Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 16:
Problems on Coplanar Non-Concurrent Force Systems

Delivered By: Dr. T. Valsa Ipe

Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

- Describe Varignon's Theorem
- Apply the method of resolution and find the resultant of coplanar non concurrent force system
- Solve the unknown forces given the resultant of non concurrent force system
- Calculate the angles of applied forces given the resultant of concurrent force system

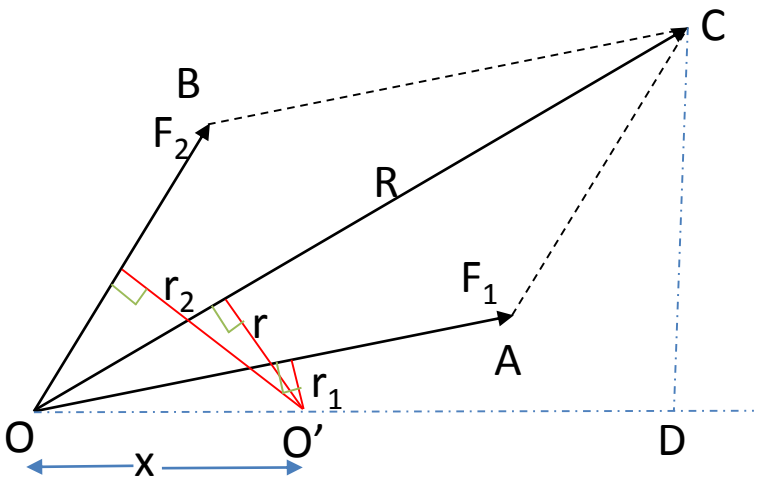
Contents

Varignon's theorem, coplanar non concurrent force system, problems on coplanar non concurrent force system

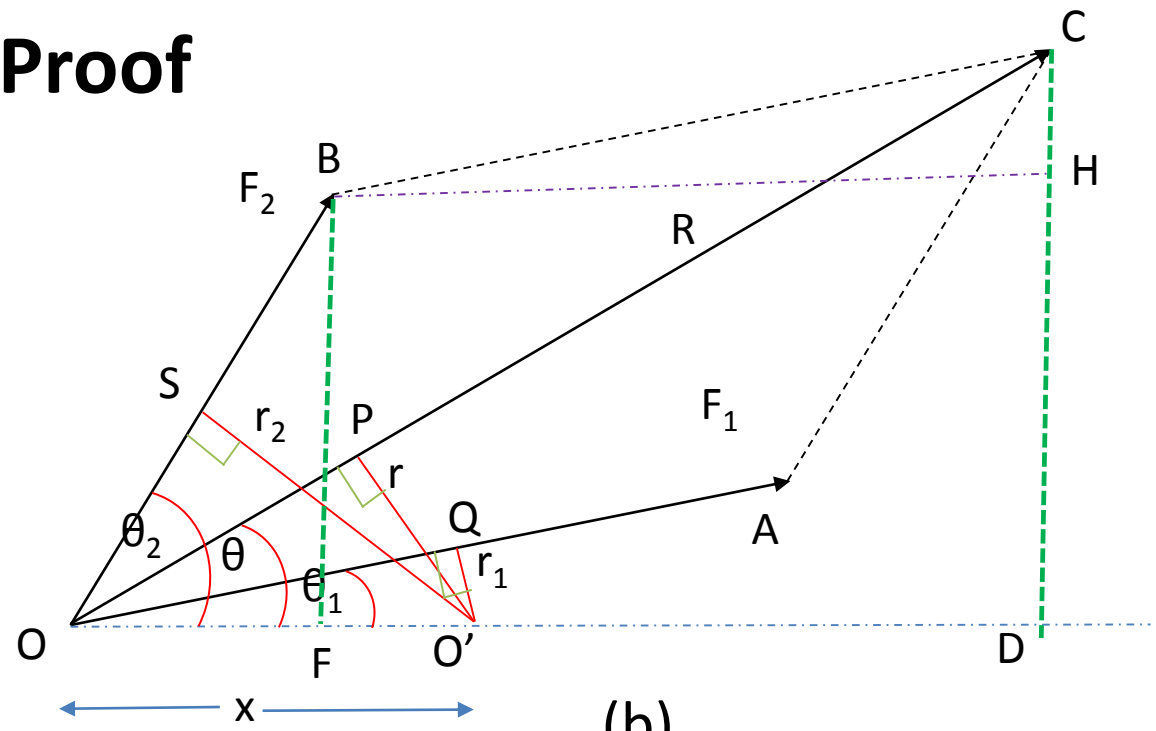
Varignon's Theorem

- This is also known as the principle of moments
- Principle of moments states that "the algebraic sum of the moments of individual forces of a force system about a point is equal to the moment of their resultant about the same point"
- Or The moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

