Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 33:
Centre of gravity/ Centroid of Planes

Delivered By: Dr. T. Valsa Ipe

Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

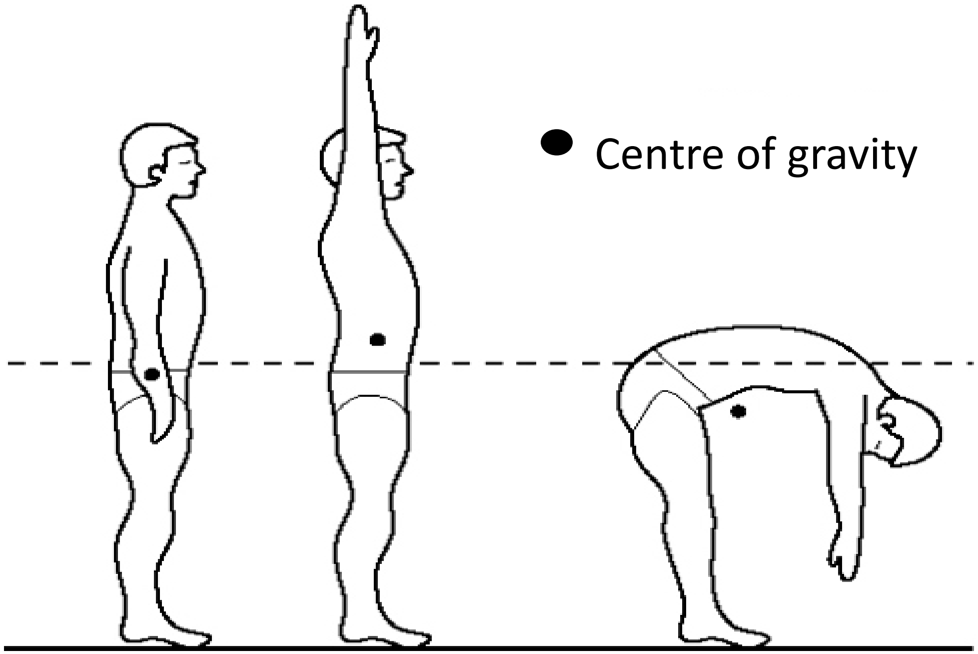
- Define centre of gravity and centroid
- Derive the centroid of lines, planes and volumes
- Solve the problems on CG of structural sections

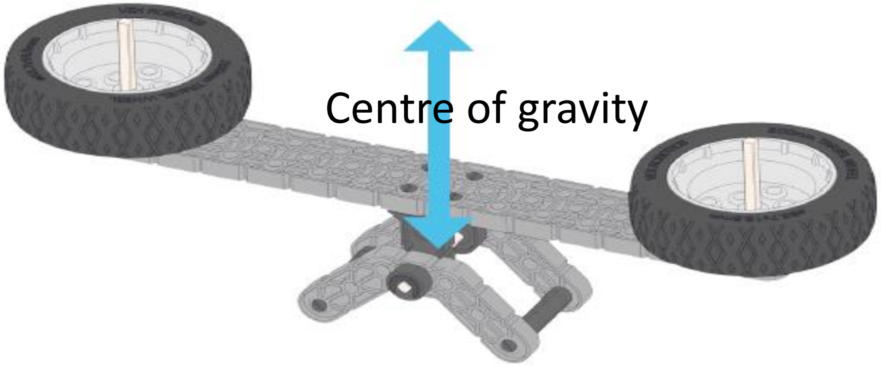
Contents

Center of Gravity, Determination of Center of Gravity, Centroid, Centroid of area, wire or rod, volume, Axis of Symmetry, centroid of rectangular area.

Centre of Gravity

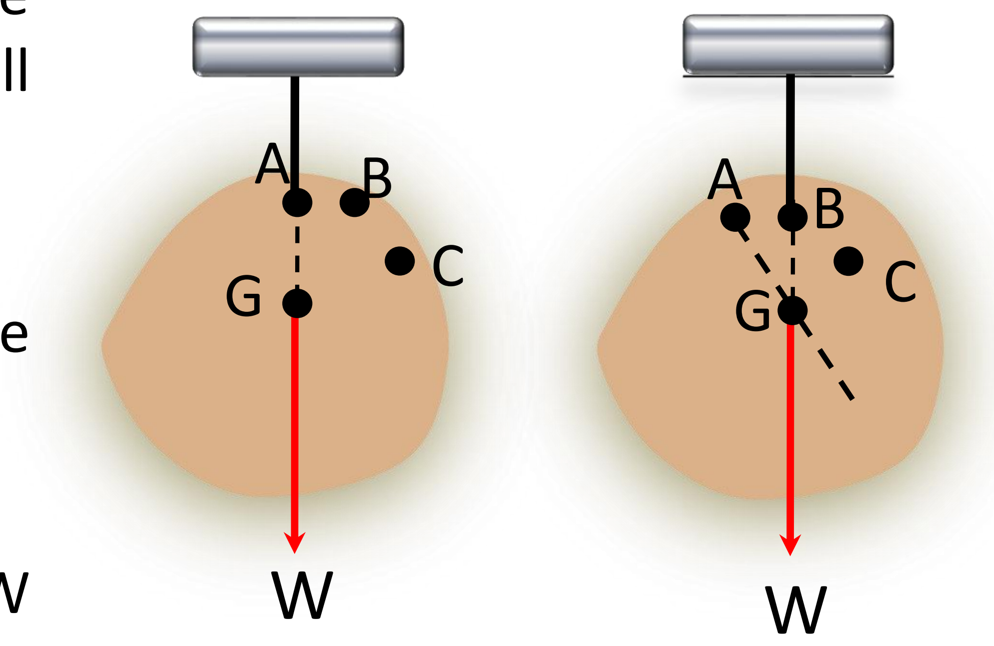
- Centre of gravity of a body is a point through which the whole weight of the body acts.
- A body will have only one centre of gravity for all positions of the body. It is represented by C.G. or G





