Lecture -21
Mutually Induced E.M.F

Lecture Delivered by

Topics

- Mutually Induced E.M.F
- Mutual Inductance
- Coupling Coefficient

Objectives

At the end of this lecture, student will be able to:

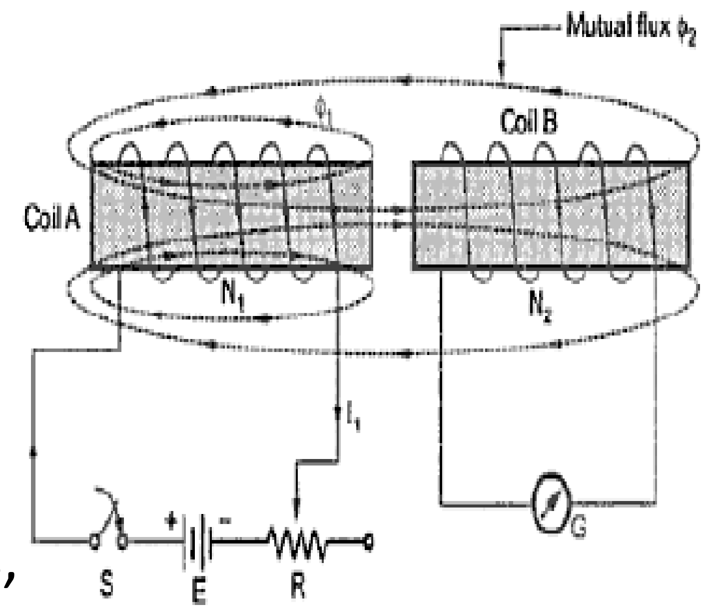
- Explain Mutual Induced E.M.F
- Derive and explain mutual inductance
- Analyze the coupling coefficient for coupled magnetic circuits
- Explain dot convention and dot rules

Video

Mutually Induced E.M.F

- The flux produced by one coil is getting linked with another coil and due to change in this flux produced by the first coil, there is induced e.m.f in the second coil, then such an e.m.f is called mutually induced e.m.f.

Magnitude of Mutually Induced E.M.F



N1 = Number of turns in coil A
N2 = Number of turns in coil B
I1= Current flowing through coil A
ϕ1= Flux Produced due to current I1 in W
Φ2 = Flux linking with coil B
From Faraday's Law, the induced e.m.f in coil B is,

$$e_2 = -N_2 \frac{d\emptyset_2}{dt}$$

Mutually Induced E.M.F

$$\text{Now } \emptyset_2 = \frac{\emptyset_1}{I_1} \times I_1$$

- If permeability of the surroundings is assumed constant then $\emptyset_2 \propto I_1$ and hence $\frac{\emptyset_2}{I_1}$ is constant
- Rate of change of $\emptyset_2 = \frac{\emptyset_2}{I_1} \times$ Rate of change of current I_1

$$\frac{d\emptyset_2}{dt} = \frac{\emptyset_2}{I_1}\frac{dI_1}{dt}$$

$$e_2 = -N_2 \times \frac{\emptyset_2}{I_1} \times \frac{dI_1}{dt}$$

Here **M** $= \frac{N_2\emptyset_2}{I_1}$ is called coefficient of mutual inductance .

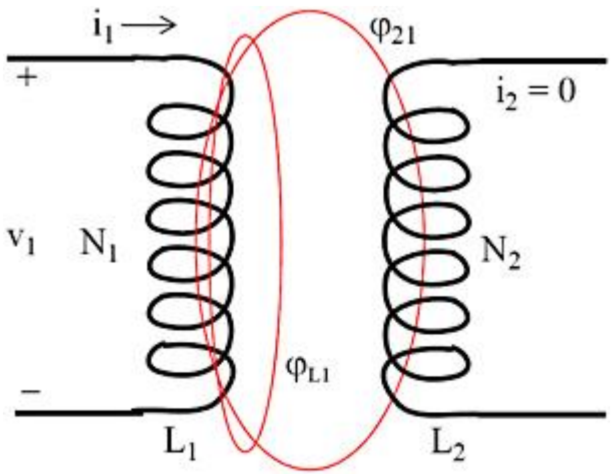
$$e_2 = -M\frac{dI_1}{dt} \text{ volts}$$

- Coefficient of mutual inductance is defined as the property by which e.m.f gets induced in the second coil because of change in current through first coil. It is measured in henries.

