Lecture-32
Ideal Transformer

Lecture delivered by:

Topics

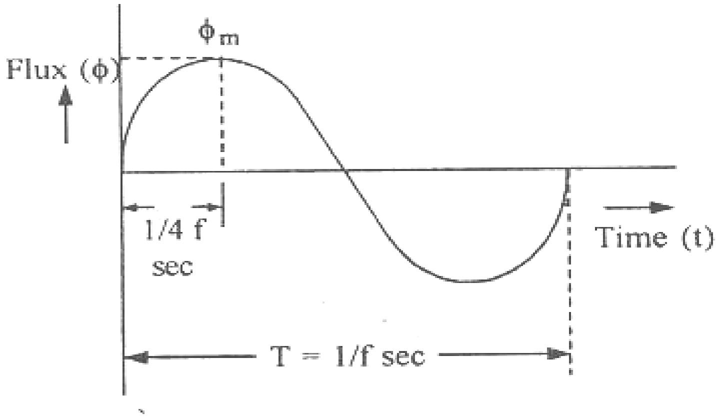
- EMF Induced in Transformer
- Transformer Rating
- Ideal Transformers
- Ideal Transformer under No-Load
- Practical Transformer

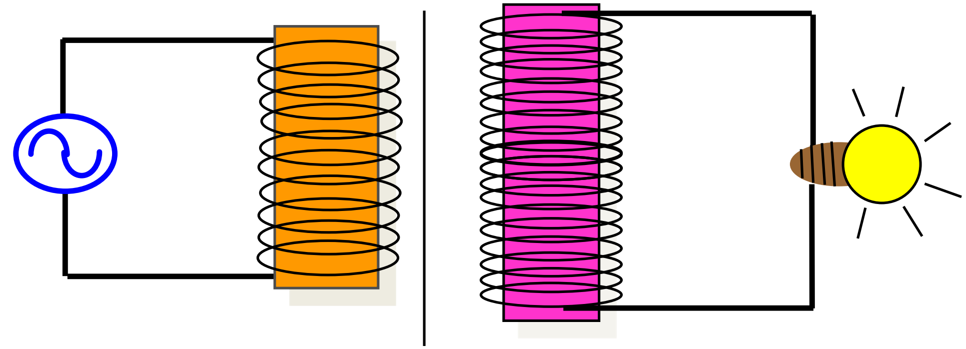
Objectives

At the end of this lecture, student will be able to:

- Derive the EMF equation of Transformer and use it in calculations
- Identify the transformer ratings based on its name plate details
- Describe the properties of Ideal transformer
- Differentiate the Ideal and practical transformers
- Solve for primary voltage, secondary voltage, primary current and number of turns in the secondary given various transformer values

EMF Induced in Transformer





Flux ϕ is produced which is given by an equation

$$\Phi = N_p i_p / S \qquad ....(1)$$

where S is the reluctance

EMF Induced in Transformer Cont..,

According to Faraday law of electromagnetic induction

$$v_{\mathrm{p}} = \mathrm{N}_{\mathrm{p}} \frac{\mathrm{d}\varphi}{\mathrm{dt}} \qquad \text{......(2)}$$

Substitute $\Phi = \mathrm{N}_p i_{\mathrm{p}}/\mathrm{S}$ into the above equation , then

$$v_{\mathrm{p}} = \frac{\mathrm{N}_{\mathrm{p}}{}^{2}}{S} \times \frac{\mathrm{d}}{\mathrm{dt}}(i_{\mathrm{p}}) \qquad \text{......(3)}$$

EMF Induced in Transformer Cont..,

If i_p is sinusoidal, the flux produced also sinusoidal, i.e

$$\Phi = \Phi_m \sin 2\pi ft \qquad ......(4)$$

therefore $$v_p = N_p \frac{d(\Phi_m \sin 2\pi ft)}{dt}$$

$$v_p = N_P 2\pi f\Phi_m \cos 2\pi ft = N_P 2\pi f\Phi_m \sin(2\pi ft + \pi/2) \qquad ......(5)$$

The peak value = $V_{pm} = N_P 2\pi f\Phi_m$ ......(6)

and v_p is leading the flux by p/2.

