# Course Code: ESC106A
# Course Title: Construction Materials and Engineering Mechanics

## Lecture No. 54:
## Introduction to Dynamics

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# Lecture Intended Learning Outcomes

- Describe the concept of projectiles
- Discuss the equation for the path of a projectile
- Explain the motion of projectiles as a combination of horizontal and vertical components of velocity
- Solve numerical examples on rectilinear and circular motion of bodies and projectiles


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- Projectile motions


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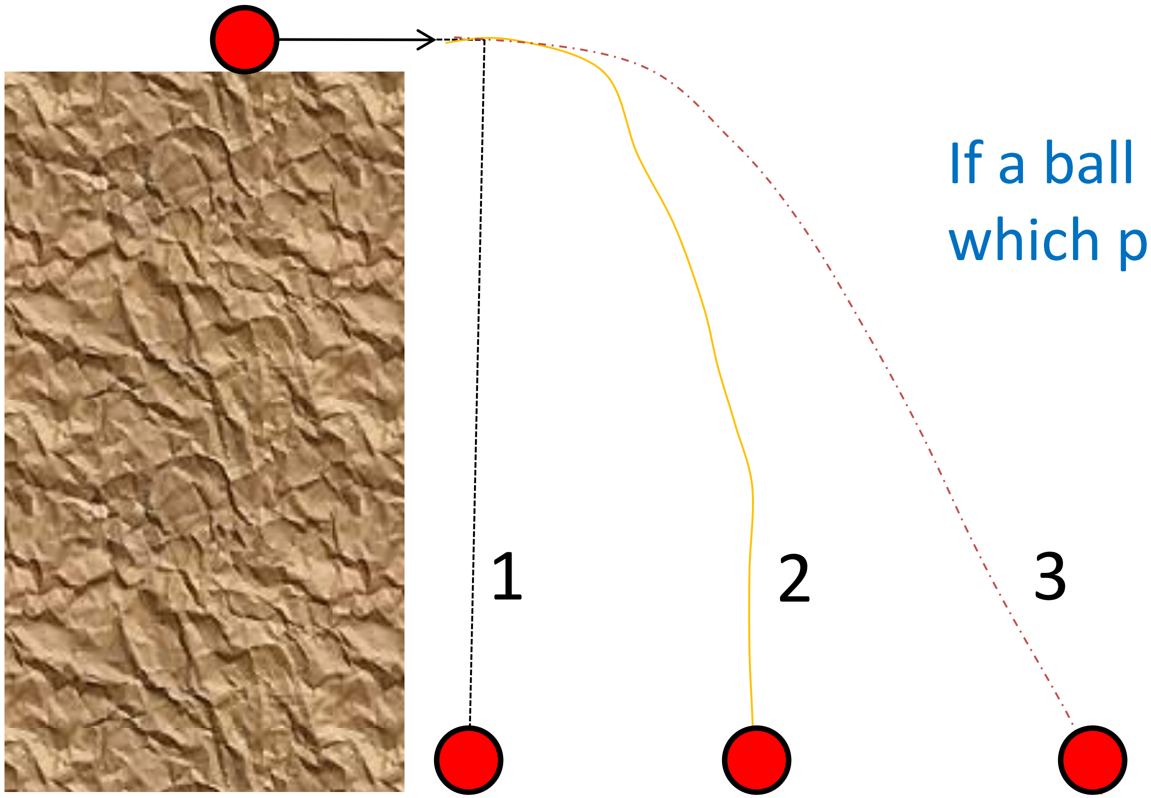
# Projectiles




If a ball is rolled down a cliff, which path will it take?