Centre of Gravity

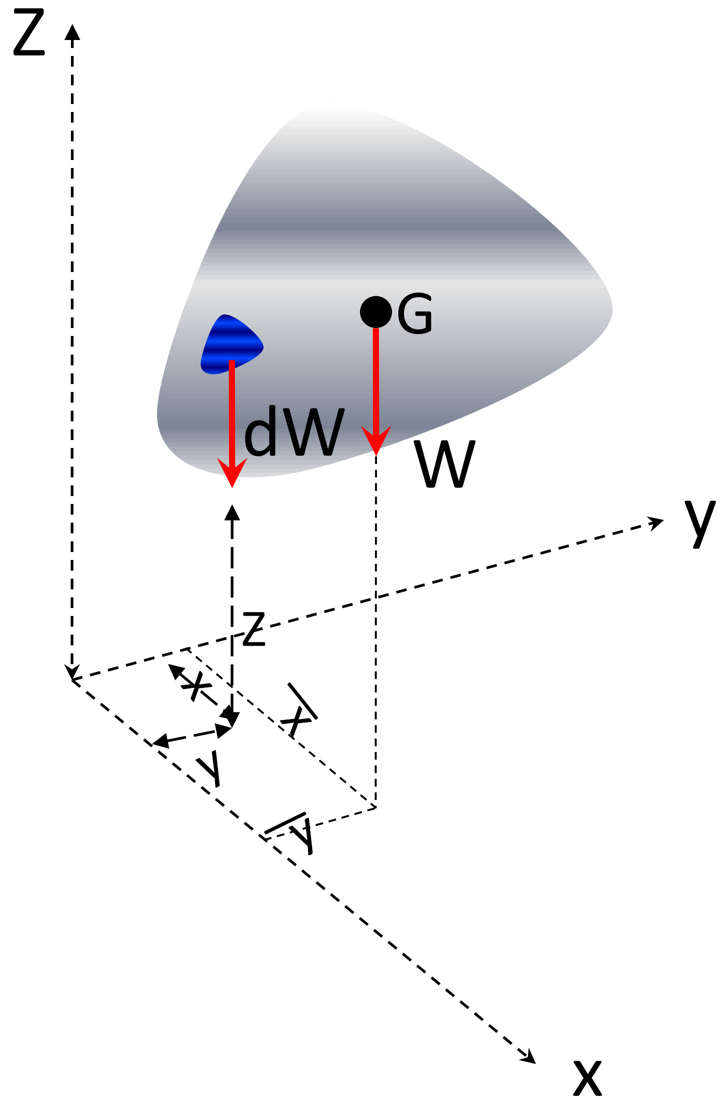
- A body of mass m in equilibrium is under the action of tension in the cord, and resultant W of the gravitational forces acting on all particles of the body.
- The resultant is collinear with the cord.
- The unique point through which W acts is the centre of gravity C.G or G.



Body is suspended at different points

Determination of Centre of Gravity

- Apply principle of moments
  - Moment of resultant gravitational force W about any axis equals sum of the moments about the same axis of the gravitational forces dW acting on all particles treated as infinitesimal elements