The rms value $$V_p = \frac{V_{pm}}{\sqrt{2}} = 0.707 \times N_P 2\pi f\Phi_m = 4.44 N_P f\Phi_m \qquad ......(7)$$

EMF Induced in Transformer Cont..,

From (2) and (8) we get

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \text{.......(9)}$$

$$N_p I_p = N_s I_s$$

rearrange $$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

Transformer Rating

- If a transformer carries the 10kVA, 1100/110volts information on its name-plate. **What are the meanings of these ratings?**
- Voltage ratio indicates that the transformer has two windings, the high-voltage winding is rated for 1100 Volts and

low-voltage winding for 110 volts.

- The kVA rating means that each winding is designed for 10 kVA.

current rating for the high-voltage winding = 10000/1100 = 9.09A

Current rating for low voltage winding = 10000/110 = 90.9 A

Problem

Worked Example No.1

A 250 kVA,11000V/400V, 50Hz single –phase transformer has 80 turns on the secondary. Calculate

(a) The appropriate values of the primary and secondary currents;

(b) The approximate number of primary turns;

(c) the maximum value of the flux.

(a) Full-load primary current $$I_p = \frac{P}{V_p} = \frac{250 \times 10^3}{11000} = 22.7A$$

Full-load secondary current $$I_s = \frac{\mathrm{P}}{\mathrm{V_s}} = \frac{250 \times 10^{-3}}{400} = 625A$$

Problem

(b) Number of primary turns

recall $$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\longrightarrow \quad N_P = \frac{N_s}{V_s} \times V_P = \frac{80}{400} \times 11000 = 2200$$

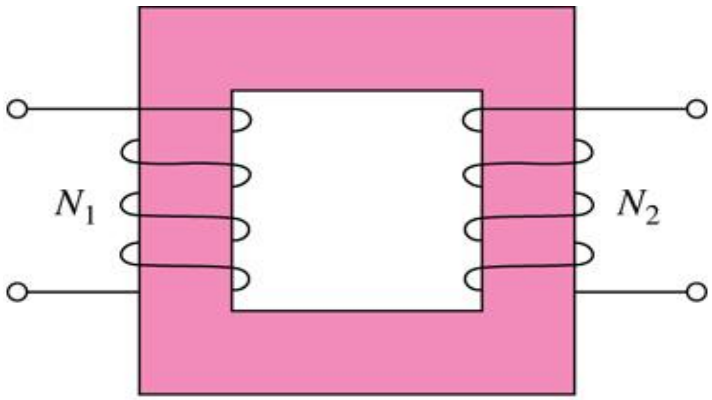
(c) Maximum flux

recall $$E = 4.44\, N\, f\, \Phi_m$$

$$\Phi_m = \frac{E_s}{4.44 N_s\, f} = \frac{400}{4.44 \times 80 \times 50} = 22.5 mWb$$

Ideal Transformers

- Ideal Transformer is a unity coupled, lossless transformer in which the primary and secondary coils have infinite self inductances.



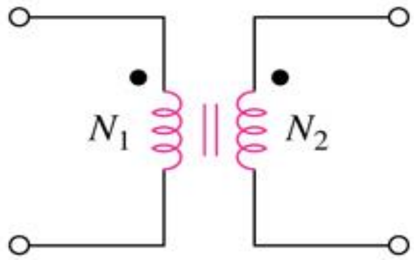
Ideal transformer

Transformer is ideal if:

1) Large reactance coils; $L_1,\ L_2,\ M \to \infty$

2) Unity Coupling k=1.