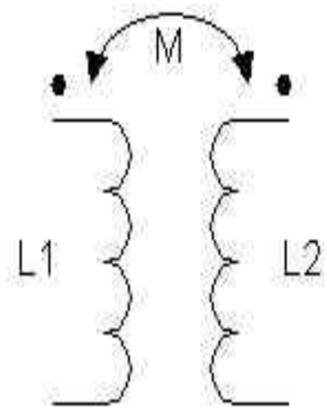
Mutual Inductance

- When two coils are brought close to each other, part of the flux produced by one coil links with the other and vice-versa
- Induced e.m.f in the coil due to change in the current of other coil also called as mutually induced e.m.f, and the coils are said to be mutually coupled

Mutual Inductance



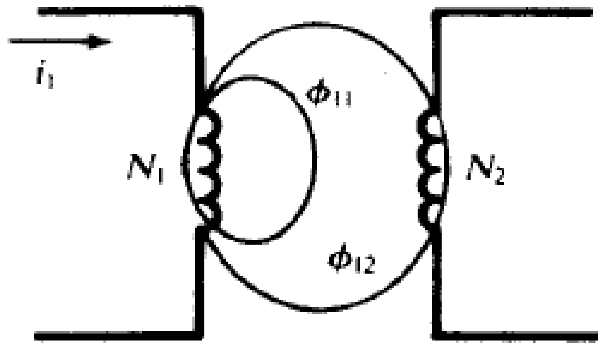
A pair of coupled coils showing self and mutually linking flux



Circuit symbol for two coils with coupling

Coupling co-efficient

- Two coils which are coupled together



ϕ_{11}-leakage flux

ϕ_{12}-linking flux

- E.M.F induced in coil 2 due to i_1

$$e = M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt} \Rightarrow M = N_2\frac{d\phi_{12}}{di_1}$$

Coupling co-efficient

- Also, as the coupling is bilateral,

$$M = N_1 \frac{d\phi_{21}}{di_2}$$

- The coupling coefficient, k, is defined as the ratio of linking flux to total flux:

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

Coupling co-efficient

- k depends only on the geometry of the system,

$$M^2 = \left(N_2 \frac{d\phi_{12}}{di_1}\right)\left(N_1 \frac{d\phi_{21}}{di_2}\right)$$

$$= \left(N_2 \frac{d(k\phi_1)}{di_1}\right)\left(N_1 \frac{d(k\phi_2)}{di_2}\right)$$

$$M^2 = k^2 \left(N_1 \frac{d\phi_1}{di_1}\right)\left(N_2 \frac{d\phi_2}{di_2}\right)$$

$$= k^2 L_1 L_2$$

Coupling co-efficient

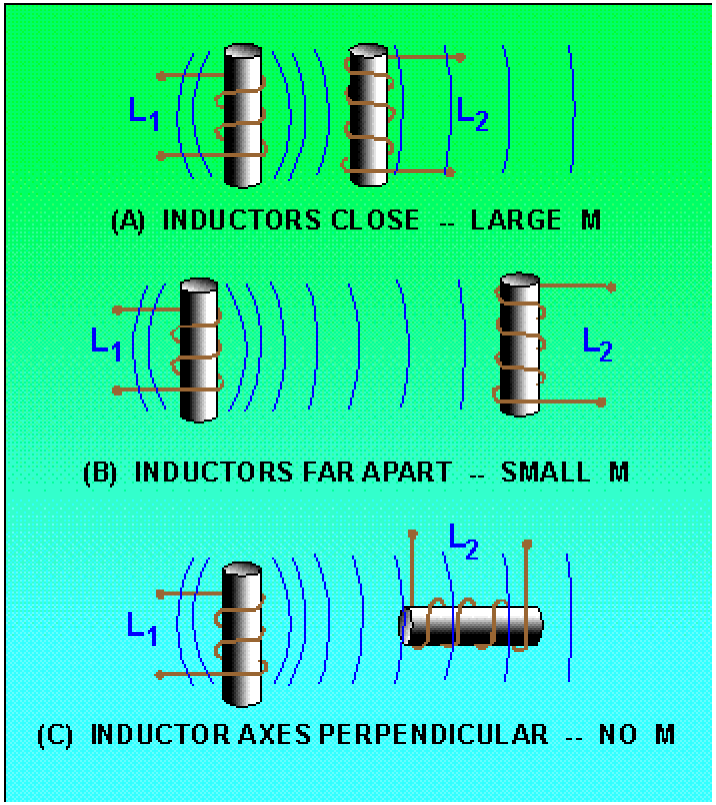
$$\Rightarrow M = k\sqrt{L_1 L_2} \quad \text{or}$$