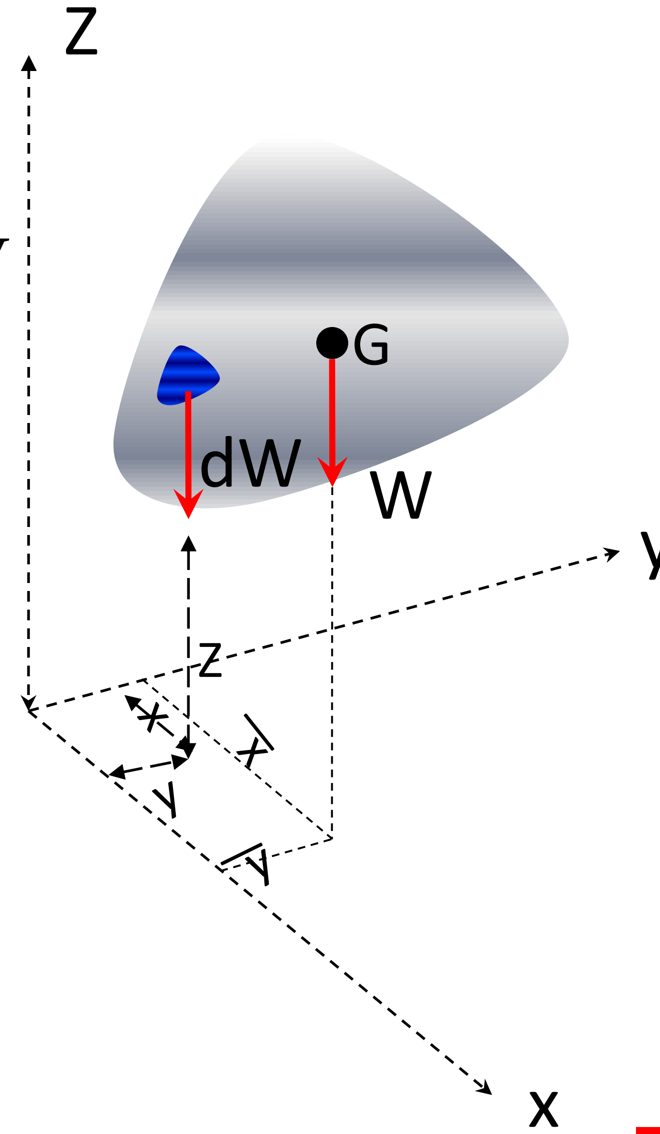
Determination of Centre of gravity

Weight of the body , $W = \int dW$

Moment of *dW* about x-axis, $= ydW$

Sum of moment of all elements $= \int ydW$

From principle of moments, $\int ydW = \bar{y}W$



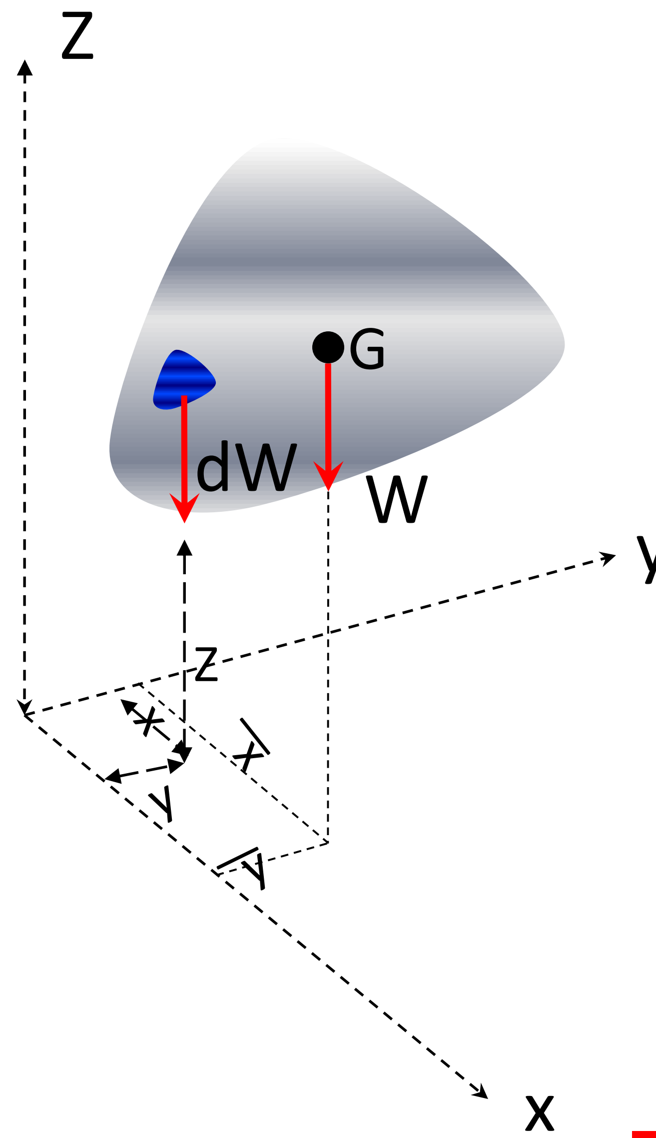
$$\bar{x} = \frac{\int xdW}{W}; \bar{y} = \frac{\int ydW}{W}; \bar{z} = \frac{\int zdW}{W}$$

Determination of Centre of gravity

$$\bar{x} = \frac{\int x dW}{W}; \bar{y} = \frac{\int y dW}{W}; \bar{z} = \frac{\int z dW}{W}$$

Substituting $W = mg; dW = gdm$

$$\bar{x} = \frac{\int x dm}{m}; \bar{y} = \frac{\int y dm}{m}; \bar{z} = \frac{\int z dm}{m}$$