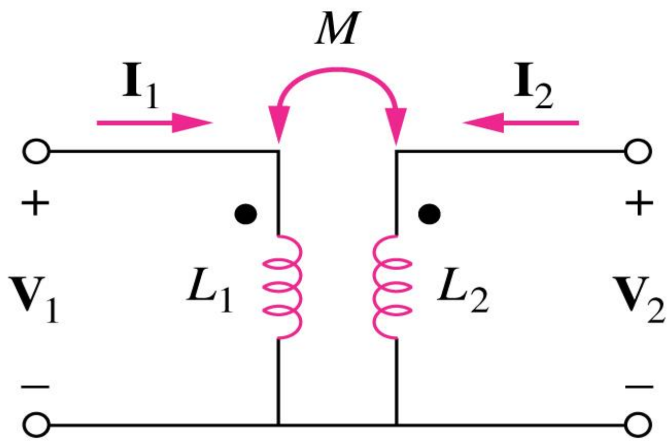
3) Coils are lossless (R_1=R_2=0)



Circuit symbol for the Ideal transformer

Variables of an Ideal Transformer

- Input and Output voltages and currents of an ideal transformer are related only by the turns ratio.



$$V_1 = j\omega L_1 I_1 + j\omega M I_2 \qquad I_1 = \frac{V_1 - j\omega M I_2}{j\omega L_1}$$

$$V_2 = j\omega M I_1 + j\omega L_2 I_2 \qquad V_2 = j\omega L_2 I_2 + \frac{M V_1}{L_1} - \frac{j\omega M^2 I_2}{L_1}$$

Perfect Coupling $k = 1$, Thus we have $M = \sqrt{L_1 L_2}$ Substitute

$$V_2 = j\omega L_2 I_2 + \frac{\sqrt{L_1 L_2}\, V_1}{L_1} - \frac{j\omega L_1 L_2 I_2}{L_1} = \sqrt{\frac{L_2}{L_1}}\, V_1 = n V_1 = \frac{N_2}{N_1} V_1$$

$$\boxed{\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \text{Turns Ratio}}$$

Turns Ratio of an Ideal Transformer

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \text{Turns Ratio}$$

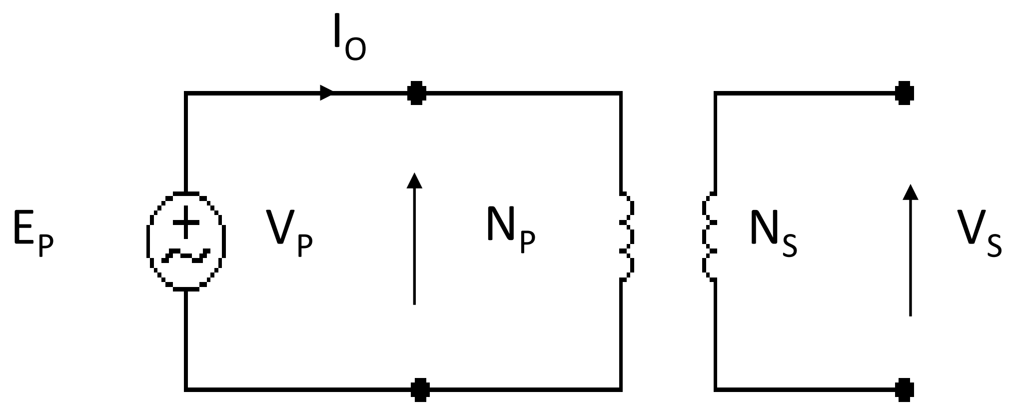
- A Ideal Transformer is called:

  1) Step-up transformer if n > 1.

  2) Step-down transformer if n < 1.

  3) Isolation transformer if n=1.

Ideal Transformer with No Load



I_o is the no load current when the secondary is open circuit.

