Circuit Analysis Techniques

Lecture 3
Superposition Theorem

Lecture delivered by:

Topics

- Linearity property
- Introduction to theorems
- Superposition theorem
- Steps to apply superposition principle
- Star delta conversion
- Delta to star transformation
- Star to delta transformation

Objectives

At the end of this lecture, student will be able to:

- Explain linearity property
- State and analyze superposition theorem for any complicated linear bilateral network
- Reduce complicated network to simple network using star delta conversions

Linearity Property

A linear element or circuit satisfies the properties of

- **Additivity:** requires that the response to a sum of inputs is the sum of the responses to each input applied separately.

  If $v_1 = i_1R$ and $v_2 = i_2R$

  then applying $(i_1 + i_2)$

  $v = (i_1 + i_2)\,R = i_1R + i_2R = v_1 + v_2$

Linearity Property

- **Homogeneity:** If you **multiply the input** (i.e. **current**) by some **constant K**, then the **output response (voltage)** is **scaled by the same constant**.

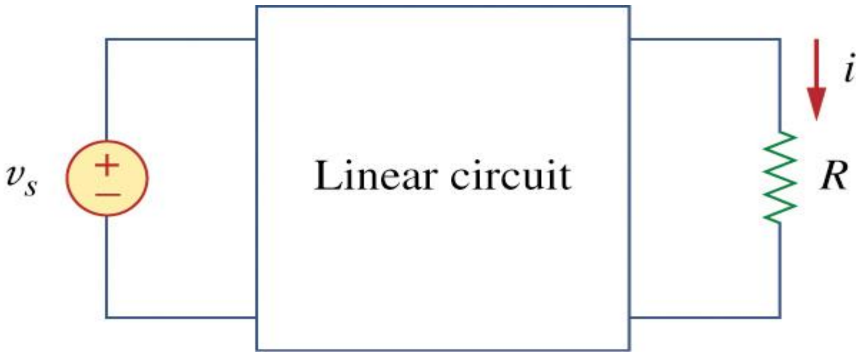
  If $v_1 = i_1 R$

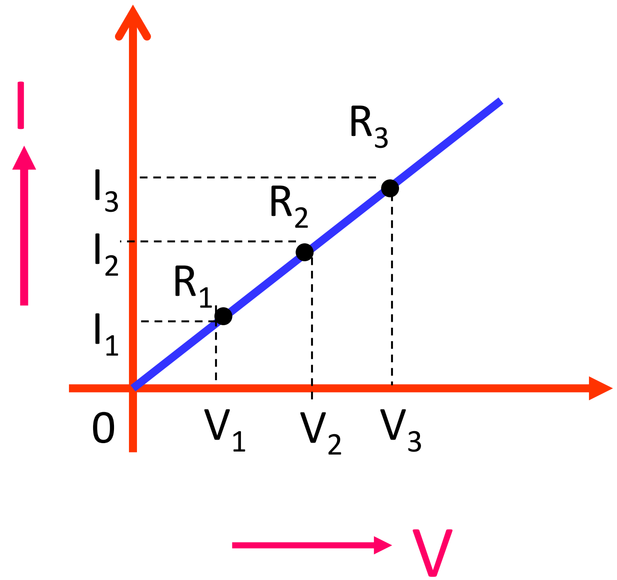
  then $K\, v_1 = K\, i_1 R$

Note: Linear circuits obey both the properties of homogeneity (scaling) and additivity.

Linearity Property

- A linear circuit is one whose output is linearly related (or directly proportional) to its input.



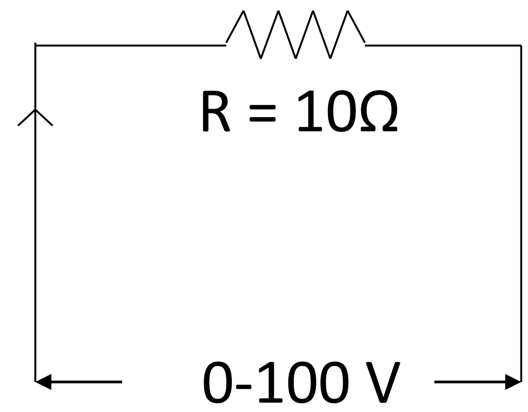


Suppose $v_s = 10$ V gives I = 2 A. According to the linearity principle, $v_s = 5$ V will give I = 1 A.

Linearity Property

Example: A resistance of 10Ω is connected across a supply of 100 volts which varies in steps of 10 volt from 0 to 100 volts. Calculate the corresponding current for each step of voltage and also draw the graph by assuming voltage on x-axis and current on y-axis.

The arrangement is shown in figure



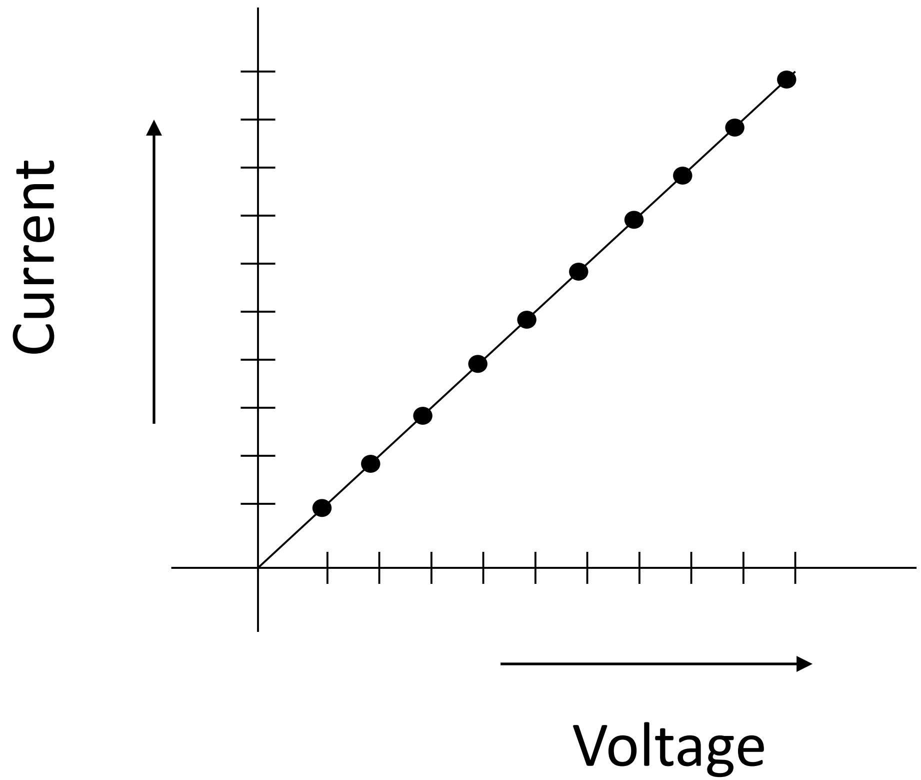
Applying Ohm's Law,

Linearity Property

V (volt)	R (Ohms)	I = V / R (Amps)
0	10	0
10	10	1
20	10	2
30	10	3
40	10	4
50	10	5
60	10	6
70	10	7
80	10	8
90	10	9
100	10	10

Linearity Property

Graph between voltage and current from the above values.