Determination of Centre of gravity

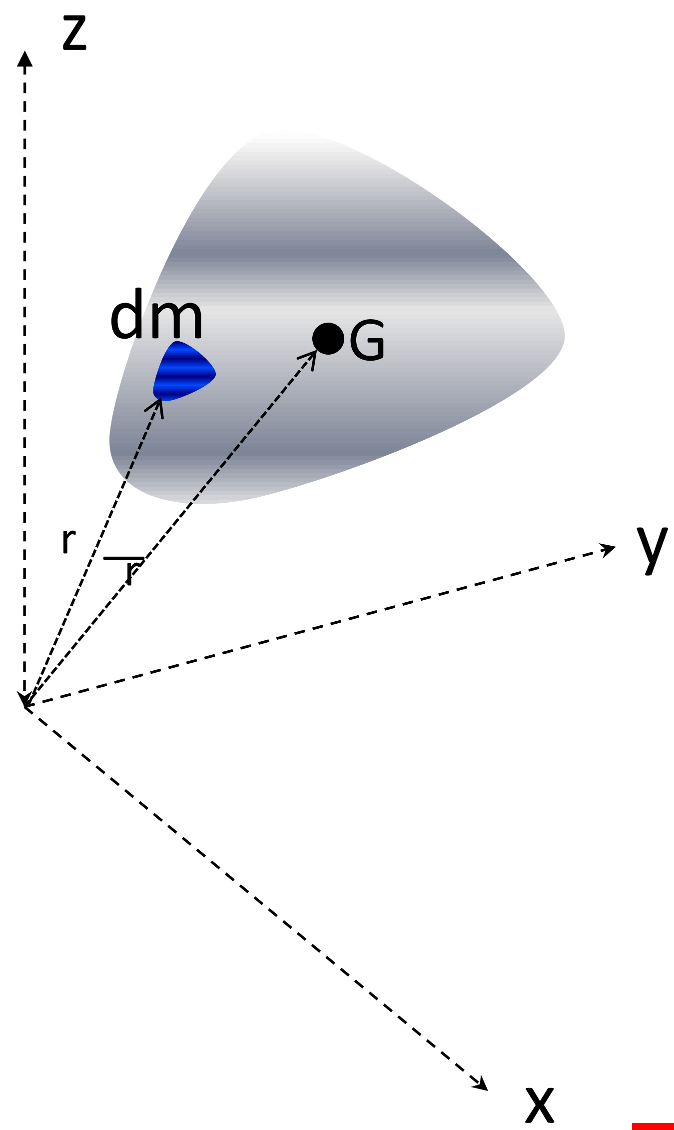
Position vector for mass dm,

$$r = xi + yj + zk$$

Position vector for mass centre G,

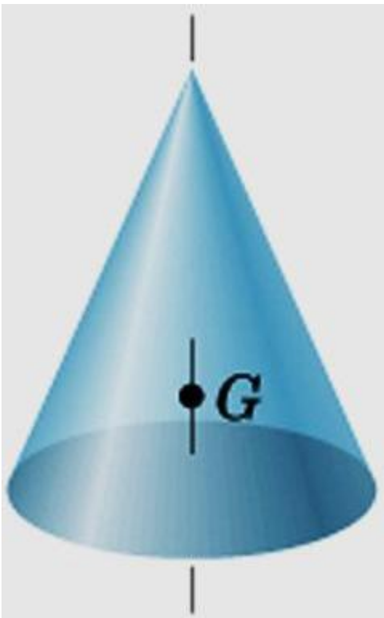
$$\bar{r} = \bar{x}\,i + \bar{y}\,j + \bar{z}\,k$$

$$\bar{r} = \frac{\int r\,dm}{m}$$



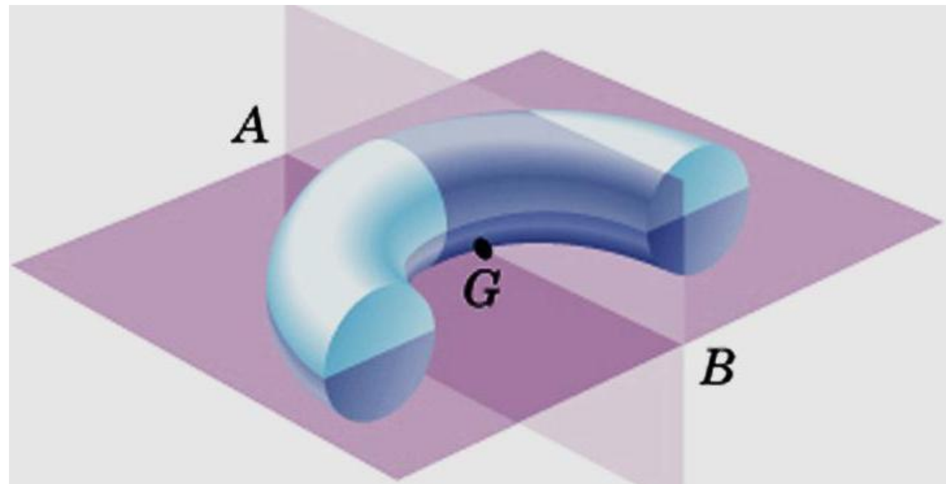
Centre of gravity

CG always lie on a line or a plane of symmetry in a homogeneous body



Right Circular Cone
CG on central axis

Half Right Circular Cone
CM on vertical plane of symmetry



Half Ring
CM on intersection of two planes of symmetry (line AB)

Centroid

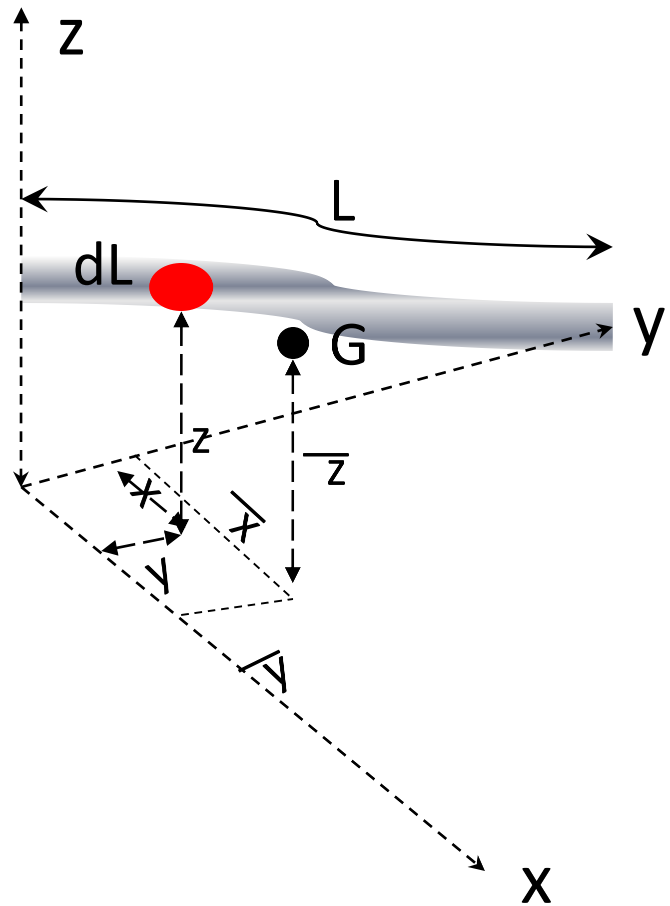
- The point at which the total area of a plane figure/area (like rectangle, triangle, quadrilateral, circle, square etc) is assumed to be concentrated is known as Centroid of that area.
- It is represented by C.G. or G.
- Centroid represents the geometric centre of a body.
- In other words, it is the geometrical property of the body.
- This will coincide with the centre of gravity only if the material of the body is uniform or homogenous.

