Course Code: ESC106A

Course Title: Construction Materials and Engineering Mechanics

Lecture No. 9:

Composition of forces

Delivered By: Mr. Shrihari K. Naik

Lecture Intended Learning Outcomes

At the end of this lecture, student will be able to:

- Define composition of forces and resultant force
- Explain and prove parallelogram law of forces
- Apply parallelogram law of forces for specific angles
- Distinguish between resolution and composition of forces

Contents

- **Engineering Mechanics**

  Laws of parallelogram, composition of forces

Resultant Force

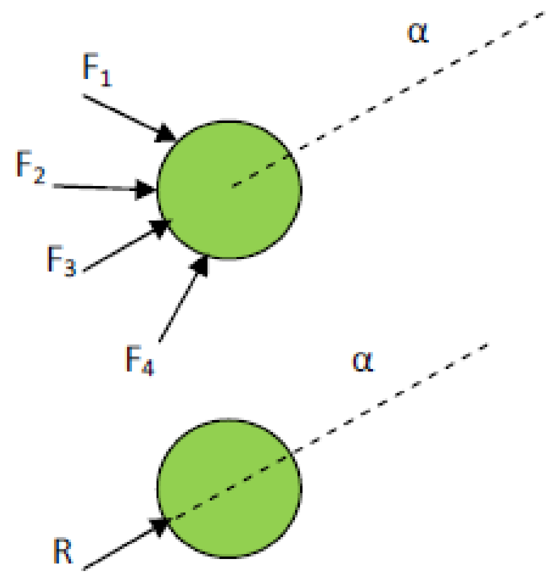
- It is possible to find a single force which will have the same effect as that of a number of forces acting on a body.

- Such a single force is called resultant force

Resultant Force of a System of Forces

- The resultant of a system of forces is a single calculated force which is capable of producing the same effect as that of system of forces on the body

- It is the vector sum of forces of the system



Composition of Forces

- The reduction of a given system of forces to the simplest system that will be its equivalent is composition of forces

Or

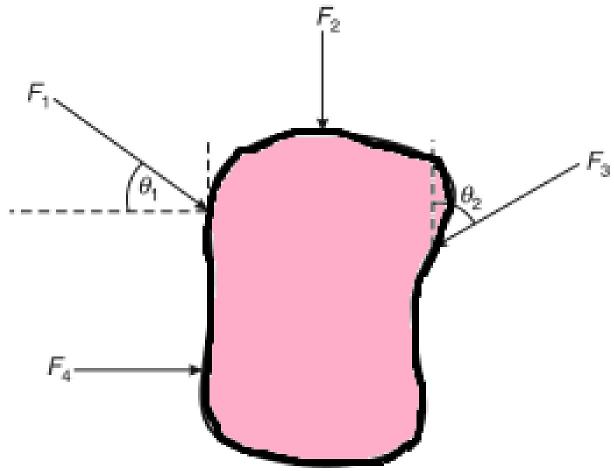
- The technique of finding the resultant of forces is called composition of forces

Composition of Forces

- It is the process of combining a number of forces into a single force such that the net effect produced by the single force is equal to the algebraic sum of the effects produced by the individual forces

- The single force in this case is called the resultant force which produces the same effect on the body as that produced by the individual forces acting together

Composition of Forces

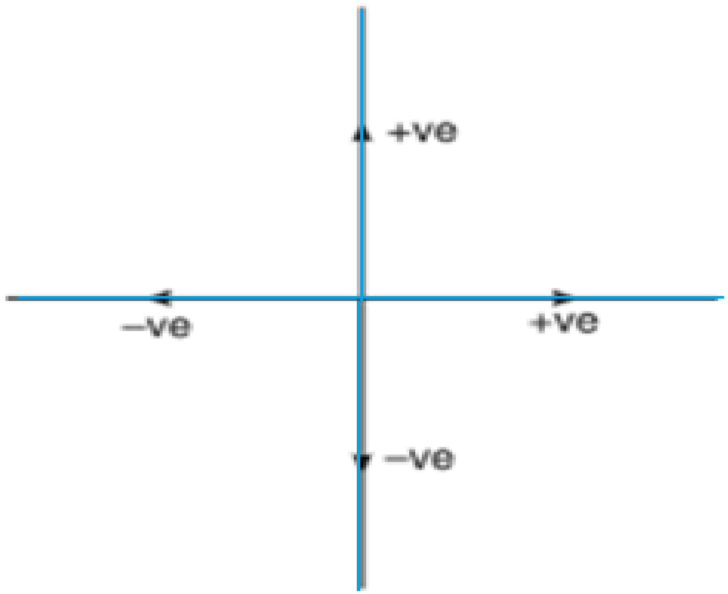


- ΣF_x = Algebraic sum of the components of the forces along the x-axis
- $\Sigma F_x = F_4 + F_1\cos\theta_1 - F_3\sin\theta_2$
- ΣF_y = Algebraic sum of the components of the forces along the y-axis
- $\Sigma F_y = -F_2 - F_1\sin\theta_1 - F_3\cos\theta_2$

Composition of Forces

Note:

- The positive and negative convention of forces used in the resolution of forces in the previous figure is as that shown in the following figure



Positive and Negative Convention of Forces

Composition of Forces

- Therefore the magnitude of the resultant

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

- and the direction of the resultant,

$$\theta = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right)$$

Two methods of Composition of forces

1) Analytical
2) Graphical

Analytical method of Composition of forces

For two forces,

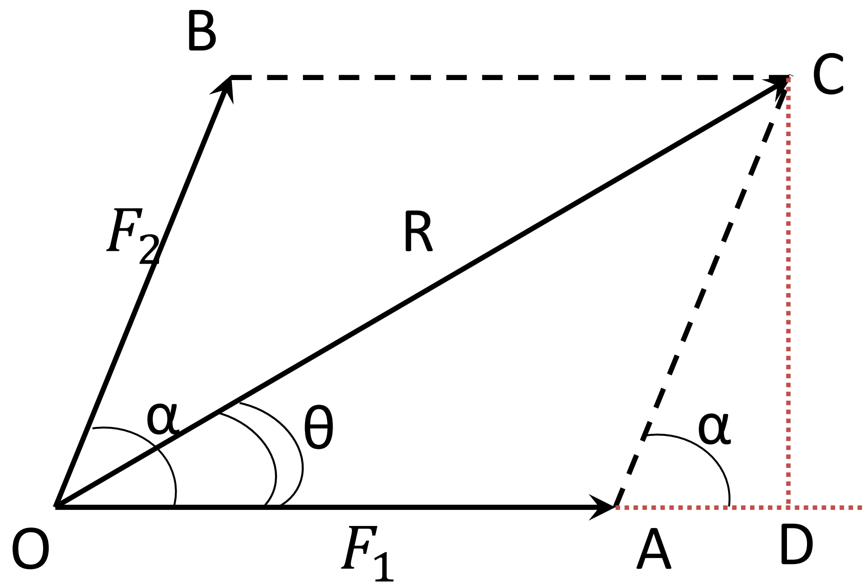
Parallelogram law of forces states that,

"If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from one of its angular points, their resultant is represented by the diagonal of the parallelogram passing through that angular point, in magnitude and direction."

Parallelogram law of forces

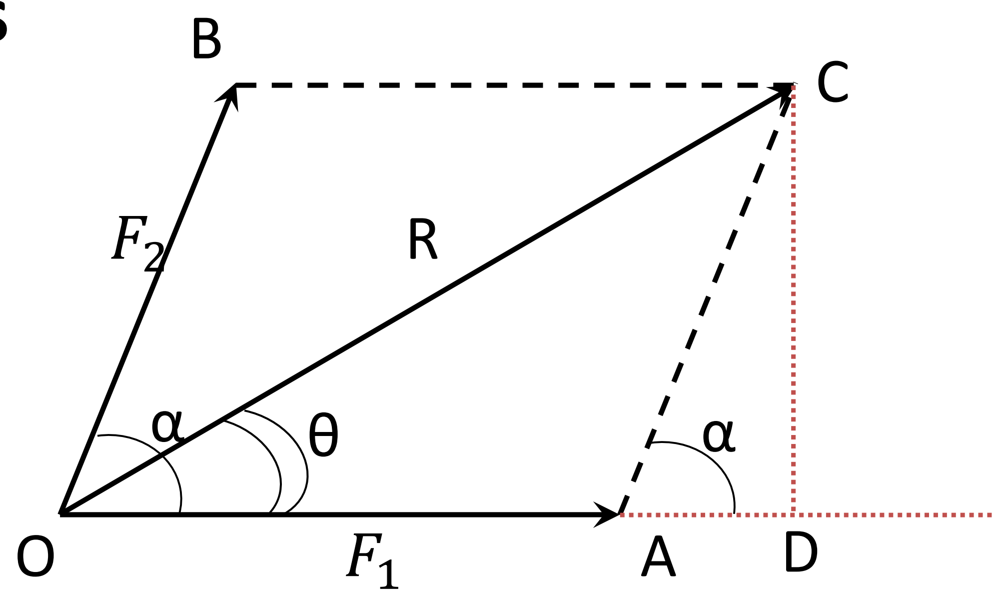
Proof:

- Consider two forces F_1 and F_2 acting at point O as shown in figure.
- Let 'α' be the angle between the two forces.
- Complete the parallelogram ACBO.
- Drop perpendicular CD to OA produced.
- Let R be the resultant force of forces
- Let 'θ' be the inclination of the resultant force with the line of action of the force.