Introduction To Theorems

A large complex circuits → **Simplify circuit analysis**

↑

Circuit Theorems

- Thevenin's theorem
- Circuit linearity
- source transformation
- Norton theorem
- Superposition
- max. power transfer

Superposition Theorem

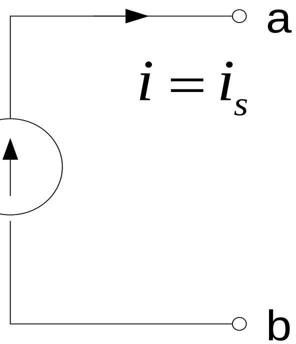
- The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Steps to apply superposition principle

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
   - Turn off voltages sources = short voltage sources make it equal to zero voltage
   - Turn off current sources = open current sources make it equal to zero current
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Turning sources off

Current source:



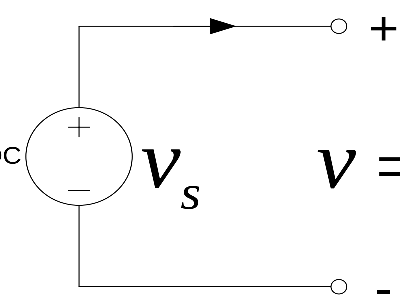
i_s $\quad i = i_s$

Replace it by a current source where

$i_s \equiv 0$

An open-circuit

Voltage source:



