Course Code: ESC106A
Course Title: Construction Materials and Engineering Mechanics

Lecture No. 39:
Moment of Inertia

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Lecture Intended Learning Outcomes

At the end of this lecture, students will be able to:

- Define Moment of Inertia
- Determine Moment of Inertia of a general element using integration method

Contents

Moment of inertia of an area under a curve

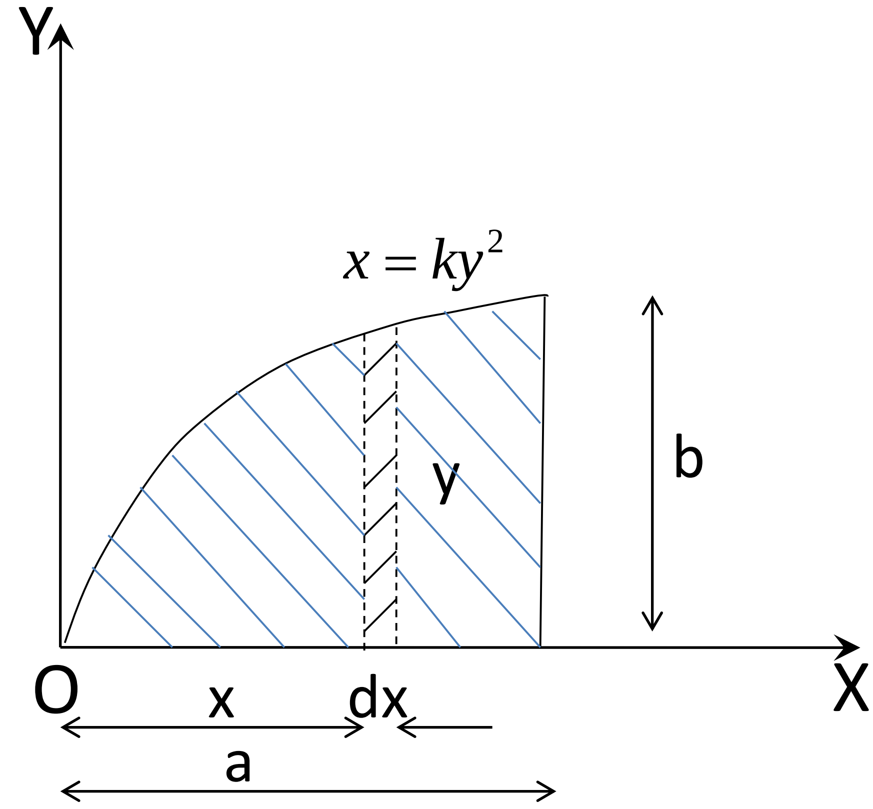
Moment of Inertia of an Area under a given curve

