Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 38:
Moment of Inertia

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Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

- Define Moment of Inertia and Radius of gyration
- Discuss theorem of Perpendicular axes
- Explain theorem of Parallel axes

Contents

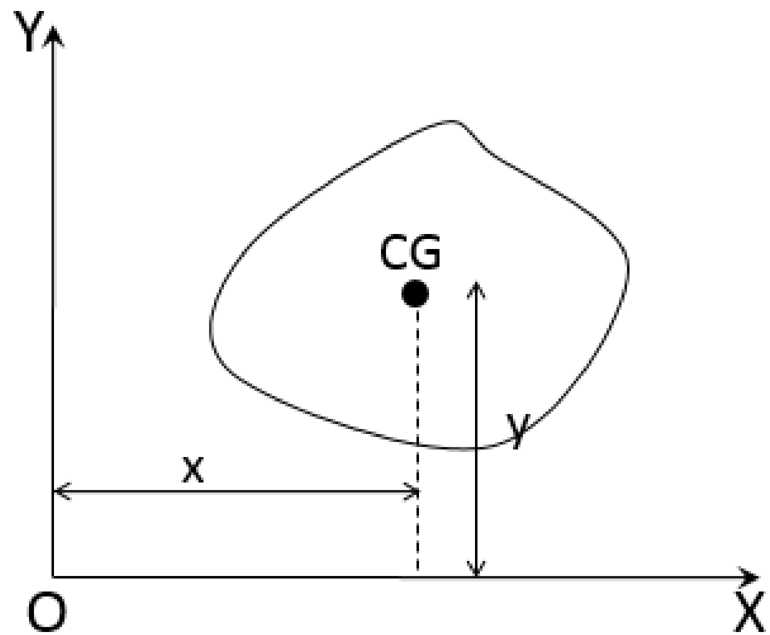
Moment of Inertia of Area, Moment of Inertia of Mass, Polar moment of inertia(mass), Radius of Gyration, Perpendicular axis theorem, Parallel axis theorem, Moment of inertial of composite sections

Moment of Inertia of Area

It measures how an area is distributed about particular axes.

Moment of area about OY=Area x Perpendicular distance of CG of area from axis OY,

$$Ax \text{ ---------------(1)}$$



Eqn (1) is known as first moment of area about axis OY

Moment of Inertia of Area

Moment of moment of area about OY=

Moment of area about axis OY x Perpendicular distance of CG of area from axis OY = $Ax \times x = Ax^2$ .... (2)

Eqn (2) is known as second moment of area or moment of inertia about axis OY

Moment of Inertia of Mass

- If mass is taken into consideration instead of area, then second moment is known as second moment of mass
- Moment of moment of mass about OY=Moment of mass about axis

  OY x Perpendicular distance of CG of mass from axis OY

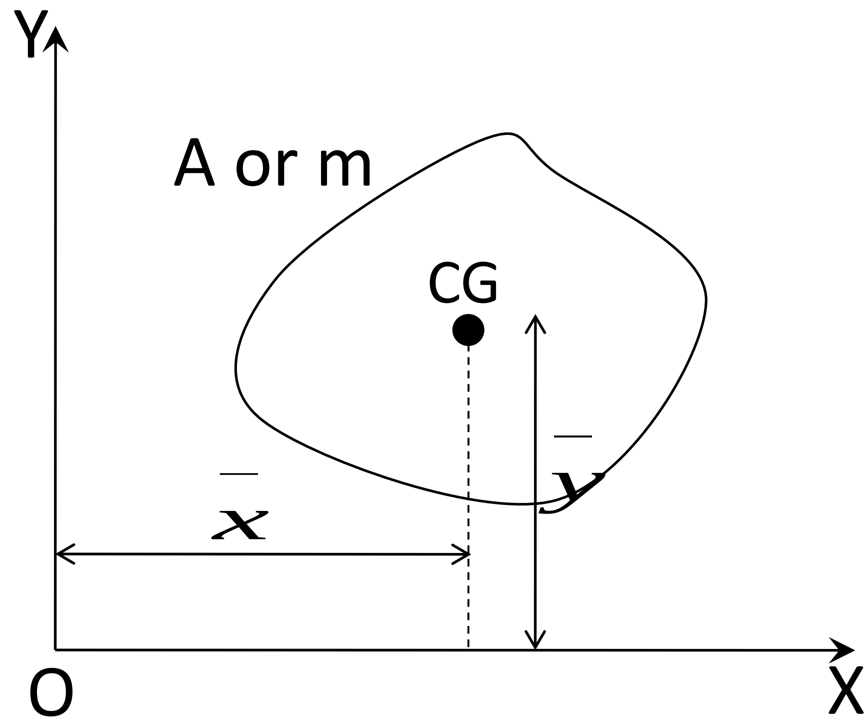
$$= mx \times x = mx^2 \quad \text{..............} \ (2)$$