$v_s \quad v = v_s$

Replace it by a voltage source

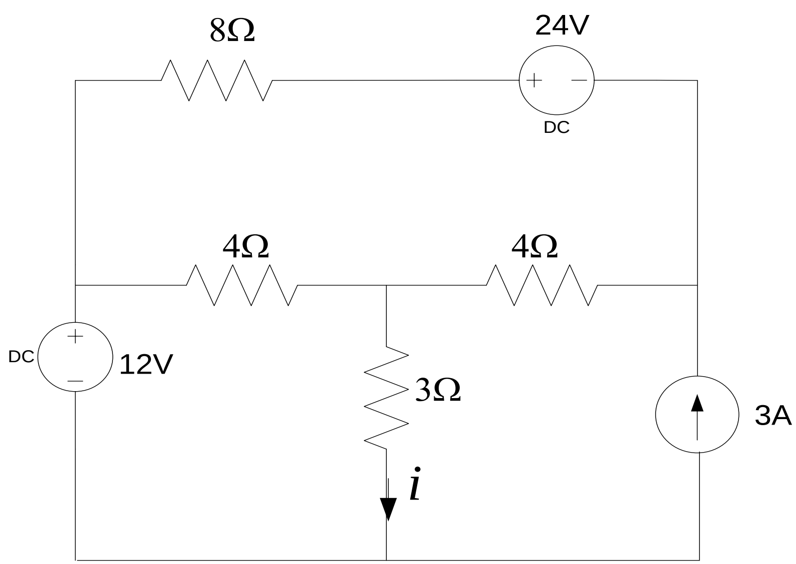
$v_s \equiv 0$

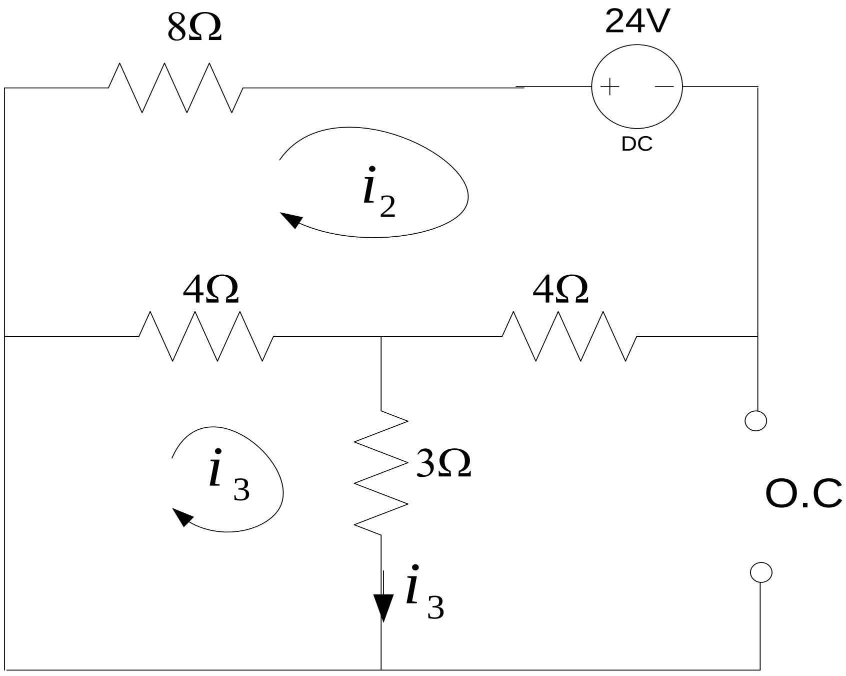
An short-circuit

Superposition Theorem

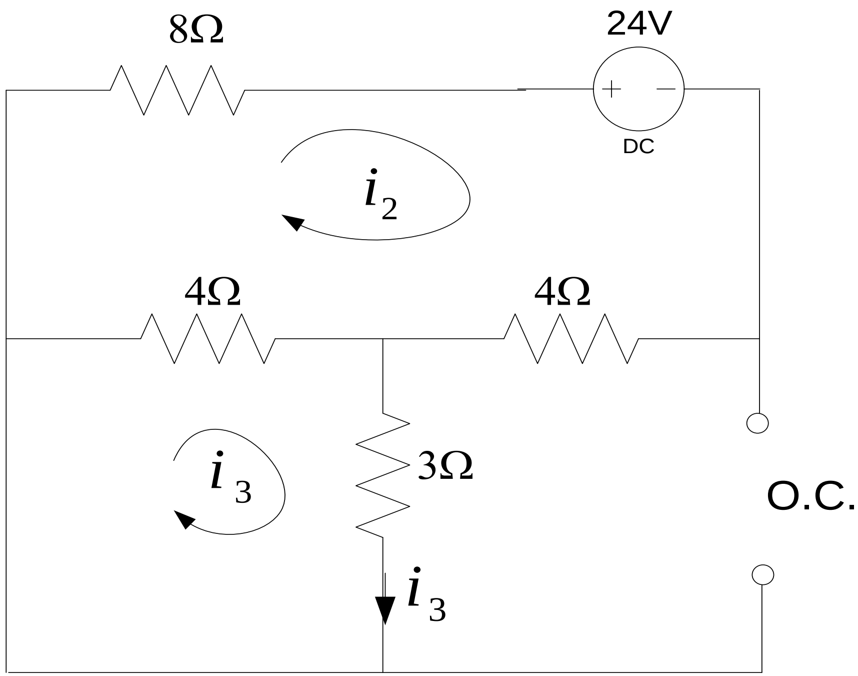
Example: In the circuit below, find the current i by superposition

Turn off the 3A & 12V sources





Superposition Theorem



$$\begin{pmatrix} 4+8+4 & -4 \\ -4 & 4+3 \end{pmatrix}\begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$$

$$16i_2 - 4i_3 = -24$$

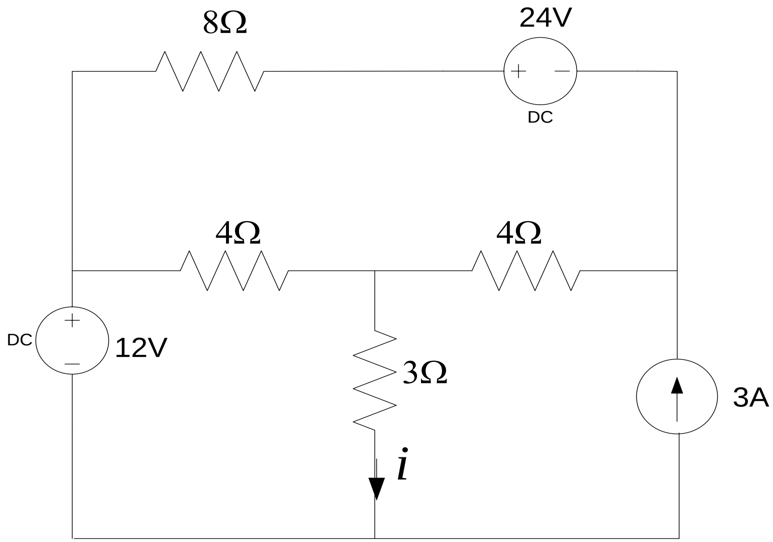
$$-4i_2 + 7i_3 = 0$$

$$i_2 = \frac{7}{4} i_3$$

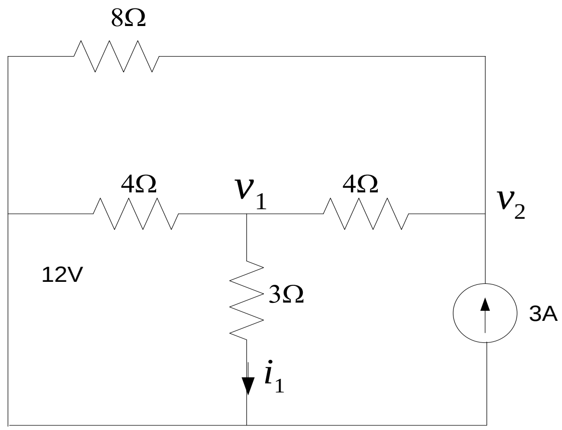
$$i_3(28-4) = -24$$

$$i_3 = -1$$

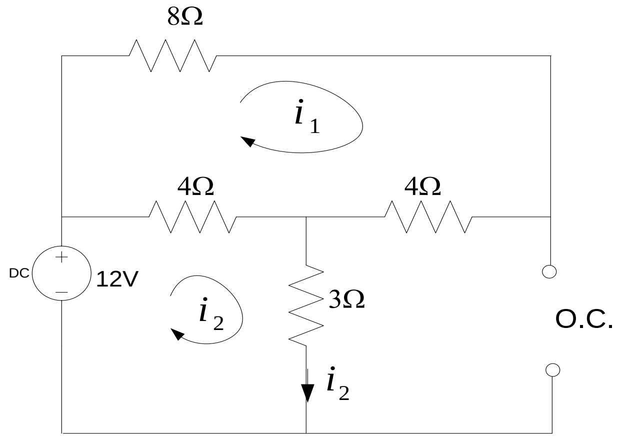
Superposition Theorem



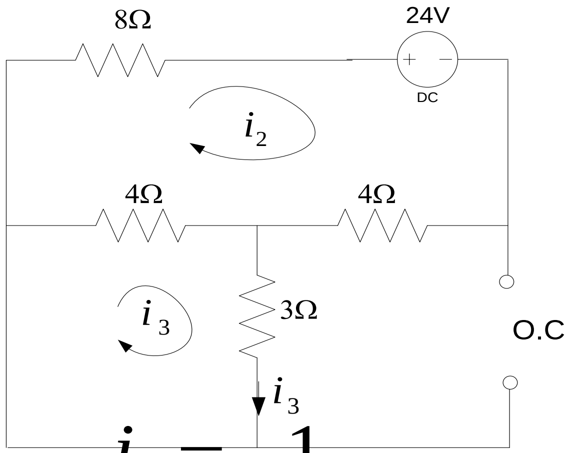
$$i = i_1 + i_2 + i_3 = 1\text{A} + 2\text{A} - 1\text{A} = 2\text{A}$$



