# Lecture-33
# Circuit model of a Transformer

Lecture delivered by:

# Topics

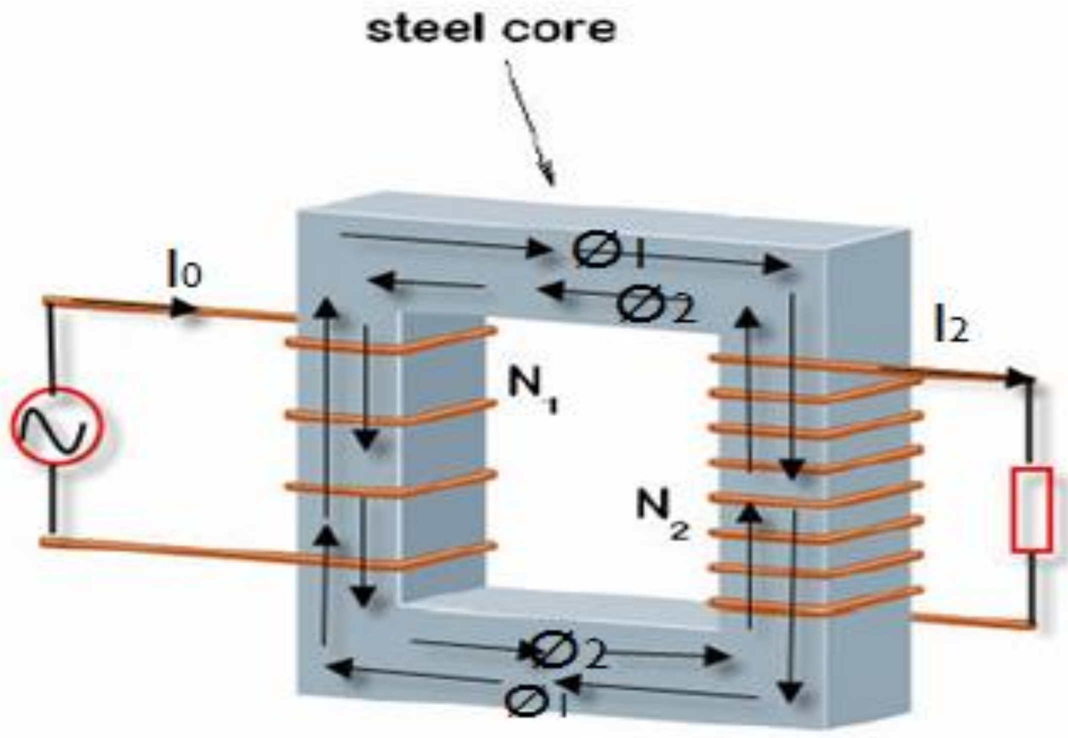
- Ideal Transformer under On Load
- Circuit Model of Transformer under No-Load condition
- Circuit Model of Transformer under Load condition

# Objectives

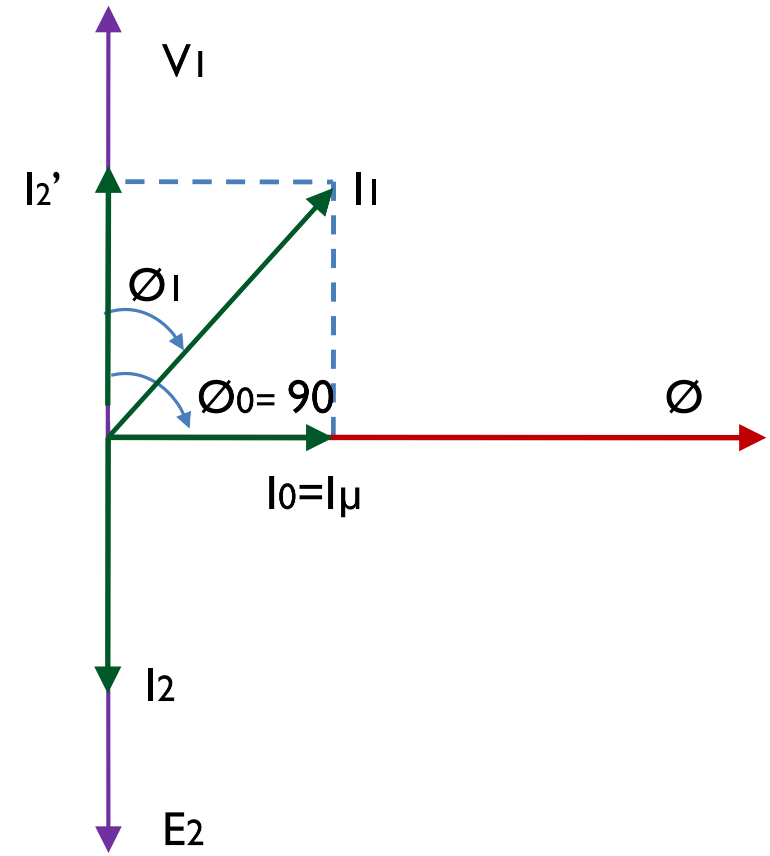
At the end of this lecture, student will be able to:

- State the meaning of a "no-load condition" and “on-load condition" relative to a transformer
- Describe the operation of ideal transformer under on-load condition
- Construct a transformer on-load phasor diagram for different loads
- Derive the equivalent resistance and reactance referred to the primary and secondary side of a transformer

# Ideal Transformer On Load

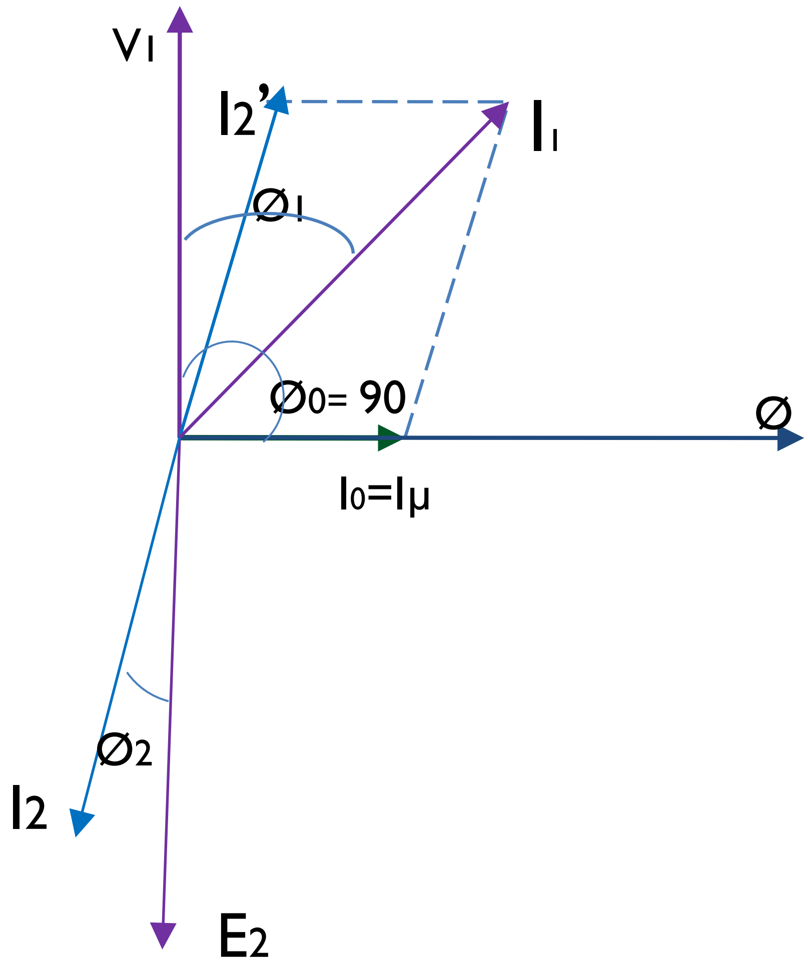


# Phasor Diagram of Ideal Transformer For Resistive Load :



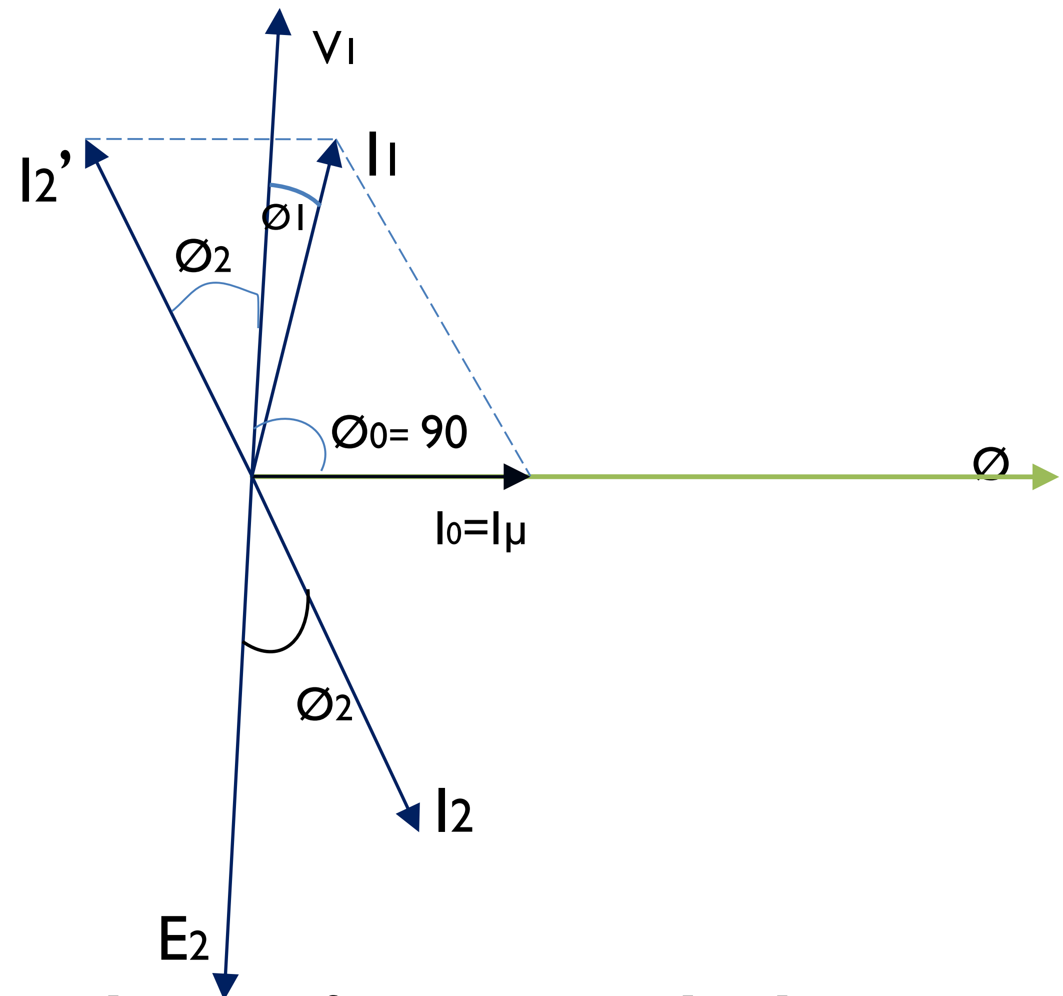

Phasor diagram for resistive load

# Phasor Diagram of Ideal transformer for inductive load




Phasor diagram for inductive load

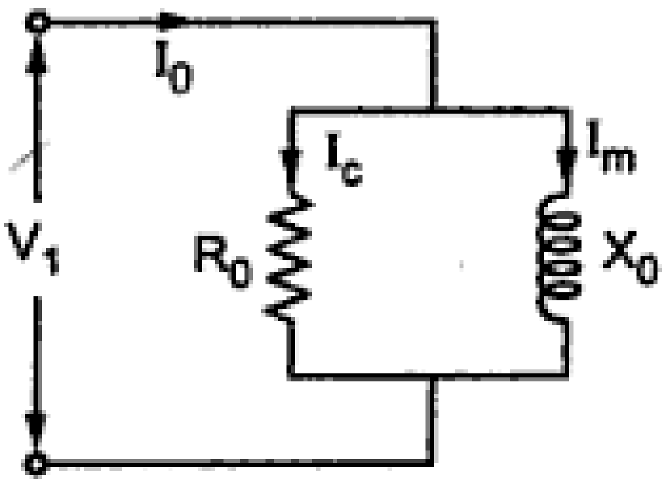
# Phasor Diagram of Ideal transformer for Capacitive load