Varignon's Theorem - Proof



(a)



(b)

- Let R be the resultant of forces F_1 and F_2 and O' be the moment centre
- Let r, r_1 and r_2 be the moment arms of forces R, F_1 and F_2 respectively , from the moment centre O'
- According to Varignon's principle,

  Moment of R about O' = Algebraic sum of moments of forces F_1 and F_2 about O'

Varignon's Theorem - Proof

$R.r = F_1 r_1 + F_2 r_2$.....(1)

From Fig

θ_1 – Angle made by F_1 with OD

θ_2 – Angle made by F_2 with OD

θ – Angle made by R with OD

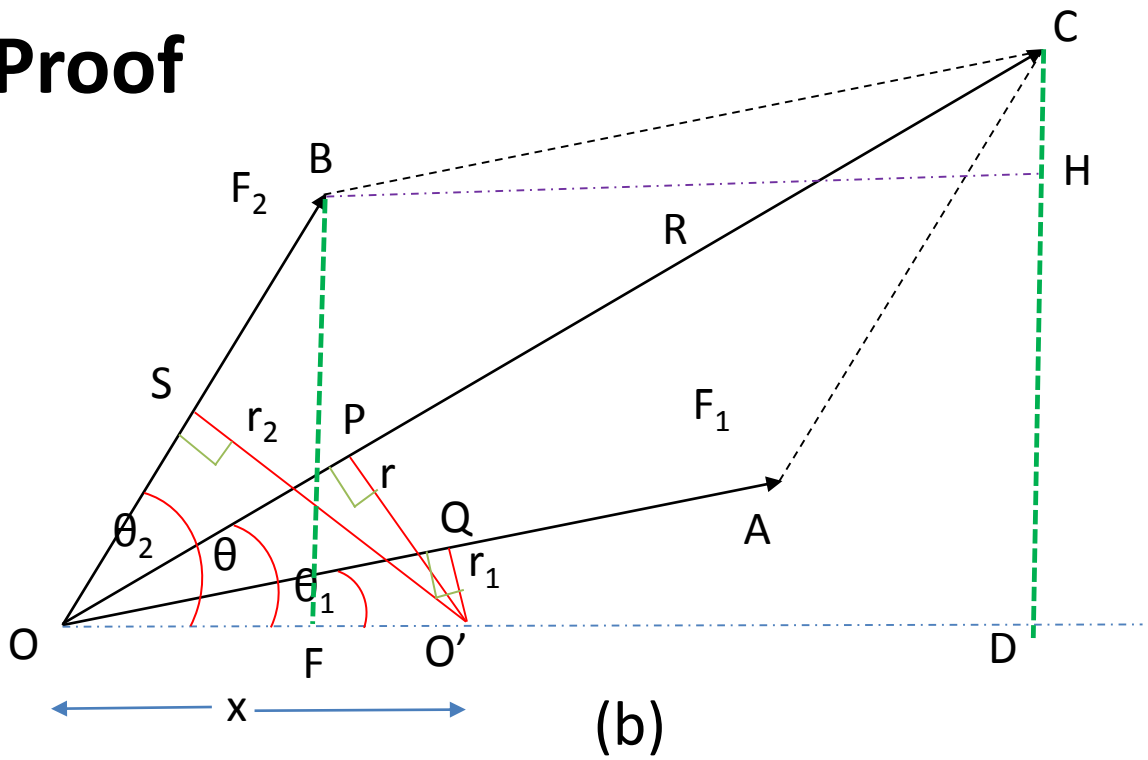
x – Distance between O and O'

From O O'P, $r = x sin\theta$,

From O O'Q, $r_1 = x sin\theta_1$

From O O'S, $r_2 = x sin\theta_2$

Substituting in (1)



