Course Code: ESC106A

Course Title: Construction Materials and Engineering Mechanics

Lecture No. 53:
Introduction to Dynamics

Delivered By: Dr. T. Valsa Ipe

Lecture Intended Learning Outcomes

At the end of this lecture, student will be able to:

- Define dynamics, kinetics and kinematics
- Describe various types of motion a body undergoes
- Discuss and explain the concepts of linear displacement, velocity, and acceleration
- Explain the concepts of linear displacement, velocity, and acceleration

Contents

- Introduction to Dynamics
- Types of motion
- Newton's laws of motion
- Rectilinear motion, Displacement-time curve, Velocity-time curve, Acceleration-time curve

Introduction to Dynamics

Dynamics is the branch of mechanics dealing with the motion of particles /bodies and the forces causing such motion.

Dynamics

- Kinematics
  - Deals with the motion of bodies without considering the cause of motion ie. Only geometrical aspects of motion is considered
- Kinetics
  - Deals with the motion of bodies considering the causes of motion of the body. ie. Analysis of forces

Types of Motion

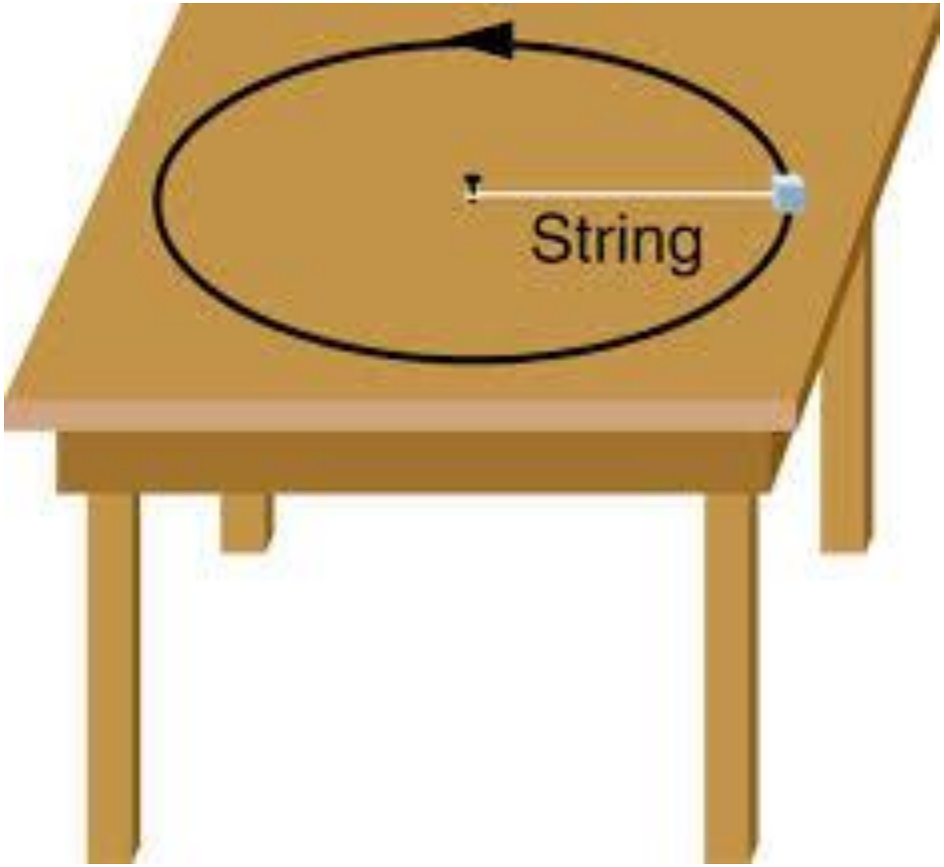
- Types of motion
  - Rectilinear motion
  - Curvilinear motion
  - Circular motion
  - Translation
  - Rotation

Rectilinear Motion

Rectilinear motion:- Motion of the body along a straight line.