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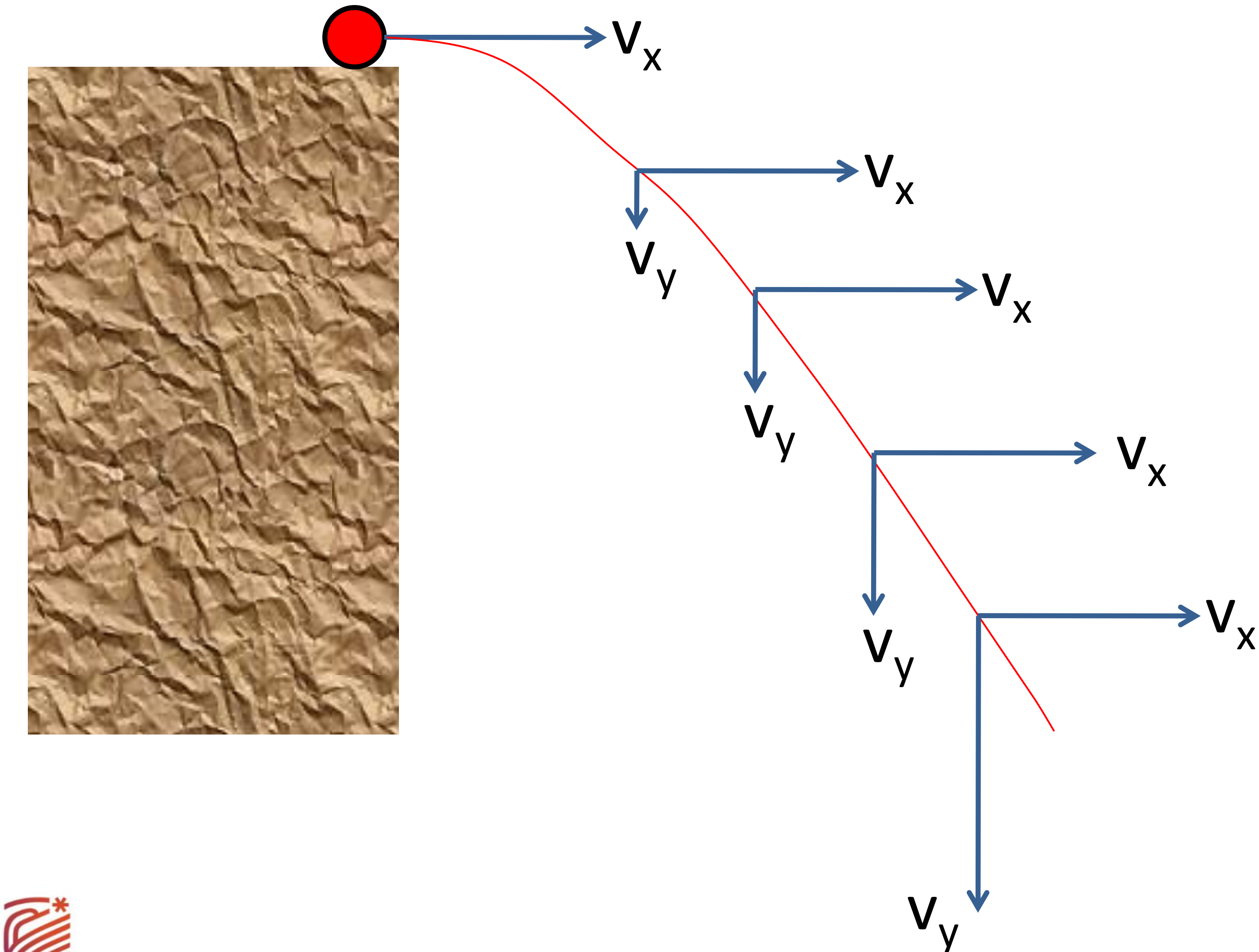
# Projectiles

Motion of a body thrown horizontally from a given height into air





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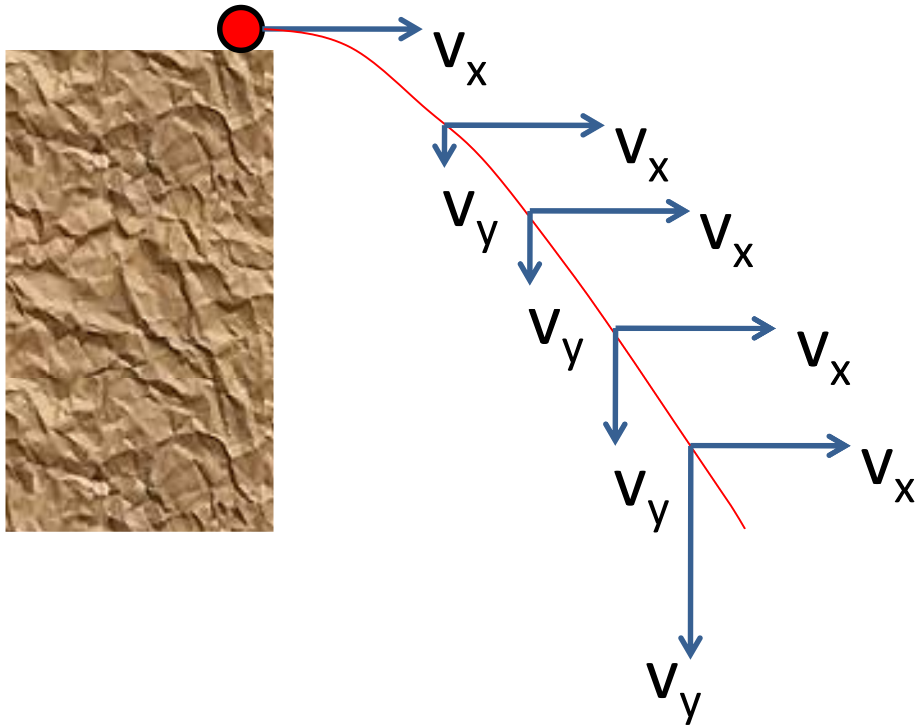
# Projectiles