Phasor diagram for capacitive load

# Equivalent Circuit of Transformer

## No Load Equivalent Circuit



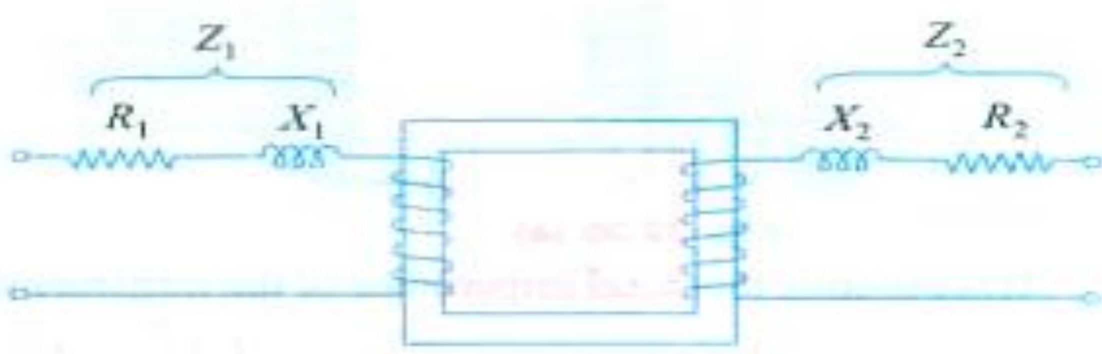

$$R_0 = \frac{V_1}{I_c}$$

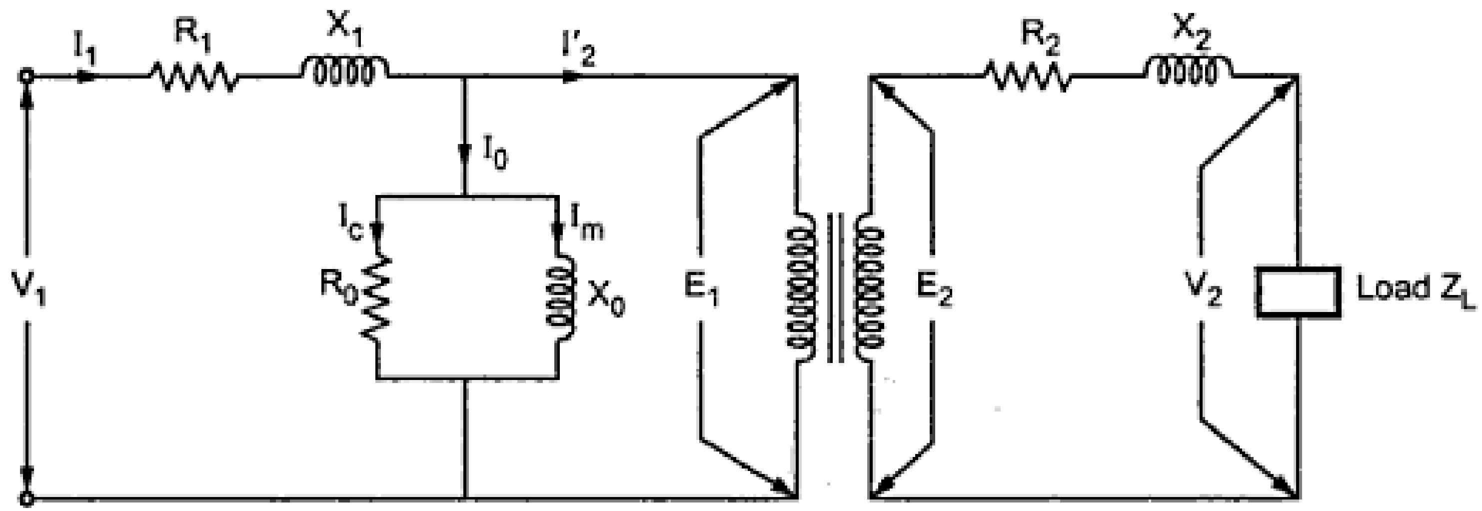
$$X_0 = \frac{V_1}{I_m}$$

$$I_m = I_0 \sin \phi_0 = \text{Magnetising component}$$

$$I_c = I_0 \cos \phi_0 = \text{Active component}$$

# Equivalent Circuit with Load



# Equivalent Circuit with Load Cont..

$$I_2^2 R_2 = I_1^2 R_2^{\,1}$$

$$\therefore R_2^{\,1} = \frac{I_2^2}{I_1^2} R_2$$

$$R_2^{\,1} = \frac{R_2}{K^2}$$

where $K = \dfrac{I_1}{I_2}$

The $R_2^{\,1}$ is called the secondary resistance referred to primary.