Ideal Transformer with No Load Cont…

V_P= emf of supply to the primary coil

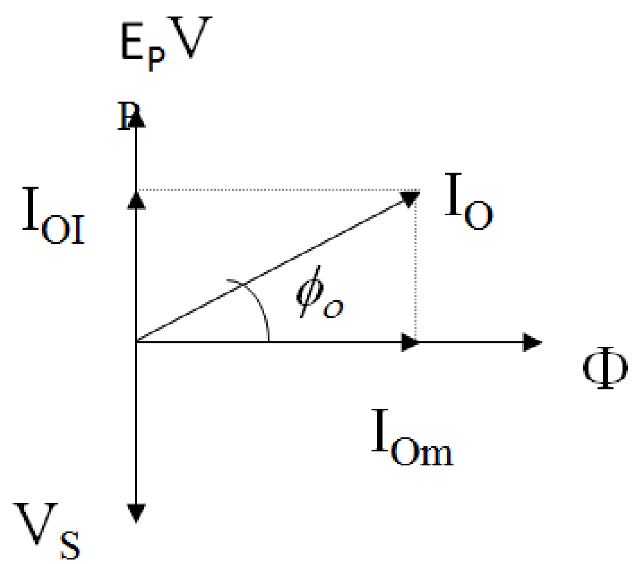
E_P=emf induced in the primary coil

V_S=emf induced in the secondary coil

I_{om}=magnetizing current

I_{o1}=current to compensate the losses due to hysteresis and eddy current.

I_o=the no load current and given by

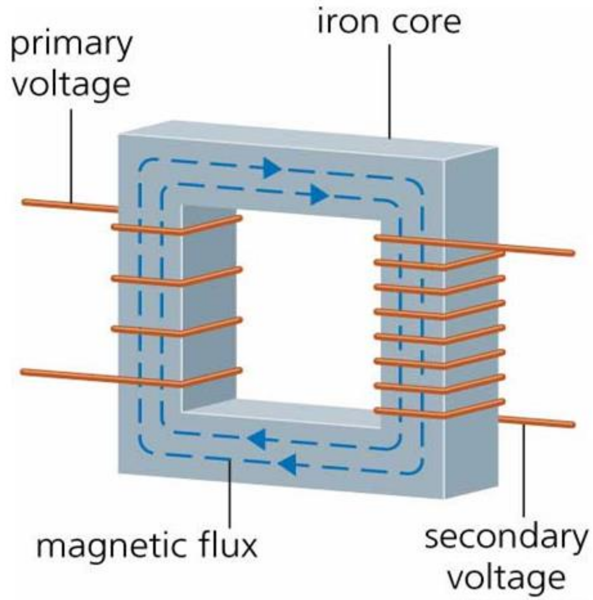


Phasor diagram for no load transfomer

$$I_o = \sqrt{I_{om}^2 + I_{o1}^2}$$

Power factor $\cos\phi_o = \dfrac{I_{o1}}{I_o}$

Practical Transformer



Practical transformer has

- Copper resistance
- Leakage flux
- Finite core permeability (i.e., finite ir
- Core loss

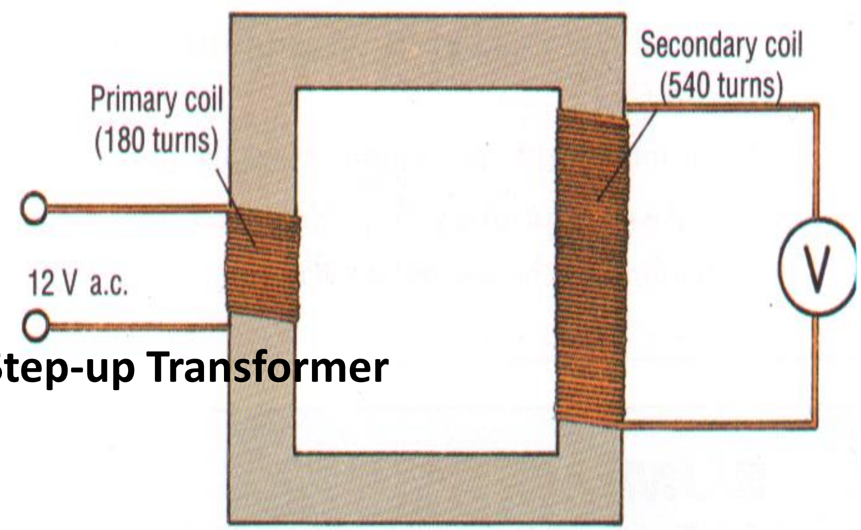
Note: Practically no transformer is ideal .knowledge of ideal transformer helps us to develop the concept of behavior of practical transformer

Problem

Worked Example No.2

The diagram shows a transformer. Calculate the voltage across the secondary coil of this transformer.