Centroid of a rod or wire

Consider a slender rod or wire:

Cross-sectional area = *A;* ρ & A are constant over L

$$\therefore dm = \rho A dL$$



Centroid and CG are the same points

$$\bar{x} = \frac{\int x\,dm}{m};\ \bar{y} = \frac{\int y\,dm}{m};\ \bar{z} = \frac{\int z\,dm}{m}$$

$$\bar{x} = \frac{\int x\,dL}{L};\ \bar{y} = \frac{\int y\,dL}{L};\ \bar{z} = \frac{\int z\,dL}{L}$$

$$\bar{x} = \frac{x_1 L_1 + x_2 L_2 + x_3 L_3 + \ldots\ldots\ldots}{L}$$

Centroid of an Area

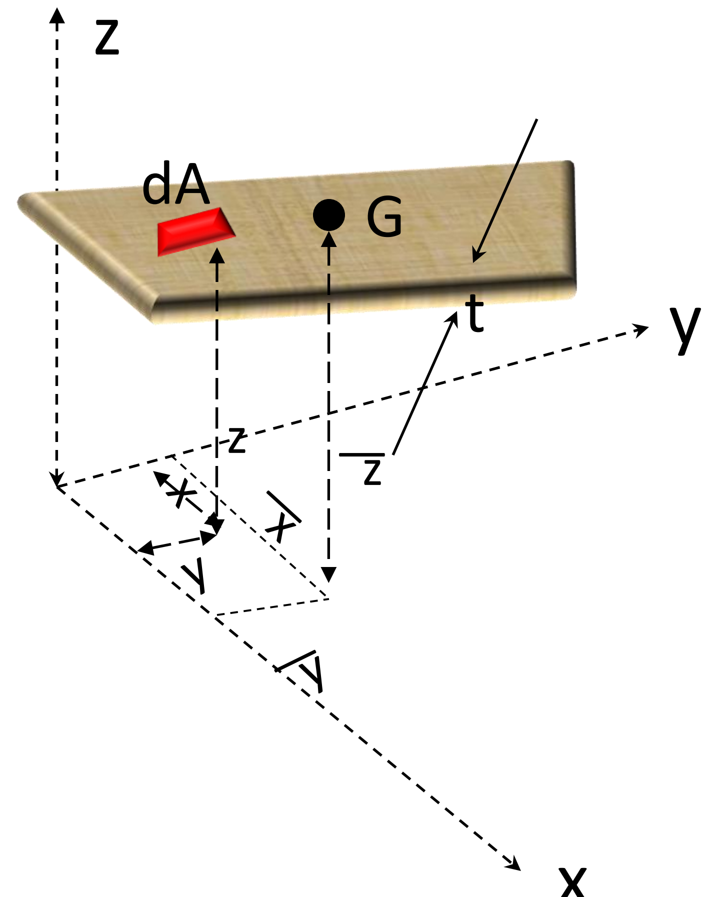
Consider an area : Body with small but constant thickness t

Cross-sectional area = *A*

ρ and A are constant over A

$$\therefore dm = \rho t dA$$

Centroid and CG are the same points



$$\bar{x} = \frac{\int x dm}{m};\ \bar{y} = \frac{\int y dm}{m};\ \bar{z} = \frac{\int z dm}{m}$$

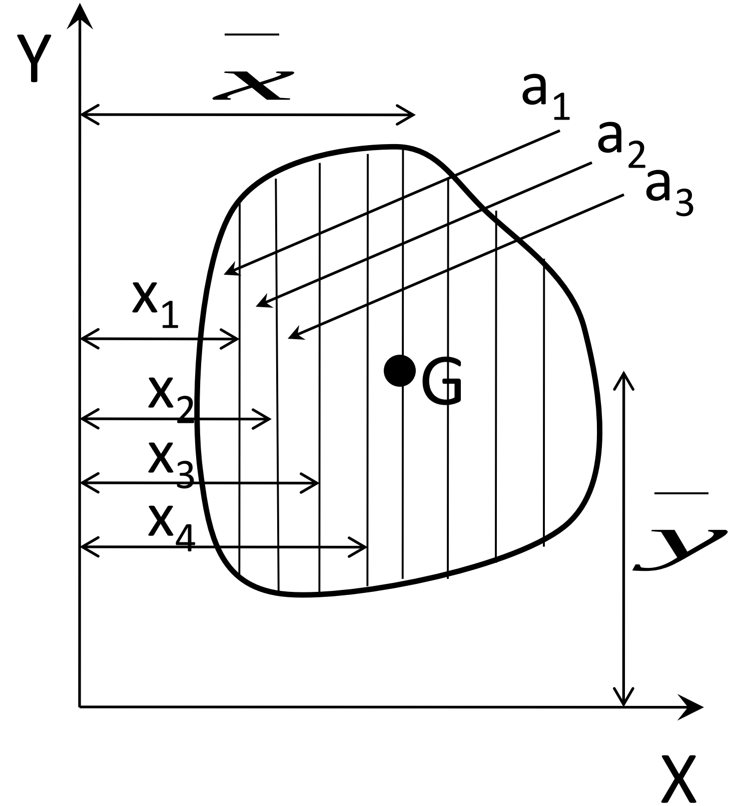
$$\bar{x} = \frac{\int x dA}{A};\ \bar{y} = \frac{\int y dA}{A};\ \bar{z} = \frac{\int z dA}{A}$$

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2 + x_3 A_3 + \ldots\ldots\ldots}{A}$$

Note: $\int x dA$ First moments of area

Centroid of Plane Areas

- Consider a plane area A lying in XY plane whose centroid lies at G
- To locate the position of G with respect to X-axis and Y-axis,
  - Divide the given area A into smaller elemental areas a_1, a_2, a_3, ...
  - Let x_1, x_2, x_3, ... be the distance of the CG of the respective elemental areas from Y-axis.