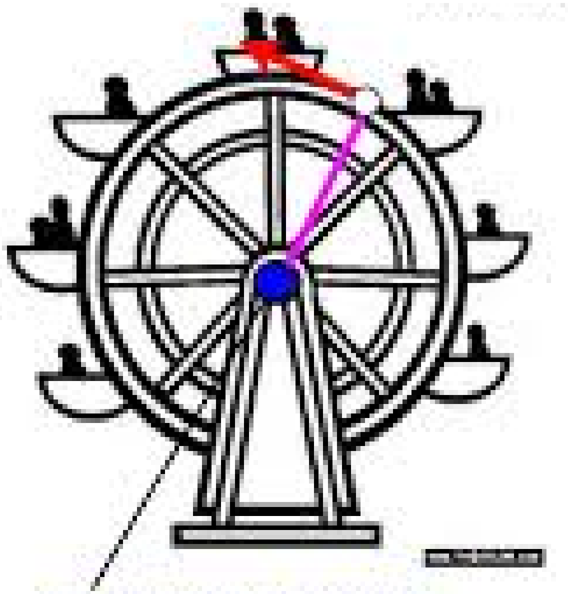
Curvilinear motion

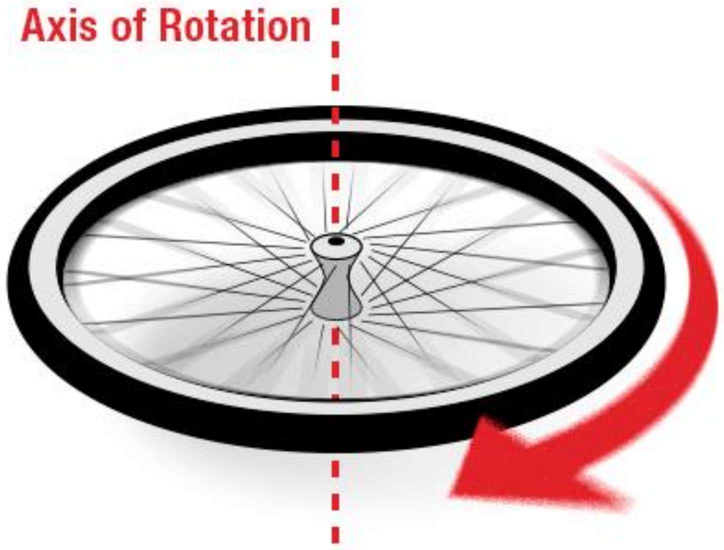
Circular Motion



Circular Motion

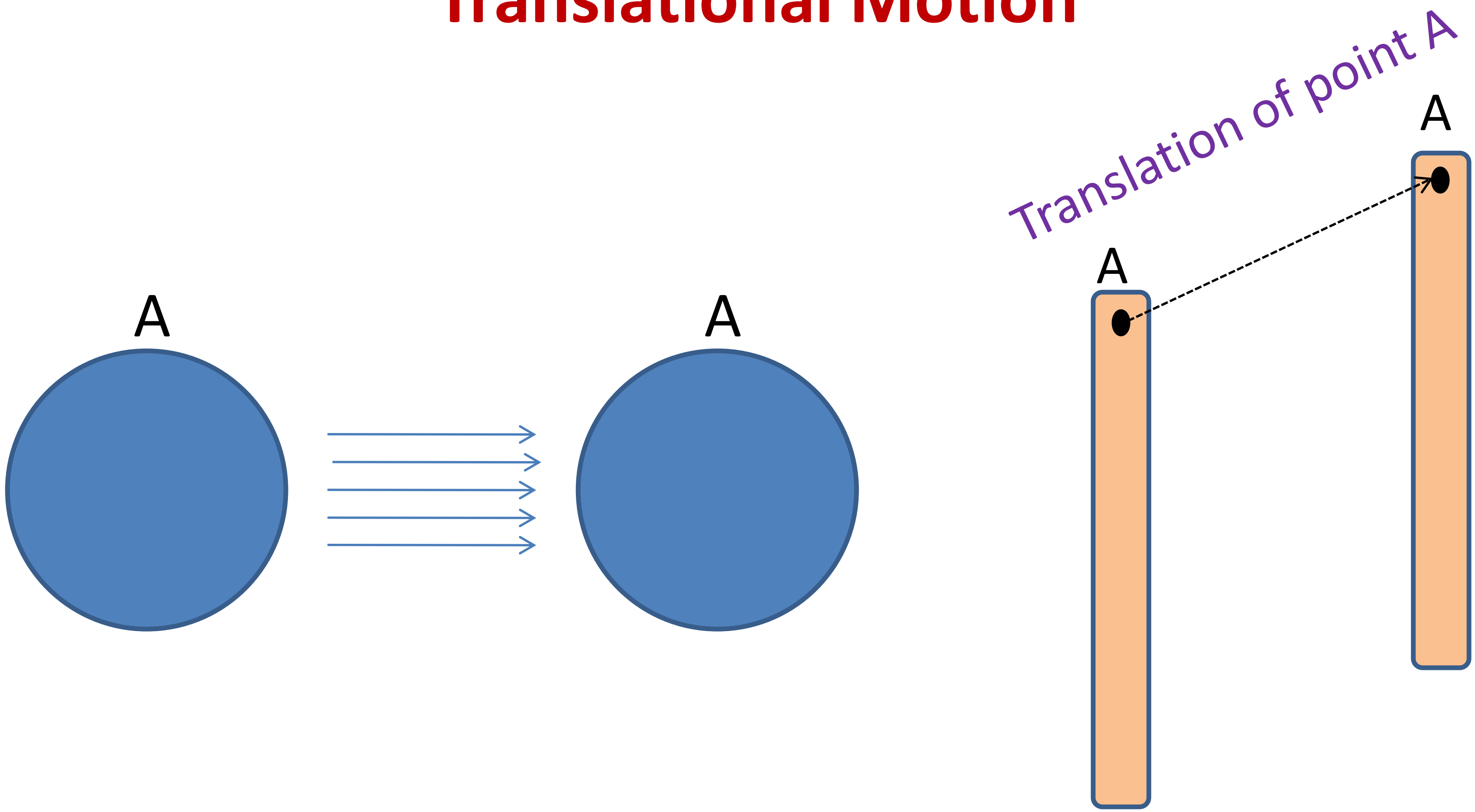
Rotational Motion





If a body rotates about a fixed point in such a way that all its particle move in circular path, it is rotational motion.

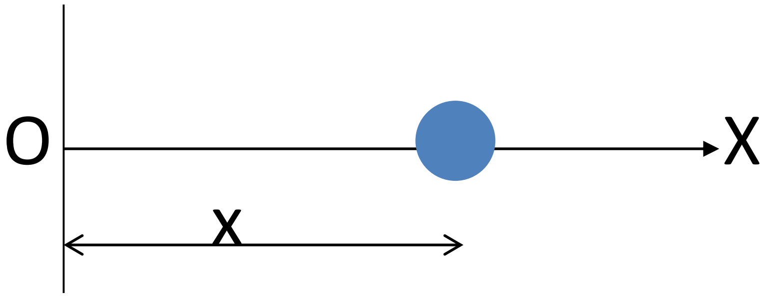
Translational Motion

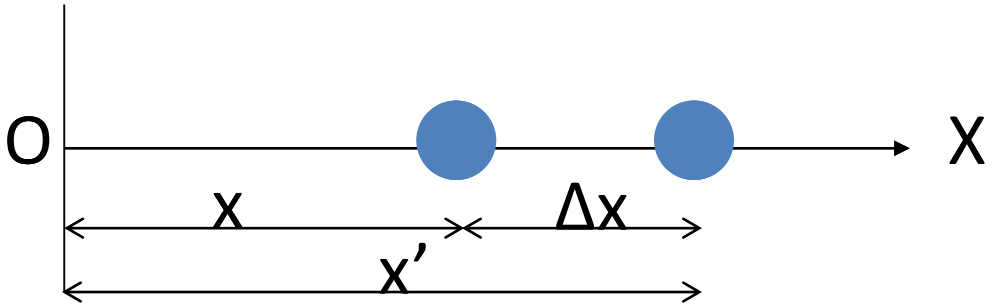


If a body moves in such a way that all its particle travel in parallel planes and cover the same distance, it is termed as translational motion

Rectilinear Motion

Kinematics of a particle is characterized by the particle's position, velocity, and acceleration at any given instant.