- When the ball leaves the cliff, it has horizontal velocity. But as there is no external force acting on the body no horizontal acceleration develops.
- However, due to gravity, a vertical force will be exerted in the ball resulting in an accelerated motion downwards.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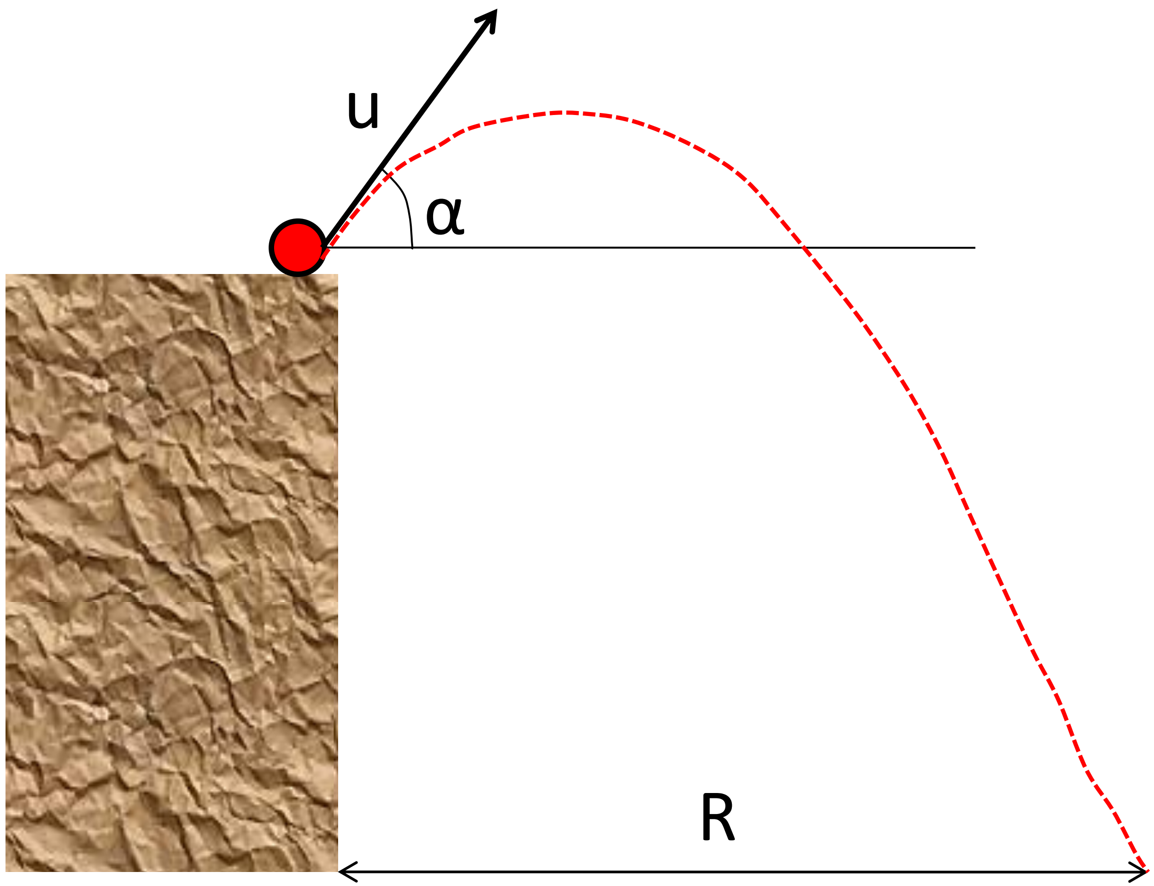
# Projectiles




Terms associated with projectiles

u – Velocity of projection

α – Angle of projection

t – Time of flight

R – Horizontal Range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# Projectiles

Velocity of projection : The velocity with which a projectile is projected into space (m/sec).

Angle of projection: The angle between the direction of projection and the horizontal.

Trajectory : The path traced by the projectile when projected in air at a certain angle.

Horizontal range: The horizontal distance between the point of projection and the point where projectile strikes the ground.

Time of flight: The total spent by projectile in space since it is projected and hits the ground again.


