Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 40:
Determination of Moment of Inertia

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Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

- Determine the Moment of Inertia for different sections by integration method
- Apply perpendicular axis theorem for circular section

Contents

Moment of inertia of rectangular section and hollow rectangular sections, Circular and hollow circular sections

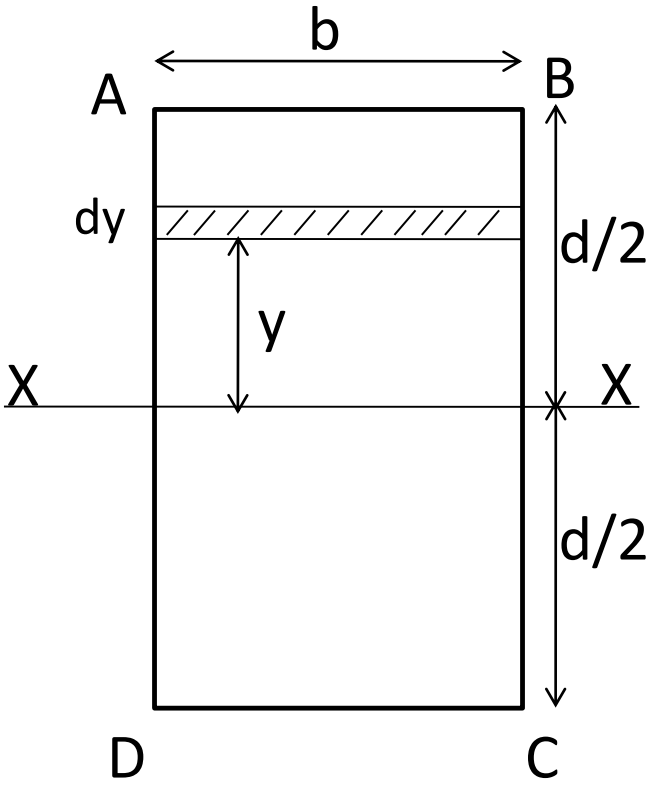
Moment of Inertia of rectangular section

Case 1: MI about XX axis passing through CG

$$(I_{XX})_{dA} = (b.dy)y^2$$

$$I_{XX} = \int_{-\frac{d}{2}}^{\frac{d}{2}} by^2 dy = b\left[\frac{y^3}{3}\right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$I_{XX} = \frac{b}{3}.\frac{2d^3}{8} = \frac{bd^3}{12}$$



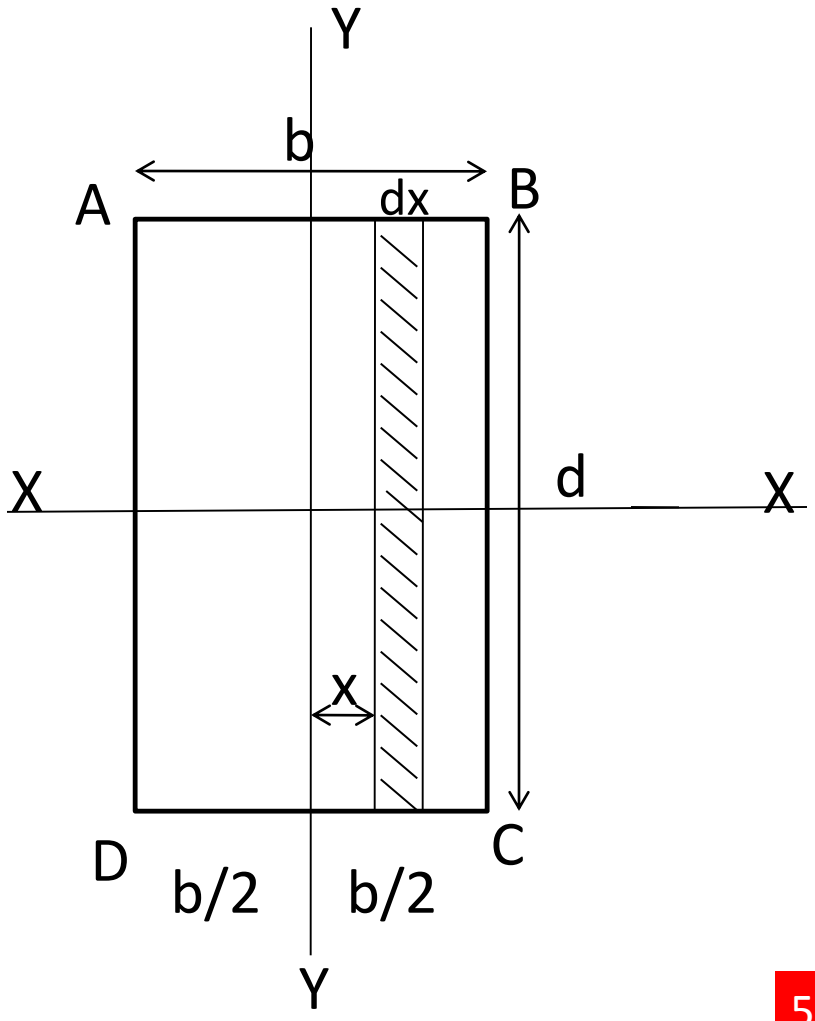
Moment of Inertia of rectangular section

