Circuit Analysis Techniques

Lecture 8
Circuit Element Impedances

Lecture delivered by:

Topics

- Maximum Power Transfer Theorem
- Sinusoidal Forcing Function
- Advantages of Sinusoidal Forcing Function
- Complex Numbers
- Complex Forcing Function
- Impedance and admittance
- Circuit element impedances

Objectives

At the end of this lecture, student will be able to:

- State and compute Maximum Power Transfer Theorem
- Describe Sinusoidal Forcing Function
- Solve Complex Forcing Function
- Define Impedance and admittance

Maximum Power Theorem

- Maximum Power Theorem states that, “The maximum amount of power will be dissipated by a load resistance when that load resistance is equal to the source resistance of the network supplying the power. If the load resistance is lower or higher than the source resistance of the source network, its dissipated power will be less than maximum”

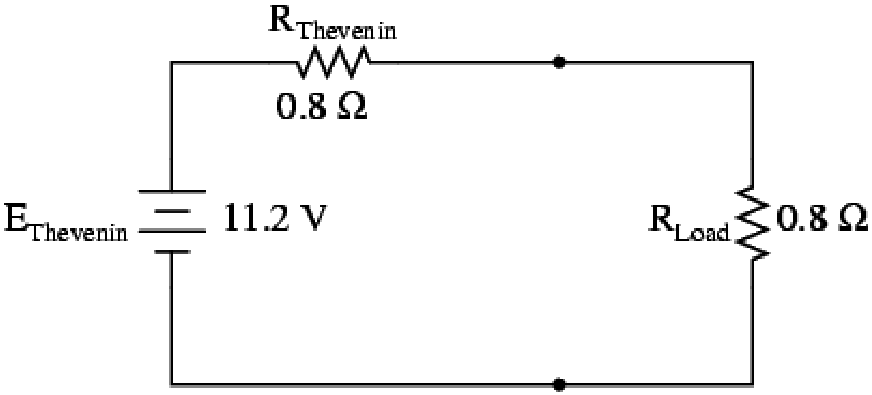
Maximum Power Theorem

- Considering Thevenin equivalent example circuit, and verify Maximum Power Transfer Theorem considering three cases
  - ✓ Load resistance equal to source resistance
  - ✓ Load resistance is greater than source resistance
  - ✓ Load resistance is less than source resistance

Load Resistance Equal To Source Resistance

- When $R_L = R_S = 0.8\Omega$

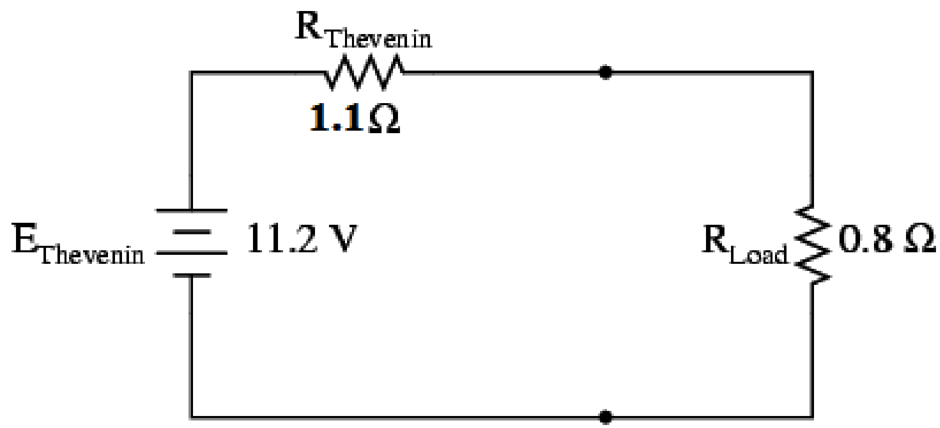
	$R_{Thevenin}$	R_{Load}	Total	
E	5.6	5.6	11.2	Volts
I	7	7	7	Amps
R	0.8	0.8	1.6	Ohms
P	39.2	**39.2**	78.4	Watts