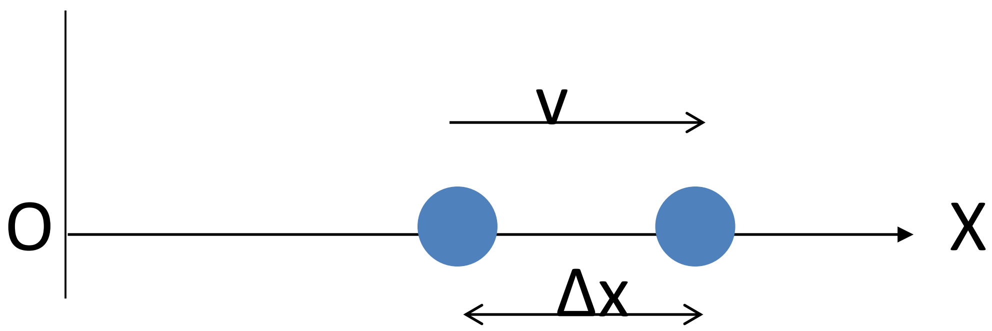
Position

Specifies the location of a particle at any given instant

Displacement

The displacement of the particle is defined as the change in its position

Rectilinear Motion



Velocity

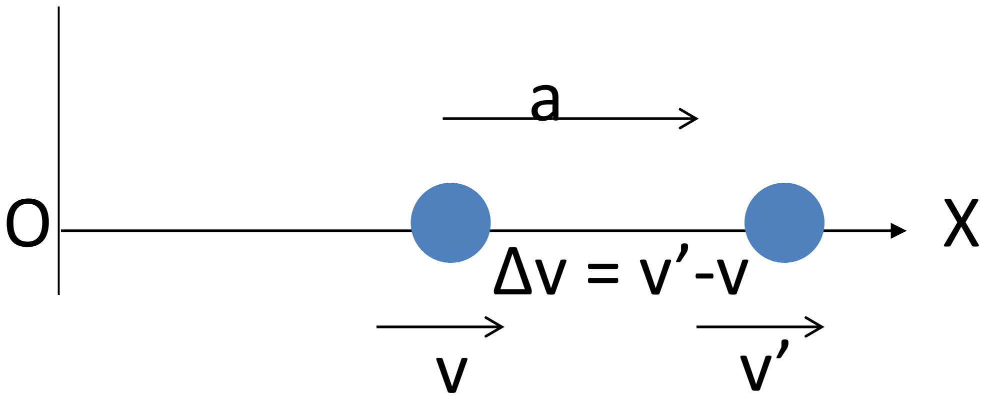
- If particle moves through a displacement Δx during an interval Δt, average velocity

$$v = \frac{\Delta x}{\Delta t}$$

- Velocity can be positive or negative, depending on the direction of motion of the body.

- Unit of velocity is m/s

Rectilinear Motion



Acceleration

Acceleration is rate of change of velocity of a body.

$$a = \frac{\Delta v}{\Delta t} \approx \frac{dv}{dt} \, when \, \Delta t \to 0$$

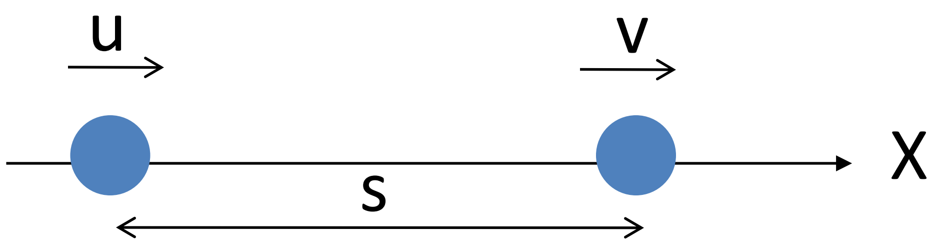
But, $v = \frac{\Delta x}{\Delta t}$

Hence, $a = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$

Acceleration can be positive or negative, depending on whether the particle is gaining speed or slowing down. Unit of acceleration is m/s^2

Rectilinear Motion

Equations of motion in straight line



u – Initial velocity of particle in m/s

v – Final velocity of particle in m/s

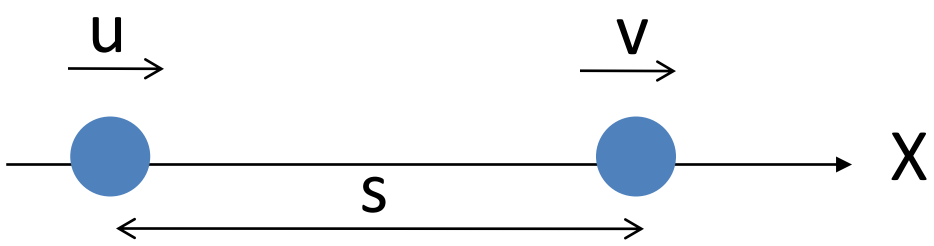
t - Time taken in seconds by the particle to change the velocity from u to v

s – Distance travelled in m by the particle in time ‘t’

a – Acceleration in m/s^2 of the particle in time ‘t’

