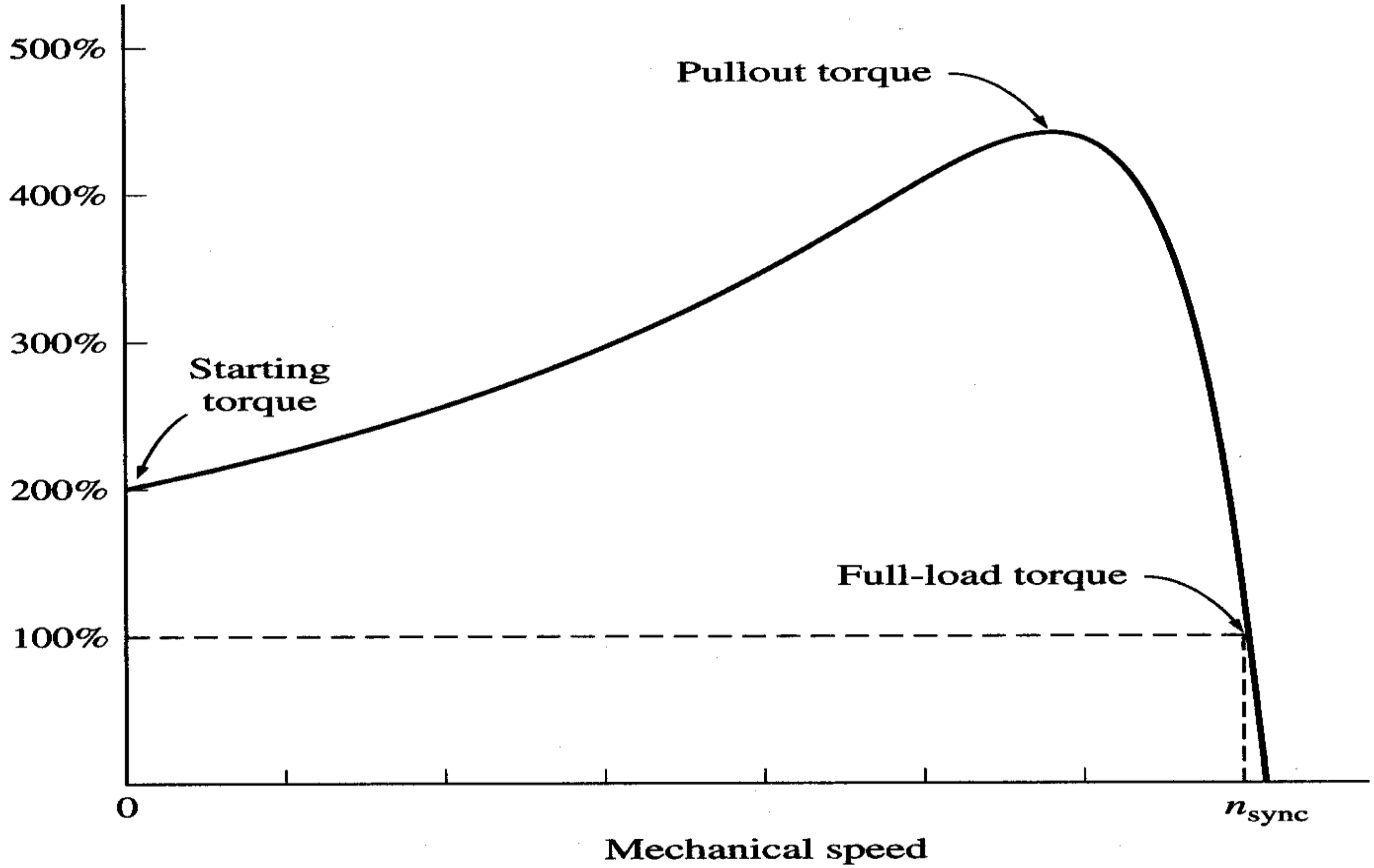


Equivalent Circuit

- We can rearrange the equivalent circuit as follows



Torque-speed characteristics



Typical torque-speed characteristics of induction motor

Torque-speed characteristics

1. Induced torque is zero at synchronous speed. Discussed earlier.
2. Curve is nearly linear between no-load and full load. In this range, the rotor current, torque increase linearly with the slip.
3. Maximum possible torque is called as *pullout torque* and is 2 to 3 times the rated full-load torque.

Torque-speed characteristics

4. Starting torque of the motor is slightly higher than its full-load torque, so the motor will start carrying any load it can supply at full load.
5. The torque of the motor for a given slip varies as the square of the applied voltage.
6. If the rotor is driven faster than synchronous speed it will run as a generator, converting mechanical power to electric power.

Torque-Equation

- Torque, can be derived from power equation in term of mechanical power or electrical power.

$$Power, P = \omega T, \; where \; \omega = \frac{2\pi n}{60} (rad/s)$$

$$Hence, T = \frac{60P}{2\pi n}$$

$$Thus,$$

$$Mechanical\ Torque, T_m = \frac{60P_m}{2\pi n_r}$$

$$Output\ Torque, T_o = \frac{60P_o}{2\pi n_r}$$

Torque-Equation

- **Note that, Mechanical torque can written in terms of circuit parameters. This is determined by using approximation method**

$$P_m = 3I_R'^2 \frac{R_R'}{s}(1-s) \; and \; P_m = \omega_r T_m$$

$$\therefore T_m = \frac{P_m}{\omega_r} = \left[\frac{3I_R'^2 \frac{R_R'}{s}(1-s)}{\omega_r} \right]$$

...