- Eqn (2) is known as second moment of mass or moment of inertia about axis OY
- About axis OX,

$$my \times y = my^2$$

Moment of Inertia of Area (Mass)

- The product of the area (mass) and the square of the distance of the centre of gravity of the area (mass) from an axis is known as moment of inertia of the area (mass) about that axis.
- Moment of inertia is represented by I.
- Moment of inertia about Y-axis – I_{YY}
- Moment of inertia about X-axis- I_{XX}



Polar Moment of Inertia of Area (Mass)

The product of the area (mass) and the square of the distance of the centre of gravity of the area (mass) from an axis perpendicular to the plane of the area is known as polar moment of inertia of the area (mass).

Polar moment of inertia is represented by J.

Moment of Inertia of Area (Mass)

- Consider a plane area which is split up into small areas a_1, a_2, a_3..... Let the CG of the small areas from a given axis be at a distance of r_1, r_2, r_3...
- Then Moment of Inertia (I) of the plane area about the given axis is

$$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \dots$$
$$I = \sum a r^2$$

Radius of Gyration

- Radius of gyration of a body about an axis is a distance such that its square multiplied by the area gives moment of inertia of the area about the given axis.
- If the whole mass or area of the body is concentrated at a distance k from the axis of reference, then MI of the whole area about the given axis is Ak^2