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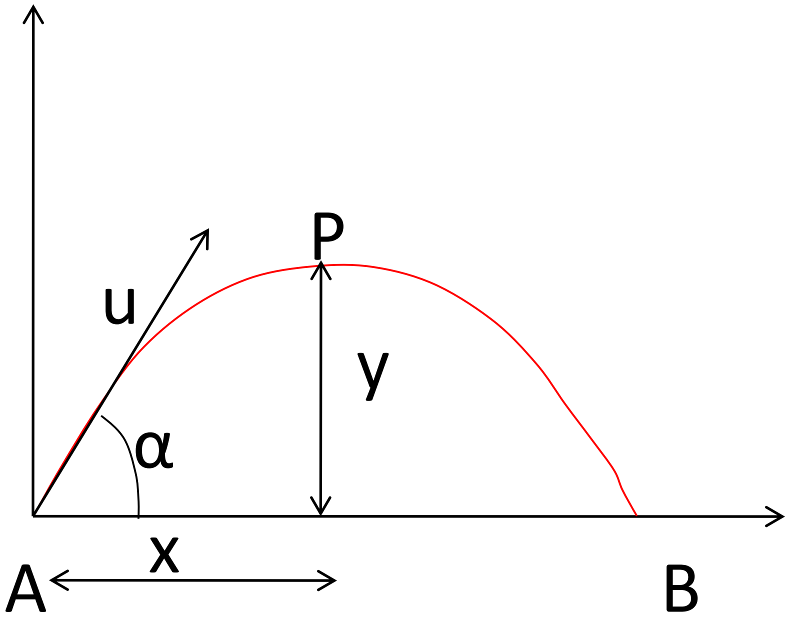
## Equation for the path of a projectile




Consider a point 'P' along the path of the projectile

$$u_x = u\cos\alpha$$

$$u_y = u\sin\alpha$$

$$x = u_x t = u\cos\alpha \times t;$$

$$t = \frac{x}{u\cos\alpha}$$

$$y = u_y t + \frac{1}{2}gt^2$$

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2$$

$$y = (u\sin\alpha)\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$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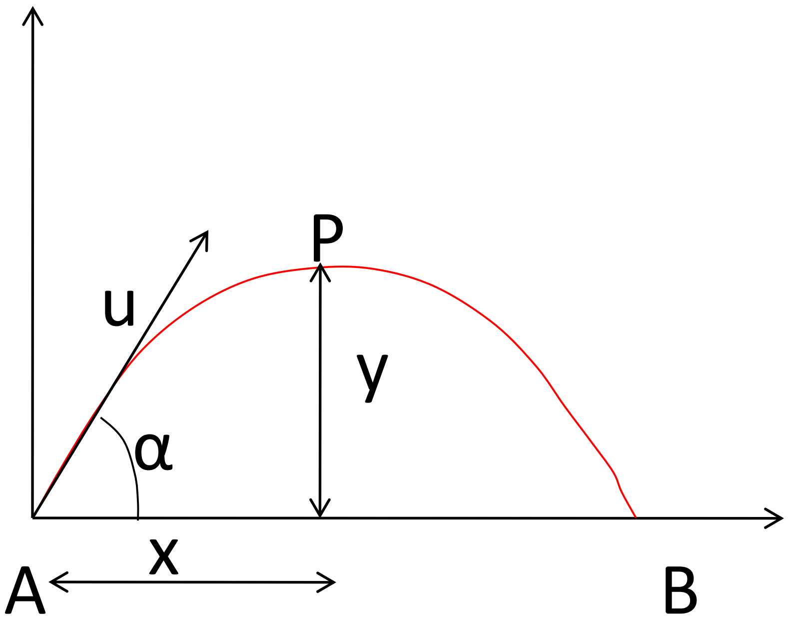
## Maximum height attained by a projectile