Load Resistance Is Greater Than Source Resistance

- When R_L = 1.1 'Ω R_S = 0.8'Ω

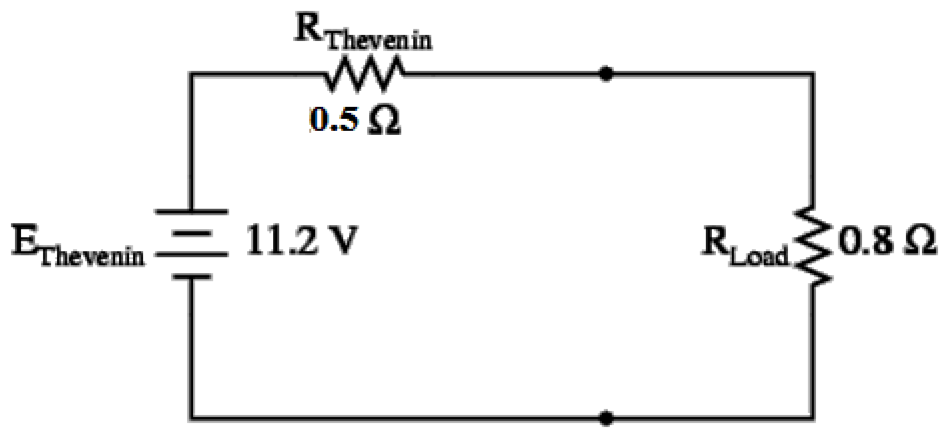
	$R_{Thevenin}$	R_{Load}	Total	
E	4.716	6.484	11.2	Volts
I	5.895	5.895	5.895	Amps
R	0.8	1.1	1.9	Ohms
P	27.80	38.22	66.02	Watts



Load Resistance Is Less Than Source Resistance

- When R_L = 0.5 ′Ω R_S = 0.8′Ω

	$R_{Thevenin}$	R_{Load}	Total	
E	6.892	4.308	11.2	Volts
I	8.615	8.615	8.615	Amps
R	0.8	0.5	1.3	Ohms
P	59.38	37.11	96.49	Watts

