Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 35:
Derivation of Centre of gravity/ Centroid of Planes

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Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

- Calculate the co-ordinates of the centre of gravity and tabulate them

Contents

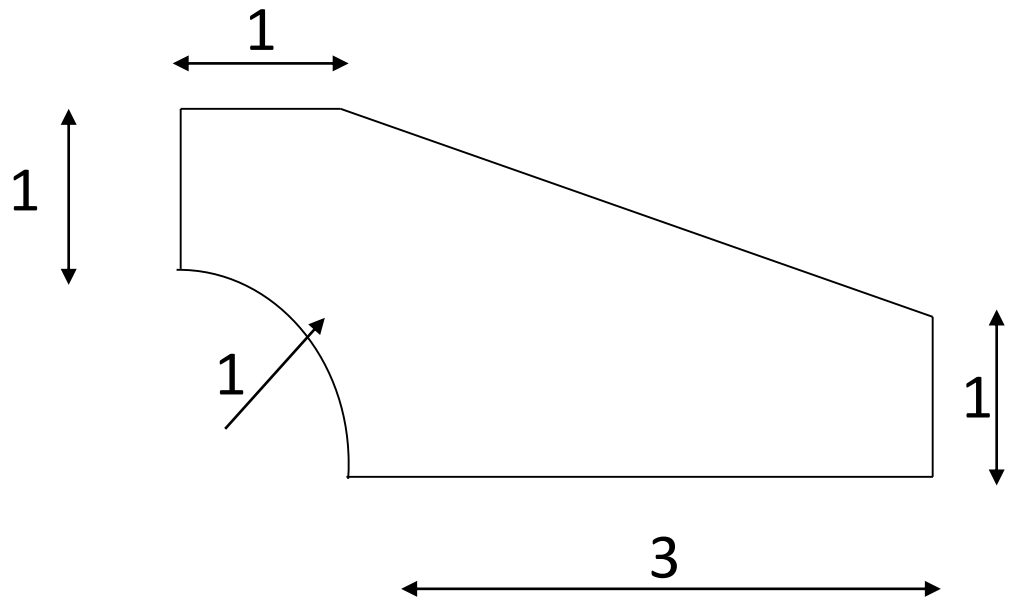
Center of Gravity, Problems on center of Gravity

Centre of gravity

- Determination of the coordinates of a given section.
- Consider the section given
- Aim is to locate the co-ordinates of the centroid

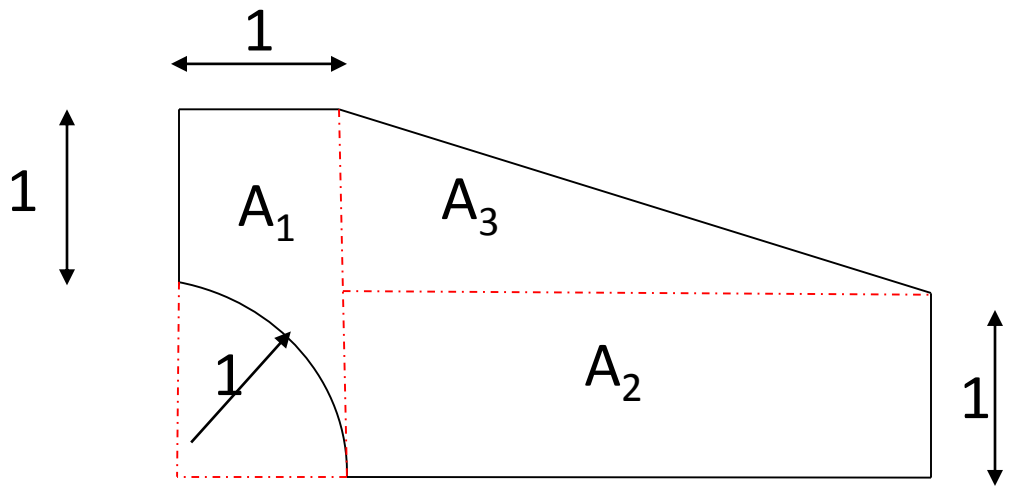
$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$