Solution



Step-up Transformer

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

Substituting

$$\frac{12}{V_S} = \frac{180}{540}$$

Crossmultiplying

$$180.V_S = 12 \text{ x } 540$$

$$\therefore V_S = \frac{12 \text{ x } 540}{180}$$

$$\therefore V_S = 36 \text{ V}$$

Problem

Worked Example No. 3

A transformer which has 1380 turns in its primary coil is to be used to convert the mains voltage of 230 V to operate a 6 V bulb. How many turns should the secondary coil of this transformer have?

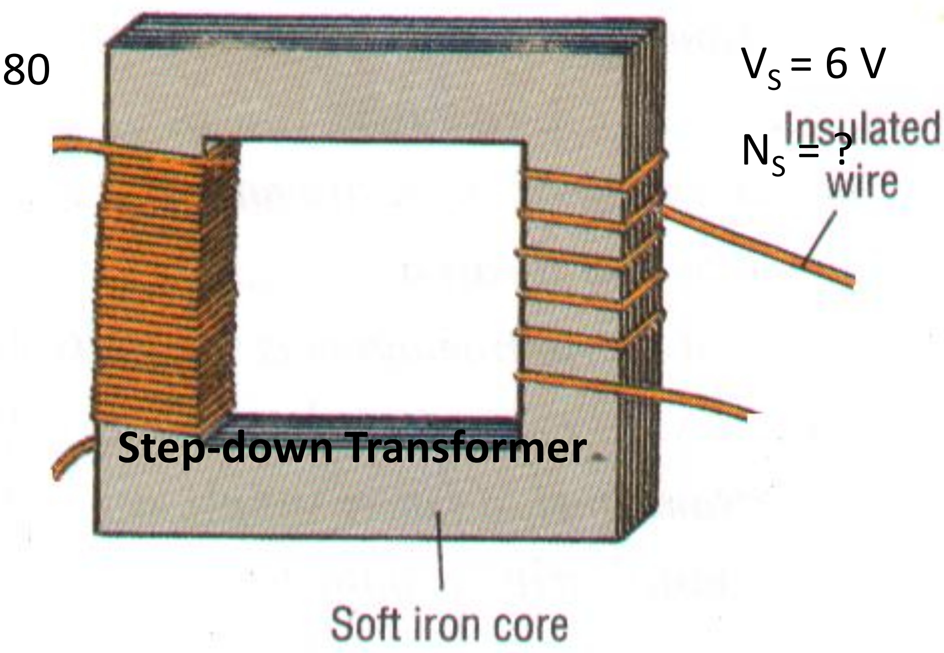
Solution

$V_P = 230$ V

$N_P = 1380$

$V_S = 6$ V

$N_S = ?$



$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

Substituting

$$\frac{230}{6} = \frac{1380}{N_s}$$

Crossmultiplying

$$2300.N_S = 6 \text{ x } 13800$$

$$\therefore N_S = \frac{6 \text{ x } 1380}{230}$$

$$\therefore N_S = 36 \text{ turns}$$

Summary

- EMF equation of Transformer is $E = 4.44\,N\,f\,\Phi_m$
- Losses are zero in an ideal transformer
- An ideal transformer divides a sinusoidal input voltage by a factor of a and multiplies a sinusoidal input current by a to obtain secondary voltage and current.
- If a transformer increases the voltage, the current decreases and vice versa.