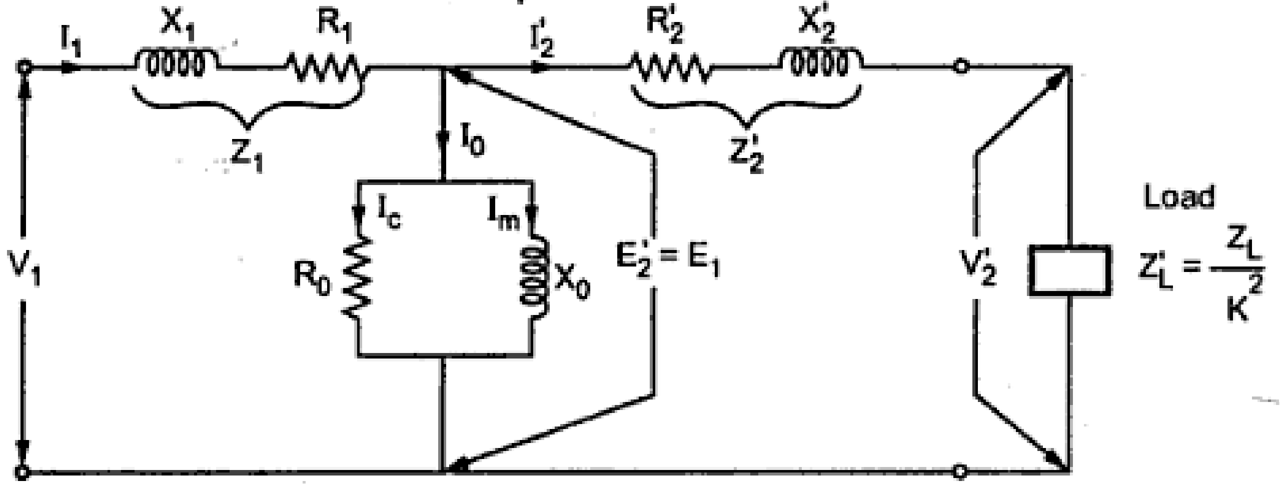
# Equivalent Circuit Referred to Primary Side

## Transferring Secondary Side Parameters to Primary Side

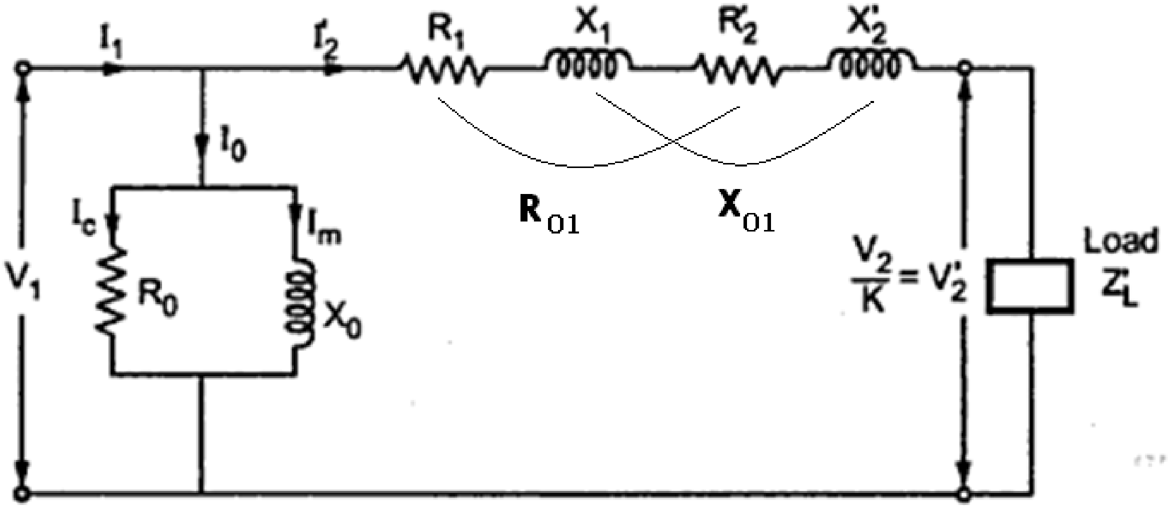
$$R_2' = \frac{R_2}{K^2}, \quad X_2' = \frac{X_2}{K^2}, \quad Z_2' = \frac{Z_2}{K^2}$$

