Lecture 29
Conformal Mapping -3

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Intended learning Outcomes

At the end of this lecture, student will be able to:

- Illustrate conformal mapping
- Discuss the properties of standard conformal mappings

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Topics

- Joukowski Transformation

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Joukowski Transformation $f(z) = z + 1/z$

- The complex function $f(z) = z + 1/z$ is conformal at all points except $z = \pm 1$ and $z = 0$. In particular, the function is conformal at all points in the upper half-plane satisfying $|z| > 1$.
- If $z = re^{i\theta}$, then $w = re^{i\theta} + (1/r)e^{-i\theta}$, and so

$$u = \left(r + \frac{1}{r}\right)\cos\theta, \quad v = \left(r - \frac{1}{r}\right)\sin\theta$$

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Example-1

The Transformation $w = z + \frac{a^2}{z}$

Consider the transformation $w = z + \frac{a^2}{z}$ −-----------(i)

Put $z = re^{i\theta}$

and $w = u + iv$ In (1)

$$u + iv = re^{i\theta} + \frac{a^2}{r}e^{-i\theta}$$

Therefore $u = \left(r + \frac{a^2}{r}\right)cos\theta,$

$v = \left(r - \frac{a^2}{r}\right)sin\theta$ -----(ii)

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Example-1……..

- From these we get
- $\frac{u^2}{r+\frac{a^2}{r}} + \frac{v^2}{r-\frac{a^2}{r}} = 1$-------------------(3)
- Consider the polar equation $r = A$, a constant , which represents a circle centered at the origin in the z-plane.
- From equation(2), we have
- $u^2 + v^2 = 4a^2$

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Example-1……….

- For $\theta = c$, a constant , equation (4) represents a hyperbola having center at the origin of the w-plane .Thus under the transformation (i) the radial line $\theta = c$ in the $z - Plane$ is transformed to the hyperbola

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Example-2

- In the transformation $(w+1)^2 = \frac{4}{z}$, the unit circle in w –Plane corresponds to a parabola in $z - plane$ and inside of the circle to the outside of the parabola?
- If $w = \tan^2\left(\frac{z}{2}\right)$, show that the strip in z –plane between $x = 0, x = \frac{\pi}{2}$ is represented on the interior of the unit circle in w –Plane with a cut along the real axis from $w = -1\ to\ w = 0$

