

**Course Code: ESC106A**  
**Course Title: Construction Materials and  
Engineering Mechanics**

**Lecture No. 39:  
Moment of Inertia**

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# Lecture Intended Learning Outcomes

**At the end of this lecture, students will be able to:**

- Define Moment of Inertia
- Determine Moment of Inertia of a general element using integration method



# Contents

Moment of inertia of an area under a curve



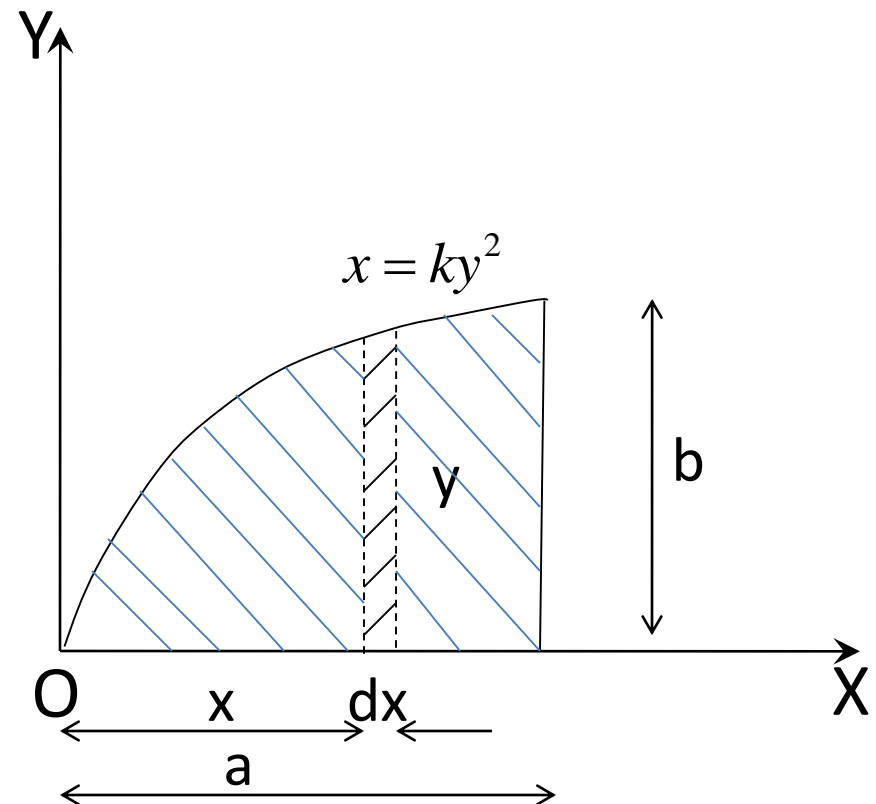
# Moment of Inertia of an Area under a given curve