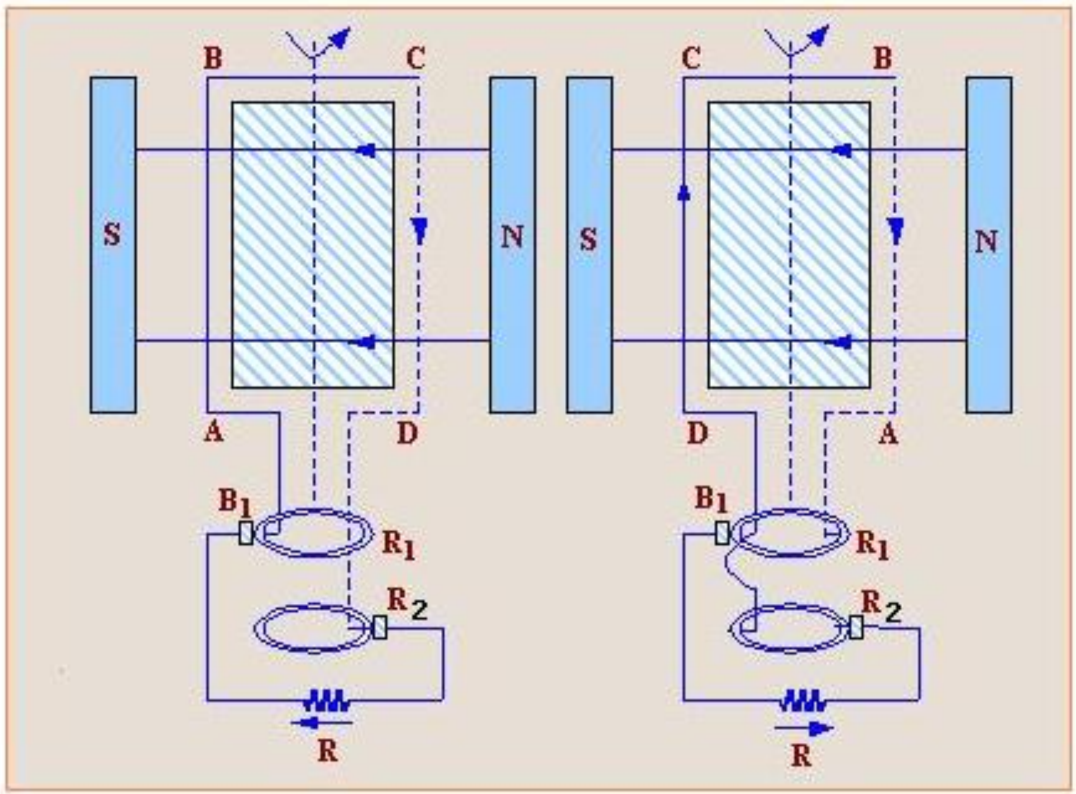


AC Dynamo

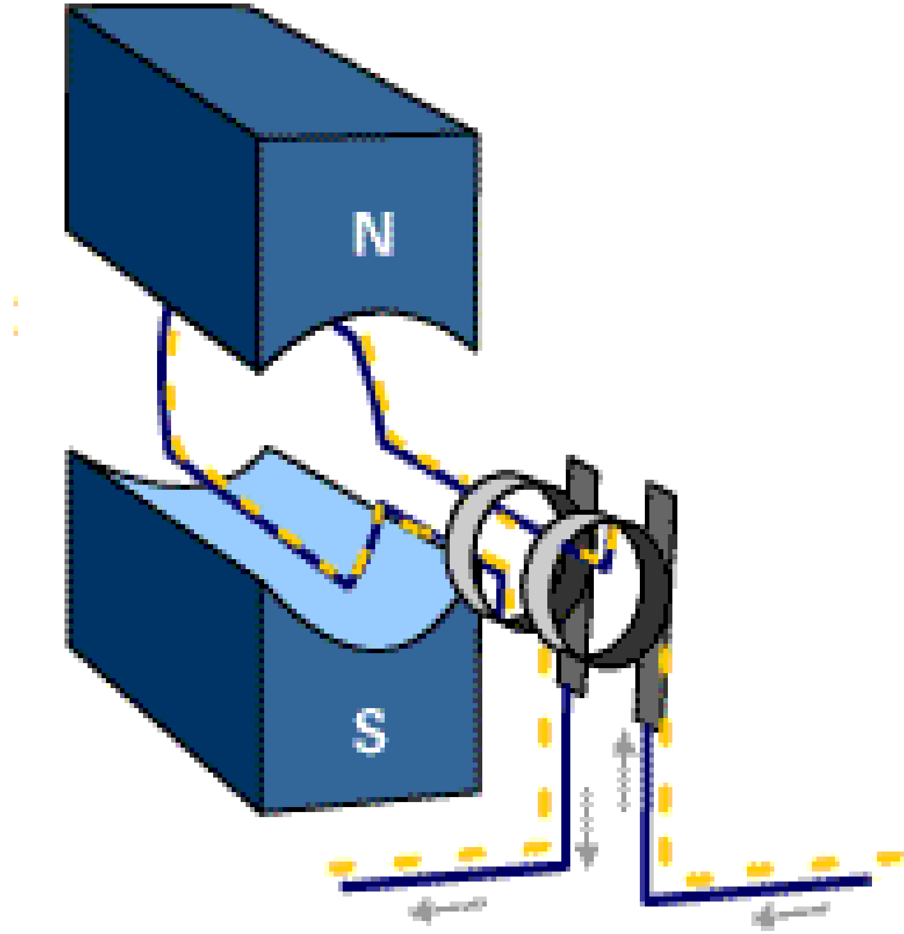
- AC Dynamo is based on the phenomenon of electromagnetic induction. That is, when the relative orientation between the coil and the magnetic field changes, the flux linked with the coil changes and this induces a current in the coil, Sinusoidal wave is generated.

