Circuit Analysis Techniques

Lecture 12
Phasor Relationship for R, L and C Elements

Lecture delivered by:

Topics

- Phasor Relationship for R, L and C Elements
- Resistor Phasor
- Inductor Phasor
- Capacitor Phasor
- Phasor voltage-current relations
- Angular Frequency (ω)

Objectives

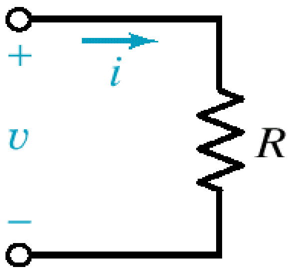
At the end of this lecture, student will be able to:

- Analyze phasor Relationship for R, L, and C Elements
- Define phasor voltage-current relations
- Define angular Frequency (ω)

Phasor Relationship for R, L and C Elements

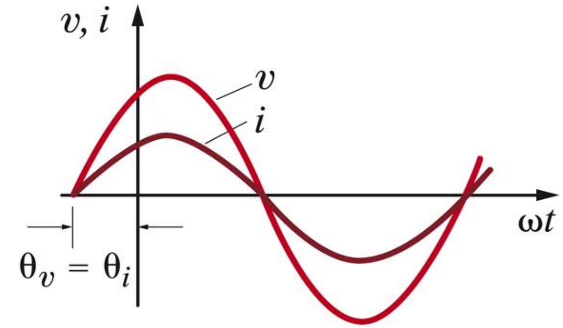
Resistor phasor

Time domain

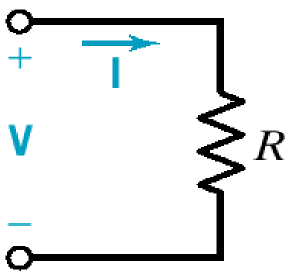


(a)

$$v = Ri$$



Frequency domain

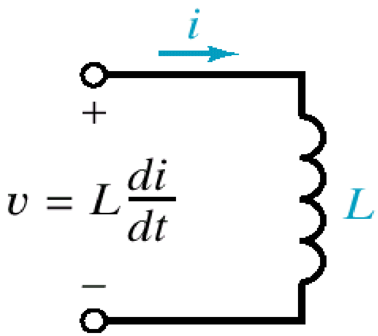


(b)

$$\mathbf{V} = R\mathbf{I} \quad or \quad \mathbf{I} = \frac{\mathbf{V}}{R}$$

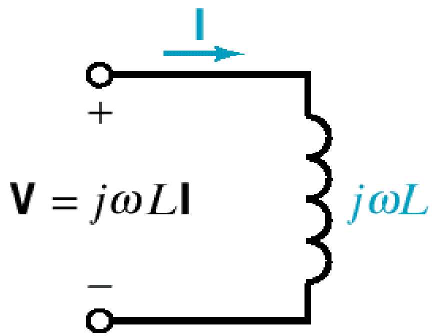
Voltage and current are in phase

Inductor Phasor



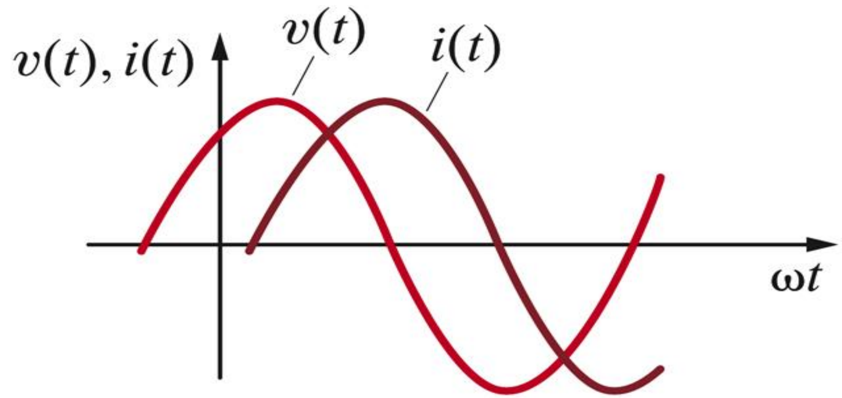
(a)

Time domain



(b)