$$u_x = u\cos\alpha$$

$$u_y = u\sin\alpha$$

At maximum height, $u_y = 0$

Let $h_{max}$ be the maximum height

$$v^2 - u^2 = 2as$$

$$\because v = u_y = 0$$

$$-(u\sin\alpha)^2 = 2(-g)h_{\max}$$

$$h_{\max} = \frac{u^2 \sin^2\alpha}{2g}$$



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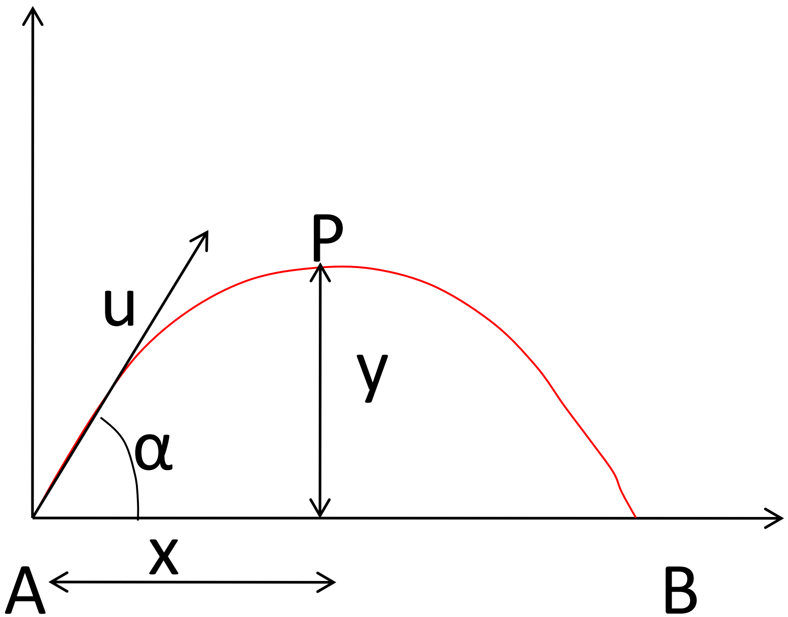
# Projectiles

## Time of flight




$$u_x = u\cos\alpha$$

$$u_y = u\sin\alpha$$

Let 'T' be the time taken to travel from A to B.

$$y = (u\sin\alpha)\times t - \frac{1}{2}gt^2$$

$$y = 0; t = T$$

$$0 = (u\sin\alpha)\times T - \frac{1}{2}gT^2$$

$$T = \frac{2u\sin\alpha}{g}$$

## Time to reach highest point

At highest point, $u_y = 0$

$$0 = u\sin\alpha - gt; \therefore t = \frac{u\sin\alpha}{g}$$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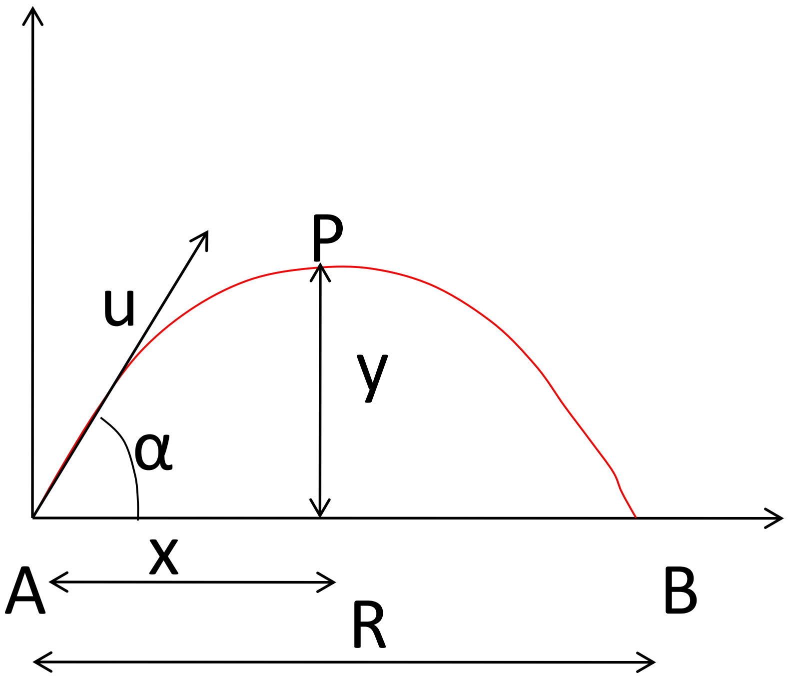
# Projectiles

## Horizontal range of a projectile




$$u_x = u\cos\alpha$$

$$u_y = u\sin\alpha$$

Let 'R' be the horizontal range.

$$R = (u\cos\alpha) \times T$$

$$R = u\cos\alpha \times \left(\frac{2u\sin\alpha}{g}\right)$$

$$R = \frac{u^2}{g}\sin 2\alpha$$

$$If \alpha = 45°, \sin 2\alpha = 1$$

$$R_{\max} = \frac{u^2}{g}$$



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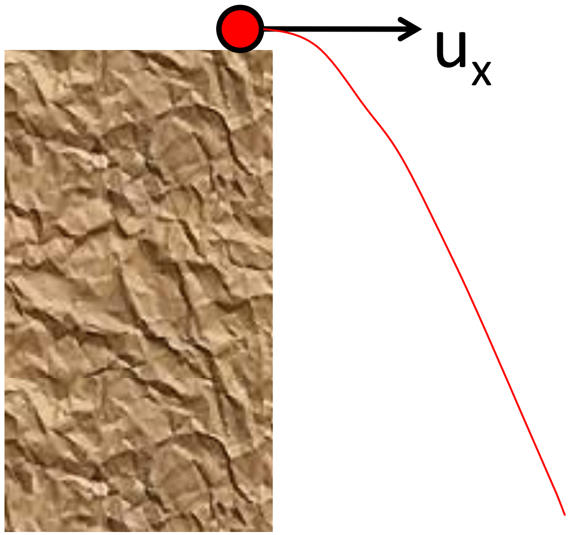
# Numerical Problems on Projectiles




1.A ball is thrown horizontally from the top of a cliff with a velocity of 8m/s. The cliff is 40 m high. Determine

1. Determine the time of flight
2. Determine the horizontal range
3. Determine the vertical velocity with which the ball strikes the ground.
4. Determine the total velocity with which the ball hits the ground.