$$Ak^2 = I$$

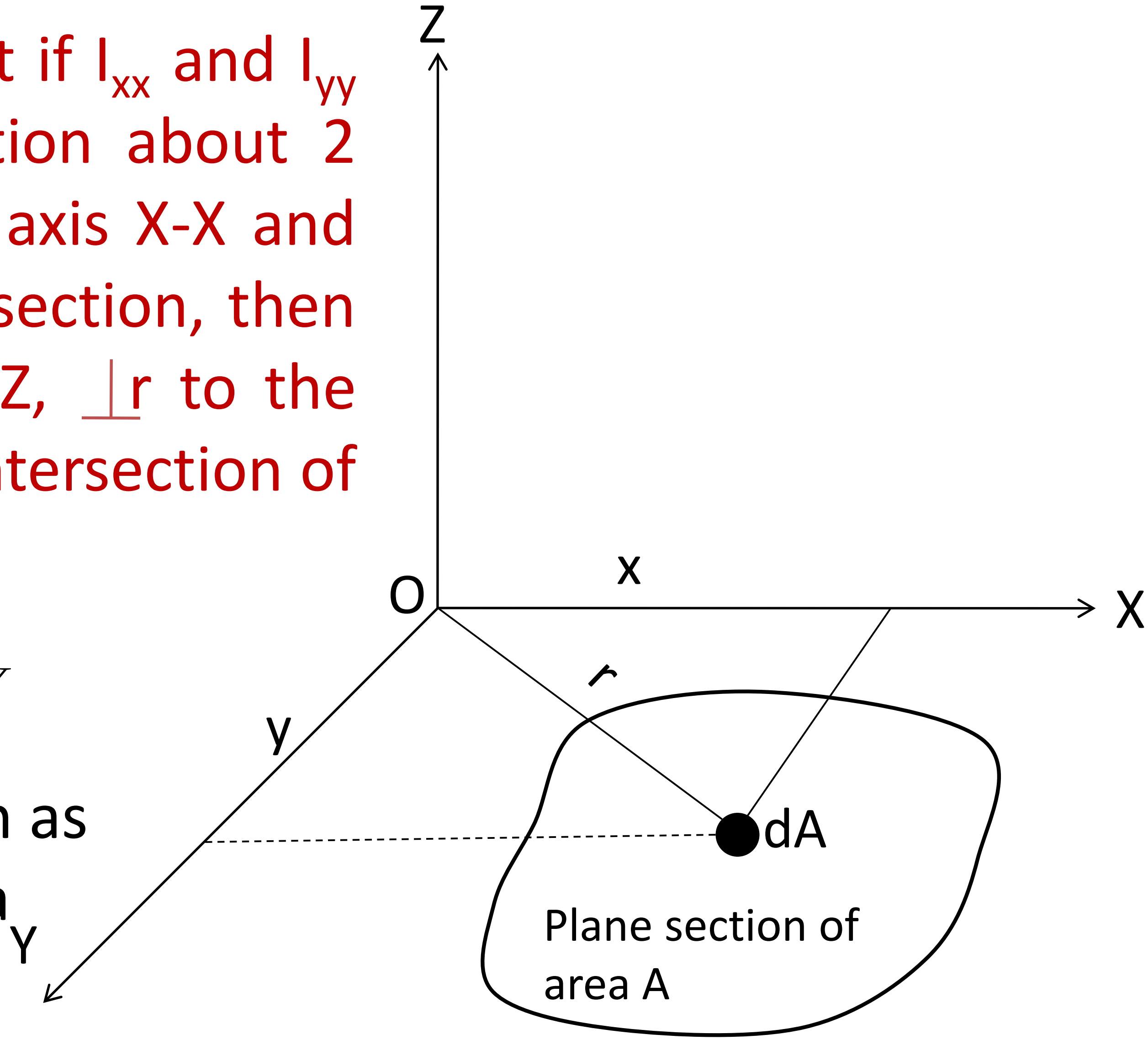
Radius of gyration about the given axis, $k = \sqrt{\frac{I}{A}}$

Theorem of the Perpendicular axis

This theorem states that if I_{xx} and I_{yy} are MI of a plane section about 2 mutually perpendicular axis X-X and Y-Y in the plane of the section, then MI I_{zz} about the axis Z-Z, $\perp$r to the plane passing through intersection of X-X and Y-Y is given by