Centroid of Plane Areas

Taking moments of the elemental areas about Y-axis

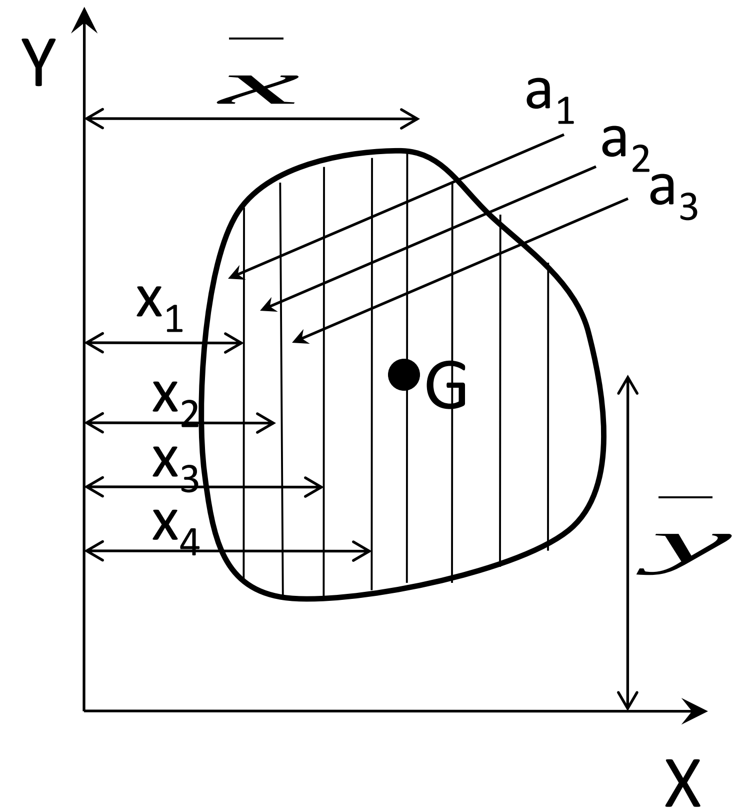
$$M_{y-axis} = a_1x_1 + a_2x_2 + a_3x_3 + ...$$

Taking moment of the entire area about Y-axis

$$M_{y-axis} = A\bar{x}$$

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + ...$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + ...}{A} = \frac{\sum a_nx_n}{A}$$



$$A\bar{y} = a_1y_1 + a_2y_2 + a_3y_3 + ...$$

Similarly $$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + ...}{A} = \frac{\sum a_ny_n}{A}$$

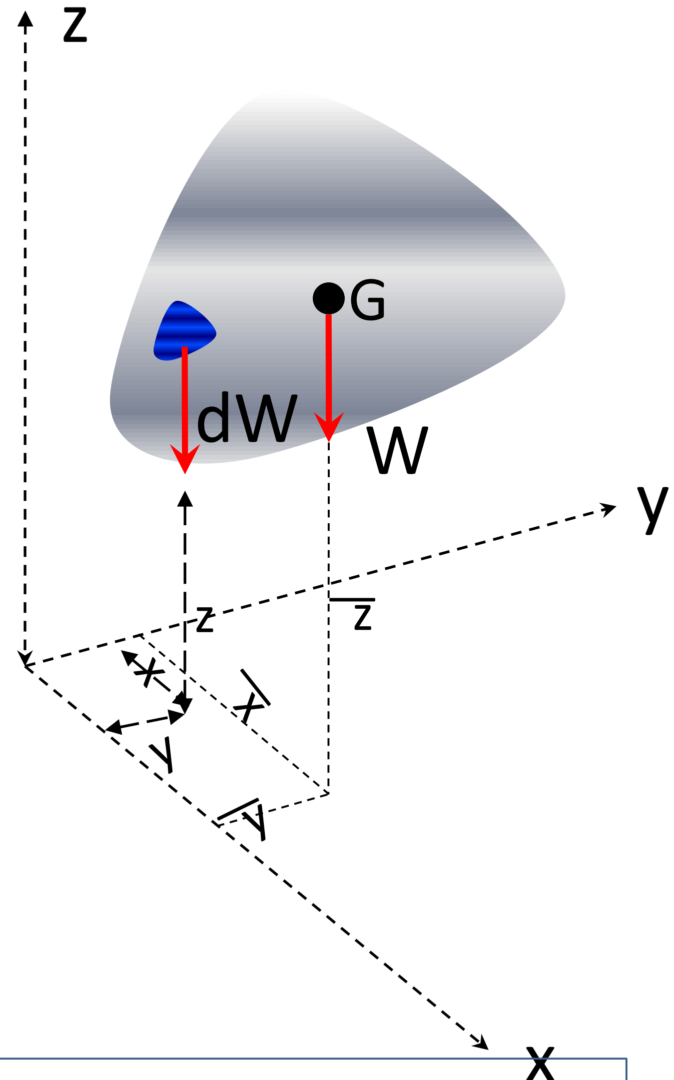
Centroid of a Volume

consider a volume: body with volume v

ρ is constant over v

$$\therefore dm = \rho dV$$

Centroid and CG are the same point



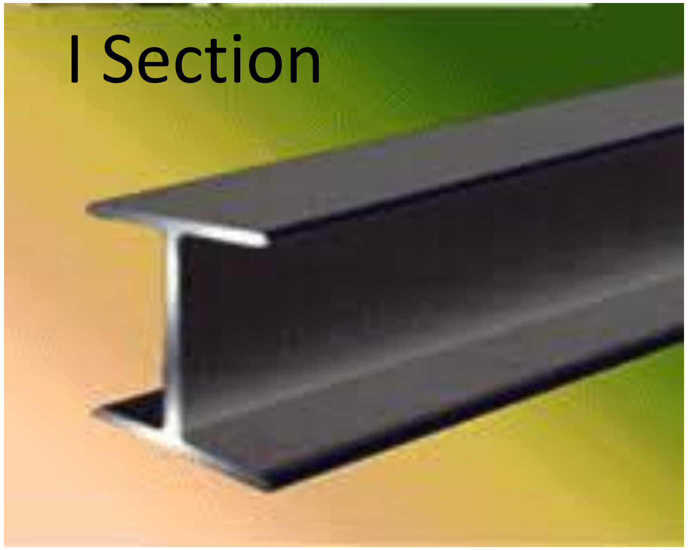
$$\bar{x} = \frac{\int x dm}{m}; \bar{y} = \frac{\int y dm}{m}; \bar{z} = \frac{\int z dm}{m}$$