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# Numerical Problems on Projectiles

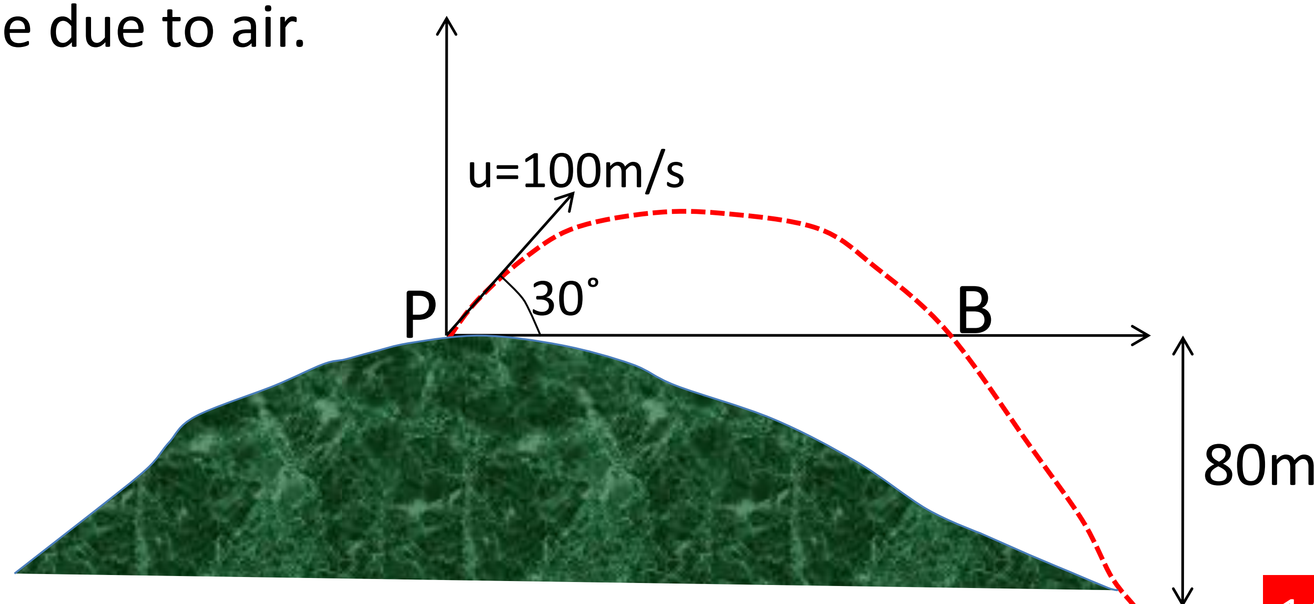
2. A bullet if fired upwards at an angle of 30˚ to the horizontal from a point P on a hill and it strikes a target which is 80m lower than B. The initial velocity of the bullet is 100m/s. Calculate:

a) The maximum height to which the bullet will rise above the horizontal

b) The actual velocity with which it will strike the target

c) The total time required for the flight of the bullet.

Neglect the resistance due to air.





