Circuit Analysis Techniques

Lecture 2
Mesh and Node Analysis

Lecture delivered by:

Topics

- Source transformation
- Mesh analysis
- Node analysis

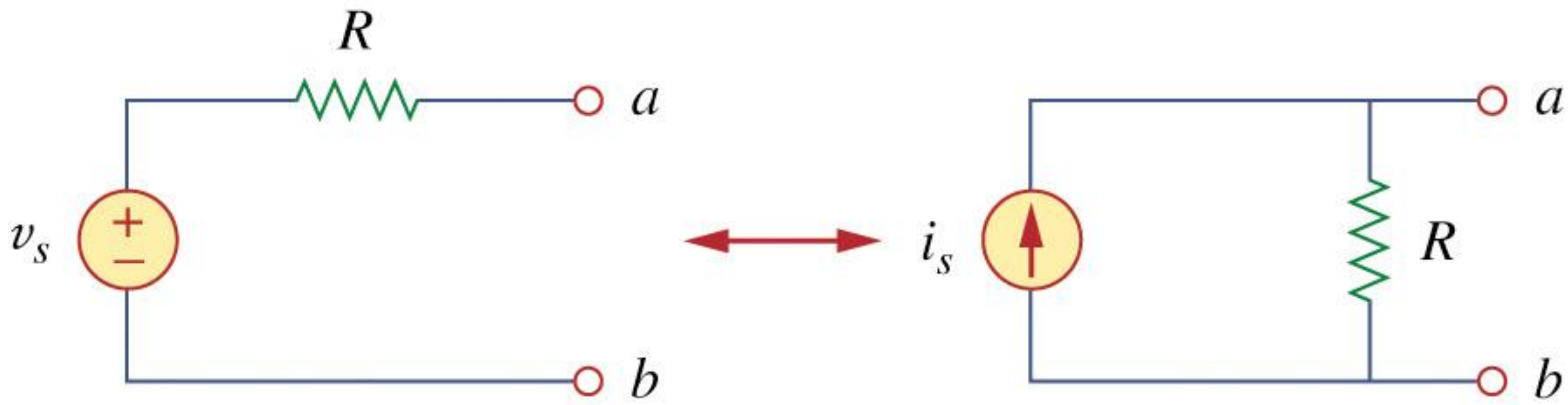
Objectives

At the end of this lecture, student will be able to:

- Perform source transformation, Voltage source to current source and vice versa
- Analyse mesh circuits using KVL
- Analyse nodal circuits using KCL

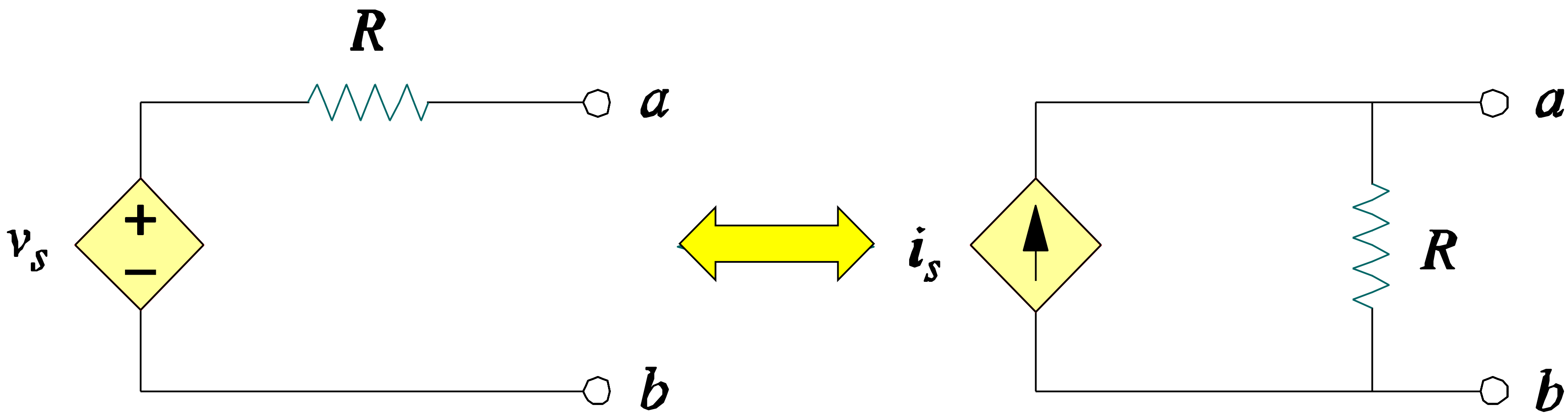
Source Transformation

- Source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Source Transformation