Consider a curve whose equation is parabolic and is given by $x = ky^2$

x varies from 0 to a

y varies from 0 to b

Consider a strip of thickness dx at a distance x from y-axis



Moment of Inertia of an Area under a given curve

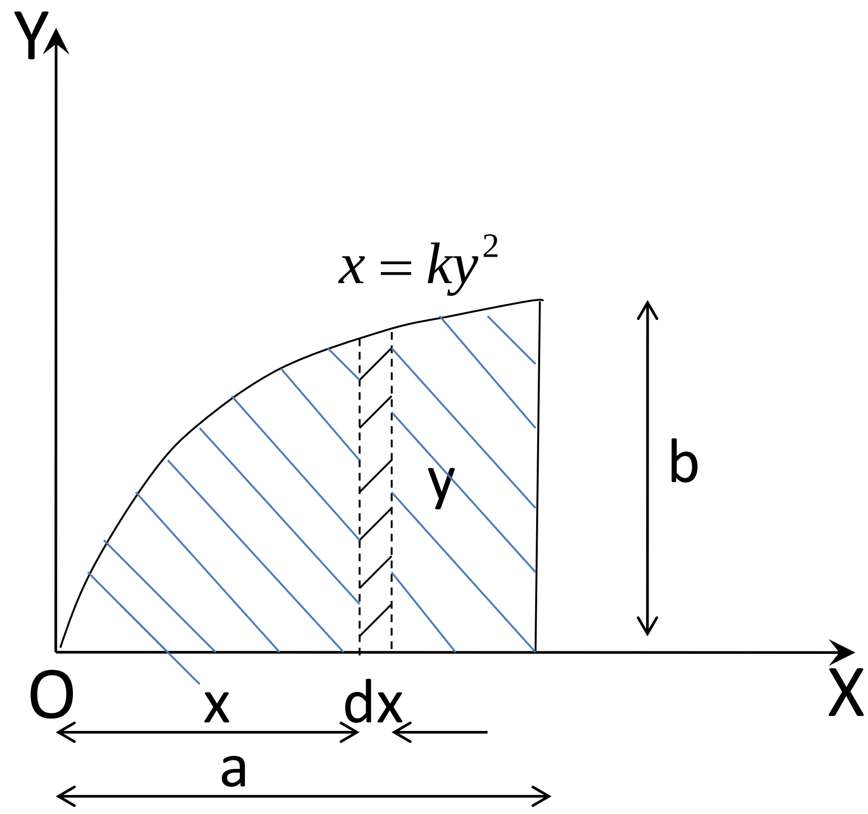
Area of the strip, dA = ydx

From the equation of the curve,

$x = ky^2$

Find the value of k using the given

values of x and y



Moment of Inertia of an Area under a given curve