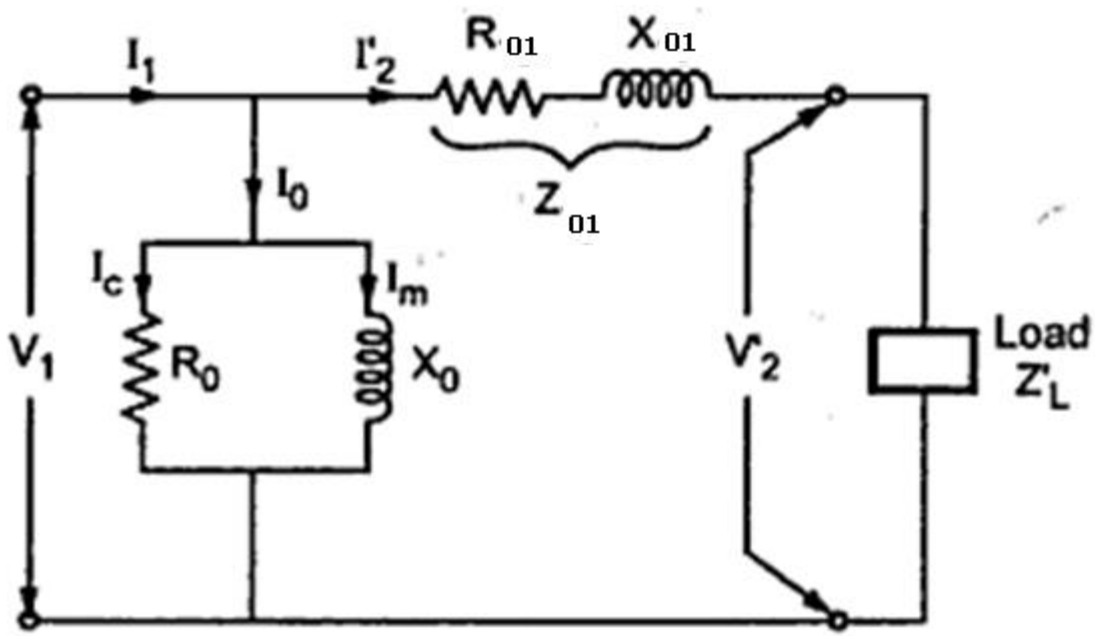
While $$E_2' = \frac{E_2}{K}, \quad I_2' = KI_2$$

where $$K = \frac{N_2}{N_1}$$



# Problem




where

$$R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}$$

$$X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2}$$

$$Z_{01} = R_{01} + j\,X_{01}$$

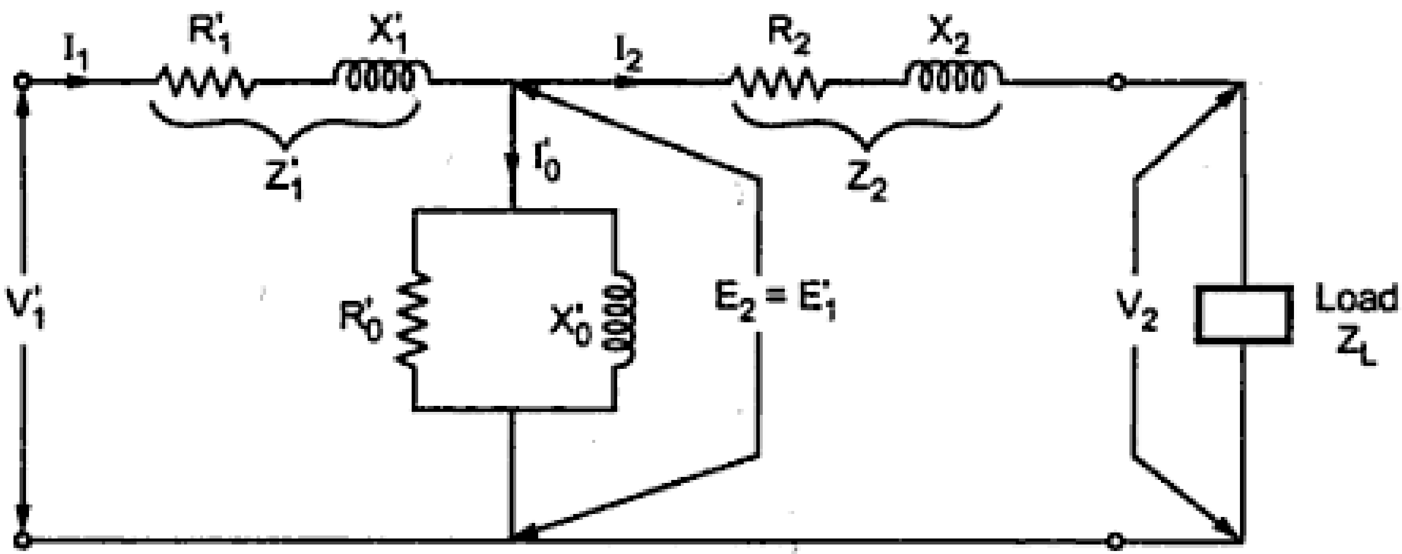
# Equivalent Circuit Referred to Secondary Side