Centre of gravity

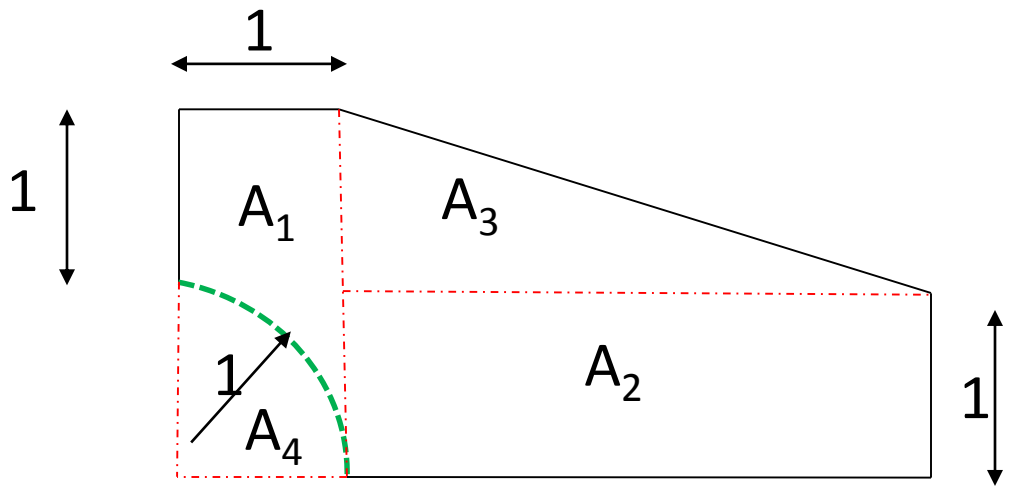
- Divide the given section into a series of regular shapes.
- Divide the left portion into a rectangle of area A_1
- Divide the right portion into another rectangle A_2
- The top portion is divided into a triangle A_3



$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- To remove the area enclosed by the quadrant of the circle, an area equivalent to quarter circle is considered as A_4

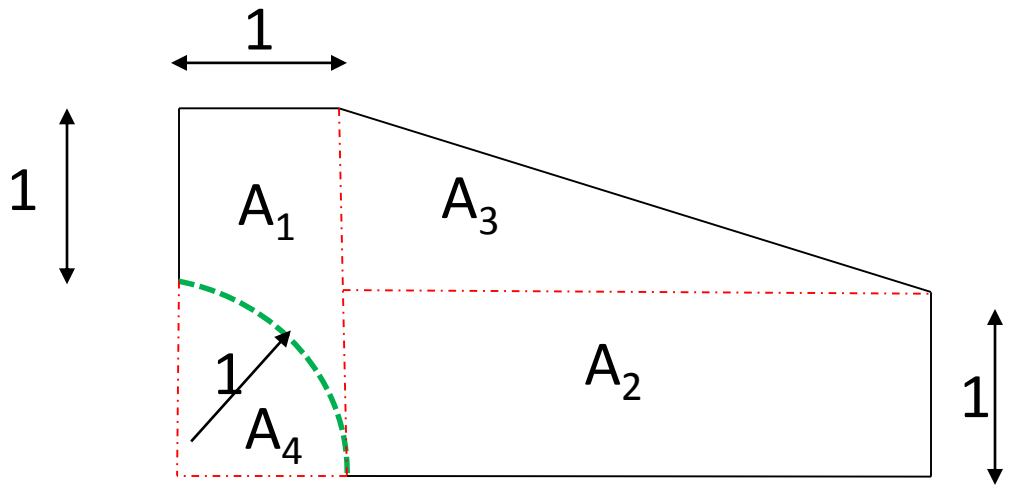


$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Create a table to tabulate the details of each shape for the calculation of CG co-ordinates