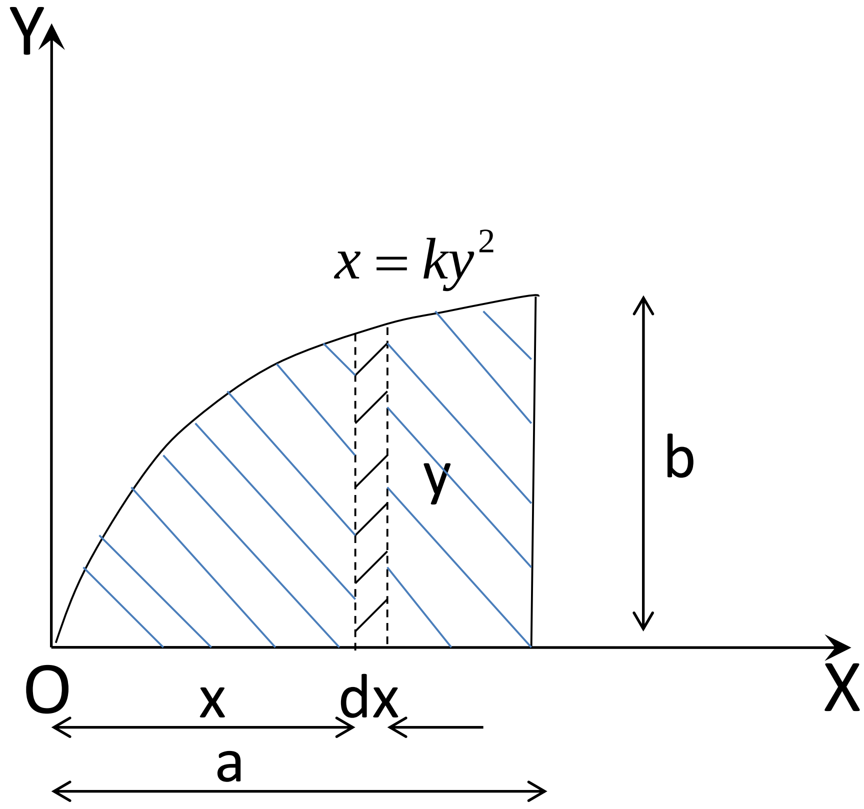
When y=b, x=a

$$a = kb^2$$

$$k = \frac{a}{b^2}$$

Substituting the value of k

$$x = \frac{a}{b^2} y^2$$

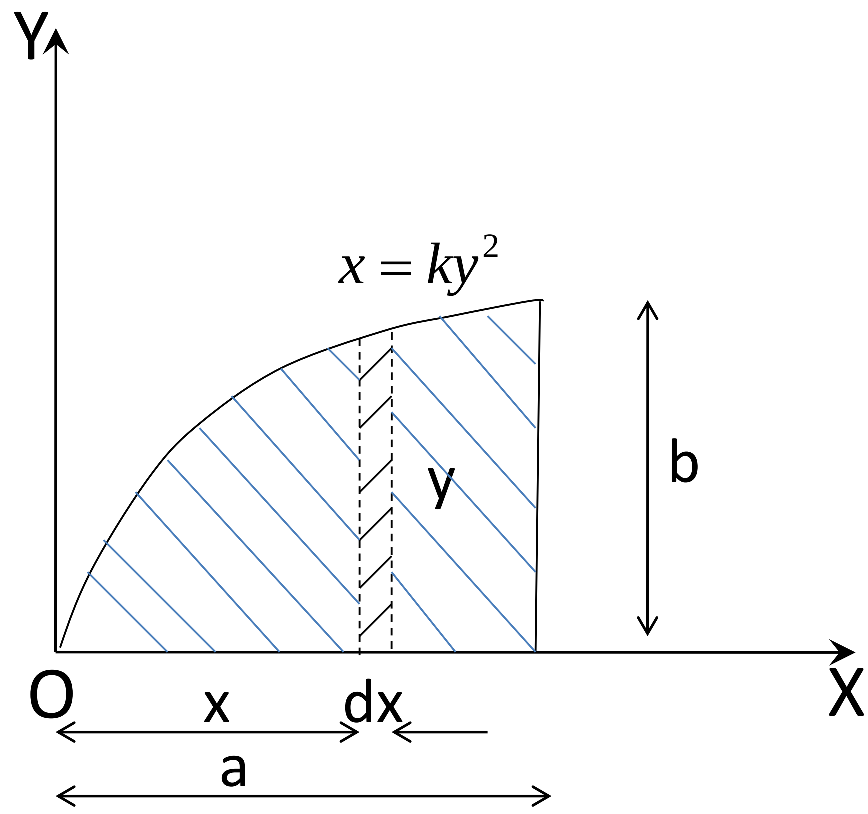


Moment of Inertia of an Area under a given curve

$$\Rightarrow$$

$$y^2 = \frac{b^2 x}{a}$$

$$y = b\sqrt{\frac{x}{a}}$$



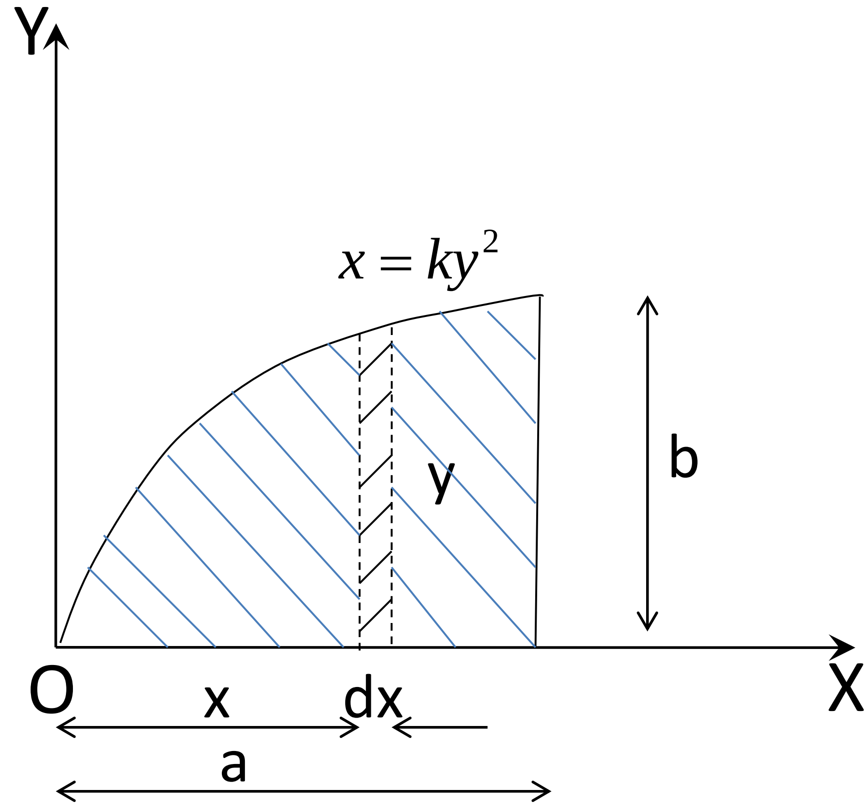
Substituting this value in the equation of dA