Working of AC Dynamo

- http://www.youtube.com/watch?v=WXPvysew69Y

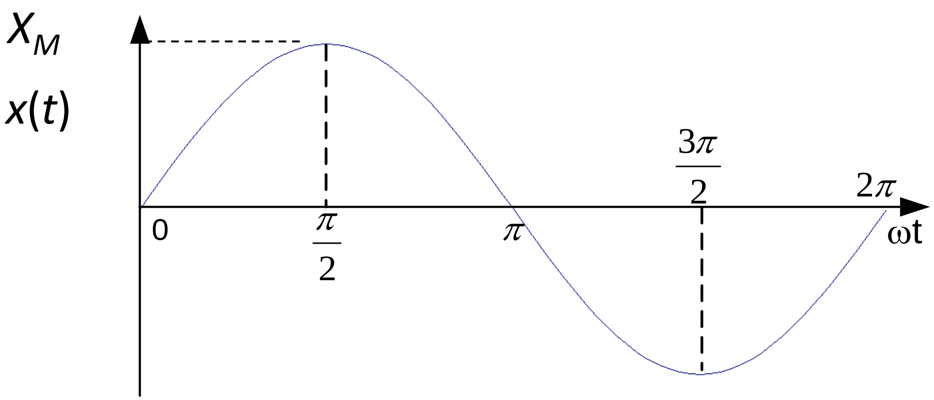


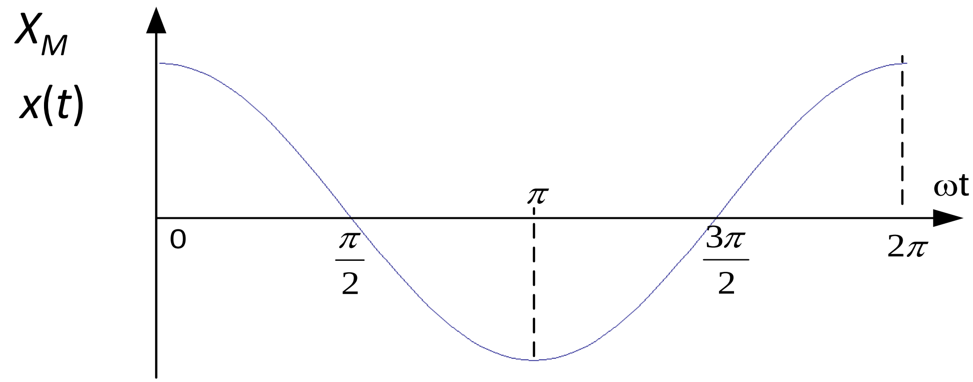
Simple Loop Generator



Sinusoidal Forcing Function

- Sinusoidal forcing function is a signal that has the form of the sine or cosine function





$$x(t) = X_M \sin \omega t$$

$$x(t) = X_M \cos \omega t$$

$$\text{period} \quad T = 2\pi/\omega$$

$$\sin\frac{\pi}{2} = 1 \qquad \cos\frac{\pi}{2} = 0$$

Advantages of Sinusoidal Forcing Function

- A sinusoidal current or voltage is usually referred to as an alternating current or voltage and circuits excited by sinusoids are called ac circuits
- Sinusoids are important for several reasons

  I. Nature itself is characteristically sinusoidal
  II. A sinusoidal current or voltage is easy to generate.
  III. Through Fourier Analysis, any practical periodic signal can be represented by a sum of sinusoids.
  IV. A sinusoid is easy to handle mathematically; its derivative and integral are also sinusoids.
  V. When a sinusoidal source is applied to a linear circuit, the steady-state response **is also sinusoidal**, and we call the response the **sinusoidal steady-state response.**