Frequency domain

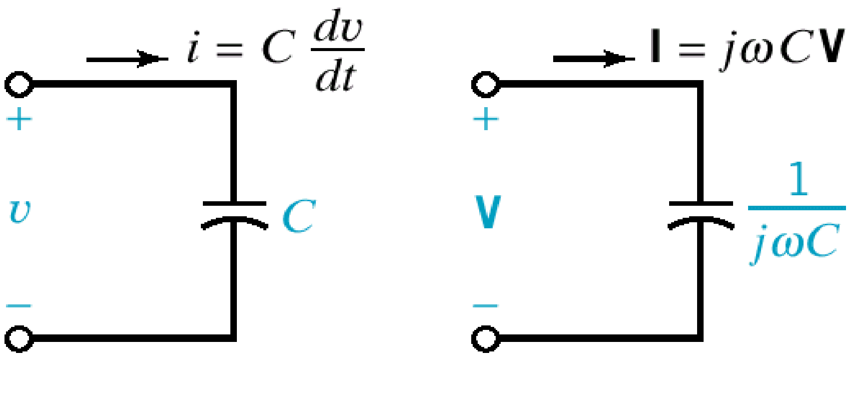


$$v = L\frac{di}{dt}$$

$$\mathbf{V} = j\omega L\mathbf{I} \quad or \quad \mathbf{I} = \frac{\mathbf{V}}{j\omega L}$$

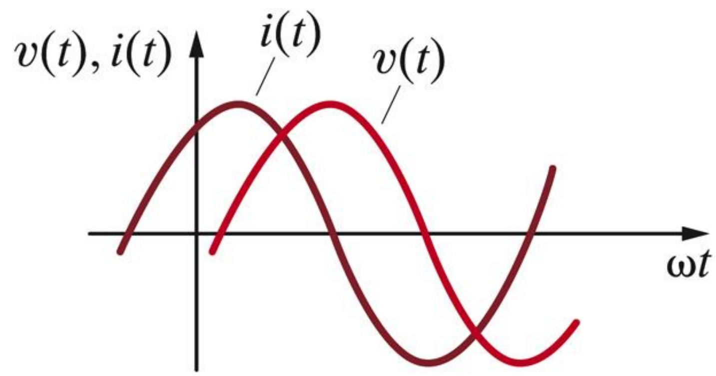
Voltage *leads* current by $90°$

Capacitor Phasor



(a) (b)

Time domain Frequency domain

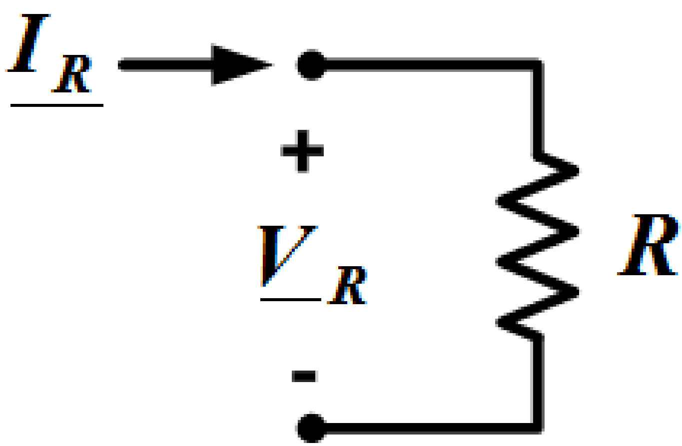


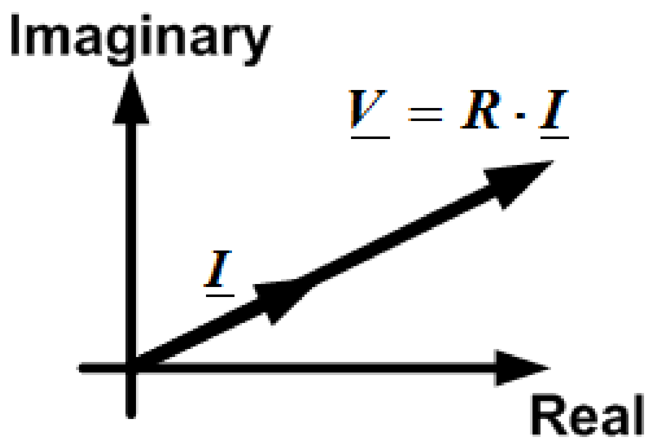
$$i = C\frac{dv}{dt}$$

$$\mathbf{I} = j\omega C\mathbf{V} \quad or \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