Moment of Inertia of an Area under a given curve

$$dA = b\sqrt{\frac{x}{a}}.dx$$

The moment of this elemental area dA about the OY axis is

$$dA.x$$

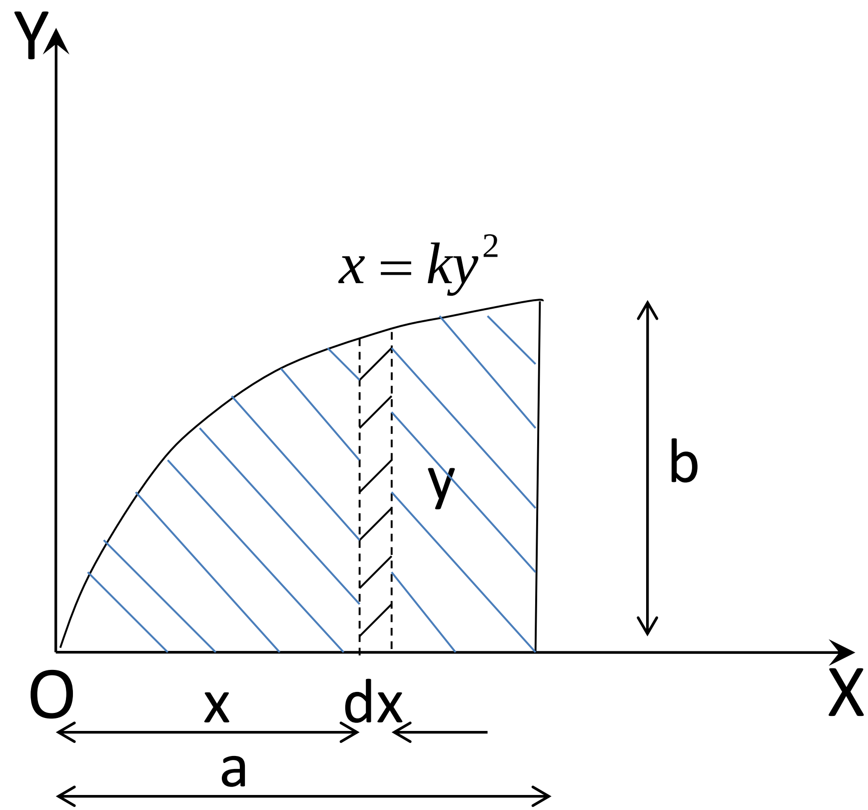


Moment of Inertia of an Area under a given curve

The moment of inertia of this elemental area dA about the OY axis is

$$dA.x.x = dA.x^2$$

$$= x^2.b.\sqrt{\frac{x}{a}}.dx$$