Case 2: MI about YY axis passing through CG

$$(I_{YY})_{dA} = (d.dx)x^2$$

$$I_{YY} = \int_{-\frac{b}{2}}^{\frac{b}{2}} dx^2 dx = d\left[\frac{x^3}{3}\right]_{-\frac{b}{2}}^{\frac{b}{2}}$$

$$I_{YY} = \frac{d}{3}.\frac{b^3}{4} = \frac{db^3}{12}$$



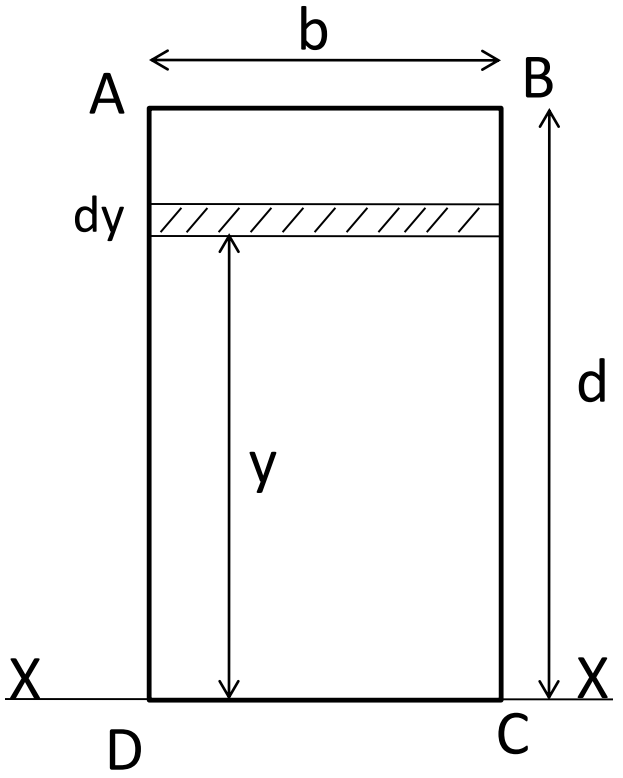
Moment of Inertia of rectangular section

