Lecture 4
Cauchy's Mean Value Theorem

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Intended Learning Outcomes

At the end of this Lecture, student will be able to:

- State Cauchy’s mean value theorem
- Apply Cauchy’s mean value theorem to specific problems

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Topics

- Cauchy mean value theorem
- Examples of Cauchy mean value theorem

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Mathematical Statement of Cauchy Mean Value Theorem

Let $f(x)$ and $g(x)$ be a real function defined in $[a, b]$ such that

- $f(x)$ and $g(x)$ are continuous in the closed interval $[a, b]$
- $f(x)$ and $g(x)$ are differentiable in the open interval (a, b).
- Suppose that $g'(x) \neq 0$, then there is a point c in the open interval (a, b), such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

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Example 1

- Verify the Cauchy Mean value theorem for the following functions $f(x) = e^{x},\ g(x) = e^{-x}$ in $[a, b]$

Solution:

Given that $f(x) = e^{x},\ g(x) = e^{-x}$

$$\Longrightarrow \mathrm{f}'(\mathrm{x}) = e^{x}\ \ g(x) = -e^{x}$$

We notice that

- $f(x)$ and $g(x)$ are continuous in the closed interval $[a, b]$
- $f(x)$ and $g(x)$ are differentiable in the open interval (a, b)
- $g'(x) \neq 0$ for all x

Hence $f(x)$ and $g(x)$ satisfy all the three conditions of the Cauchy's Mean Value Theorem in the $[a, b]$

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Example-1(Conti.)

Cauchy Mean value theorem , we have

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$$\Rightarrow \frac{e^c}{-e^{-c}} = \frac{e^b - e^a}{e^{-b} - e^{-a}}$$

$$\Rightarrow -e^{2c} = \frac{e^b - e^a}{\dfrac{1}{e^b} - \dfrac{1}{e^a}} = \frac{(e^b - e^a)e^b e^a}{e^a - e^b} = -e^{a+b}$$

$$\Rightarrow c = (a+b)/2$$

Thus c lies between a and b

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Example-2

Verify the Cauchy Mean value theorem for the following functions

$$f(x) = x^3, \qquad g(x) = x^2 \quad in \quad [1,2]$$

$$f'(x) = 3x^2 \qquad g'(x) = 2x$$

Solution:

Thus f(x) and g(x) are differentiable and therefore continuous ,

and $g'(x) \neq 0$ for all x . Hence $f(x)$ and $g(x)$ satisfy all the three conditions of the Cauchy's Mean Value Theorem in the $[a,b]$.

Now the result $$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)} \Rightarrow \frac{3c^2}{2c} = \frac{f(2) - f(1)}{g(2) - g(1)}$$

$$\Rightarrow 3c^2 / 2c = 7/3 \Rightarrow c = 14/9$$

Hence c lies in the interval (1,2)

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Summary

- If $f(x)$ and $g(x)$ are two real valued functions in $[a, b]$ that satisfies the following three conditions:

  – $f(x)$ and $g(x)$ are continuous on $[a, b]$
  – $f(x)$ and $g(x)$ are differentiable on $[a, b]$
  – $g(a) \neq g(b)$ and $g'(x) \neq 0, \forall x \in (a, b)$

then there exits at least a point $c \in (a, b)$, such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

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