$$X_M = k\sqrt{X_1 X_2}$$

where $0 \leq k \leq 1$

- Energy stored in the coupled inductor

$$E = \frac{1}{2}L_1 I_1^{\,2} + \frac{1}{2}L_2 I_2^{\,2} \pm M I_1 I_2$$

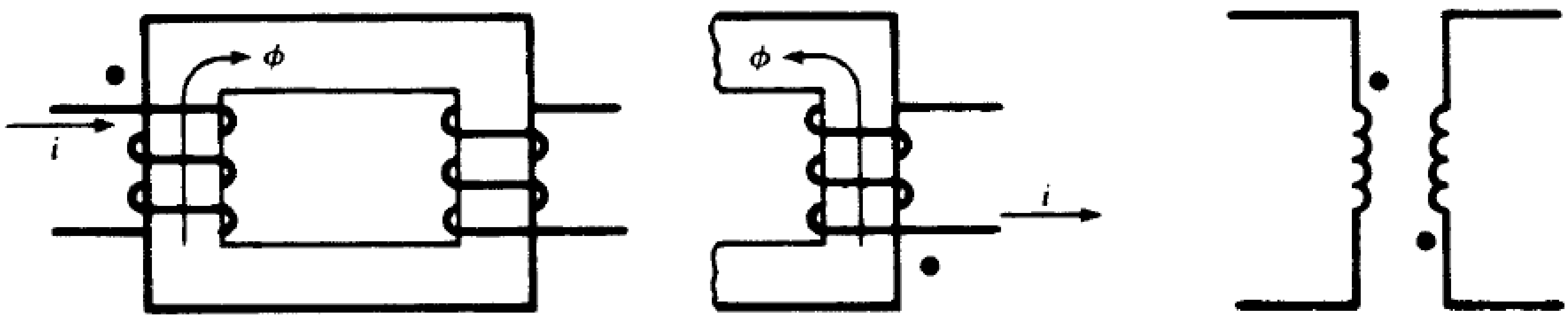


Dot Convention

- Sign of the induced E.M.F depends on the winding sense
- Dot convention is used as showing detailed winding diagram is difficult
- Coils are marked with dots at the terminals which are instantaneously of the same polarity.

Dot Convention

- Select a current direction in one coil and place a dot at the terminal where this current enters the winding
- Place a dot at the terminal of the second winding where the natural current leaves the winding