Consider a curve whose equation is parabolic and is given by  $x = ky^2$

$x$  varies from 0 to  $a$

$y$  varies from 0 to  $b$

Consider a strip of thickness  $dx$  at a distance  $x$  from  $y$ -axis



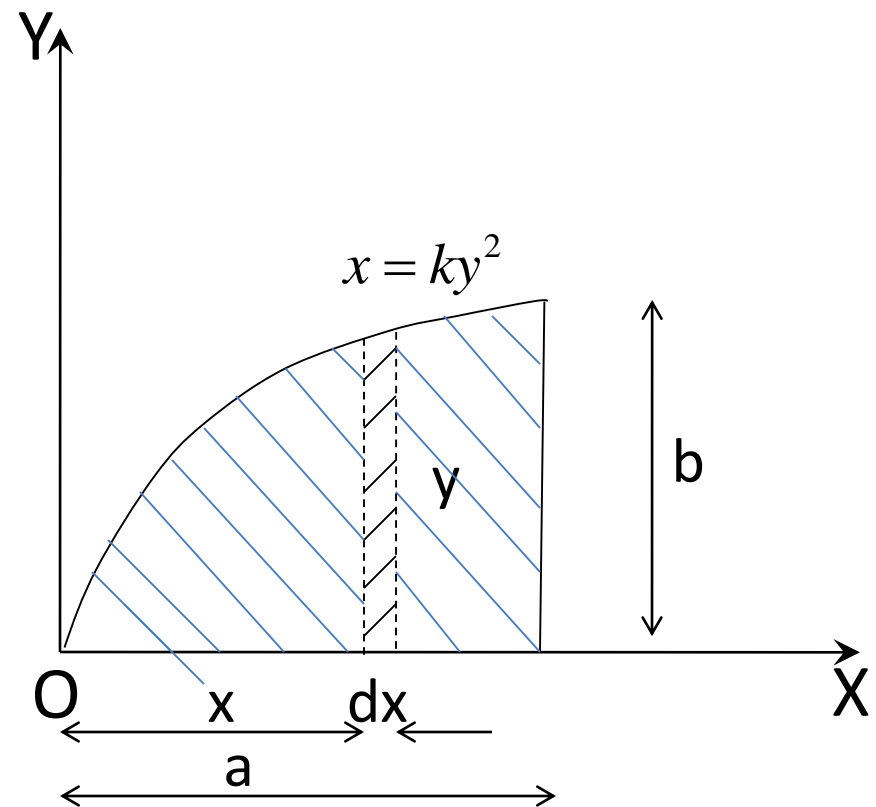
# Moment of Inertia of an Area under a given curve