ID	Area (cm^2)
A1	2
A2	3
A3	1.5
A4	-0.7853

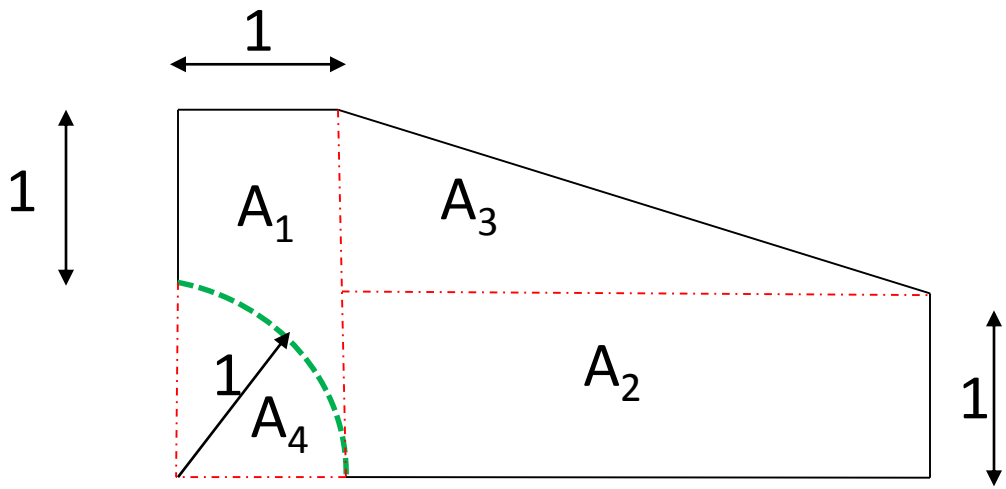


$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Next calculate the distance to the centroids of each of the considered areas

ID	Area (cm^2)	X_i in cm
A1	2	0.5
A2	3	2.5
A3	1.5	2
A4	-0.7853	0.4244

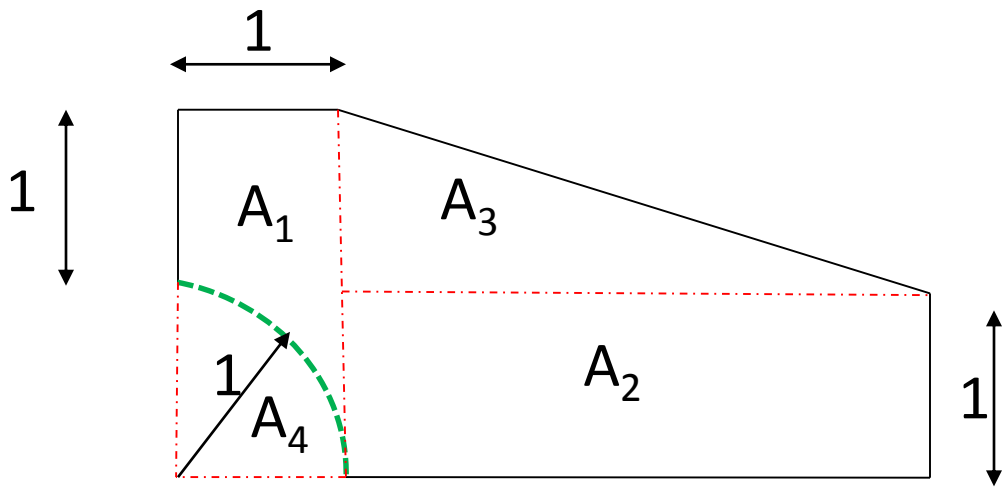


$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Calculate the product of area and centroid distance

ID	Area (cm^2)	x_i in cm	A. x_i
A_1	2	0.5	1
A_2	3	2.5	7.5
A_3	1.5	2	3
A_4	-0.78	-0.42	-0.33