Rectilinear Motion

Equation for final velocity



Change in velocity = $v - u$

Rate of change of velocity = $\dfrac{(v-u)}{t}$

Acceleration, a = Rate of change of velocity

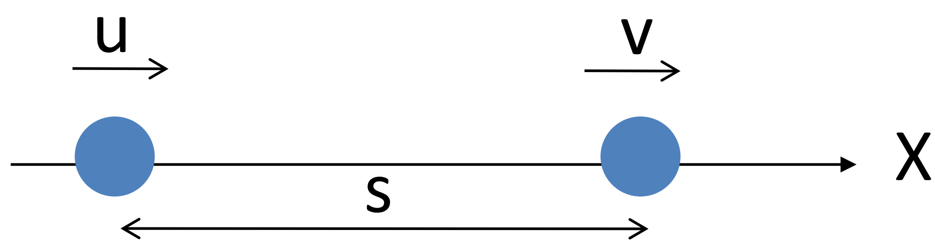
a = $\dfrac{(v-u)}{t}$

$$at = v - u$$

$$v = u + at$$

Rectilinear Motion

Equation of motion
for distance travelled



Average velocity $= \dfrac{u+v}{2}$

Distance covered, s $= \dfrac{(u+v)}{2} \times t$

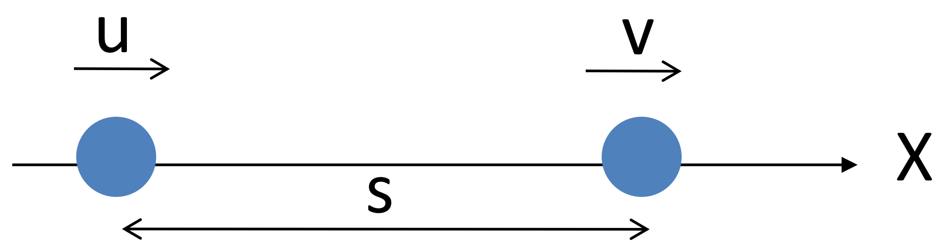
$$= \frac{(u+u+at)}{2} \times t$$

$$s = \left(\frac{2u+at}{2} \right) \times t$$

$$s = ut + \frac{1}{2} at^2$$

Rectilinear Motion

Derivation of $v^2 - u^2 = 2as$



$$s = ut + \frac{1}{2}at^2$$

$$at = v - u$$

$$t = \frac{(v-u)}{a}$$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2 = (v-u)\left[\frac{(2u+v-u)}{2a}\right]$$

$$s = \frac{(v-u)(v-u)}{2a}$$

$$v^2 - u^2 = 2as$$

Note: In all the equations it is assumed that acceleration is positive. If the case considered is of retardation or deceleration 'a' should be taken as negative.

Rectilinear Motion

Derivation of equations of motion by integration

To derive $s = ut + \frac{1}{2}at^2$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

$$d\left(\frac{ds}{dt}\right) = adt$$

$$\int d\left(\frac{ds}{dt}\right) = \int adt$$

$$\frac{ds}{dt} = at + C_1$$

Where C_1 is constant of integration

$$\left(\frac{ds}{dt}\right)_{t=t} = v; \left(\frac{ds}{dt}\right)_{t=0} = u$$

$$u = a \times 0 + C_1; \Rightarrow C_1 = u$$

$$\therefore \frac{ds}{dt} = at + u$$

$$\int ds = \int (at + u)dt$$

$$s = \frac{at^2}{2} + ut = ut + \frac{1}{2}at^2$$

Rectilinear Motion

Derivation of equations of motion by integration

To derive $v = u + at$

$$\frac{ds}{dt} = at + u$$

But, $\frac{ds}{dt} = v$

$$\therefore v = u + at$$

To derive $v^2 = u^2 + 2as$

$$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$$

$$vdv = ads$$

$$\int vdv = \int ads$$

$$\frac{v^2}{2} = as + C_2$$

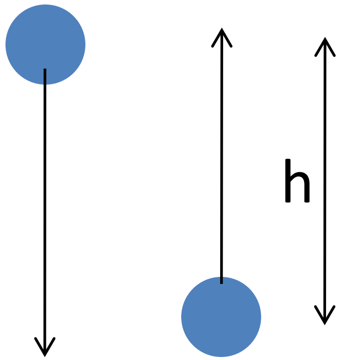
$$v_{(t=0)} = u; s_{(t=0)} = 0; C_2 = \frac{u^2}{2}$$

$$\frac{v^2}{2} = as + \frac{u^2}{2}$$

$$v^2 = u^2 + 2as$$

Rectilinear Motion

Equation of motions due to gravity



When a body falls freely due to gravity, the equations of motion are modified by substituting acceleration due to gravity ‘g’ in place of ‘a’. The distance ‘s’ is replaced by ‘h’.