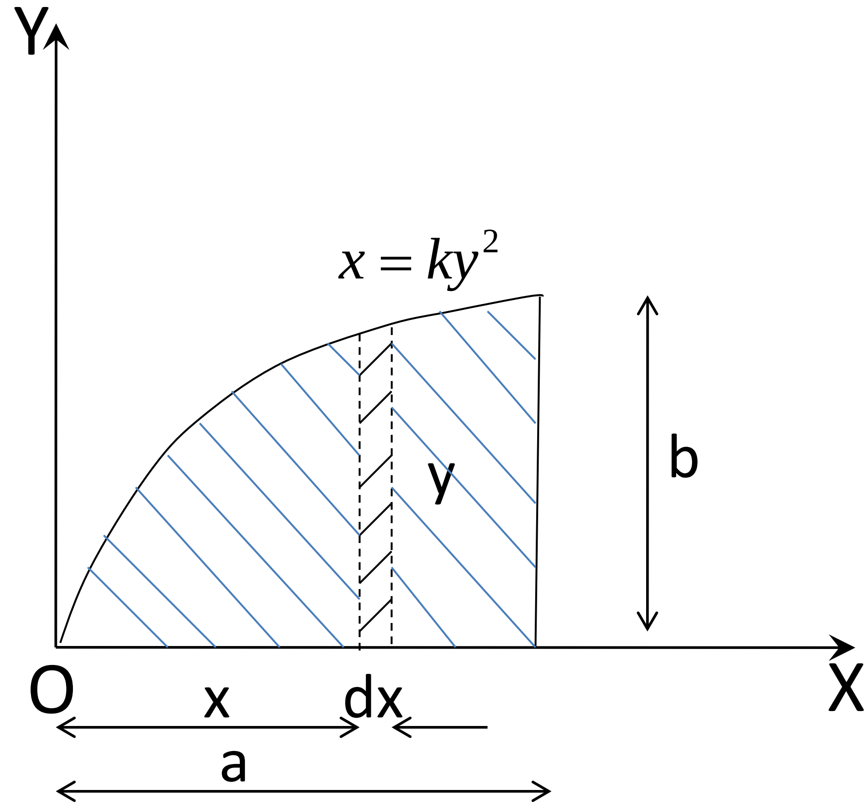


Moment of Inertia of an Area under a given curve

Moment of inertia of the total area about y-axis is obtained by integrating the above equation within the limits 0 to a.

$$dA.x.x = dA.x^2$$

$$= x^2.b.\sqrt{\frac{x}{a}}.dx$$



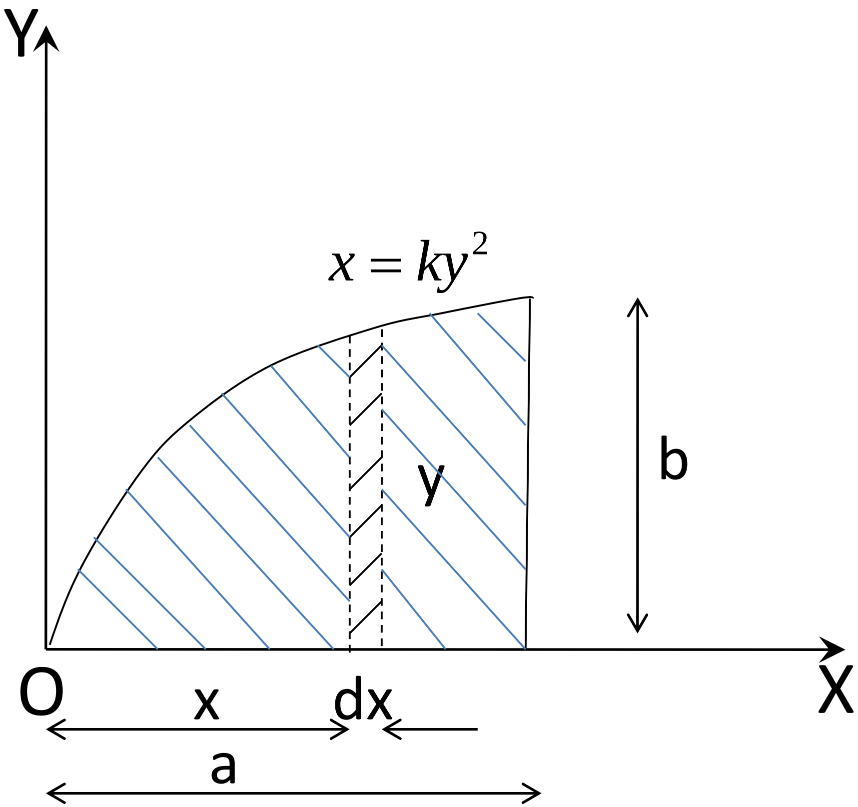
Moment of Inertia of an Area under a given curve