Area of the strip,  $dA = ydx$

From the equation of the curve,

$$x = ky^2$$

Find the value of  $k$  using the given values of  $x$  and  $y$



# Moment of Inertia of an Area under a given curve

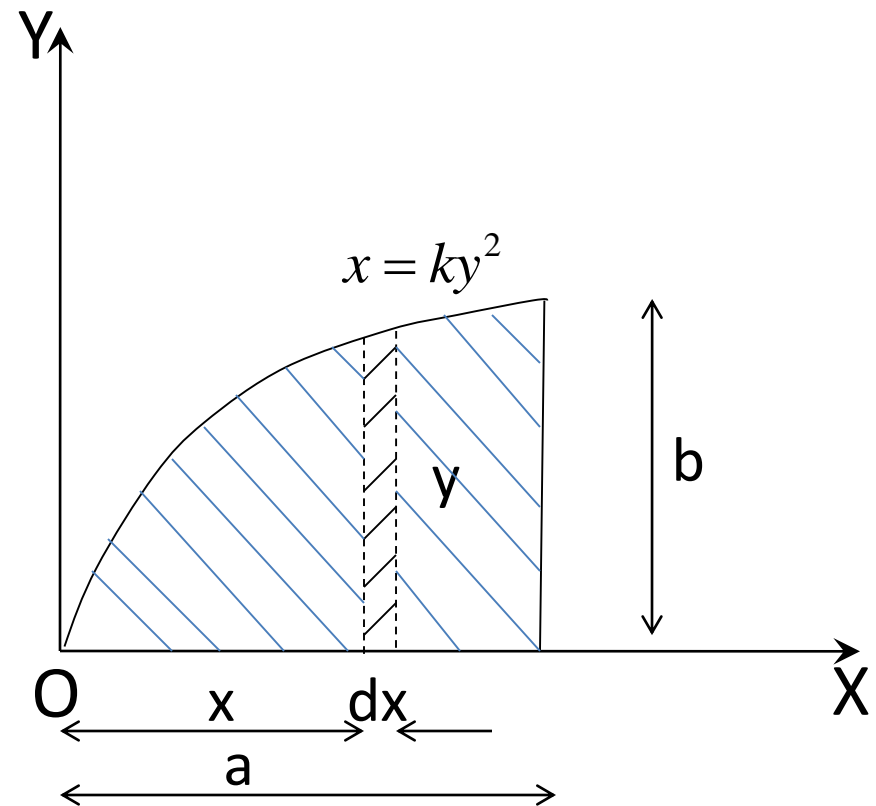
When  $y=b$ ,  $x=a$

$$a = kb^2$$

$$k = \frac{a}{b^2}$$

Substituting the value of  $k$

$$x = \frac{a}{b^2} y^2$$



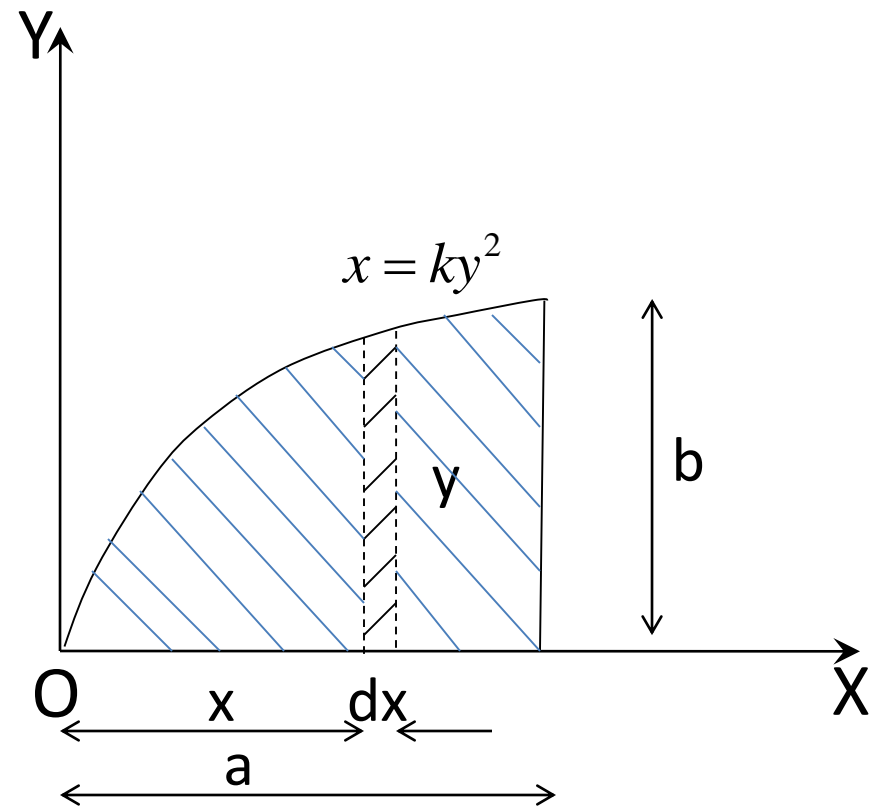
# Moment of Inertia of an Area under a given curve