$$i_1 = 1$$



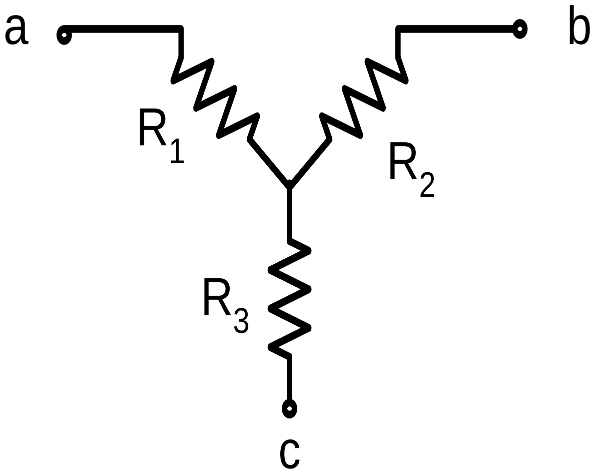
$$i_2 = 2$$



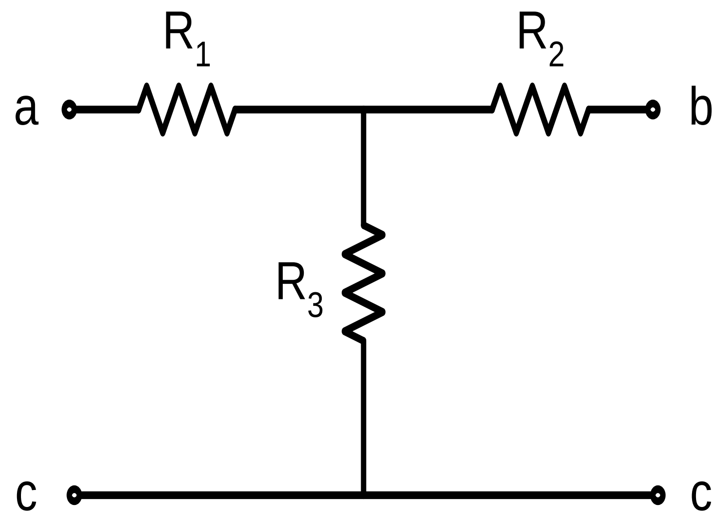
$$i_3 = -1$$

Star Delta Conversion

- Same type of connections



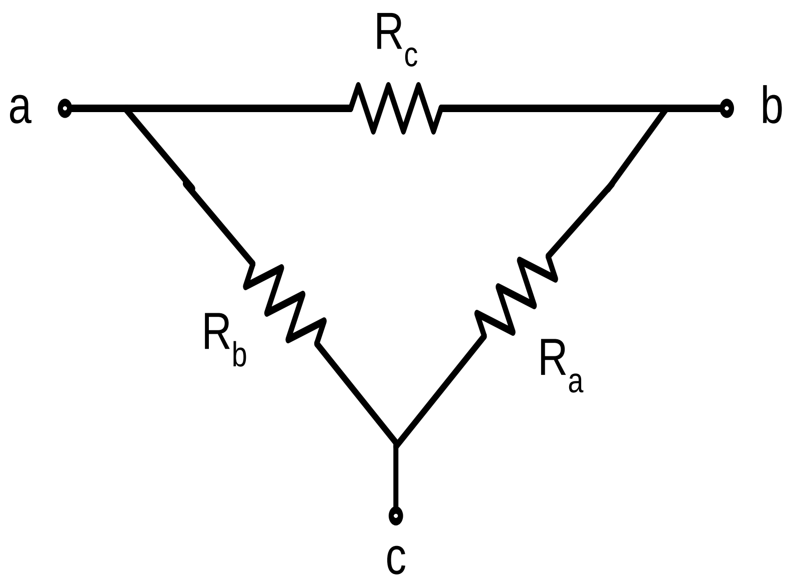
(a) Wye



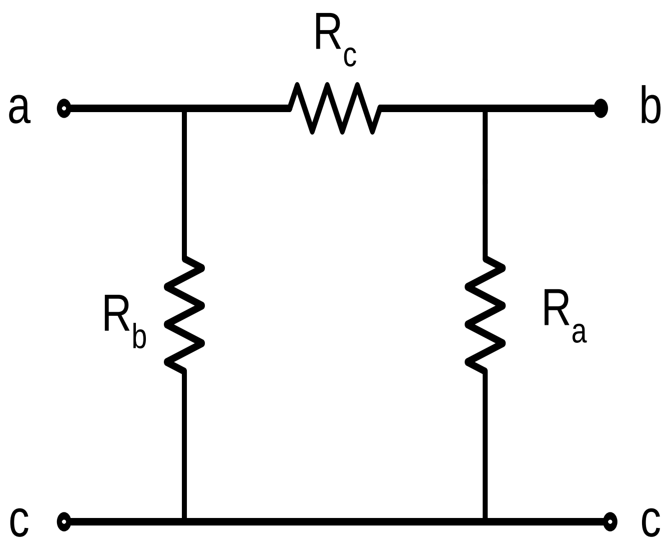
(b) Tee

Delta/Pi Circuit

- Same type of Connections

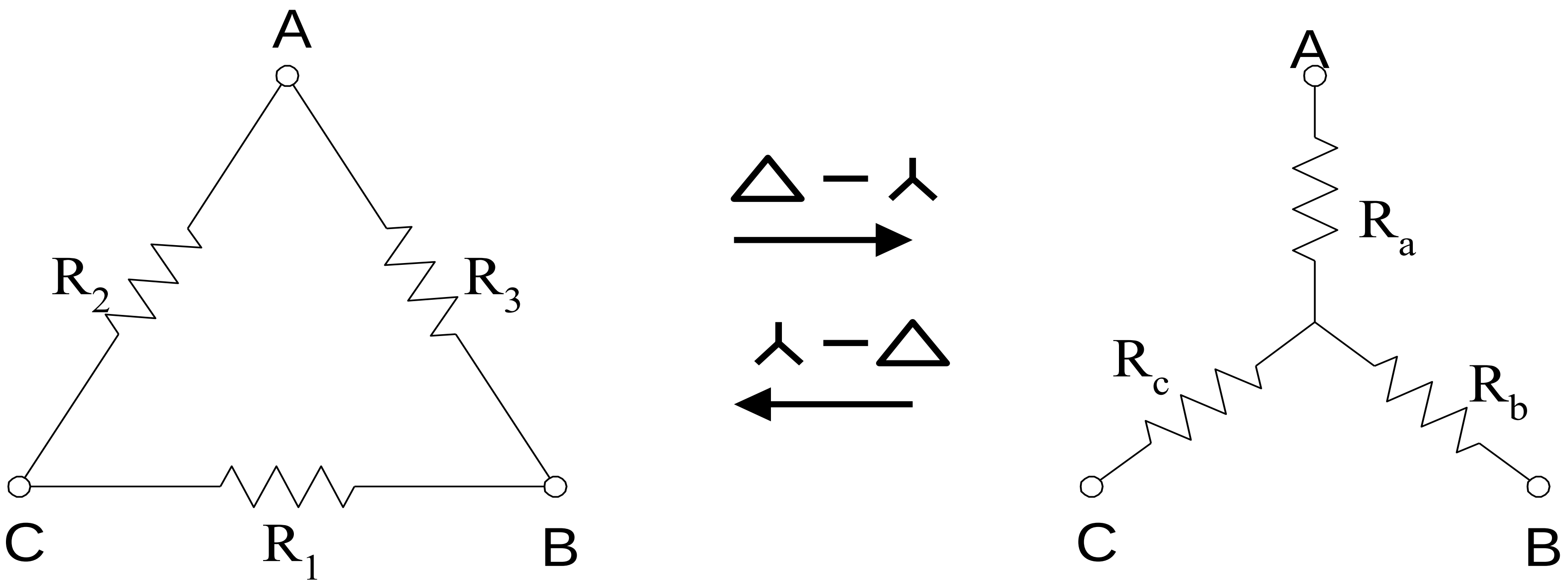


(a) Delta



(b) Pi

Delta – Star / Star - Delta Transformation



Delta To Star Transformation

- From delta cct , impedance sees from AB $R_{AB} = \dfrac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3}$
- From star cct , impedance sees from AB $R_{AB} = R_a + R_b$
- Thus equating