Case 3: MI about XX axis passing through base

$$(I_{XX})_{dA} = (b.dy)y^2$$

$$I_{XX} = \int_0^d by^2 dy = b\left[\frac{y^3}{3}\right]_0^d$$

$$I_{XX} = \frac{bd^3}{3}$$



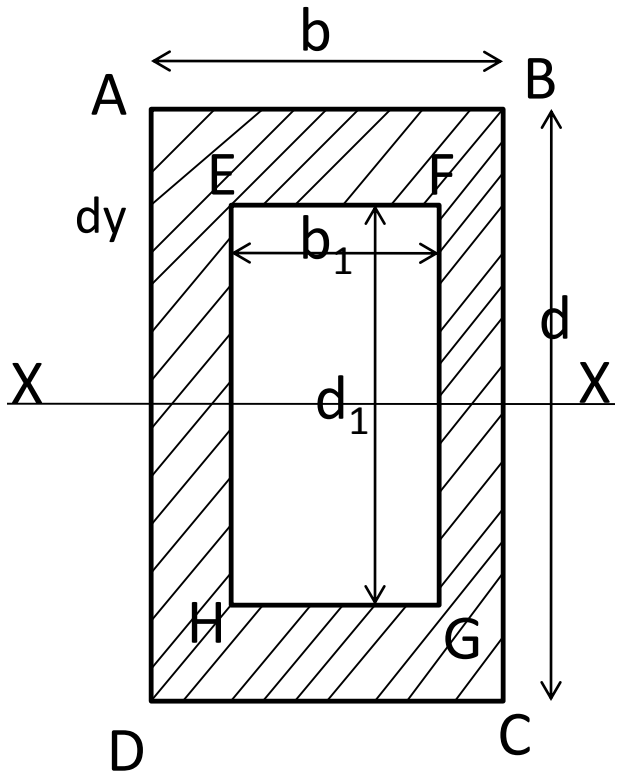
Moment of Inertia of hollow rectangular section

Case 4: MI of hollow rectangular section

$$(I_{XX})_{ABCD} = \frac{bd^3}{12}$$

$$(I_{XX})_{EFGH} = \frac{b_1 d_1^3}{12}$$

$$(I_{XX})_{Hollow} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}$$



Moment of Inertia of circular section

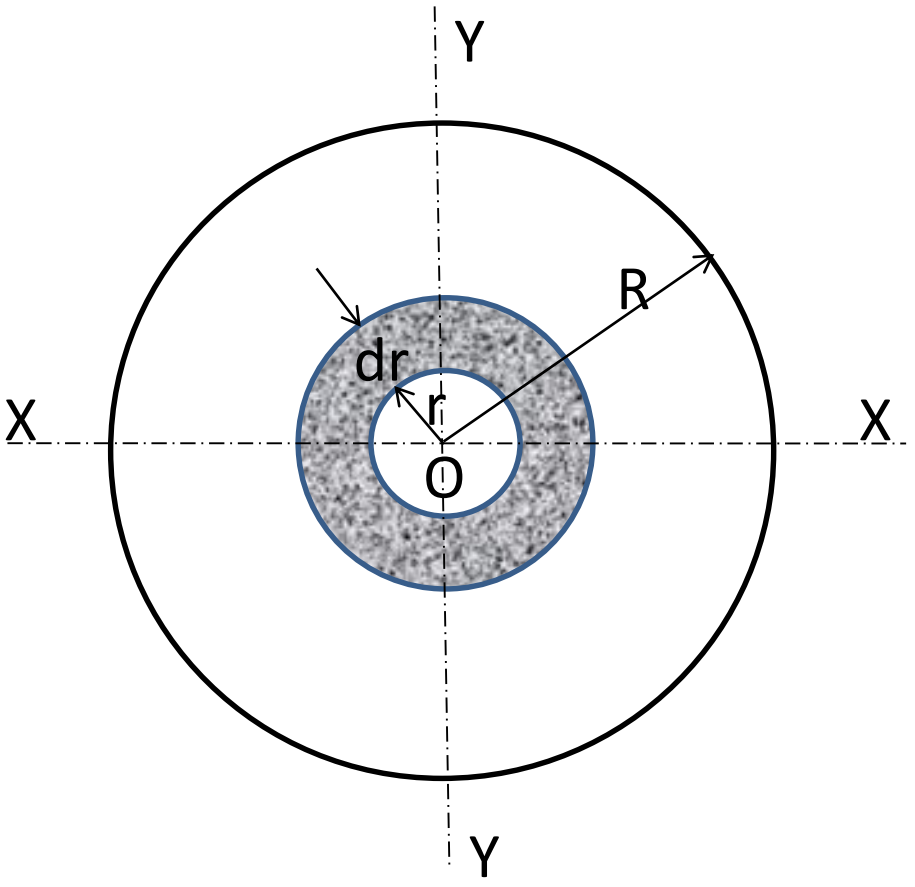
- Consider a circular section of radius R
- MI of the ring about an axis passing through O and perpendicular to the plane of paper is :