$\Rightarrow$

$$y^2 = \frac{b^2 x}{a}$$

$$y = b \sqrt{\frac{x}{a}}$$

Substituting this value in the equation of dA

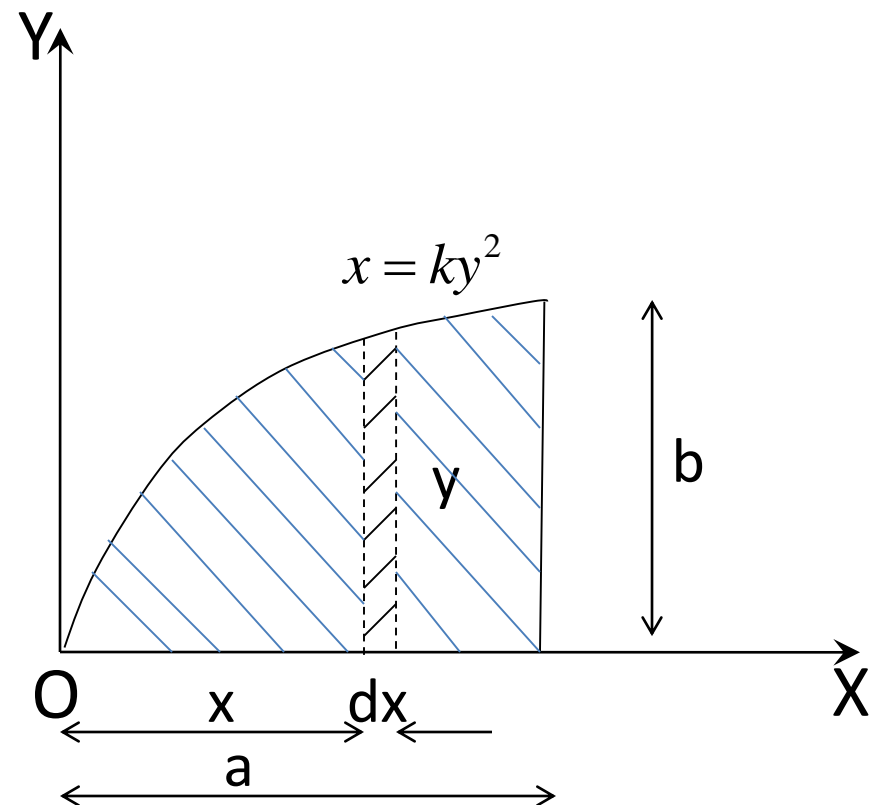


# Moment of Inertia of an Area under a given curve

$$dA = b \sqrt{\frac{x}{a}} \cdot dx$$

The moment of this elemental area  $dA$  about the OY axis is

$$dA \cdot x$$

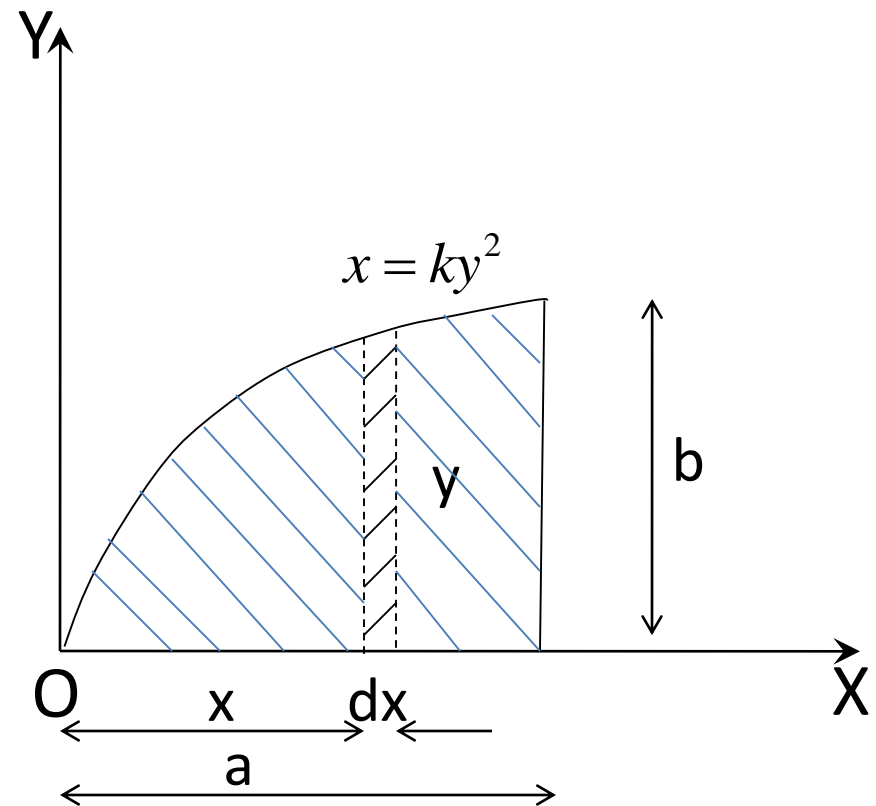




# Moment of Inertia of an Area under a given curve

The moment of inertia of this elemental area  $dA$  about the OY axis is