Transferring Primary Side Parameters to Secondary Side

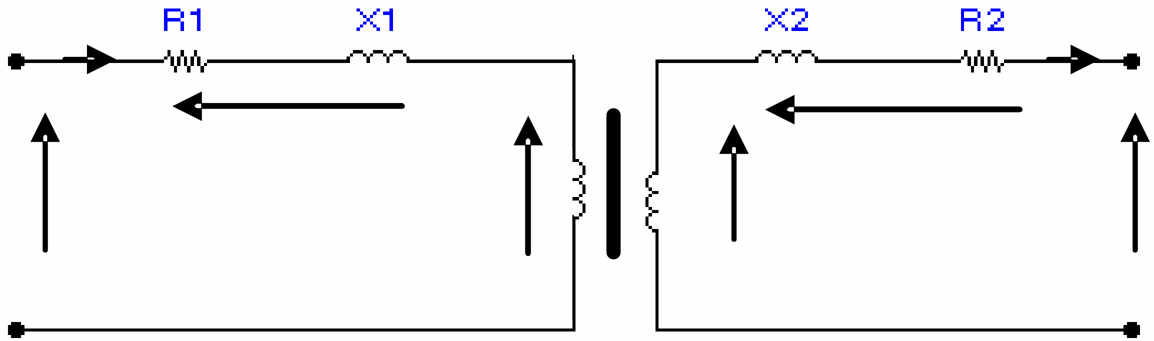
$$R_1' = K^2 R_1, \quad X_1' = K^2 X_1, \quad Z_1' = K^2 Z_1$$

$$E_1' = K E_1, \quad I_1' = \frac{I_1}{K}, \quad I_0' = \frac{I_0}{K}$$

Similarly exciting circuit parameters are also transferred to secondary as $R_o$' and $X_o$'



# Approximate Equivalent Circuit




$$I_1^2 R_{2'} = I_2^2 R_2$$

$$R_{2'} = R_2 \left( \frac{I_2}{I_1} \right)^2 \approx R_2 \left( \frac{V_1}{V_2} \right)^2$$

Similarly $$X_{2'} = X_2 \left( \frac{N_1}{N_2} \right)^2 \approx X_2 \left( \frac{V_1}{V_2} \right)^2$$

# Equivalent Circuit

$$R_e = R_1 + R_{2'} = R_1 + R_2\left(\frac{V_1}{V_2}\right)^2$$

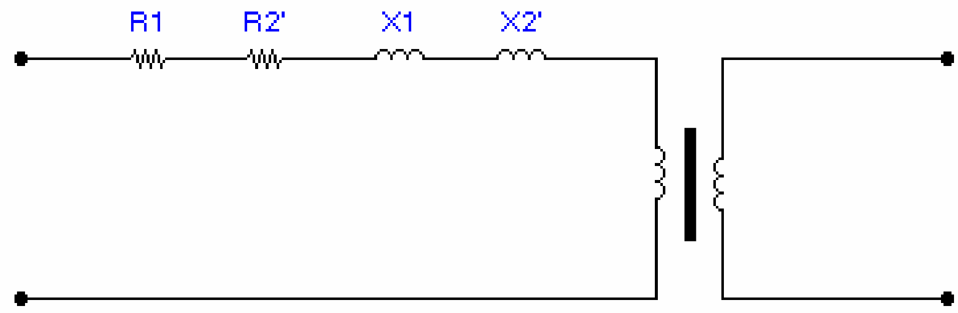
$$X_e = X_1 + X_{2'} = X_1 + X_2\left(\frac{V_1}{V_2}\right)^2$$