$$R_a + R_b = \frac{R_1R_3 + R_1R_2}{R_1 + R_2 + R_3} \qquad \text{(a)}$$

- Similarly from BC

$$R_b + R_c = \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3} \qquad \text{(b)}$$

- From AC

$$R_a + R_c = \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3} \qquad \text{(c)}$$

(b) – (c)

$$R_a - R_c = \frac{R_2R_3 - R_1R_2}{R_1 + R_2 + R_3} \qquad \text{(d)}$$

Delta To Star Transformation

By adding (a) and (d) ; (b) and (d) ;and (c) and (d) and then divided by two yield

$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3} \qquad \text{(e)}$$

$$R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3} \qquad \text{(f)}$$

$$R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3} \qquad \text{(g)}$$

Delta to star transformation

- Dividing (e) by (f)

$$\frac{R_a}{R_b} = \frac{R_2}{R_1} \quad \text{(i)}$$

Therefore

$$R_2 = \frac{R_1 R_a}{R_b} \quad \text{(j)}$$

- Similarly, dividing (e) by (g)

$$\frac{R_a}{R_c} = \frac{R_3}{R_1} \quad \text{(j)}$$

$$R_3 = \frac{R_1 R_a}{R_c} \quad \text{(k)}$$

Delta to star transformation

- Substitute R2 and R3 into (e)

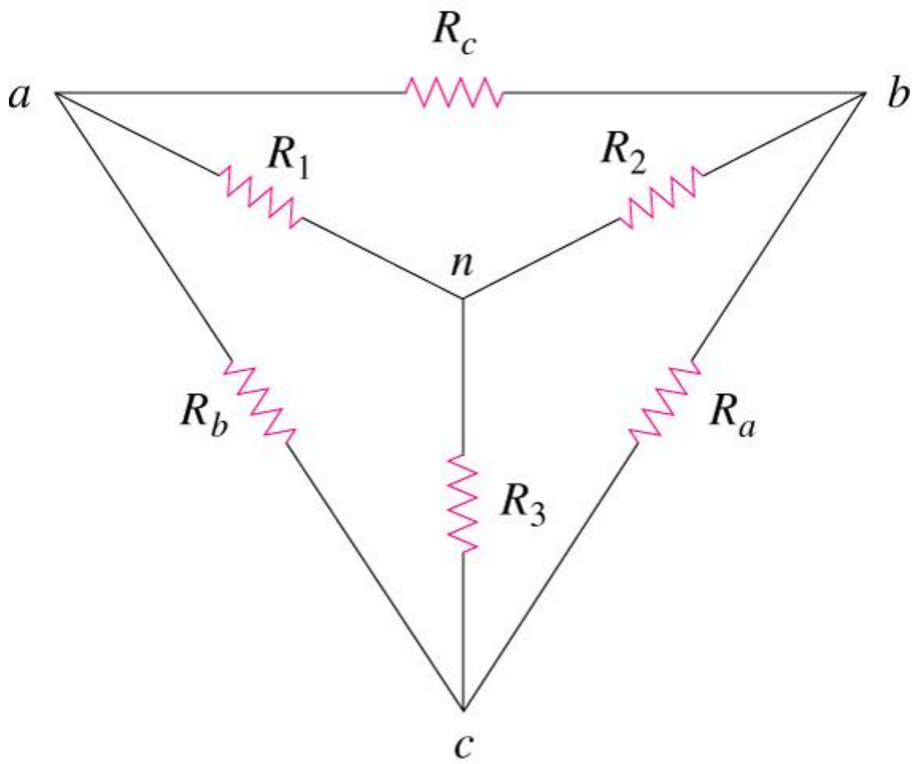
Similarly

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} \quad \text{(l)}$$

$$R_2 = R_c + R_a + \frac{R_c R_a}{R_b} \quad \text{(m)}$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} \quad \text{(n)}$$

Delta – Star / Star - Delta Transformation



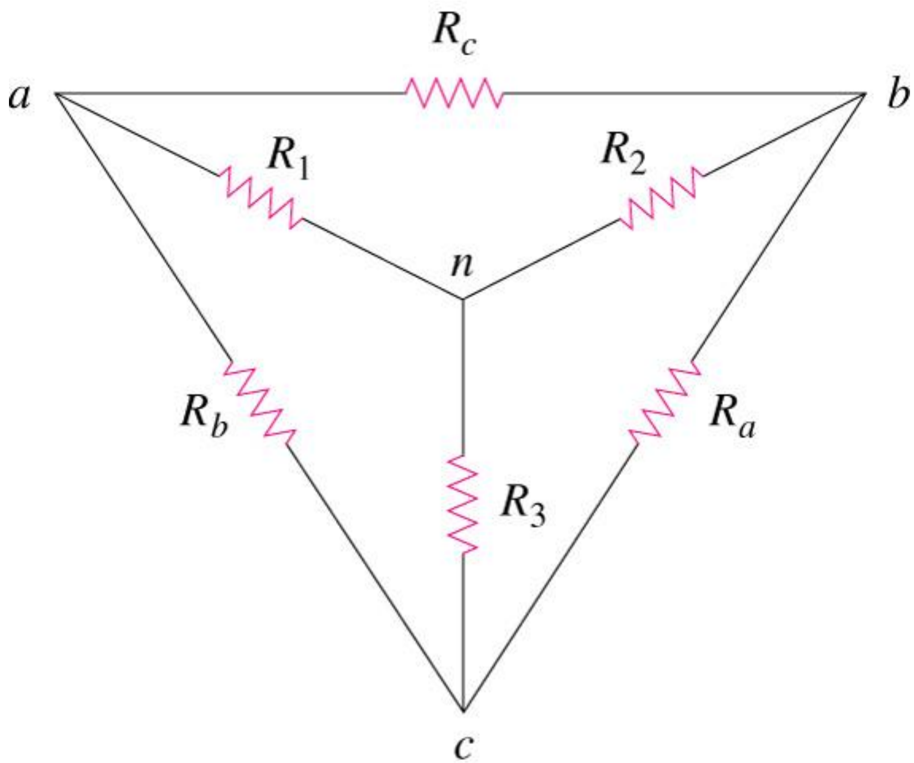
Star -> Delta

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Delta – Star / Star - Delta Transformation