$$I_{yy} = \int_0^a x^2 b.\sqrt{\frac{x}{a}}.dx$$

$$I_{yy} = \frac{b}{\sqrt{a}} \int_0^a x^{\frac{5}{2}} dx$$

$$I_{yy} = \frac{b}{\sqrt{a}} \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right]_0^a$$

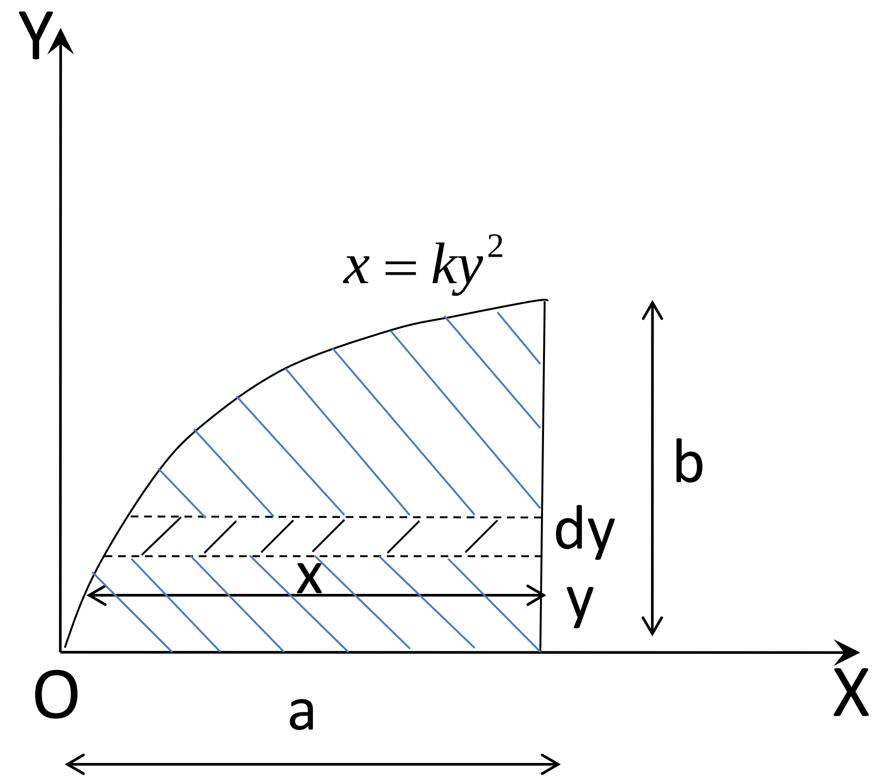
$$I_{yy} = \frac{2b}{7\sqrt{a}}.a^{\frac{7}{2}} = \frac{2}{7} ba^2$$



Moment of Inertia of an Area under a given curve

Consider a strip of thickness dy and length x parallel to x-axis.