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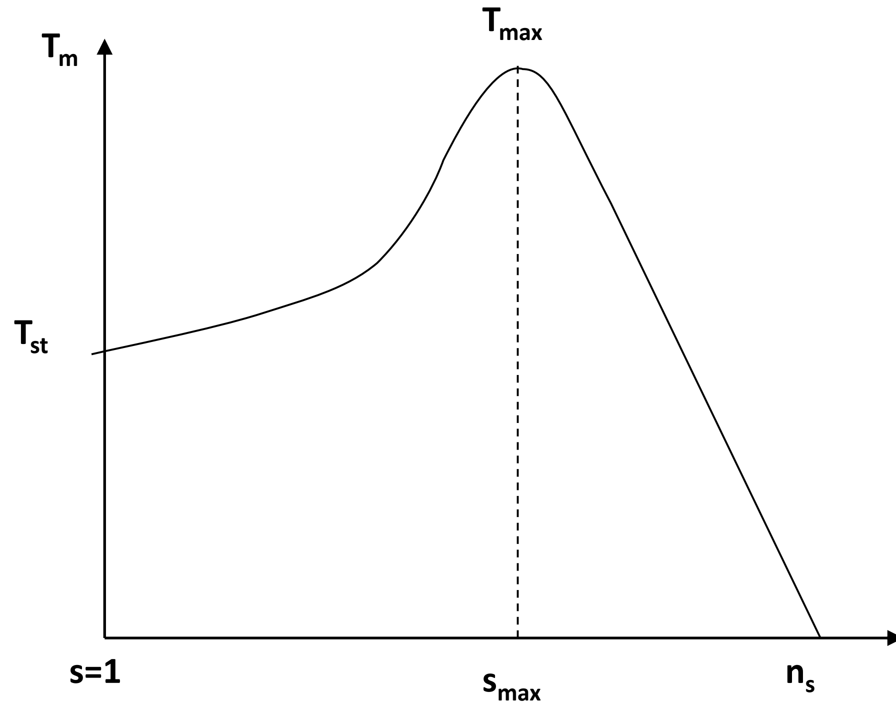
$$\therefore T_m = \left[\frac{3(V_{RM\phi})^2}{2\pi n_s} \right] \left[\frac{sR_R'}{(R_R')^2 + (sX_R')^2} \right]$$

Hence, Plot T_m vs s



s_{max} is the slip for T_{max} to occur

Torque-Equation

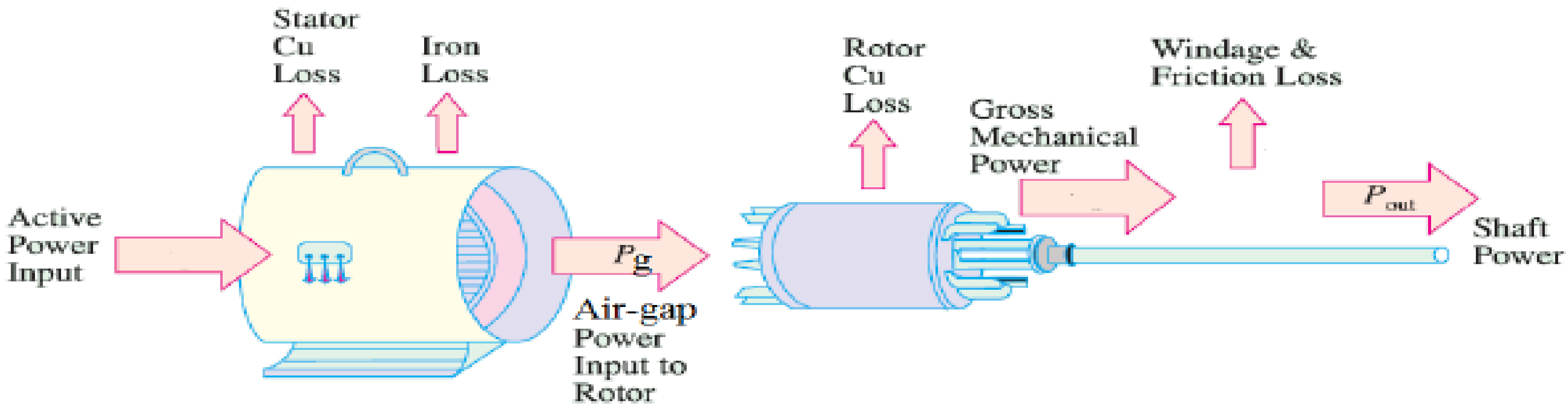
$$Starting\ Torque, s = 1$$

$$\therefore T_{st} = \left[\frac{3(V_{s\phi})^2}{2\pi\left(\frac{n_s}{60}\right)}\right]\left[\frac{R_R'}{(R_s + R_R')^2 + (X_s + X_R')^2}\right]$$

$$s_{\max} = \pm\left[\frac{R_R'}{\sqrt{(R_s)^2 + (X_R')^2}}\right]$$

$$T_{\max} = \left[\frac{3(V_{s\phi})^2}{2\left[2\pi\left(\frac{n_s}{60}\right)\right]}\right]\left[\frac{1}{R_s + \sqrt{(R_s)^2 + (X_s + X_R')^2}}\right]$$

Power Flow of Induction Motor



Summary

- Equivalent Circuit of three phase Induction Motor is developed.
- Behavior of Torque-speed characteristics are analyzed.
- Power Flow of Induction Motor