Delta -> Star

$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)}$$

$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)}$$

$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)}$$

Special Case of Δ-Y Transformation

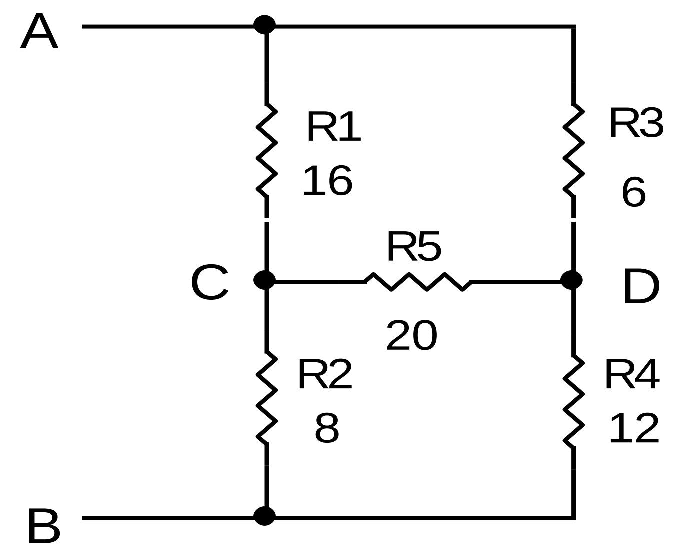
- Special case occur when $R_1 = R_2 = R_3 = R_Y$ or $R_a = R_b = R_c = R_\Delta$ under which the both networks are said to be *balanced*.
- Hence the transformation formulas will become:

$R_Y = R_\Delta/3$ or $R_\Delta = 3R_Y$

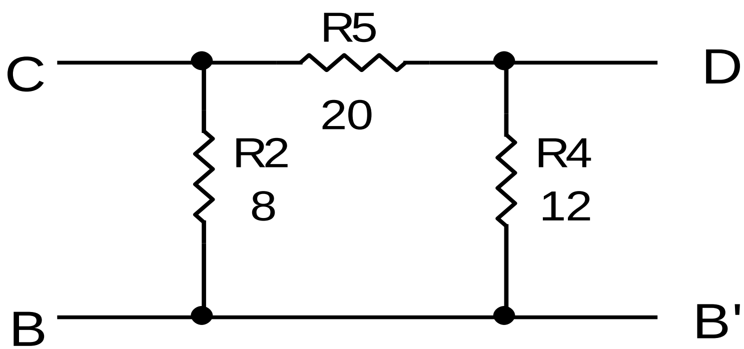
- By applying Delta/Wye transformations, we may find that this final process leads to series/parallel connections in some parts of the circuit.

Example

- Find the effective resistance at terminal between A and B of the network on the right side



Solution

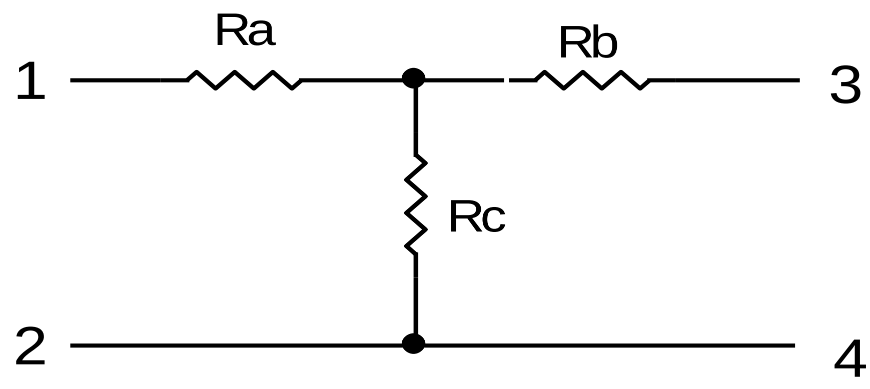


$\Sigma R = R2 + R4 + R5 = 40\ \Omega$

$Ra = R2 \times R5/\Sigma R = 4\ \Omega$

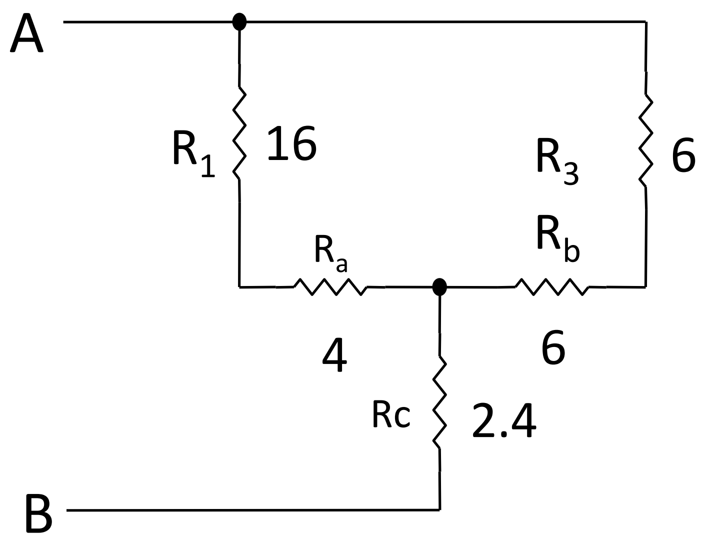
$Rb = R4 \times R5/\Sigma R = 6\ \Omega$

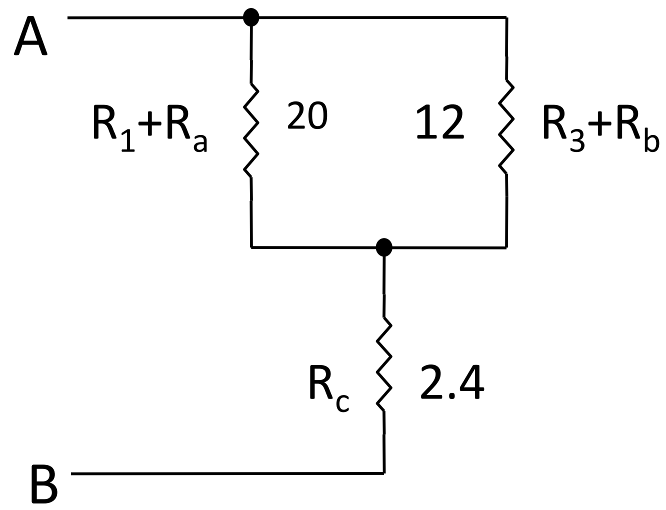
$Rc = R2 \times R4/\Sigma R = 2.4\ \Omega$