$$(I_{ZZ})_{dr} = (2\pi r dr) r^2 = 2\pi r^3 dr$$

$$I_{ZZ} = \int_0^R 2\pi r^3 dr = 2\pi \left(\frac{r^4}{4} \right)_0^R = \frac{\pi R^4}{2} \Rightarrow I_{ZZ} = \frac{\pi D^4}{32}$$

$$I_{ZZ} = I_{XX} + I_{YY}$$

$$I_{XX} = I_{YY} = I_{ZZ}/2 = \frac{\pi D^4}{64}$$



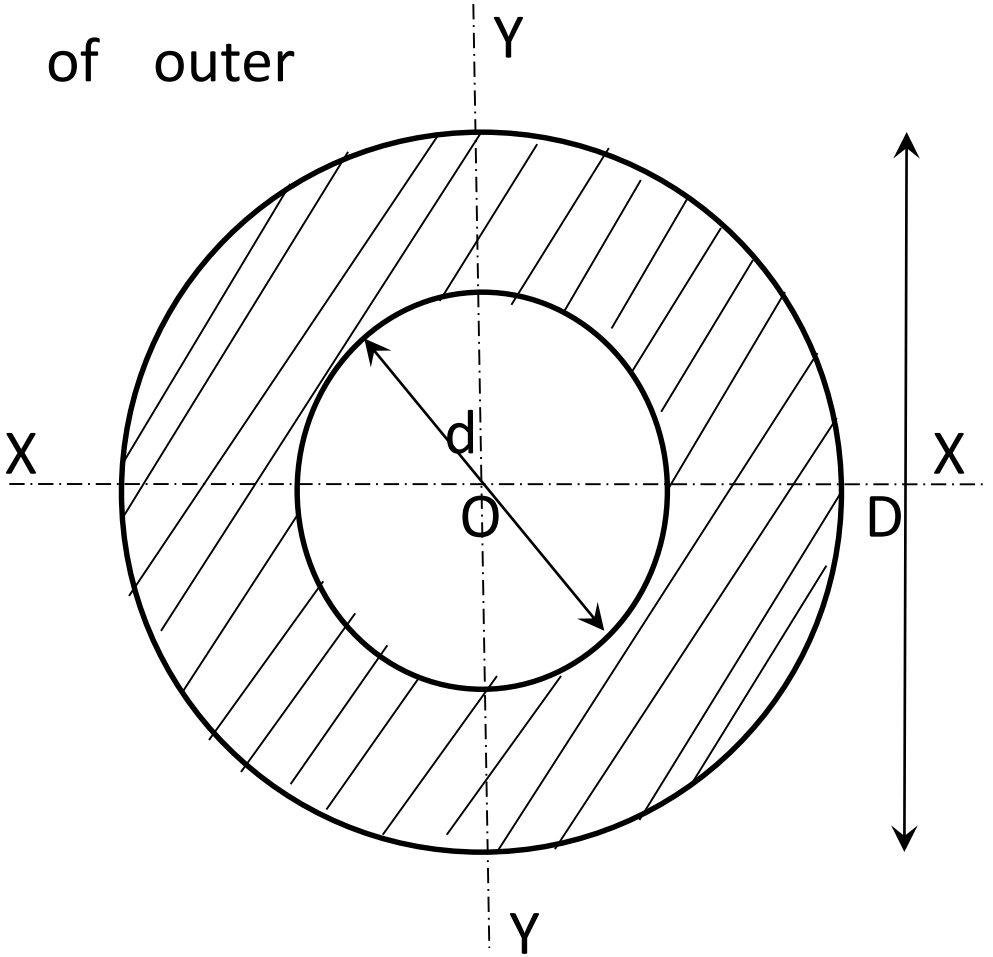
Moment of Inertia of hollow circular section

Consider a hollow circular section of outer diameter D and inner diameter d.

$$\left(I_{XX}\right)_{outer} = \frac{\pi D^4}{64}$$

$$\left(I_{XX}\right)_{inner} = \frac{\pi d^4}{64}$$

$$\left(I_{XX}\right)_{hollow} = \frac{\pi}{64}\left(D^4 - d^4\right)$$



Summary

- Moment of inertia of area (mass) of various sections about the considered reference axis is obtained by integration method.

- Perpendicular axis theorem is applied for finding out the moment of inertia of circular sections.