For downward motion

$$a = +g$$

$$v = u + gt$$

$$h = ut + \frac{1}{2} g t^2$$

$$v^2 - u^2 = 2gh$$

For upward motion

$$a = -g$$

$$v = u - gt$$

$$h = ut - \frac{1}{2} g t^2$$

$$v^2 - u^2 = -2gh$$

Rectilinear Motion

Points to be remembered

i. If a body starts from rest, its initial velocity is zero ie u=0
ii. If a body comes to rest, its final velocity is zero ie. v=0
iii. If a body is thrown vertically upwards, the final velocity of the body at the highest point is zero, ie. v=0
iv. If a body starts moving vertically downwards, its initial velocity is zero ie.u=0
v. When is body is moving vertically downwards, g is taken as positive. But if it is moving vertically upwards, g is taken as negative

Problems on Rectilinear motion

1. A body is moving with a velocity of 15m/s. After 4 seconds the velocity of the body becomes 25m/s. Find the acceleration of the body. Find the distance travelled by the body in 4 seconds.

Problems on Rectilinear motion

2. A bullet, moving at a rate of 200m/s is fired into a log of wood. The bullet penetrates to a depth of 50cm. If the bullet moving with the same velocity is fired into a similar piece of wood 25cm thick, with what velocity would it emerge? Take the resistance to be uniform in both the cases.

Problems on Rectilinear motion

3. A stone is thrown vertically upwards with a velocity of 19.6m/s from the top of a tower 24.5m high. Calculate:

   a) Time required for the stone to reach the ground

   b) Velocity of the stone in its downward travel at the point in the same level as the point of projection

   c) The maximum height to which the stone will rise in its flight.

Problems on Rectilinear motion

4. A particle moves along a straight line so that its displacement in metre from a fixed point is given by

$$s = t^3 + 3t^2 + 4t + 5$$

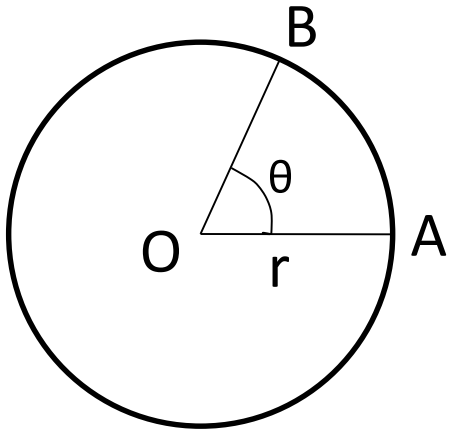
Find:

(i) Velocity at start and after 4 seconds

(ii) Acceleration at start and after 4 seconds

Curvilinear, Circular, Rotational and Translational Motion

Angular velocity – Rate of change of angular displacement of a body. Unit is rad/s



Body moving in a circle

Angular velocity $\omega = \dfrac{d\theta}{dt}$

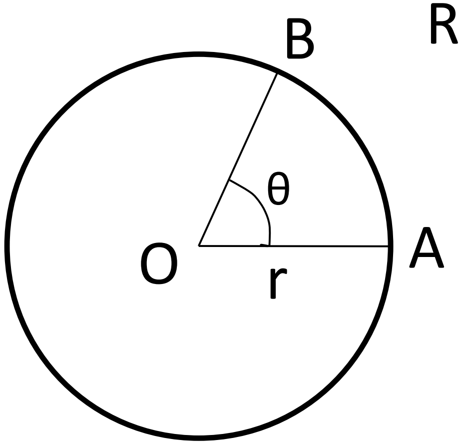
Relation between Angular velocity and linear velocity