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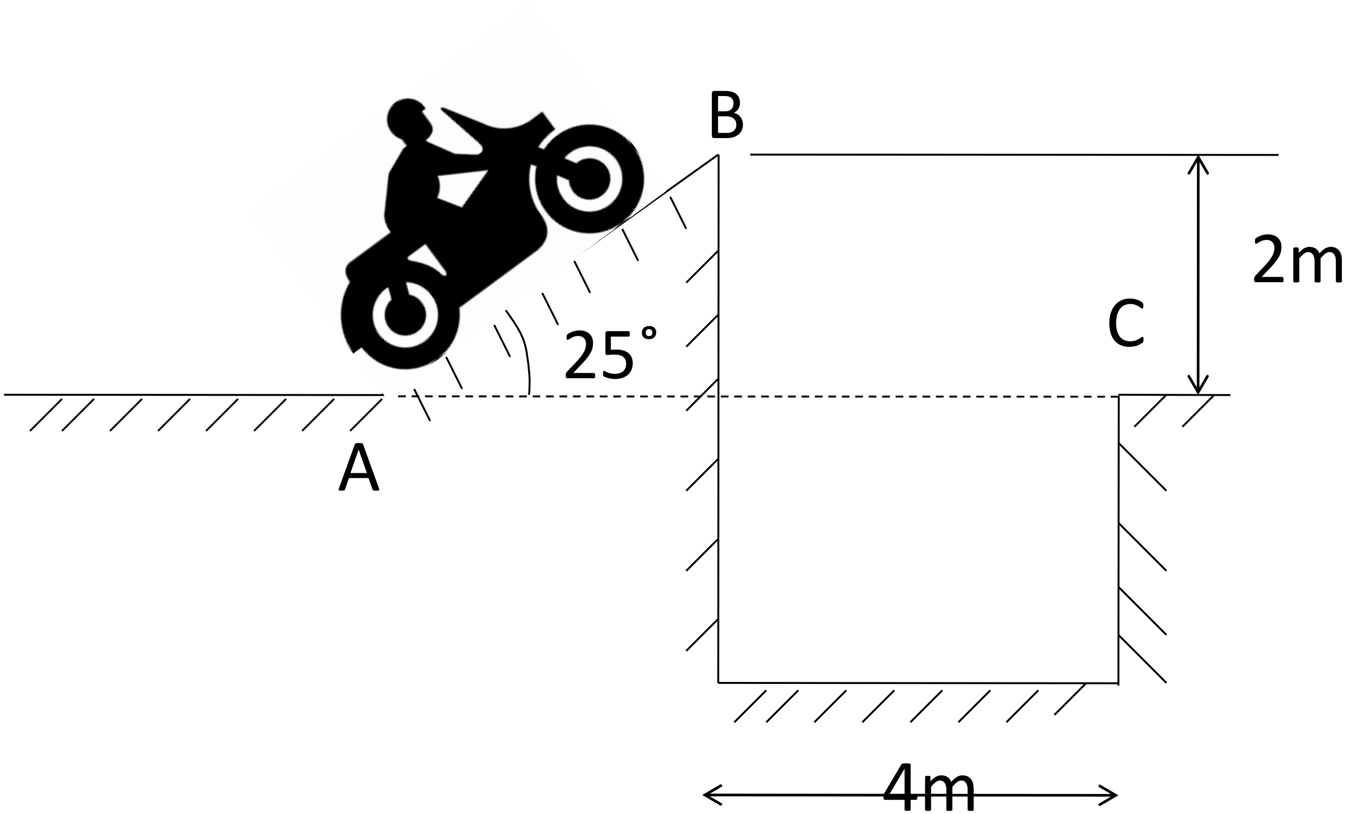
# Numerical Problems on Projectiles

3. A motorcyclist wants to clear a ditch as shown in Figure. If the ramp at B is of 25˚, determine the minimum speed of the motor cycle at B.





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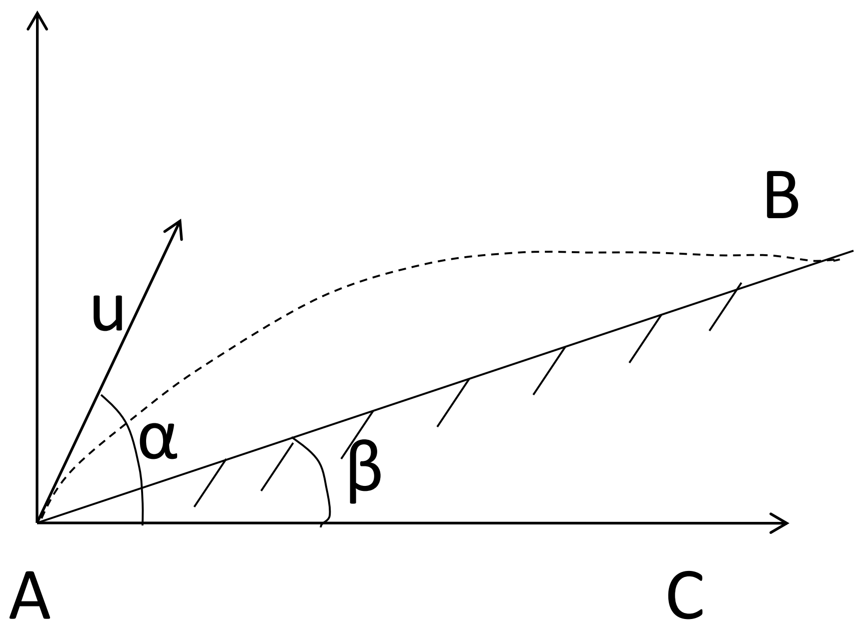
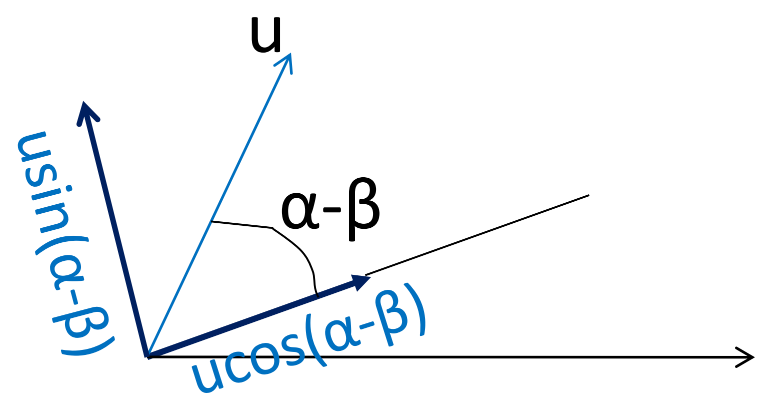
# Projectiles