Area of the strip , $dA = x.dy$

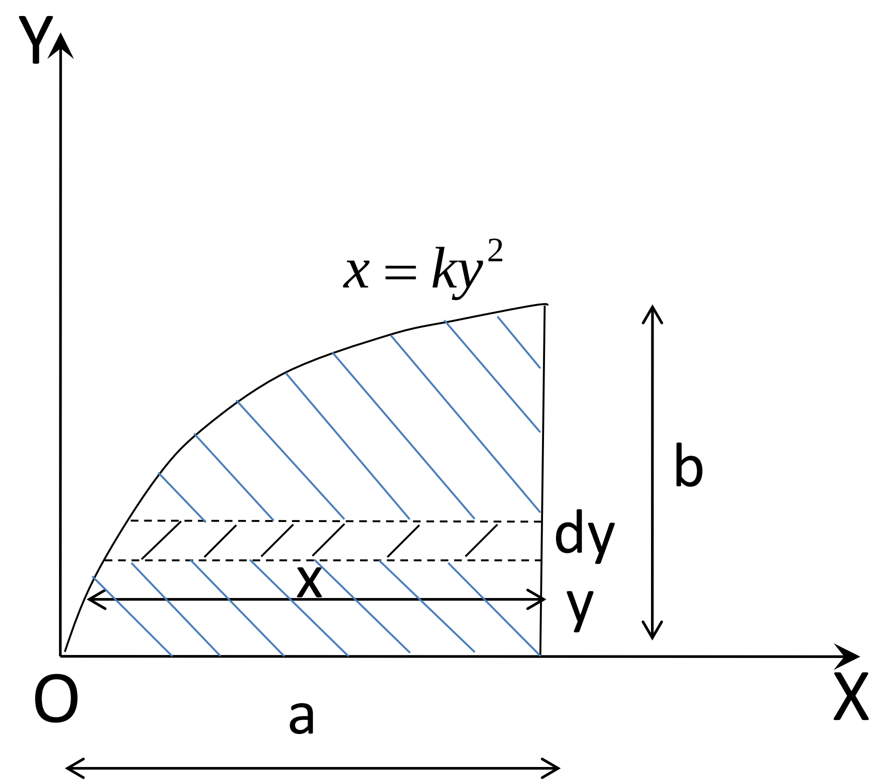


Moment of Inertia of an Area under a given curve

Moment of this elemental area dA about the OX axis is given by,

$dA.y$

Moment of inertia of the total area about x-axis is obtained by integrating the moment of moment of area within the limits 0 to b.