Linear displacement $AB = r \times \theta$

Linear velocity $v = \dfrac{r \times \theta}{t} = r\omega$

Curvilinear, Circular, Rotational and Translational Motion

Angular acceleration – Rate of change of angular velocity of a body. Unit is rad/s^2



Body moving in a circle

Rate of change of angular velocity $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

Relation between Angular and linear acceleration

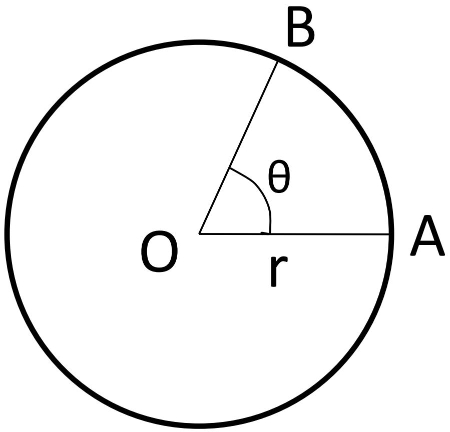
Linear velocity $v = \frac{r \times \theta}{t} = r\omega$

Differentiating, $\frac{dv}{dt} = \frac{d(\omega r)}{dt} = r\frac{d\omega}{dt}$

$$a = r\alpha$$

Curvilinear, Circular, Rotational and Translational Motion

Equations of motion along a circular path



Body moving in a circle

Angular displacement in rad θ

Initial Angular velocity in rad/s ω_0

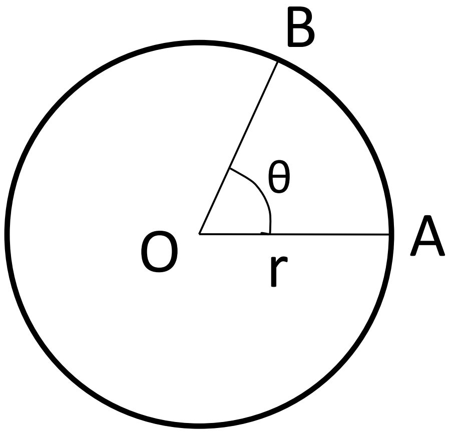
Final Angular velocity in rad/s ω

Angular acceleration in rad/s^2 α

Time taken to change velocity in sec, t

Curvilinear, Circular, Rotational and Translational Motion

Equations of motion along a circular path



Body moving in a circle

$$\alpha = \frac{\omega_t - \omega_0}{t}$$

$$\alpha t = \omega_t - \omega_0$$

Final angular velocity $\omega_t = \omega_0 + \alpha t$

Angular displacement = Average angular velocity x time

$$\theta = \frac{\omega_0 + \omega_t}{2} \times t = \left(\frac{\omega_0 + \omega_0 + \alpha t}{2} \right) t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

Curvilinear, Circular, Rotational and Translational Motion

Equations of motion along a circular path

Angular displacement in terms of initial and final angular velocities

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_t = \omega_0 + \alpha t \therefore t = \frac{\omega_t - \omega_0}{\alpha}$$

$$\theta = \omega_0 \times \left(\frac{\omega_t - \omega_0}{\alpha}\right) + \frac{1}{2}\alpha\left(\frac{\omega_t - \omega_0}{\alpha}\right)^2$$

$$\theta = \left(\omega_t - \omega_0\right)\left[\frac{2\omega_0 + \omega_t - \omega_0}{2\alpha}\right] = \frac{\omega_t^2 - \omega_0^2}{2\alpha}$$

$$\omega_t^2 - \omega_0^2 = 2\alpha\theta$$

Curvilinear, Circular, Rotational and Translational Motion

Relation between rpm and Angular Velocity

- No. of revolutions in one minute or 60seconds = N
- In one revolution, body covers 360˚ or 2π
- Angle covered by body in 1 second = $2\pi \times \frac{N}{60}$
- But angle covered per second = angular velocity $\omega = \frac{2\pi N}{60}$

Problems on Circular motion

5. A flywheel is rotating at 200rpm an after 10 seconds it is rotating at 160rpm. If the retardation is uniform, determine number of revolutions made by flywheel and the time taken by the flywheel before it comes to rest from the speed of 200 rpm.

Summary

- Dynamics is the study of motion of bodies
- There are different types of motion: Rectilinear motion, Curvilinear motion, Circular motion, Rotation and Translation
- The rate of change of displacement is termed as velocity
- The rate of change of velocity is acceleration