then $$Z_e = \sqrt{R_e^2 + X_e^2}$$

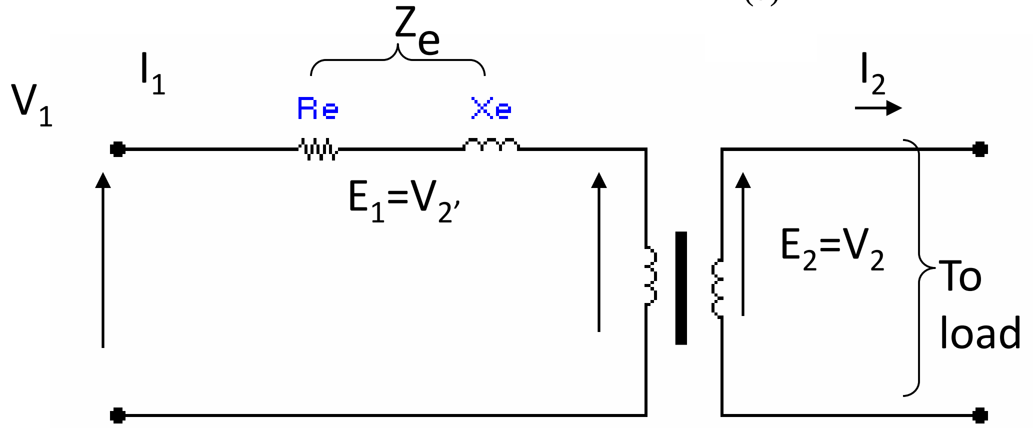
where $$R_e = Z_e \cos\phi_e$$

$$X_e = Z_e \sin\phi_e$$

and $$\tan\phi_e = \frac{\mathrm{X_e}}{\mathrm{R_e}}$$




(b)




Transformer simplified circuit

# Example

- A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3A at a power factor 0.2 lagging when secondary current is 280A at a power factor of 0.8 lagging. Calculate the primary current and the power factor. Assume the voltage drop in the windings to be negligible.

Solution:

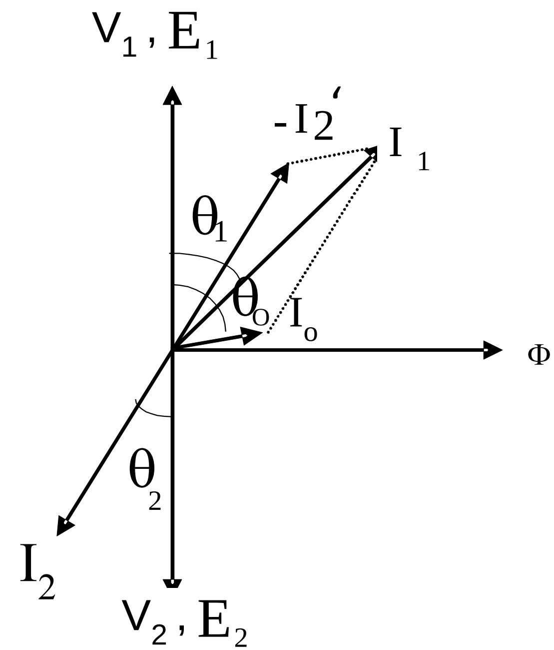
$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$I_P = \frac{N_S}{N_P} \times I_S = \frac{200}{1000} \times 280 = 56A$$

# Problem

$$\cos\phi_{2'} = 0.8 \quad \therefore \quad \sin\phi_{2'} = 0.6$$

$$\cos\phi_o = 0.2 \quad \therefore \quad \sin\phi_o = 0.98$$




Solve for horizontal and vertical components

$$I_1\cos\phi_1 = I_{2'}\cos\phi_{2'} + I_o\cos\phi_o$$

$$= (56\times 0.8) + (3\times 0.2) = 45.4A$$

$$I_1\sin\phi_1 = I_{2'}\sin\phi_{2'} + I_o\sin\phi_o$$

$$= (56\times 0.6) + (3\times 0.98) = 36.54A$$

$$I_1 = \sqrt{(45.4)^2 + (36.54)^2} = 58.3A$$

$$\tan\phi_1 = \frac{36.54}{45.4} = 0.805 \longrightarrow \phi_1 = 38^o 50'$$

Power factor $\cos\phi_1 = \cos 38^o 50' = 0.78\ lagging$

# Summary

- Operation of ideal transformer under on-load condition
- Phasor diagrams of the transformer at different loads
- Development of equivalent circuit of a transformer referred to primary side
- Development of equivalent circuit of a transformer referred to secondary side