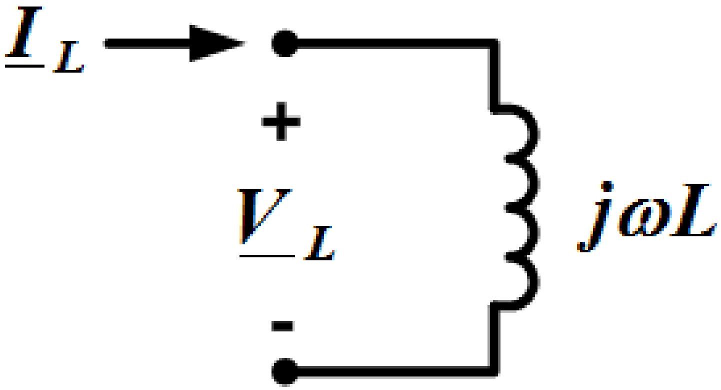
Voltage *lags* current by $90°$

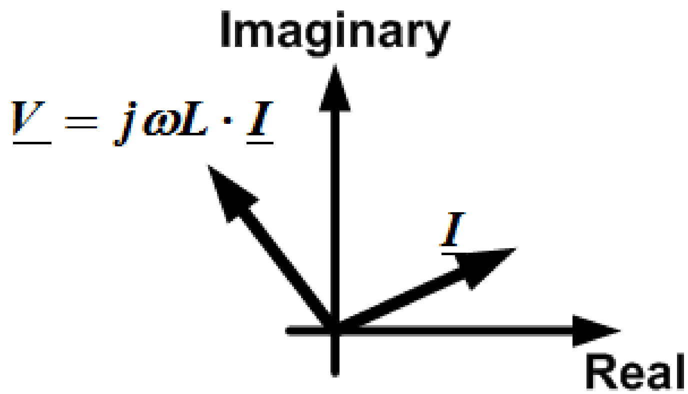
Phasor Voltage-Current Relations of Resistor



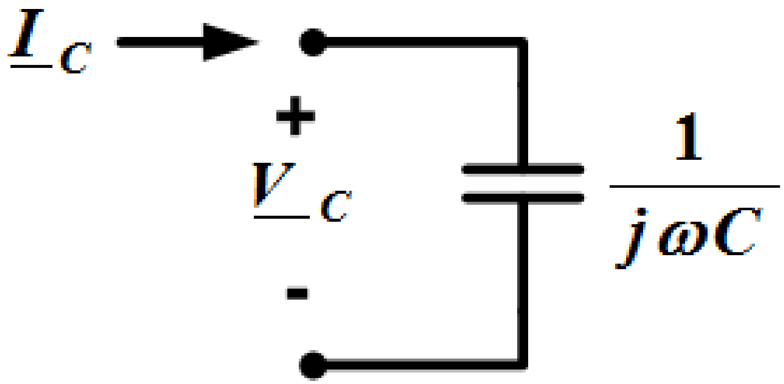


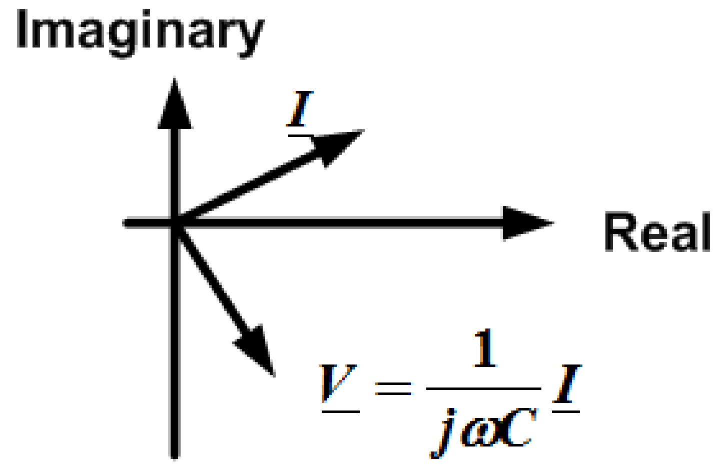
Phasor Voltage-current Relations Of Inductor