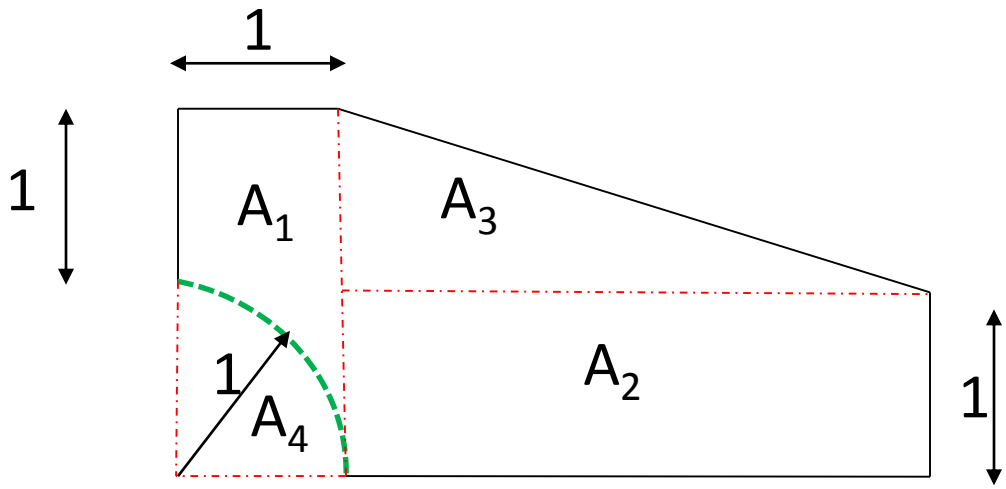


$$\sum A = 5.72 cm^2 \qquad \sum Ax = 11.17 cm^3$$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre Of Gravity

- The term $\sum Ax = 11.17 cm^3$ gives the area moment of the composite section
- Dividing the sum of area moments of the composite section by the total area gives the x-coordinate of the centroid of the section.

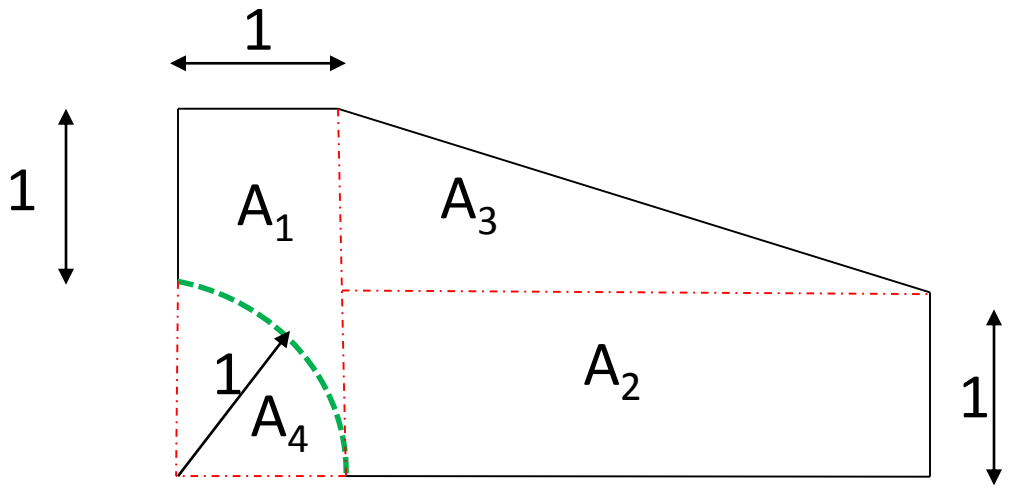


$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{11.17}{5.72} = 1.95 cm$$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Obtain the y-coordinate of the centroid of the section adopting the same procedure.
- In this case, only the distance to the centroid of each area changes.
- The area of each shape and the total area remains unchanged.