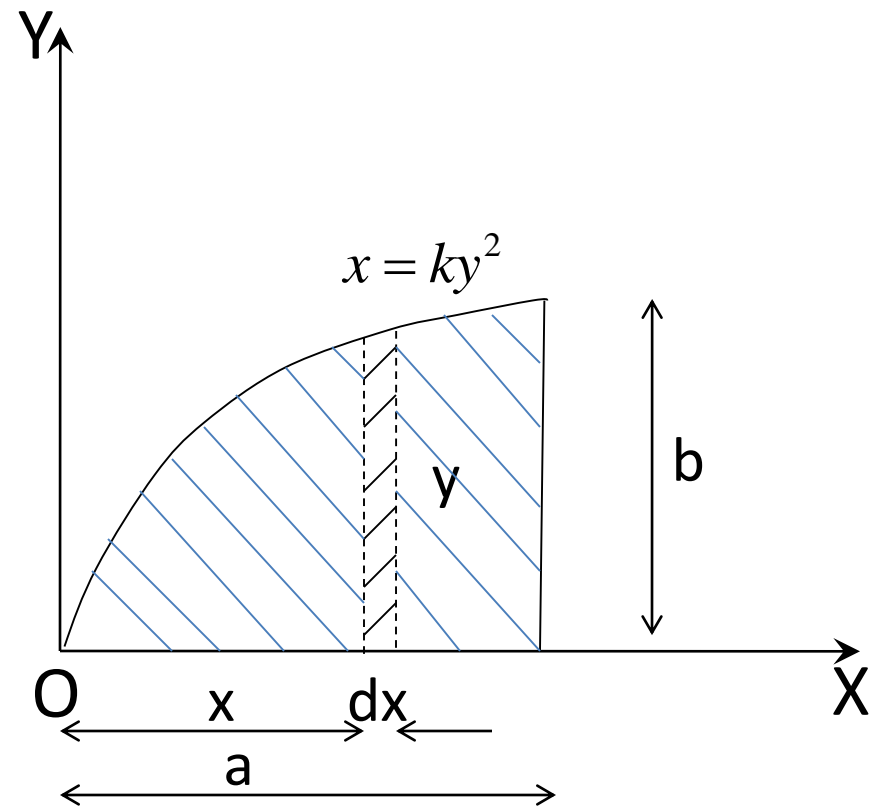
$$\begin{aligned}dA \cdot x \cdot x &= dA \cdot x^2 \\&= x^2 \cdot b \cdot \sqrt{\frac{x}{a}} \cdot dx\end{aligned}$$



# Moment of Inertia of an Area under a given curve

Moment of inertia of the total area about y-axis is obtained by integrating the above equation within the limits 0 to a.

$$\begin{aligned}dA.x.x &= dA.x^2 \\ &= x^2.b.\sqrt{\frac{x}{a}}.dx\end{aligned}$$



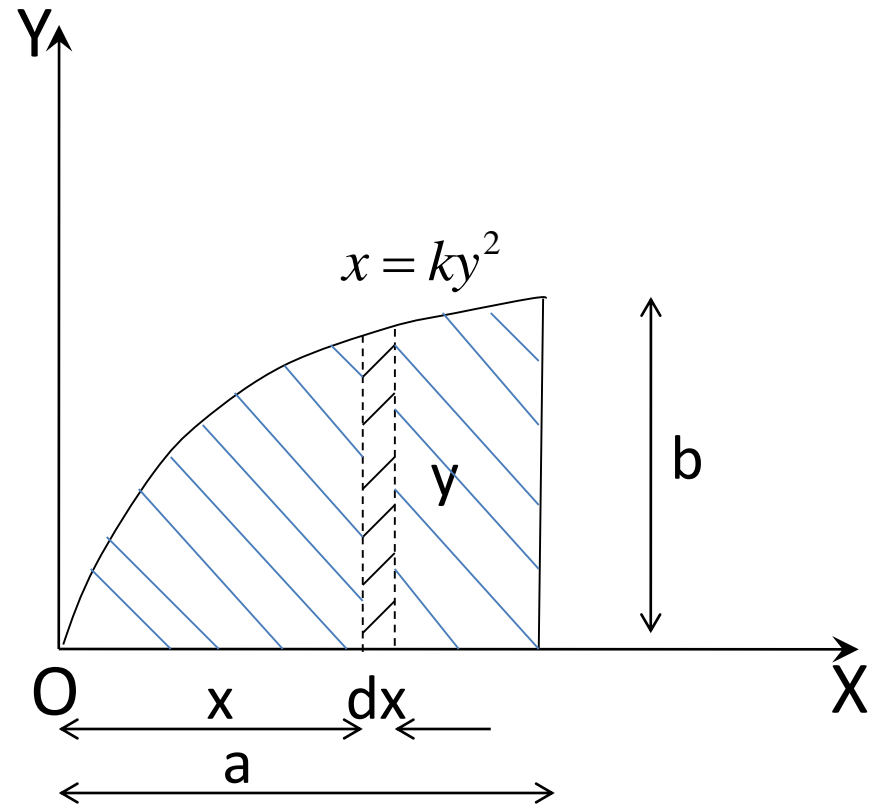
# Moment of Inertia of an Area under a given curve