Phasor voltage-current relations of capacitor



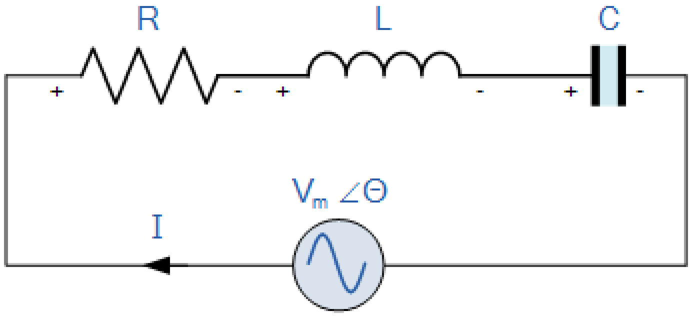


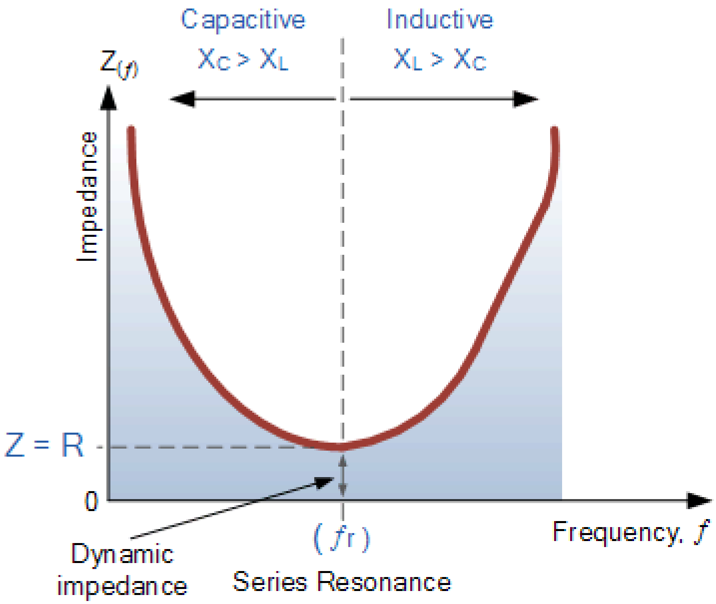
Circuit Element Phasor Relations

Element	V/I Relation	Phasor Relation	Phase
Capacitor	$I = C\, dV/dt$	$\mathbf{I} = j\,\omega\, C\, \mathbf{V}$ $= \omega CV \angle 90°$	I leads V by 90°
Inductor	$V = L\, dI/dt$	$\mathbf{V} = j\,\omega\, L\, \mathbf{I}$ $= \omega LI \angle 90°$	V leads I by 90°
Resistor	$V = I\, R$	$\mathbf{V} = R\, \mathbf{I}$ $= R\, I \angle 0°$	In-phase

Series RLC Resonance

- Series RLC circuit there becomes a frequency point were the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor
- In other words, $X_L = X_C$ The point at which this occurs is called the **Resonant Frequency** point, (f_r) of the circuit





Series RLC Resonance

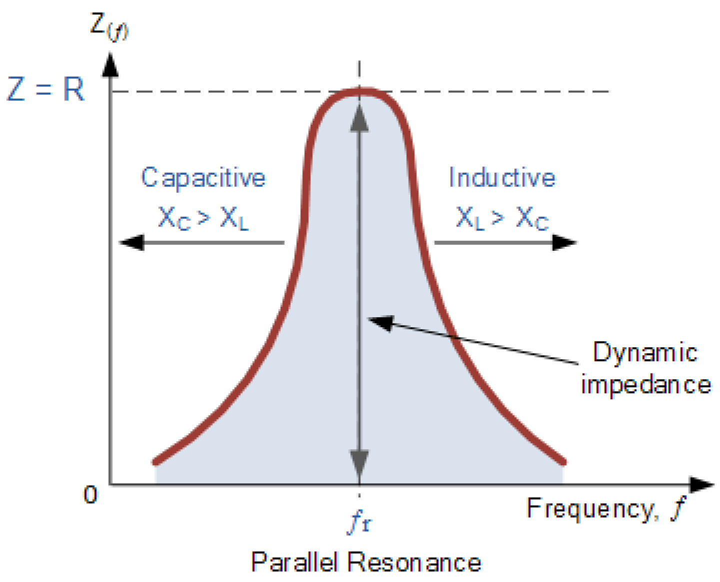
- Resonant Frequency $F_r = \dfrac{1}{2\pi\sqrt{LC}}$ Hz
- Lower cut-off Frequency $F_1 = F_r - \dfrac{R}{4\pi L}$ Hz
- Upper cut-off Frequency $F_2 = F_r + \dfrac{R}{4\pi L}$ Hz
- Band-width $BW = F_2 - F_1$ Hz
- Quality Factor $Q = \dfrac{F_r}{F_2 - F_1}$