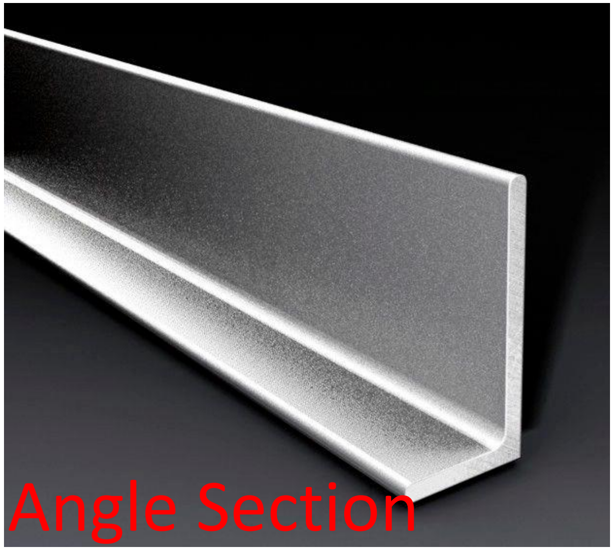
$$\bar{x} = \frac{\int x dV}{V}; \bar{y} = \frac{\int y dV}{V}; \bar{z} = \frac{\int z dV}{V}$$

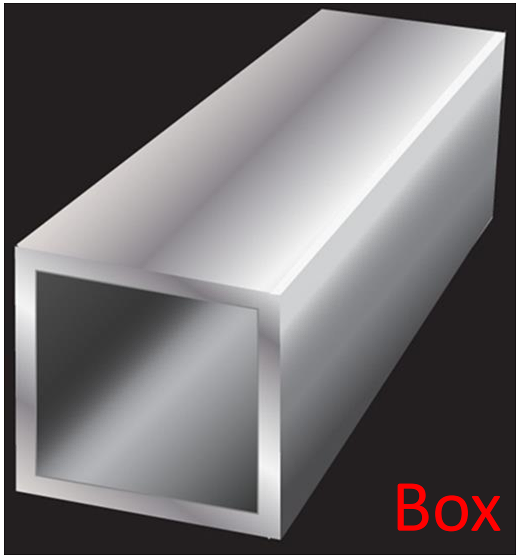
$$\bar{x} = \frac{x_1 V_1 + x_2 V_2 + x_3 L V_3 + \ldots\ldots\ldots}{V}$$