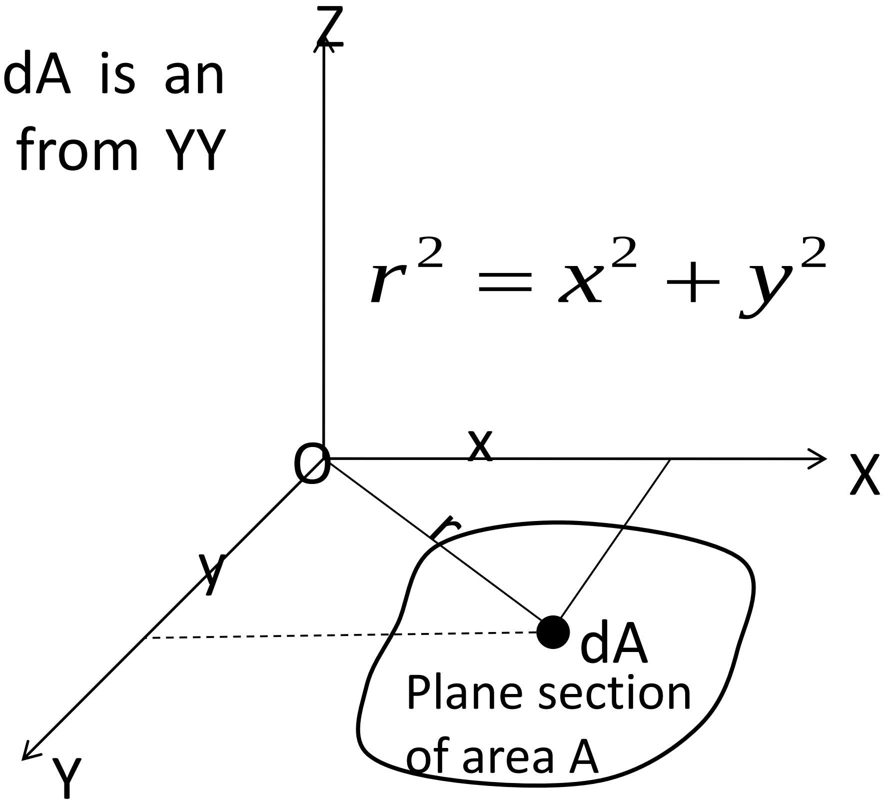
$$I_{ZZ} = I_{XX} + I_{YY}$$

The MI, I_{zz} is also known as polar moment of inertia



Proof of Theorem of the Perpendicular axis

Consider an area A lying on XY plane. dA is an elemental area whose CG is at x and y from YY and XX axes respectively.



$$r^2 = x^2 + y^2$$

MI of dA about X axis = $dA.y^2$

MI of A about X axis, $I_{XX} = \sum dA.y^2$

MI of A about Y axis, $I_{YY} = \sum dA.x^2$

MI of A about Z axis,

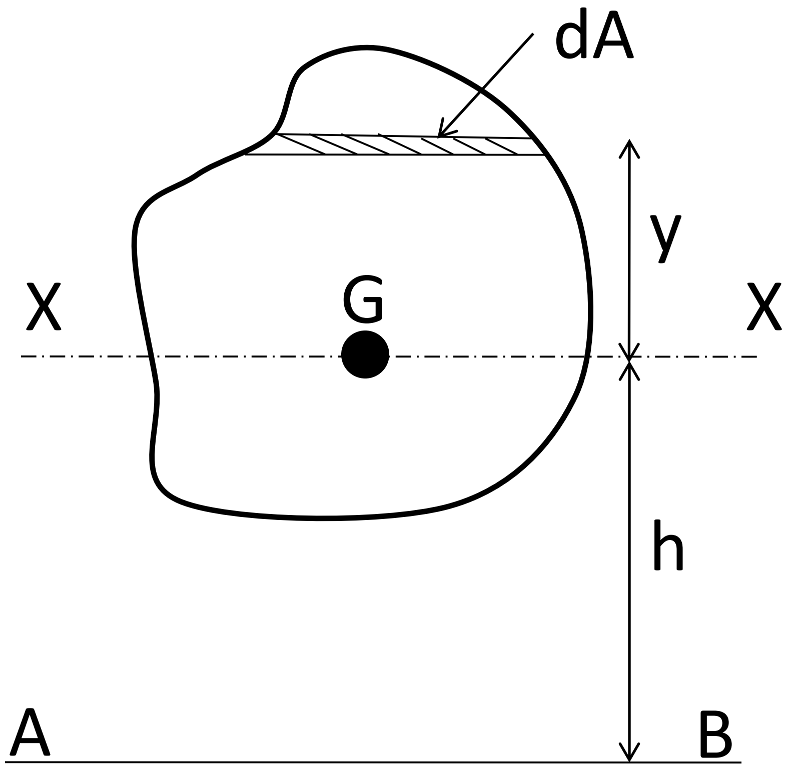
$$I_{ZZ} = \sum dA.r^2 = \sum dA\left(x^2 + y^2\right) = \sum dAx^2 + \sum dAy^2$$

$$I_{ZZ} = I_{XX} + I_{YY}$$

Theorem of the Parallel axis

It states that if the MI of a plane area about an axis in the plane of area through the CG of the plane area is I_{GG}, then the MI of the given plane area about a parallel axis AB in the plane of area at a distance h from the CG of the area is given by

$$I_{AB} = I_G + Ah^2$$



Theorem of the Parallel axis

Consider a strip parallel to XX at a distance y.