$$I_{yy} = \int_0^a x^2 b \cdot \sqrt{\frac{x}{a}} \cdot dx$$

$$I_{yy} = \frac{b}{\sqrt{a}} \int_0^a x^{\frac{5}{2}} dx$$

$$I_{yy} = \frac{b}{\sqrt{a}} \left[ \frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^a$$

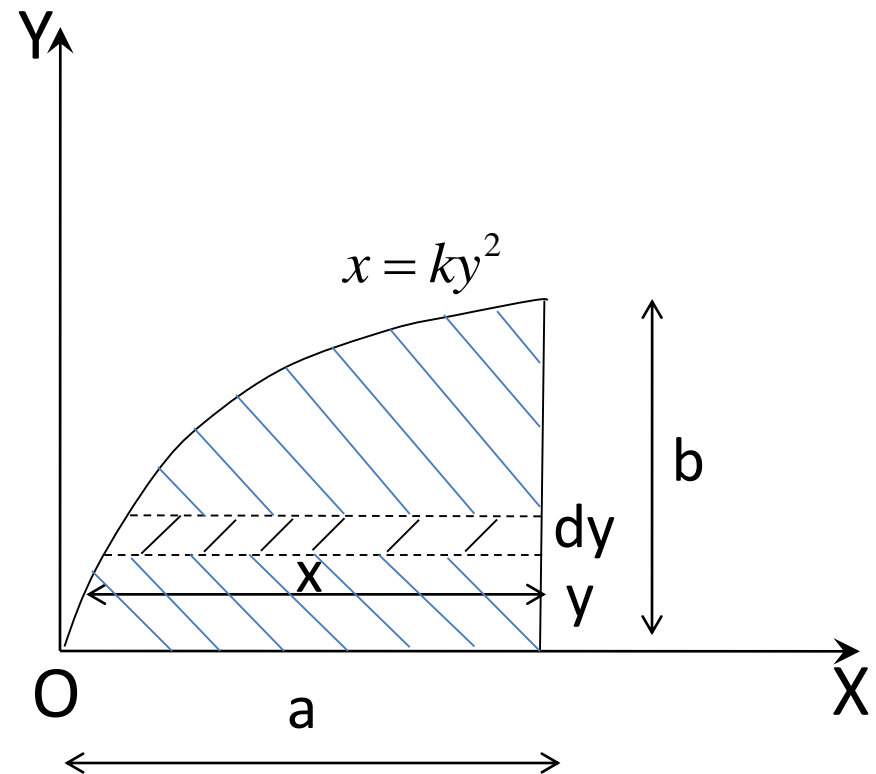
$$I_{yy} = \frac{2b}{7\sqrt{a}} \cdot a^{\frac{7}{2}} = \frac{2}{7} ba^2$$



# Moment of Inertia of an Area under a given curve

Consider a strip of thickness  $dy$  and length  $x$  parallel to  $x$ -axis.