(b)

$$R.xsin\theta = Rsin\theta.x$$

$$\text{But, } \frac{CD}{R} = sin\theta$$

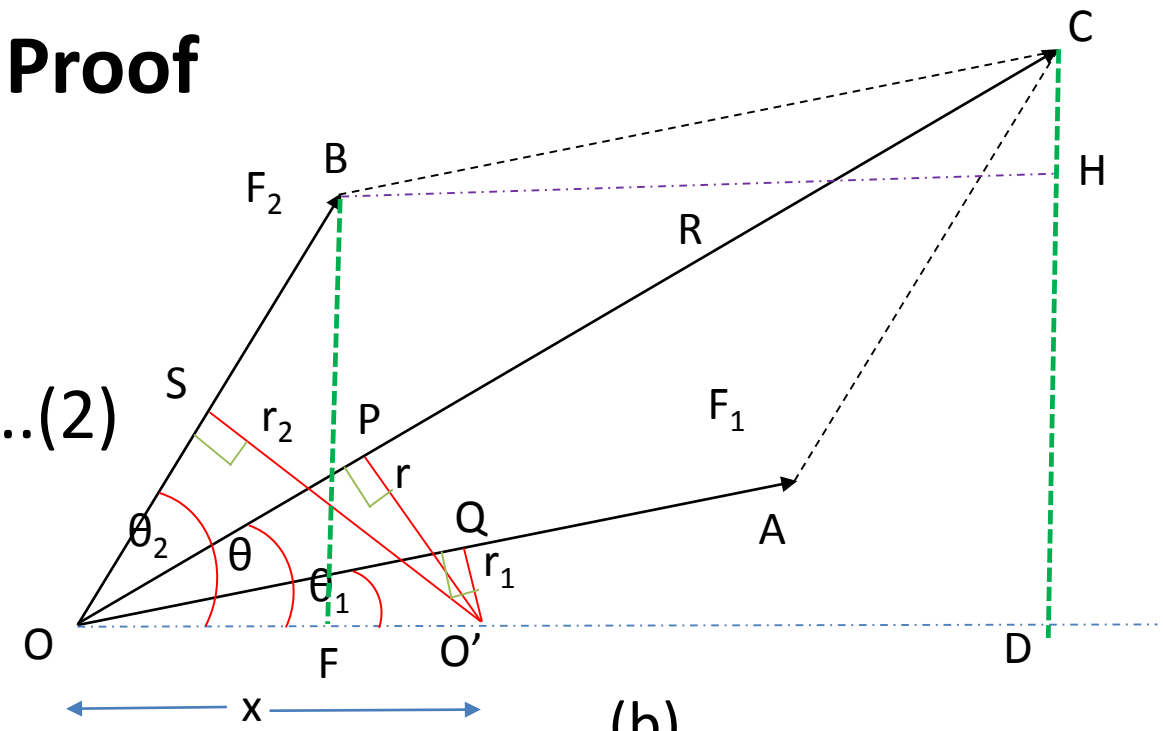
$$CD = Rsin\theta$$

Varignon's Theorem - Proof

$$R.r = Rsin\theta.x$$
$$R.r = CD.x$$
$$R.r = (CH + HD).x \text{ .....(2)}$$

From BHC, $CH = F_1 sin\theta_1$

HD = BF



(b)

From OFB, $BF = F_2 sin\theta_2 = HD$

Substituting in (2)

$$R.r = (F_1 sin\theta_1 + F_2 sin\theta_2).x$$
$$R.r = (F_1 sin\theta_1.x + F_2 sin\theta_2.x)$$
$$R.r = (F_1 sin\theta_1.x + F_2 sin\theta_2.x)$$
$$R.r = (F_1 r_1 + F_2 r_2)$$