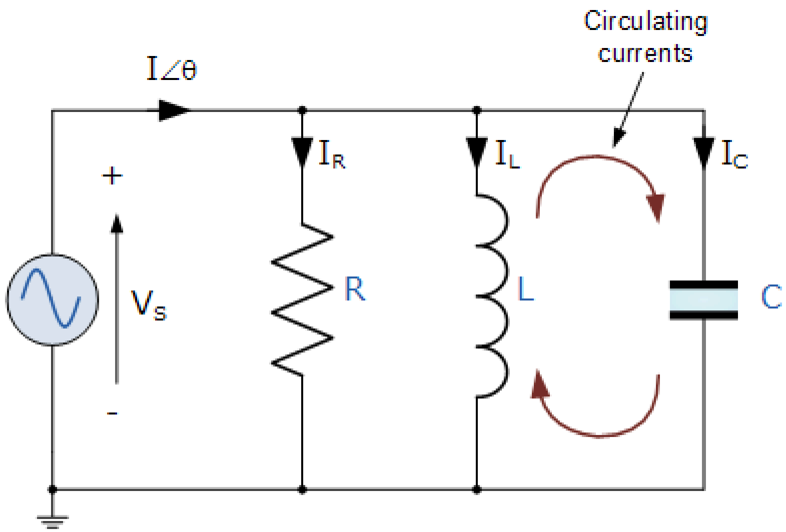
Parallel RLC Resonance

- In many ways a **parallel resonance** circuit is exactly the same as the series resonance circuit
- Both are 3-element networks that contain two reactive components making them a second-order circuit
- Both are influenced by variations in the supply frequency and have a frequency point where their two reactive components cancel each other

Parallel RLC Resonance

- Parallel resonance circuit is influenced by the currents flowing through each parallel branch within the parallel LC tank circuit





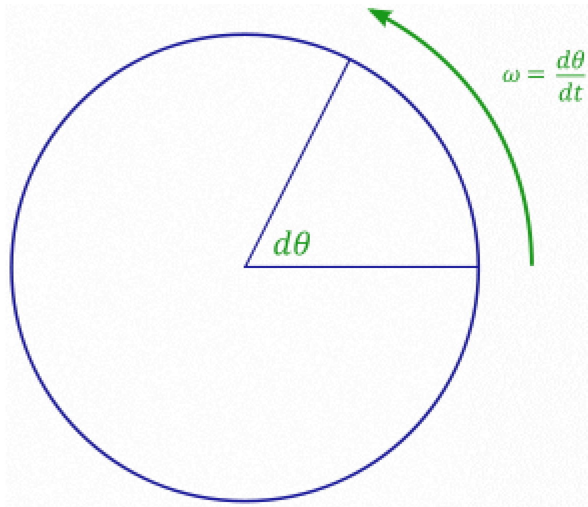
Parallel RLC Resonance

- Resonant Frequency $$F_r = \frac{1}{2\pi\sqrt{LC}}$$ Hz
- Lower cut-off Frequency $$F_1 = F_r - \frac{1}{2\pi[(\frac{-1}{2RC}) + \sqrt{(\frac{1}{2RC})^2 + (\frac{1}{LC})}]}$$ Hz
- Upper cut-off Frequency $$F_2 = F_r + \frac{1}{2\pi[(\frac{1}{2RC}) + \sqrt{(\frac{1}{2RC})^2 + (\frac{1}{LC})}]}$$ Hz
- Band-width $$BW = F_2 - F_1$$ Hz
- Quality Factor $$Q = \frac{F_r}{F_2 - F_1}$$

Angular Frequency(ω)

- **Angular Frequency** is the number of orbits an object makes around another object is a certain time

$$\omega = \frac{d\Theta}{dt}$$



$$\omega = \frac{2\pi}{T} \qquad \omega = 2\pi f$$

- This formula represents angular frequency for an oscillation with period equal T, in this case we deal with one revolution which is equal to 2 Pi radians

Summary

- Phasor Relationship for R, L and C Elements is been discussed
- Phasor voltage-current relations
  - ✓ **Voltage and current are in phase for resistive circuit**
  - ✓ **Voltage *leads* current by 90° for inductive circuit**
  - ✓ **Voltage *lags* current by 90° for capacitive circuit**
- Angular Frequency is the number of orbits an object makes around another object is a certain time