## Projectile on an inclined plane




Component of velocity of projection along the plane = ucos(α-β)
Component of velocity of projection normal to the plane = usin(α-β)
Acceleration due to gravity along the plane = -gsinβ
Acceleration due to gravity normal to the plane = -gcosβ



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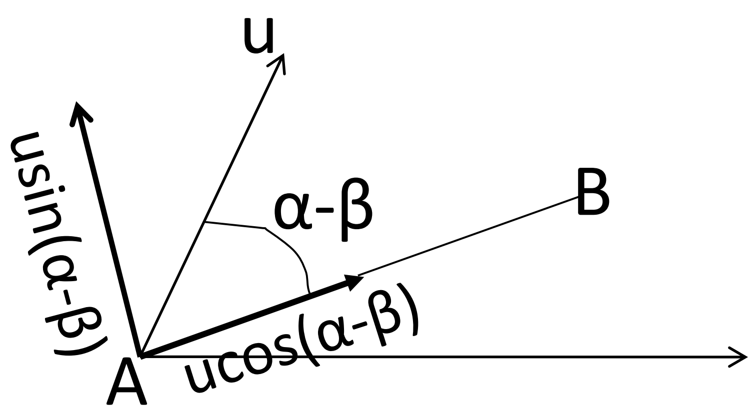
# Projectiles

## Projectile on an inclined plane

### Time of flight




T- Time of flight

Distance covered along the plane = 0

$$s = ut + \frac{1}{2}gt^2$$

$$0 = [u\sin(\alpha-\beta)] \times T + \frac{1}{2}(-g\cos\beta)T^2$$

$$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$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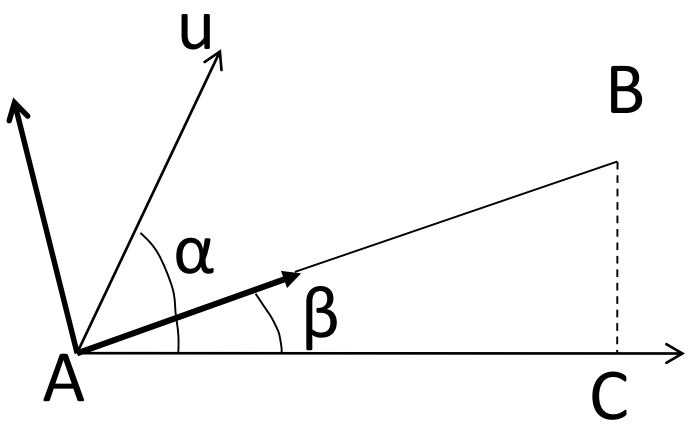
# Projectiles

## Projectile on an inclined plane

Range in inclined plane




AB – Range on inclined plane

$$\cos\beta = \frac{AC}{AB}$$

$$AB = \frac{AC}{\cos\beta}$$

AC is the horizontal component of range which is independent of gravity

$$AC = u\cos\alpha \times T$$

$$AC = u\cos\alpha \times \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$AB = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{\cos\beta \times g\cos\beta}$$

$$\because 2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$AB = \frac{u^2}{g\cos^2\beta}\left[\sin(2\alpha-\beta) - \sin\beta\right]$$





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# Summary

- A projectile is any object with an initial non-zero, horizontal velocity whose acceleration is due to gravity alone. (Air resistance is neglected. Hence it is not applicable to paper, feathers etc)
- The path traced by a particle is known as trajectory. This path is parabolic and is influenced only by the initial launch speed, angle of projection and the acceleration due to gravity.
- Applications of equations of time of flight, maximum height attained by the projectile and the horizontal range is useful in launching missiles and rockets, accurate throwing of food packets from airplanes in case of natural disasters, sports etc


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