Area of the strip ,  $dA = x \cdot dy$



# Moment of Inertia of an Area under a given curve

Moment of this elemental area

$dA$  about the OX axis is given by,

$$dA \cdot y$$

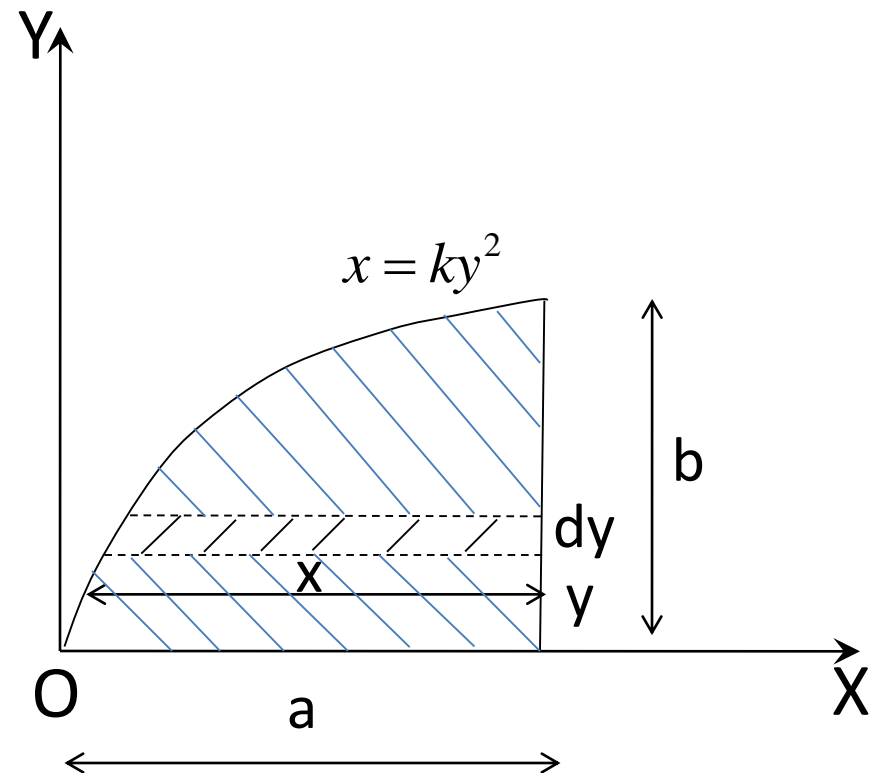
Moment of inertia of the total

area about x-axis is obtained by

integrating the moment of

moment of area within the limits

0 to  $b$ .