Problems on Coplanar Non-Concurrent Force Systems

Tips to solve the Problems

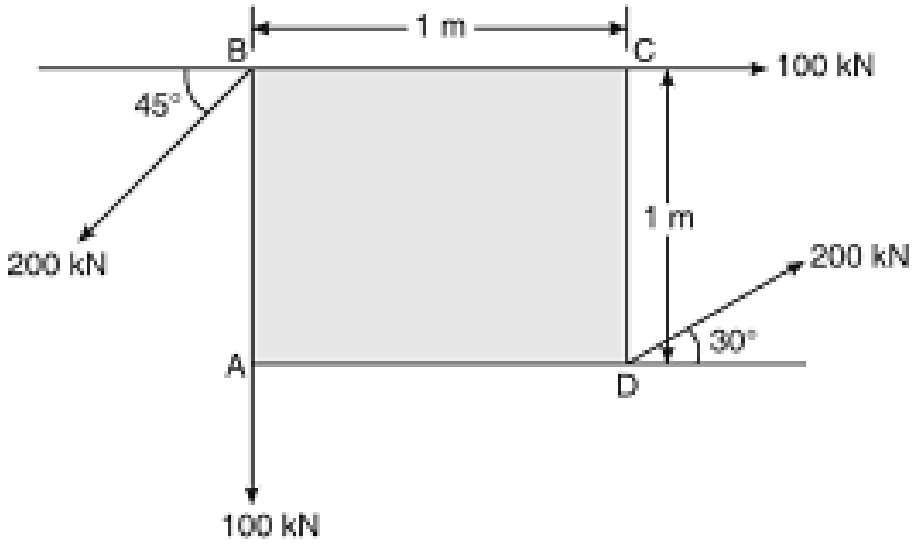
- In a coplanar non-concurrent force system, the magnitude, direction and position of resultant can be determined.
- Calculate the algebraic sum of all the forces acting in the x-direction (i.e ΣF_x) and also in the y-direction (i.e ΣF_y)
- Determine the magnitude of the resultant using the formula ,

  R= $\sqrt{\Sigma F_X{}^2 + \sum F_Y^2}$
- Determine the direction of the resultant using the formula $\theta=\tan^{-1}(\frac{\Sigma Fy}{\Sigma Fx})$
- The position of resultant can be determined by using the Varignon's theorem or using the formulae d=$\left|\frac{\Sigma M}{R}\right|$, x-intercept=$\left|\frac{\Sigma M_A}{\Sigma F_y}\right|$,

  y-intercept=$\left|\frac{\Sigma M_A}{\Sigma F_x}\right|$

Problems

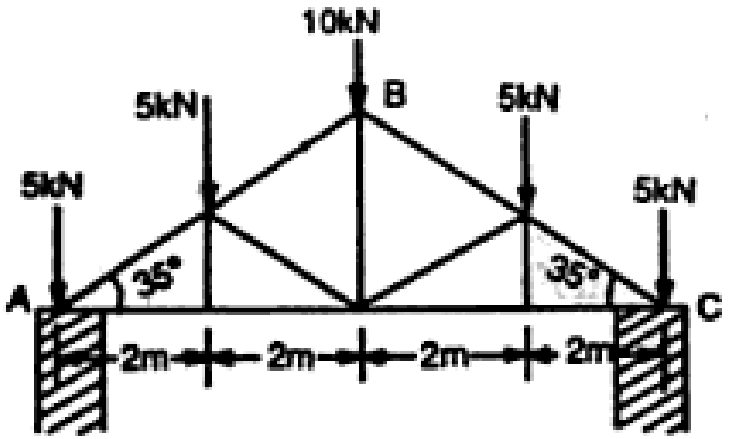
1.A rigid plate ABCD is subjected to forces as shown in figure. Compute the magnitude, direction and line of action of the resultant of the system with reference to point A



R=193.30kN
d=0.732m

Problems

2.Find the resultant of the coplanar parallel forces acting on the truss as shown in the figure.