Moment of Inertia of an Area under a given curve

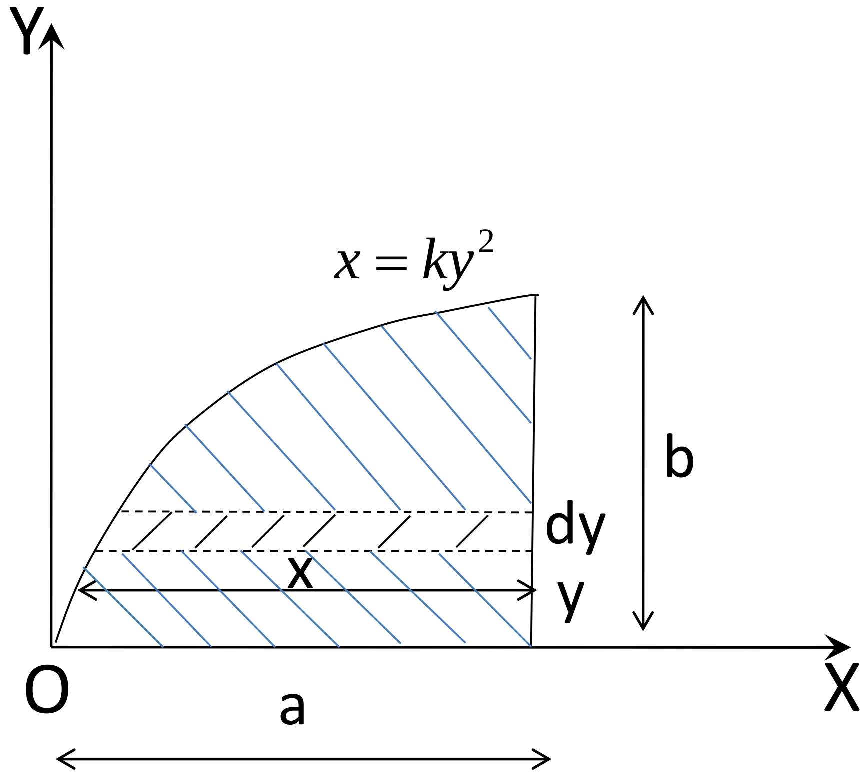
$$I_{xx} = \int_0^b dAy^2$$

$$I_{xx} = \int_0^b x.dy.y^2$$

$$I_{xx} = \int_0^b ky^2.dy.y^2$$

$$I_{xx} = \frac{a}{b^2}\int_0^b y^4.dy = \frac{a}{b^2}\left[\frac{y^5}{5}\right]_0^b$$

$$I_{xx} = \frac{a}{b^2}\frac{b^5}{5} = \frac{1}{5}ab^3$$

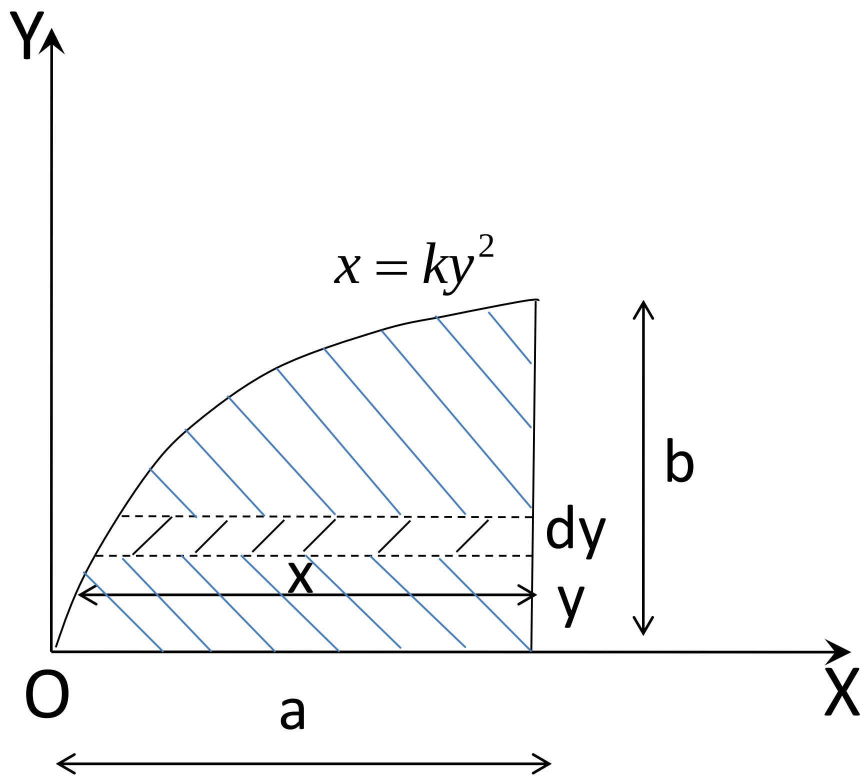


Moment of Inertia of an Area under a given curve

Moment of Inertia of an Area under a given curve $x = ky^2$

$$I_{xx} = \frac{1}{5}ab^3$$

$$I_{yy} = \frac{2}{7}a^2b$$



Summary

- Moment of inertia measures how an area is distributed about particular axes
- Moment of inertia of area (mass) about reference axis is obtained by integration method