Parallelogram law of forces

Proof:



$$OC^2 = OD^2 + CD^2$$

$$OC^2 = (OA + AD)^2 + CD^2$$

$$OA = F_1\,;\; AD = F_2\cos\alpha;\; CD = F_2\sin\alpha;\; OC = R$$

$$R^2 = (F_1 + F_2\cos\alpha)^2 + (F_2\sin\alpha)^2$$

$$R^2 = F_1^{\;2} + 2F_1F_2\cos\alpha + F_2^{\;2}\cos^2\alpha + F_2^{\;2}\sin^2\alpha$$

$$R^2 = F_1^{\;2} + 2F_1F_2\cos\alpha + F_2^{\;2}$$

$$R = \sqrt{\left(F_1^{\;2} + 2F_1F_2\cos\alpha + F_2^{\;2}\right)}$$

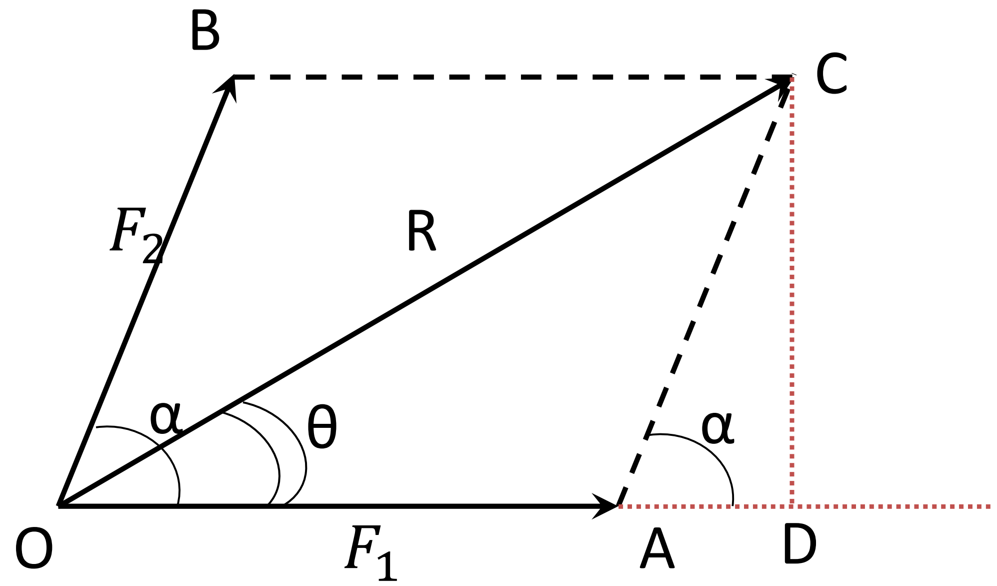
Parallelogram law of forces

Proof:

$$\tan\theta = \frac{CD}{OD}$$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}$$

$$\theta = \tan^{-1}\left(\frac{F_2 \sin\alpha}{F_1 + F_2 \cos\alpha}\right)$$



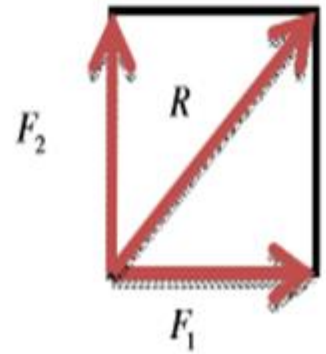
Parallelogram law of forces

Proof:

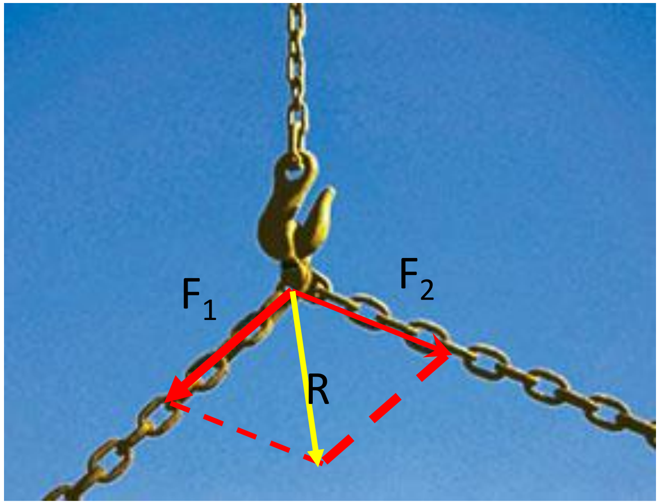
$$If\ \alpha = 90^\circ,\ R = \sqrt{F_1^{\,2} + F_2^{\,2}}$$

$$If\ \alpha = 0^\circ,\ R = F_1 + F_2$$

$$If\ \alpha = 180^\circ,\ R = F_1 - F_2$$



Parallelogram law of forces



Problems

1)Two forces of magnitudes 10 N and 8 N are acting at a point. If the angle between the two forces is 60^0, determine the direction and magnitude of the resultant force.

R=15.62N, θ=26.33°

2) Two equal forces are acting at a point with an angle of 60^0 between them. If the resultant force is equal to $20\sqrt{3}$ N find the magnitude of each force.

F=20N, θ=30°

Summary

- The reduction of a given system of forces to its equivalent simplest system is known as composition of forces
- A single force which will have the same effect as that of a number of forces acting on a body is known as a resultant
- The two methods of composition of forces are analytical and graphical

Summary

- Application of parallelogram law of forces for specific angles

- Parallelogram law of forces states that:
  - If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram
  - Resultant is represented by the diagonal of the parallelogram passing through that angular point, in magnitude and direction