# Moment of Inertia of an Area under a given curve

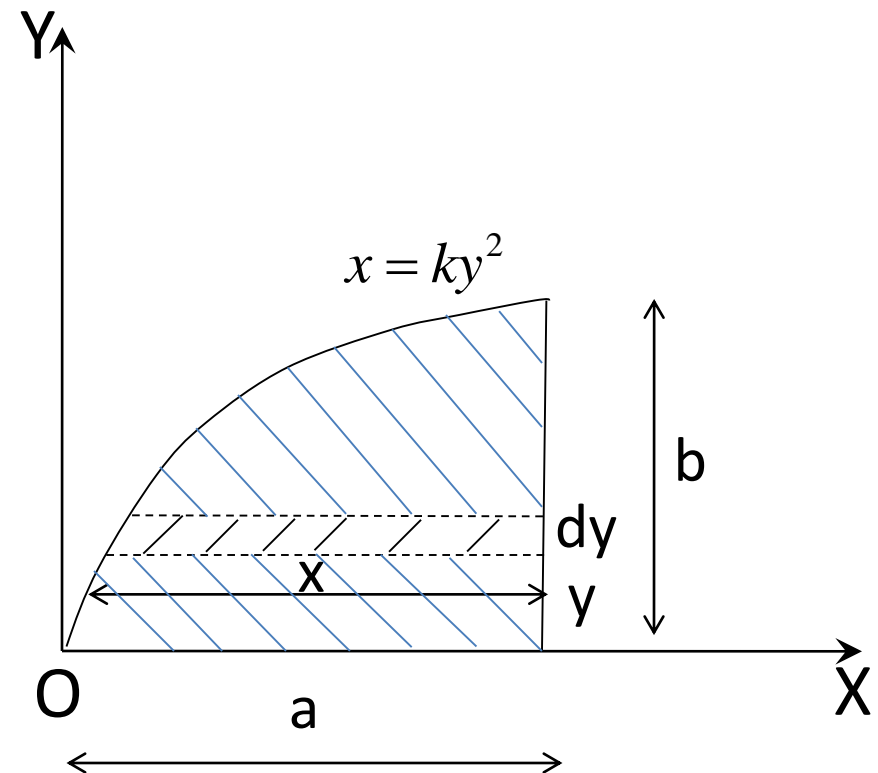
$$I_{xx} = \int_0^b dA y^2$$

$$I_{xx} = \int_0^b x \cdot dy \cdot y^2$$

$$I_{xx} = \int_0^b ky^2 \cdot dy \cdot y^2$$

$$I_{xx} = \frac{a}{b^2} \int_0^b y^4 \cdot dy = \frac{a}{b^2} \left[ \frac{y^5}{5} \right]_0^b$$

$$I_{xx} = \frac{a}{b^2} \frac{b^5}{5} = \frac{1}{5} ab^3$$

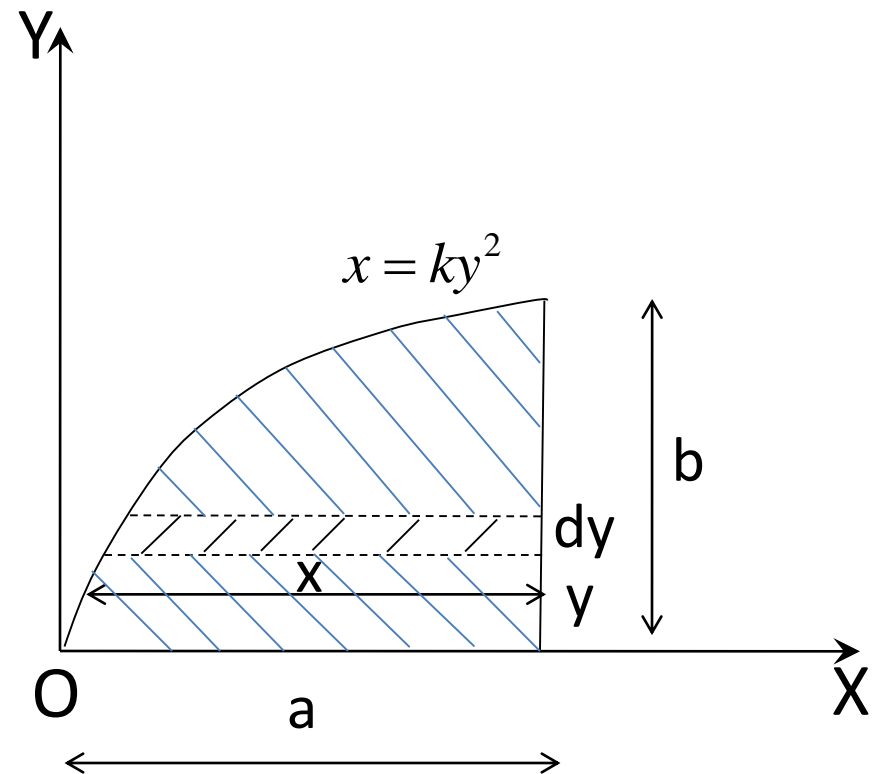


# Moment of Inertia of an Area under a given curve

Moment of Inertia of  
an Area under a given  
curve  $x = ky^2$

$$I_{xx} = \frac{1}{5} ab^3$$

$$I_{yy} = \frac{2}{7} a^2 b$$



# Summary

- Moment of inertia measures how an area is distributed about particular axes
- Moment of inertia of area (mass) about reference axis is obtained by integration method