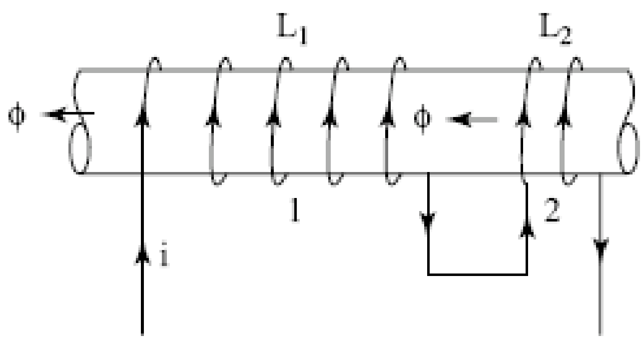


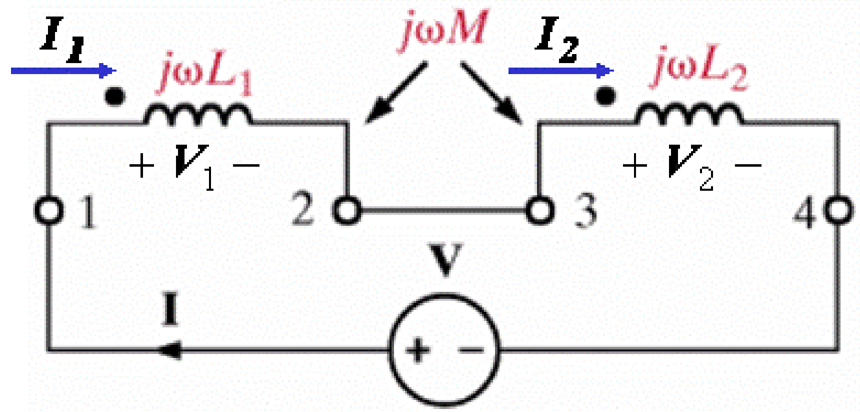
(a) (b) (c)

Dot Rule

- When the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms
- If one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms.

Coupled Inductors in Series (Case-1)





- Both currents are entering at the dotted terminal so M is +ve

$$V = V_1 + V_2 \quad \text{and} \quad I = I_1 = I_2$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \quad \text{and} \quad V_2 = j\omega L_2 I_2 + j\omega M I_1$$

$$\Rightarrow V = j\omega(L_1 + L_2 + 2M)I$$

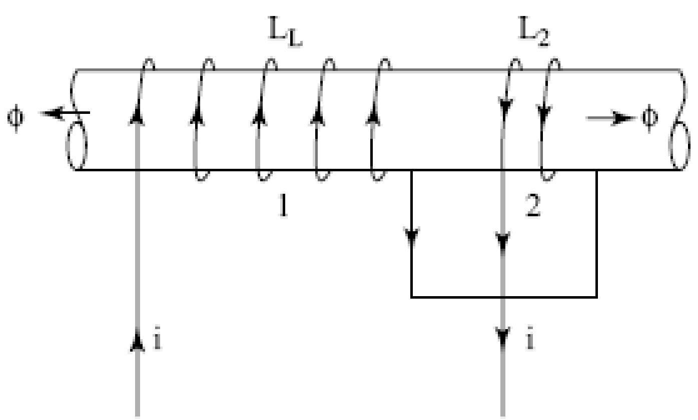
Coupled Inductors in Series (Case-1)

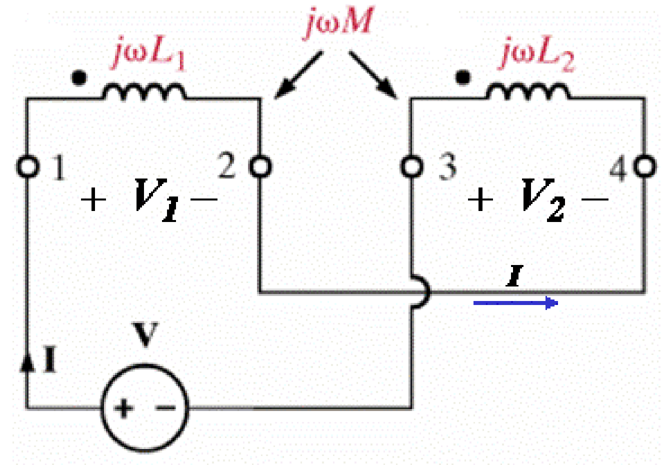
- For equivalent inductance,

$$V = j\omega L_{eq} I$$

$$\therefore L_{eq} = L_1 + L_2 + 2M$$

Coupled Inductors in Series (Case-2)





- Here the current is entering coil 1 and leaving coil 2 through dotted terminals, hence M is -ve

$$V = V_1 - V_2$$

Coupled Inductors in Series (Case-2)

$$V_1 = j\omega L_1 I - j\omega M I$$

$$V_2 = -j\omega L_2 I + j\omega M I$$

$$\Rightarrow V = j\omega(L_1 - 2M + L_2)I$$

$$= j\omega L_{eq} I$$

$$\therefore L_{eq} = L_1 + \ L_2 - 2M$$

Summary

- The flux produced by one coil is getting linked with another coil and due to change in this flux produced by the first coil, there is induced e.m.f in the second coil, then such an e.m.f is called mutually induced e.m.f
- Induced e.m.f in the coil due to change in the current of other coil also called as mutually induced e.m.f
- Coupling coefficient (k) is defined as the ratio of linking flux to total flux
- Dot Convention is used for sign of induced e.m.f