Complex Forcing Function

- Consider a sinusoidal forcing function given as a complex function:

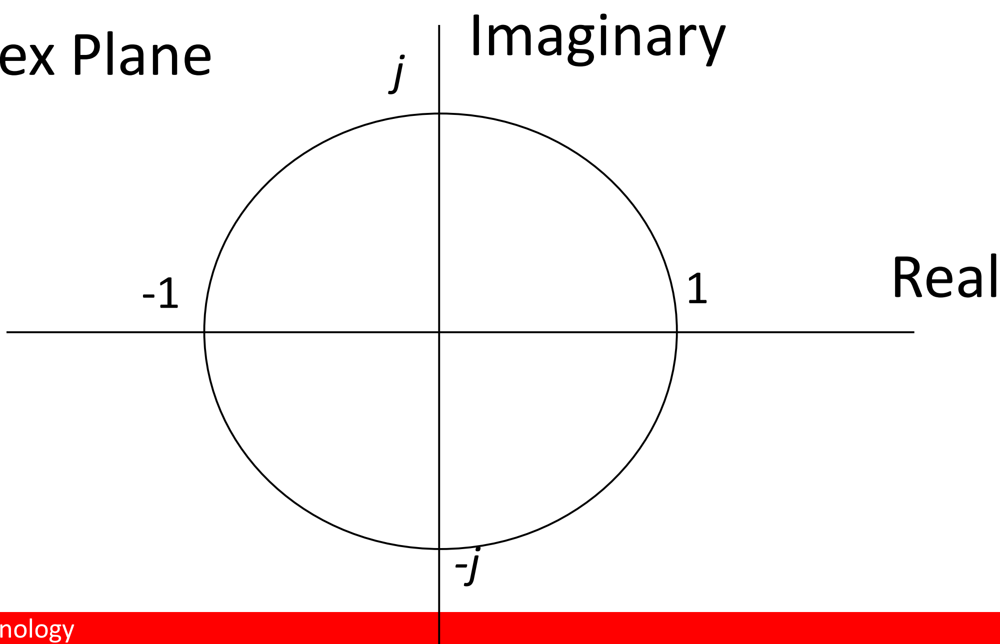
$$X_m \exp(j(\omega t+\theta)) = X_m \cos(\omega t+\theta) + jX_m \sin(\omega t+\theta)$$

What is the solution of

$X^2 = -1$

$$X = \pm\sqrt{-1} = \pm j \qquad j = \sqrt{-1} \qquad j^2 = -1$$

Complex Plane



Note:

$$\frac{1}{j} = \frac{j}{jj} = \frac{j}{j^2} = -j$$

Complex Numbers

- Following mathematical operations are important

Addition

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

Multiplication

$$z_1 z_2 = r_1 r_2 \angle(\phi_1 + \phi_2)$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle(\phi_1 - \phi_2)$$

Reciprocal

$$\frac{1}{z} = \frac{1}{r} \angle(-\phi)$$

Square Root

$$\sqrt{z} = \sqrt{r} \angle(\phi / 2)$$

Complex Conjugate

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

Complex Numbers

Example

- Evaluate the following complex numbers:

a. $[(5+j2)(-1+j4)-5\angle 60^{o}]$

b. $\dfrac{10+j5+3\angle 40^{o}}{-3+j4}+10\angle 30^{o}$

Solution:

a. –15.5 + j13.67

b. 8.293 + j2.2

Impedance

- Define the impedance $\underline{Z}$, of a circuit as:

$$\underline{Z} = \frac{\underline{V}}{\underline{I}} \Rightarrow \underline{V} = \underline{I} \cdot \underline{Z}$$