$$(I_{XX})_{dA} = dA.y^2$$

$$I_{XX} = I_G = \sum dA.y^2$$

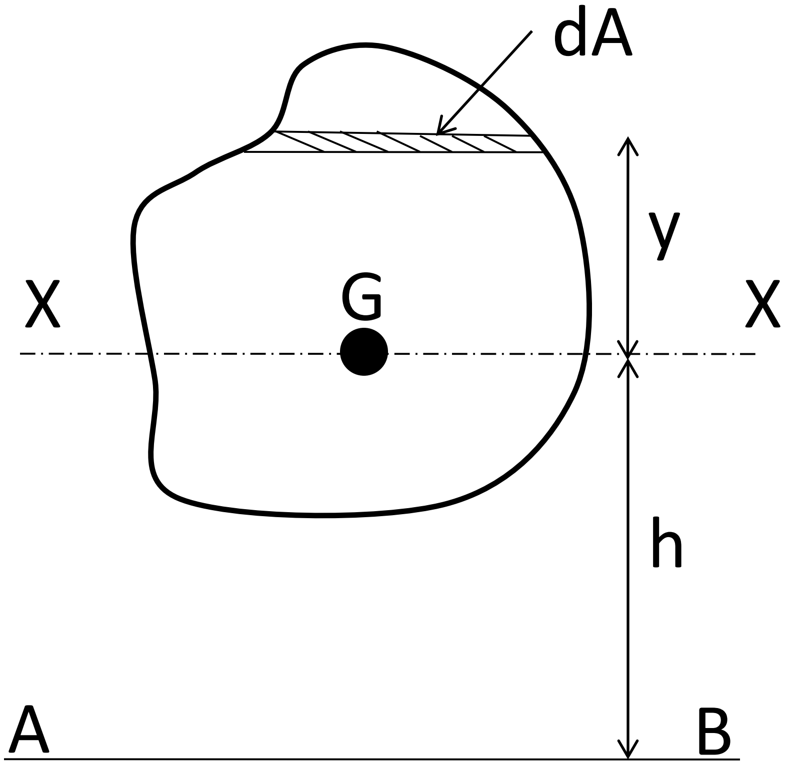
$$(I_{AB})_{dA} = dA.(y+h)^2 = dA(y^2 + h^2 + 2yh)$$