Note: $\int xdV$ First moments of volume

Centre of gravity of structural sections

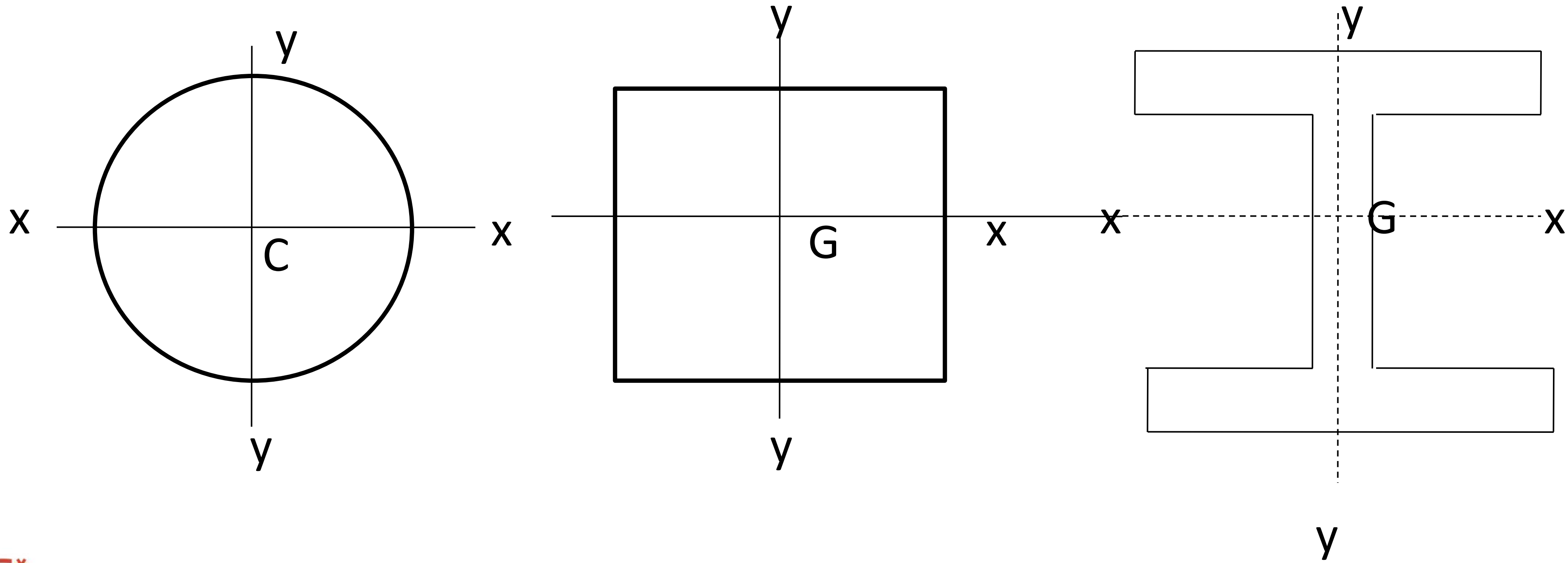






Axis of Symmetry

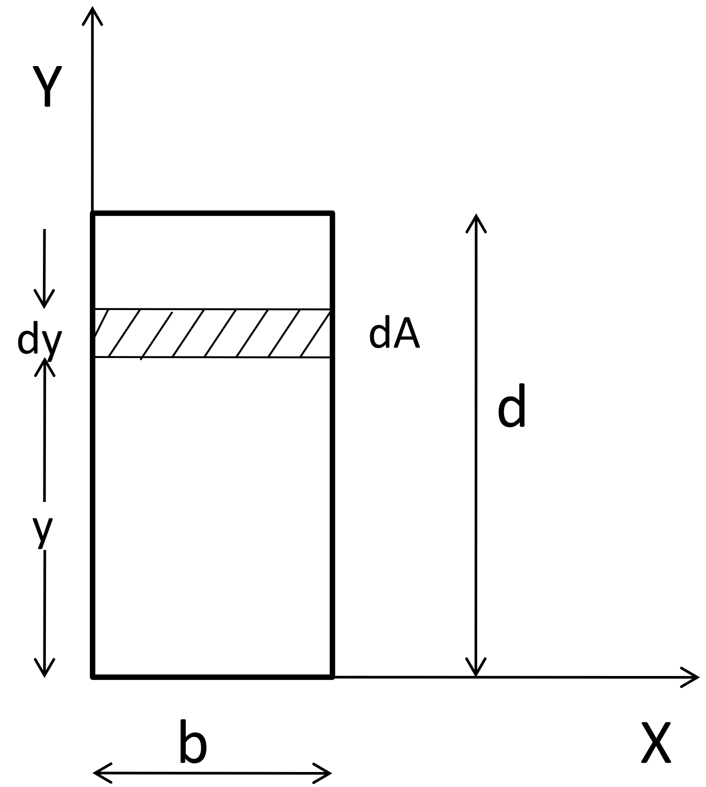
- It is a line or axis which divides the given line, area or volume into two equal and identical parts.
- Reference line about which mirror image of an object is obtained
- Centroid always lies along the axis of symmetry



Centroid of a Rectangular Area

- Consider a rectangle of breadth b and depth d
- Let G be the centroid
- Consider an elemental area dA of breadth b and thickness dy at a distance y from X-axis
- Area of the element, $dA = bdy$
- Taking moment of dA about X-axis,

$$dM_x = bdy \times y$$



Centroid of a Rectangular Area

- Moment of the entire area A about X-axis

$$M_x = \int_0^d dAy$$

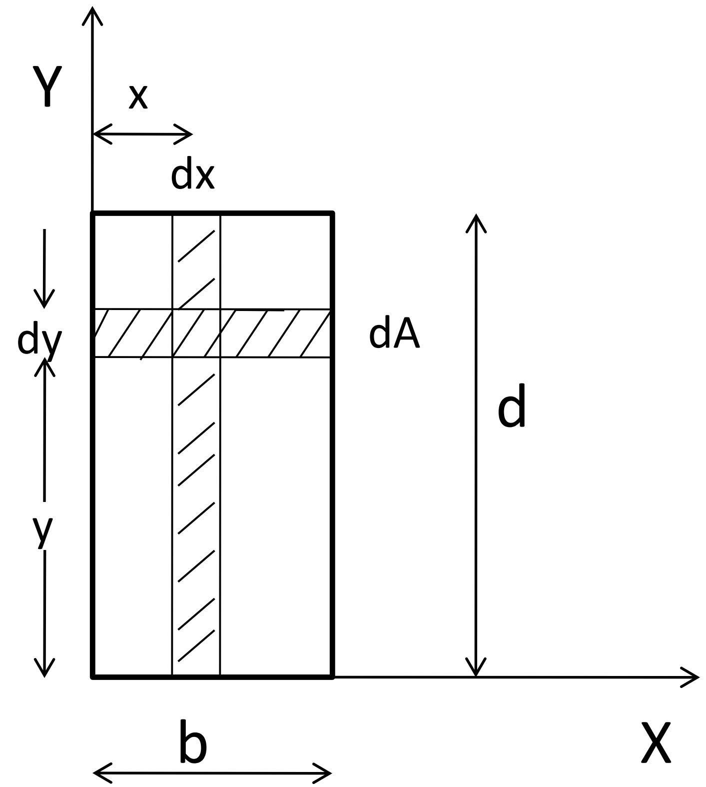
$$M_x = \int_0^d bydy$$

$$M_x = \left[\frac{by^2}{2}\right]_0^d = \frac{bd^2}{2}$$

$$M_y = \frac{b^2d}{2}$$

$$\bar{x} = \frac{M_y}{A} = \frac{b}{2}$$

$$\bar{y} = \frac{M_x}{A} = \frac{d}{2}$$



Summary

- The point through which the whole weight of the body acts is known as centre of gravity.
- The point at which the total area of a plane figure is assumed to be concentrated is known as centroid of that area. The centroid and centre of gravity coincides if the material of the body is uniform or homogenous.
- Axis of symmetry is a line or axis which divides the given line, area or volume into two equal and identical parts
- If a given section is symmetrical about X-X axis or Y-Y axis, the CG of the section will lie on the axis of symmetry.