- **Notes:**
  - Impedance defines the relationship between the voltage and current Phasor's
  - The above equations are identical in form to Ohm's Law
  - Units of impedance are ohms (Ω)

Impedance

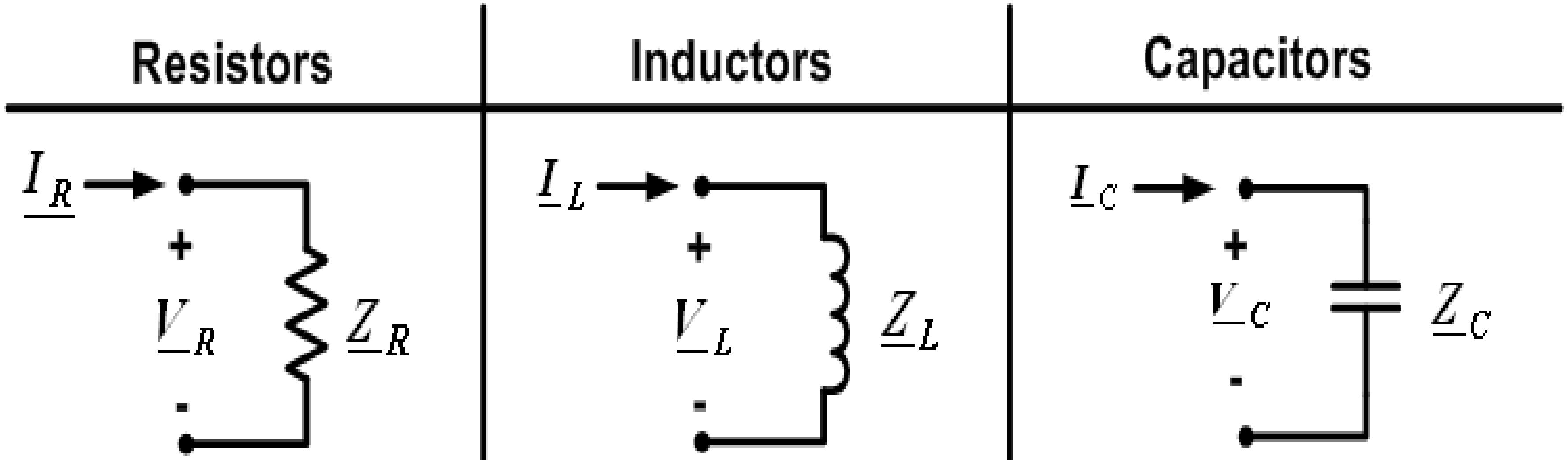
- Impedance is a **complex number**

$$Z = R + jX$$

- Where
- *R* is called the *resistance*
- *X* is called the *reactance*

Circuit element impedances

- Our phasor circuit element voltage-current relations can all be written in terms of impedances



$$Z_R = R \qquad Z_L = j\omega L \qquad Z_C = \frac{1}{j\omega C}$$

Admittance

- Admittance is the inverse of impedance

$$Y = \frac{1}{Z}$$

- Admittance is a complex number $Y = G + jB$

- Where
  - *G* is called the *conductance*
  - *B* is called the *susceptance*

Why are impedance and admittance useful?

- The analysis techniques we used for time domain analysis of resistive networks are applicable to phasor circuits
  - E.g. KVL, KCL, circuit reduction, nodal analysis, mesh analysis, Thevnin's and Norton's Theorems…

- To apply these methods:
  - Impedances are substituted for resistance
  - Phasor voltages, currents are used in place of time domain voltages and currents

Summary

- Maximum power transfer theorem is discussed
- Sinusoidal forcing function is a signal that has the form of the sine or cosine function
- Advantages of Sinusoidal Forcing Function is discussed
- A Complex Number is a combination of a Real Number and an Imaginary Number
- Impedance defines the relationship between the voltage and current Phasor's, admittance is reciprocal of impedance