$$I_{AB} = \sum dAy^2 + \sum dAh^2 + \sum 2yhdA$$

$$I_{AB} = h^2 \sum dA + \sum dAy^2 + 2h \sum ydA$$

$$I_{AB} = h^2 A + I_G + 0$$

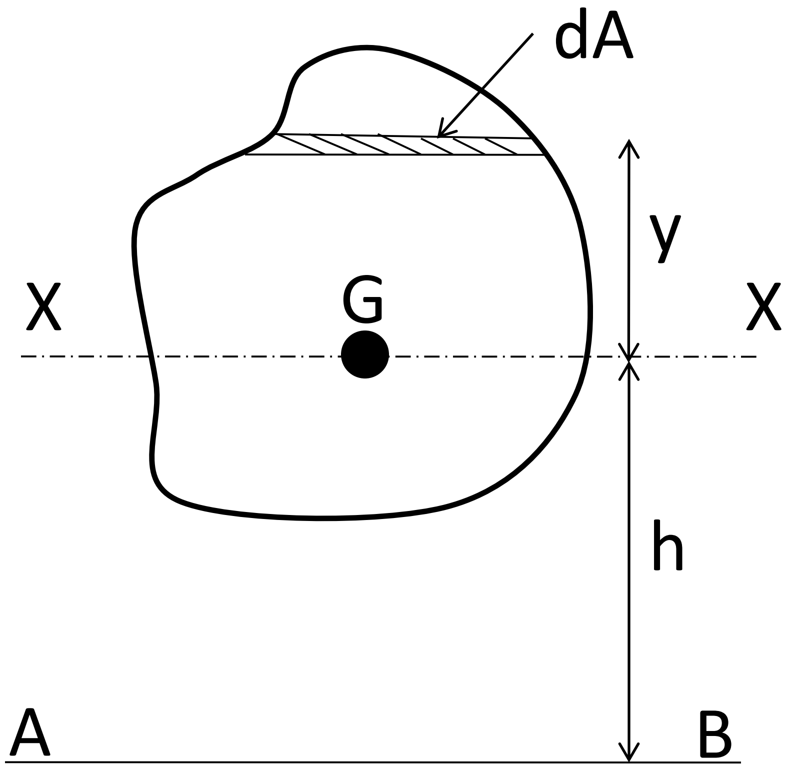
$$I_{AB} = I_G + Ah^2$$



Determination of Moment of Inertia

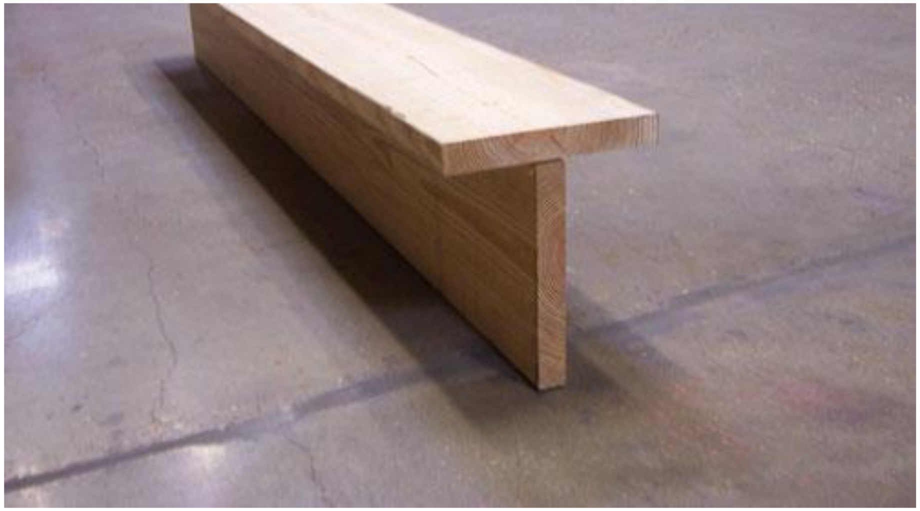
It states that if the MI of a plane area about an axis in the plane of area through the CG of the plane area is I_{GG}, then the MI of the given plane area about a parallel axis AB in the plane of area at a distance h from the CG of the area is given by

$$I_{AB} = I_G + Ah^2$$



Moment of Inertia of Composite sections

A composite area consists of a series of connected "simpler" parts or shapes, such as rectangles, triangles, and circles. The moment of inertia for the composite area about an axis equals the algebraic sum of the moments of inertia of all its parts.



1. Locate the centroid of its cross-sectional area.
2. Find the moment of inertia of the area about the centroidal axis.

Summary

- Moment of inertia of area (mass) about reference axis is obtained by integration method.
- Radius of gyration of a body about an axis is a distance such that its square multiplied by the area gives moment of inertia of the area about the given axis
- Theorem of perpendicular axis and theorem of parallel axis is used to find out moment of inertia.