Example

Substitute R_2, R_5 and R_4 with R_a, R_b dan R_c:



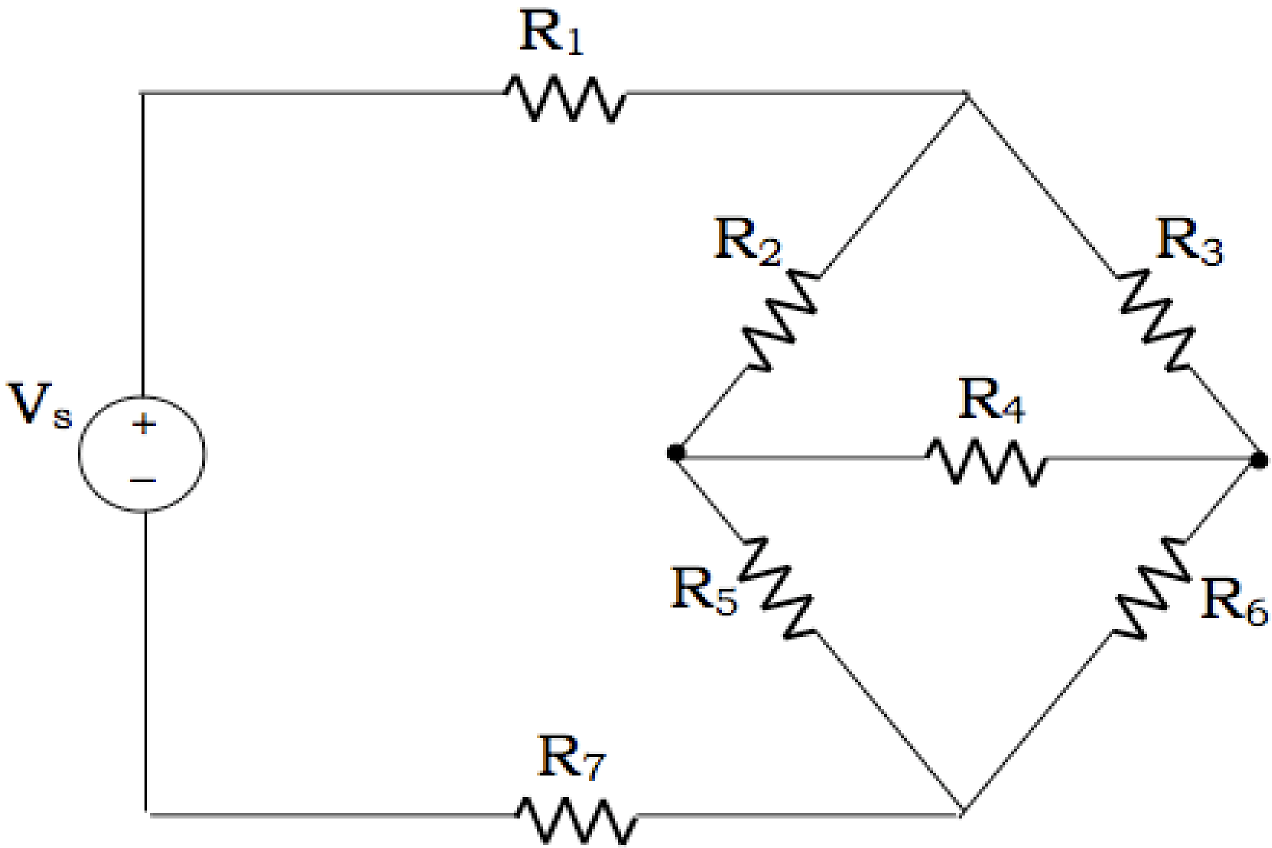


$$RAB = [(20x12)/(20+12)] + 2.4 = 9.9\ \Omega$$

Delta – Star / Star - Delta Transformation

xample

How to combine R_1 to R_7 ?