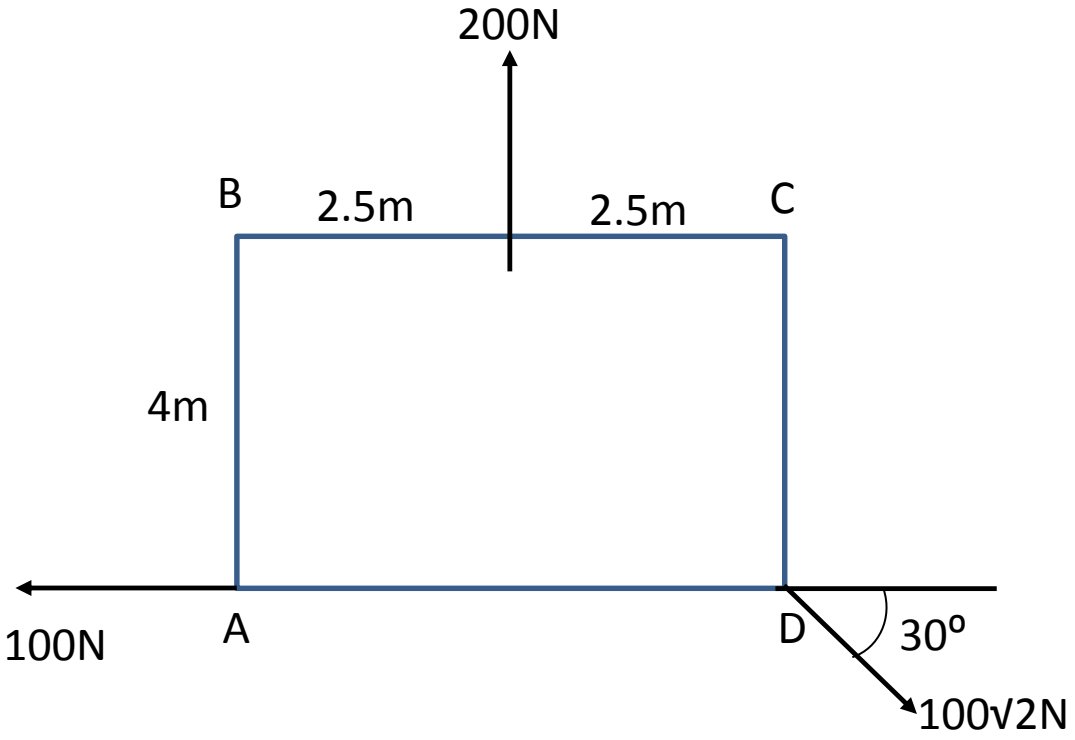


R=-30kN

d=4m

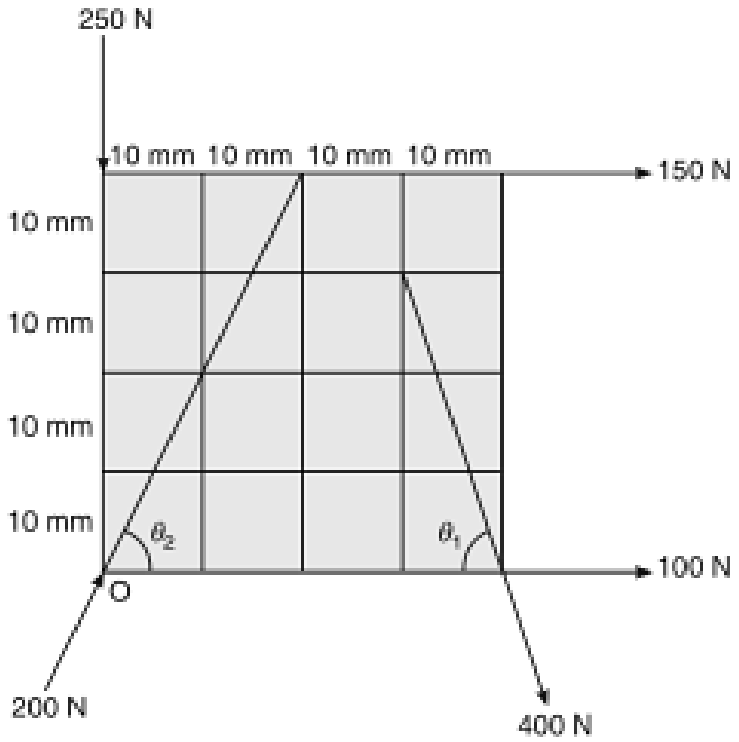
Problems

3.For the non-concurrent coplanar system shown in the figure, determine the magnitude , direction and position of the resultant force with reference to A.



Problems

4.Determine the resultant of a system of forces as shown in figure, acting on a 40mmx40mm size lamina. Each grid is of size 10mmx10mm. Determine the location also.



R=648.169N

d=33mm

Summary

- The forces can be resolved and resultant of coplanar non concurrent force system can be obtained
- Varignon's theorem:
  - The moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.