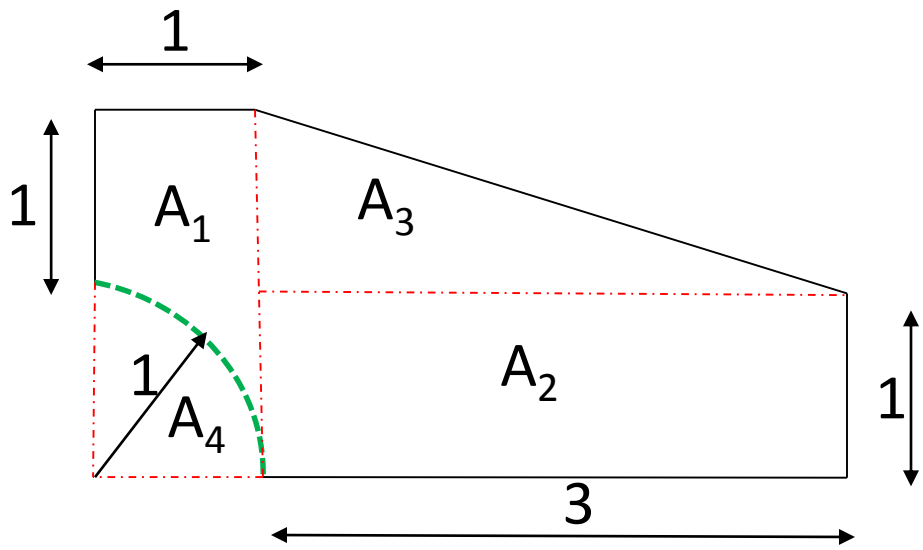
$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Add a the y distances

ID	Area cm^2	x_i in cm	A. x_i	y_i in cm
A_1	2	0.5	1	1
A_2	3	2.5	7.5	0.5
A_3	1.5	2	3	1.33
A_4	-0.78	0.42	-0.33	0.42

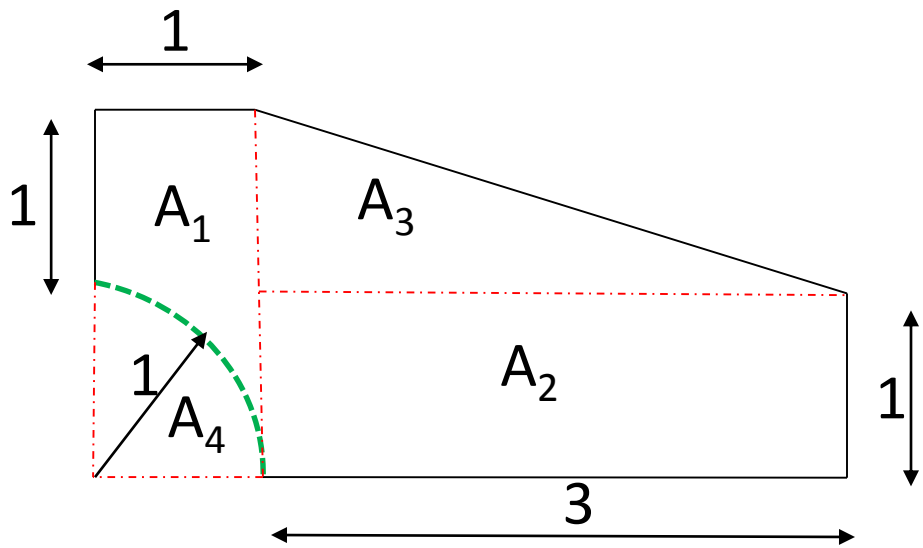
$$\sum A = 5.72 cm^2$$



$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Centre of gravity

- Obtain the moment of area about the X-X axis



ID	Area cm^2	x_i in cm	A. x_i	y_i in cm	A. y_i
A_1	2	0.5	1	1	2
A_2	3	2.5	7.5	0.5	1.5
A_3	1.5	2	3	1.33	2
A_4	-0.78	0.42	-0.33	0.42	-0.33