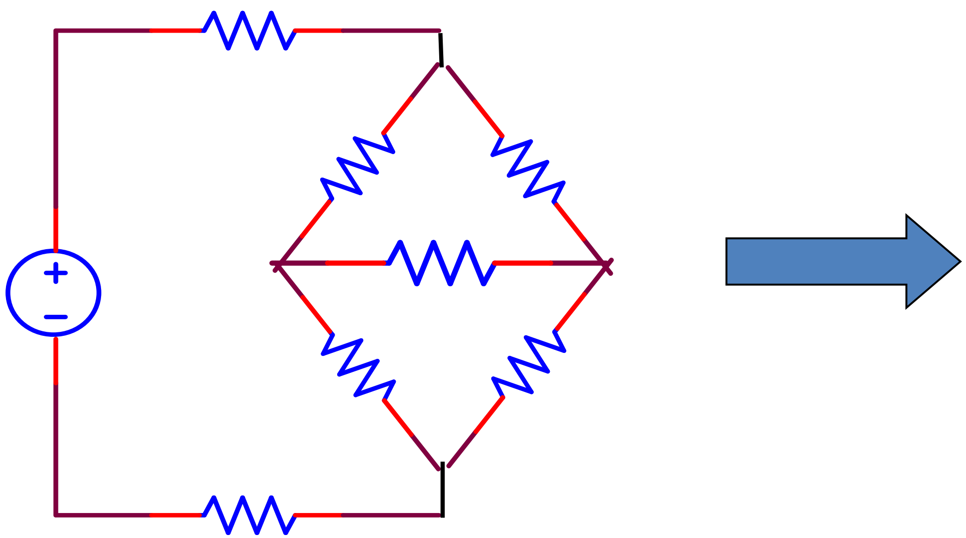


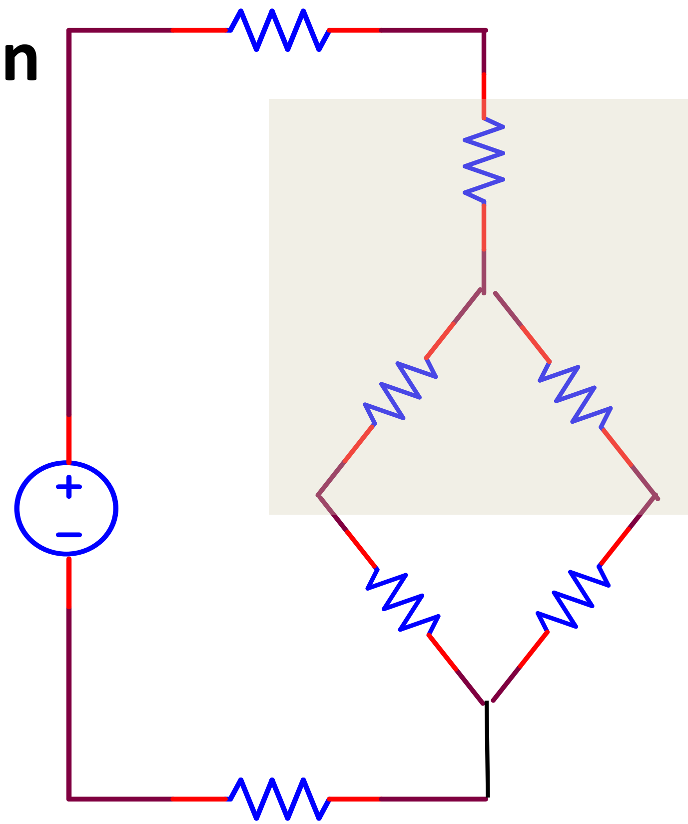
Delta – Star / Star - Delta Transformation

Example

Y ↔ Δ transformation

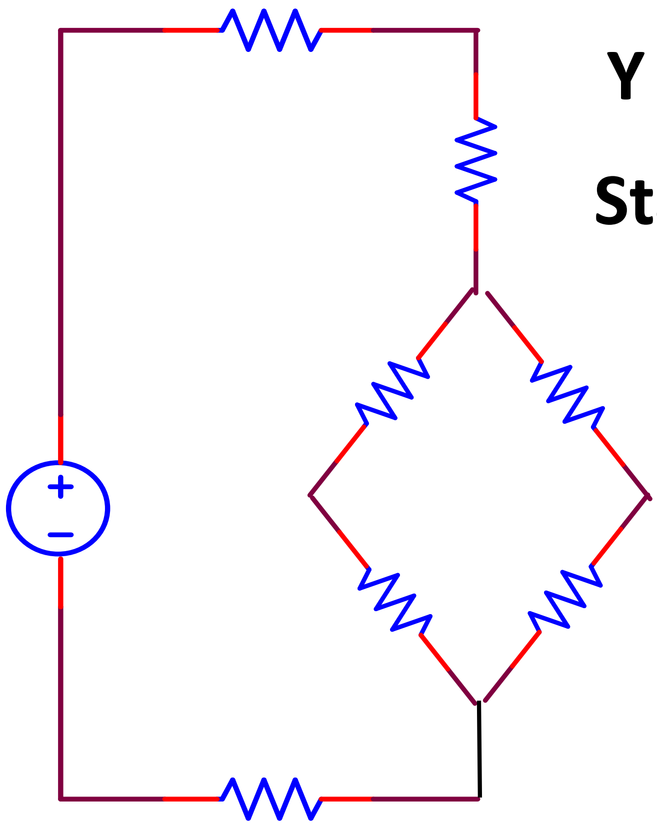
Star ↔ delta transformation





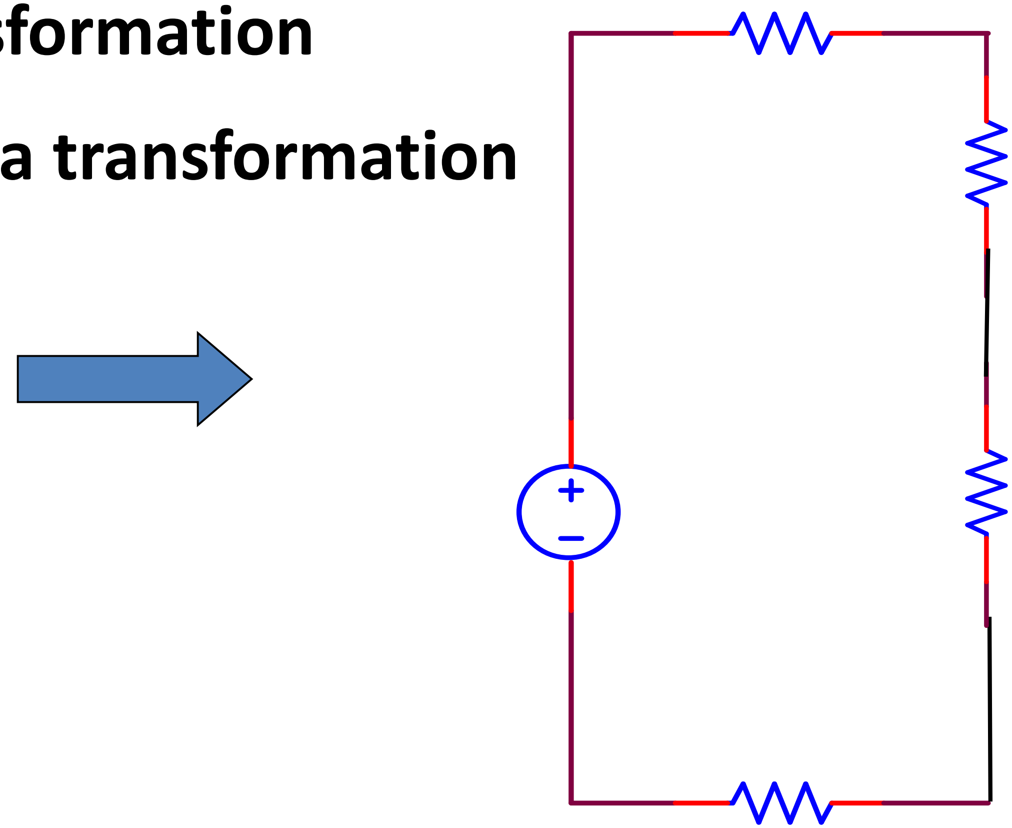
Delta – Star / Star - Delta Transformation

xample



Y ⟺ Δ transformation

Star ⟺ delta transformation



Summary

- Linearity is the behavior of a circuit,, in which the output signal varies in direct proportion to the input signal
- The **superposition principle** states that “The voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone”.
- **Star Delta Transformations** allows to convert impedances connected together from one type of connection to another. Thus making simple series, parallel or bridge type resistive networks which can be solved using KCL and KVL