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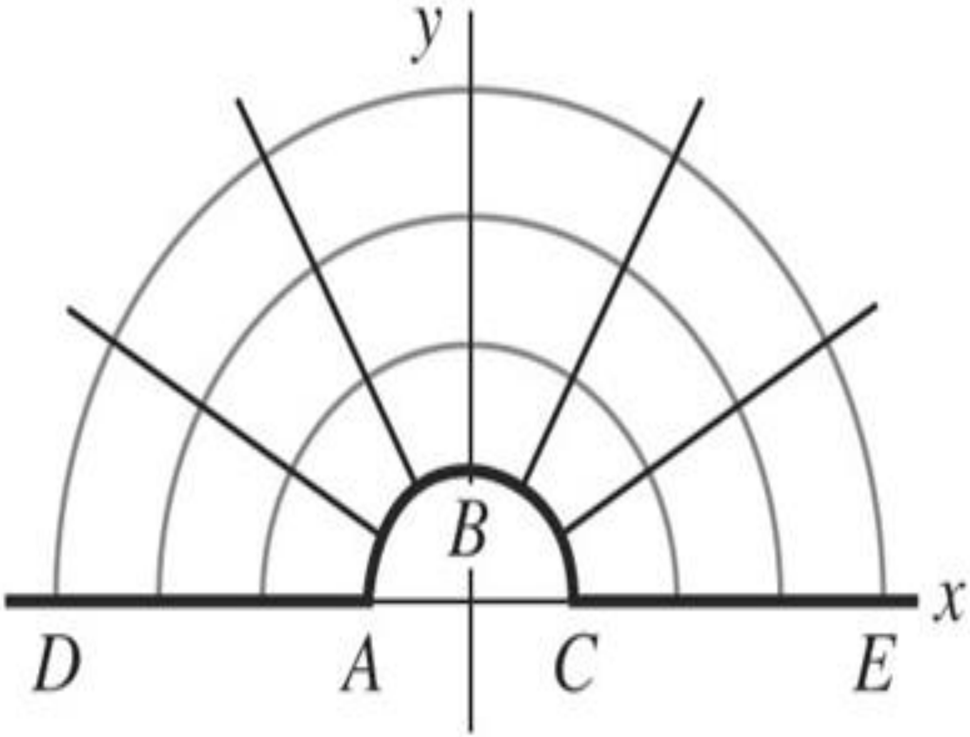
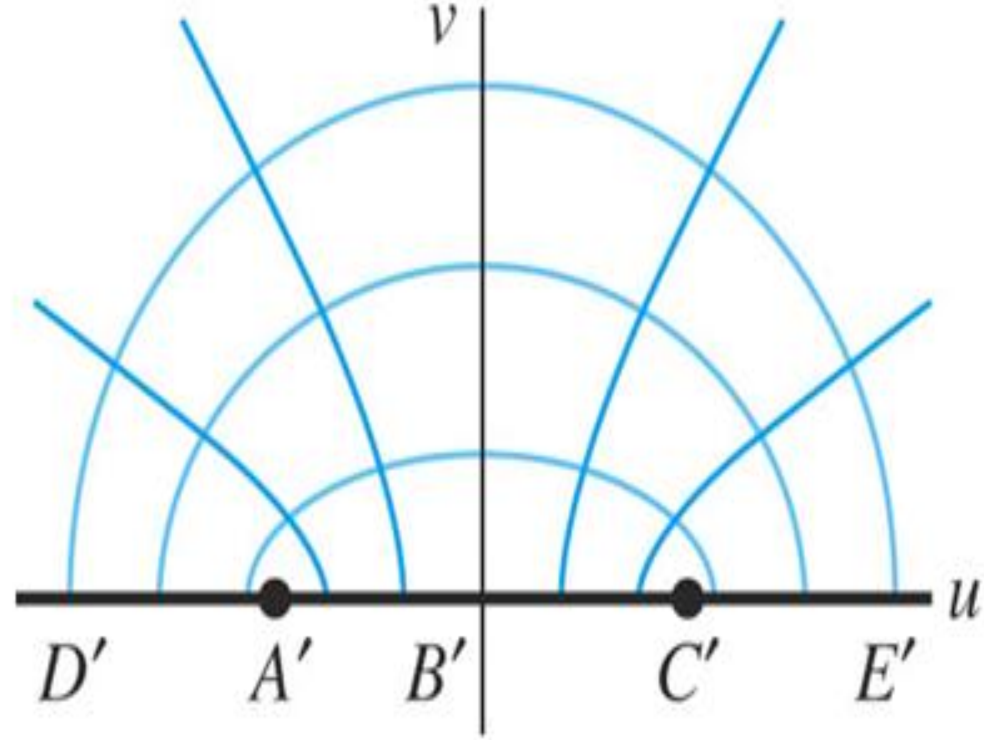


(a) **z - plane**



(b) **w - plane**

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Session Summary

- A complex function w = $f(z)$ gives a **mapping of its domain** in the complex z-plane onto its **range of values** in the complex w-plane. *If* $f(z)$ is analytic, this mapping is **conformal,** that is**, angle-preserving**, i.e., The angle between any two intersecting curves and the corresponding angle between their image curves are the same.

- **Linear fractional transformations,** also called ***Möbius transformations*** map the extended complex plane onto itself.

- They solve the problems of mapping half-planes onto half-planes or disks, and disks onto disks or half-planes.

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