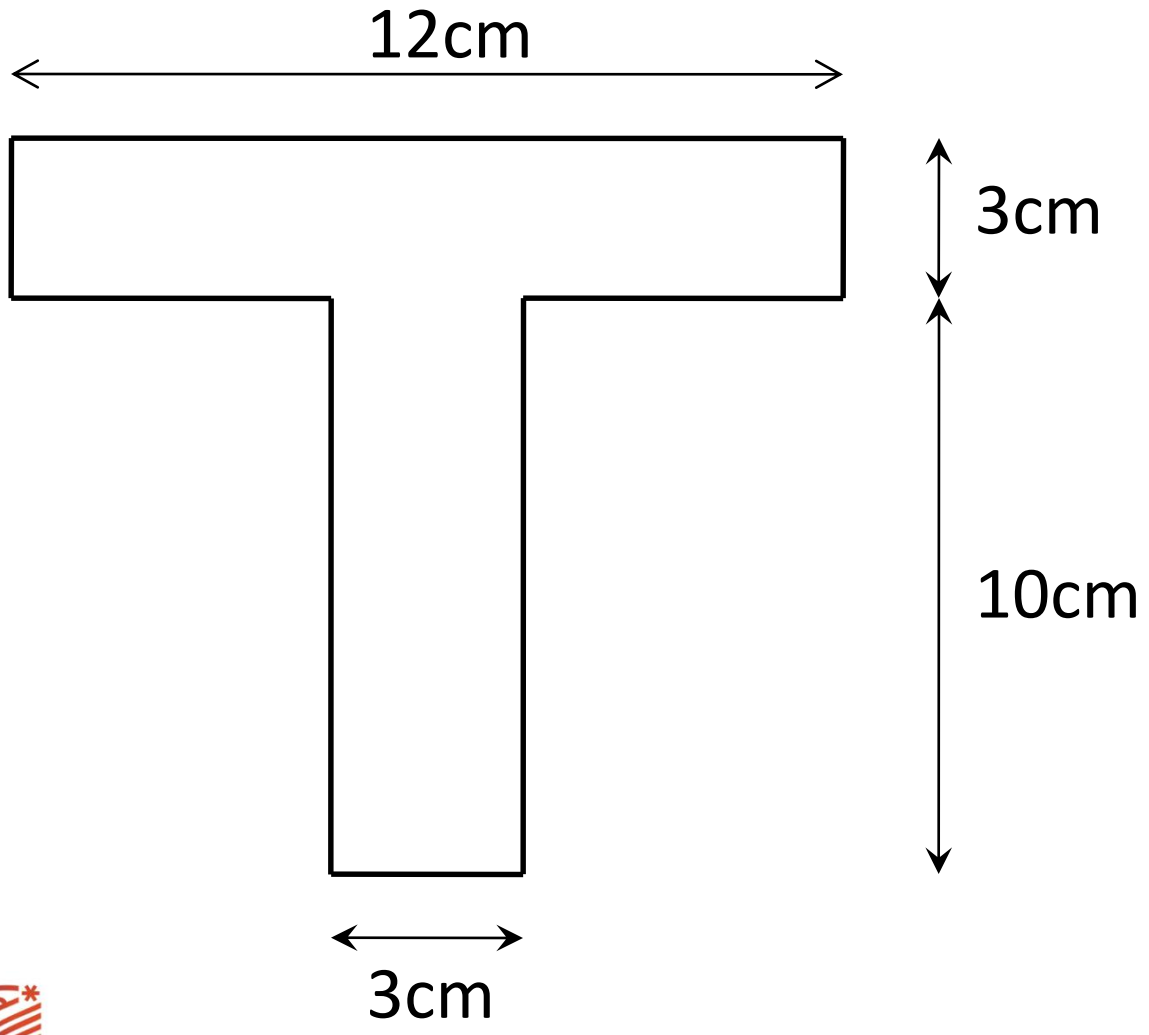
$$\sum A = 5.72 cm^2$$

$$\sum Ay = 5.17 cm^3$$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i} ; \bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$

Problems on centre of gravity

1. Find the centre of gravity of the given T section



Solution:

Step 1. Divide the composite section into simple areas

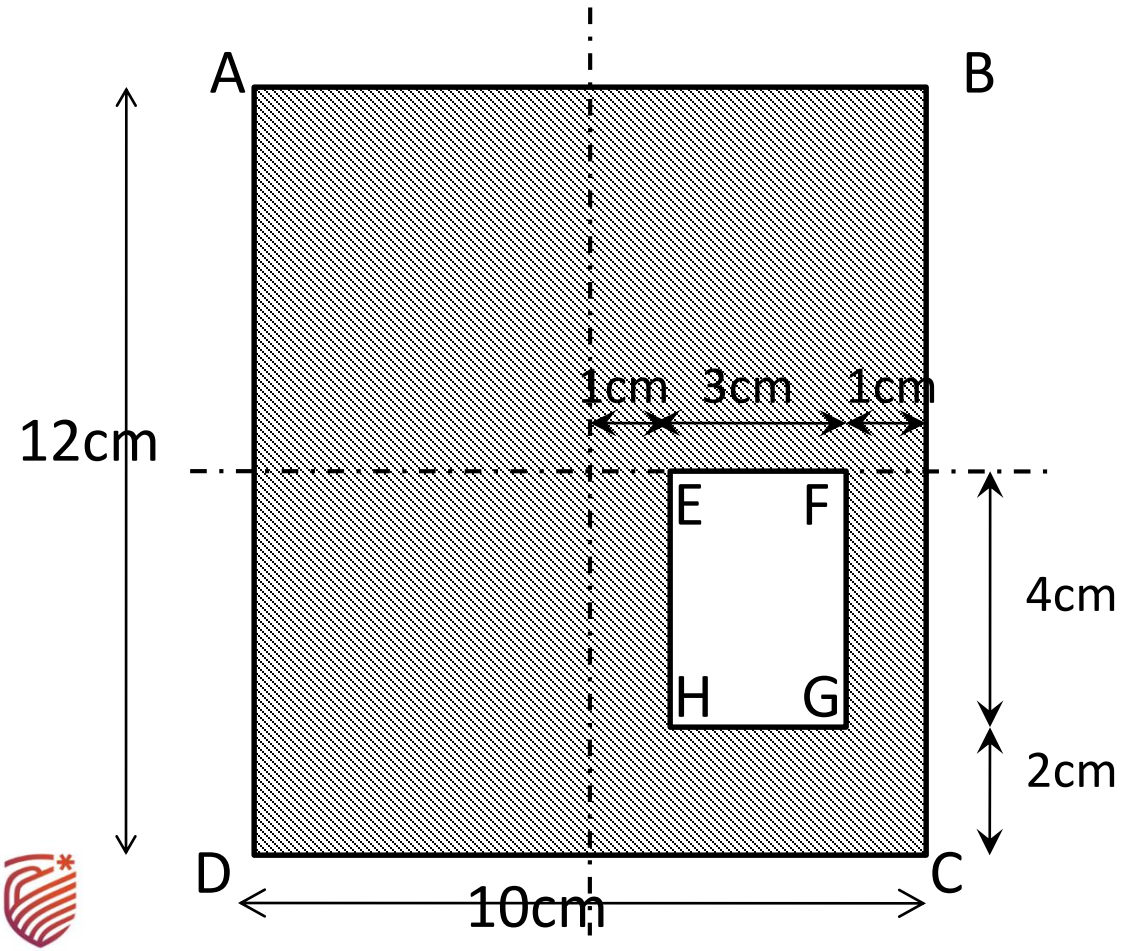
Step 2.Find the moment of the areas with respect to the axes and sum it up

Step 3. Divide the summed moment with area and get the co-ordinates of CG

Ans: $\bar{x} = 6cm;\ \bar{y} = 8.55cm$

Problems on centre of gravity

2. From a rectangular lamina ABCD 10cmx12cm a rectangular hole of 3cmx4cm is cut as shown. Find CG of the remaining lamina.



Solution:

Step 1. Find the moment of the areas with respect to the axes

Step 2. To find the CG of remaining lamina, divide the effective sum of moments with effective area and get the co-ordinates of CG

Ans: $\bar{x} = 4.72cm; \quad \bar{y} = 6.22cm$

Summary

- Centroid is determined using a moment balance of geometric elements such as line, area, or volume segments.