$$V_s = R_s I_s$$

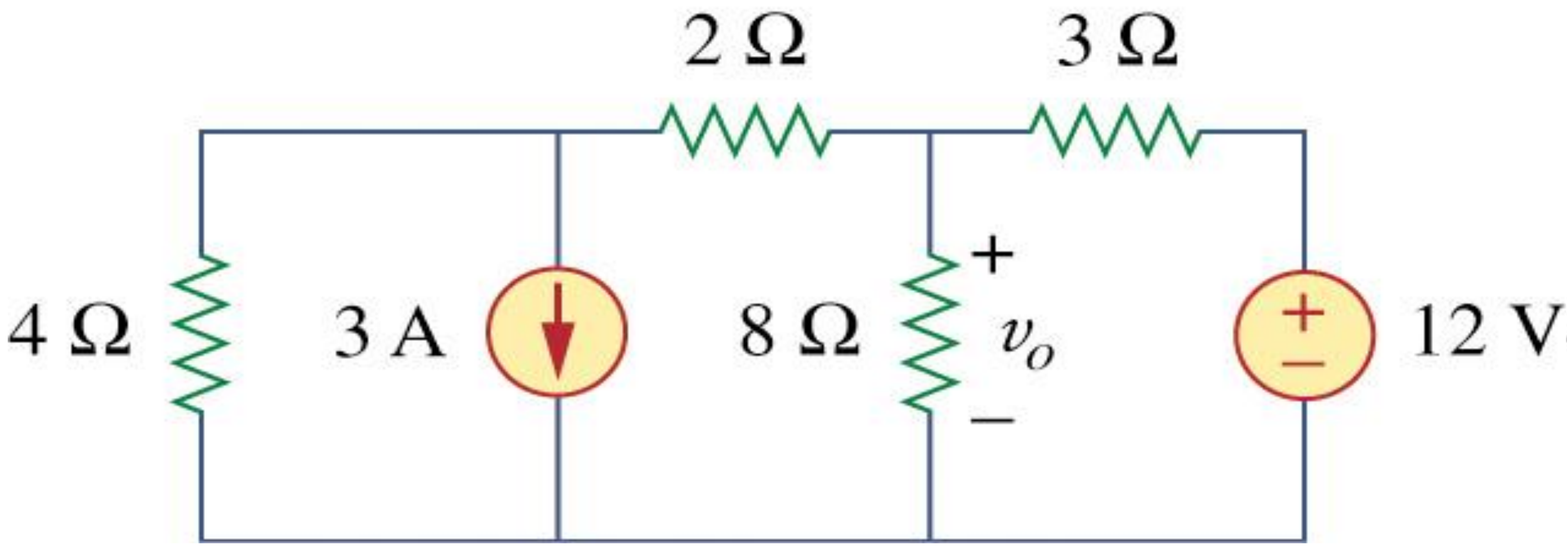
$$I_s = \frac{V_s}{R_s}$$

Source Transformation

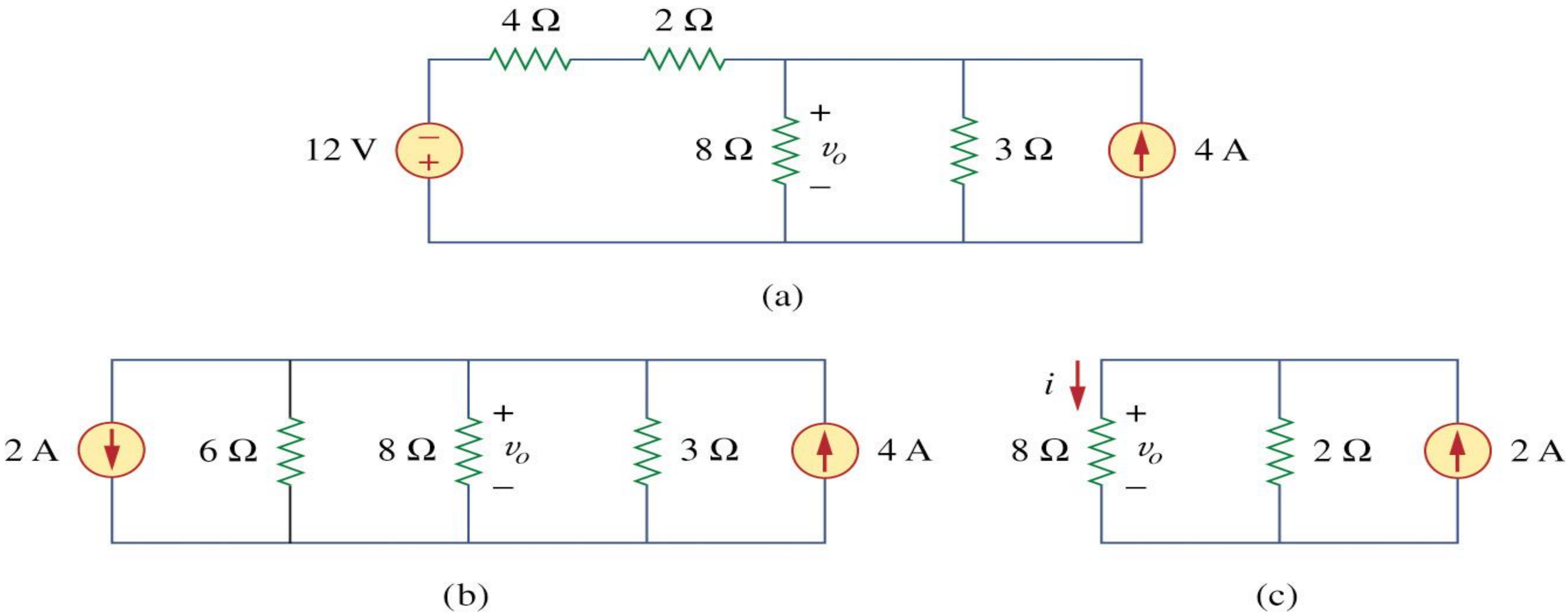
- Equivalent sources can be used to simplify the analysis of some circuits
- Voltage source in series with a resistor is transformed into a current source in parallel with a resistor
- Current source in parallel with a resistor is transformed into a voltage source in series with a resistor

Example

- Use source transformation to find v_o in the circuit in Fig.



Example Cont..



Use current division in Fig.(c) to get

$$i = \frac{2}{2+8}(2) = 0.4\text{A} \quad \text{and} \quad v_o = 8i = 8(0.4) = 3.2\text{V}$$

Mesh Analysis

- Mesh analysis method a current is assigned to each window of the network such that the currents complete a closed loop
- They are also referred to as loop currents
- Each element and branch therefore will have an independent current
- If a branch has two of the mesh currents, the actual current is given by their algebraic sum
- The assigned mesh currents may have either clockwise or counterclockwise directions

Mesh Analysis

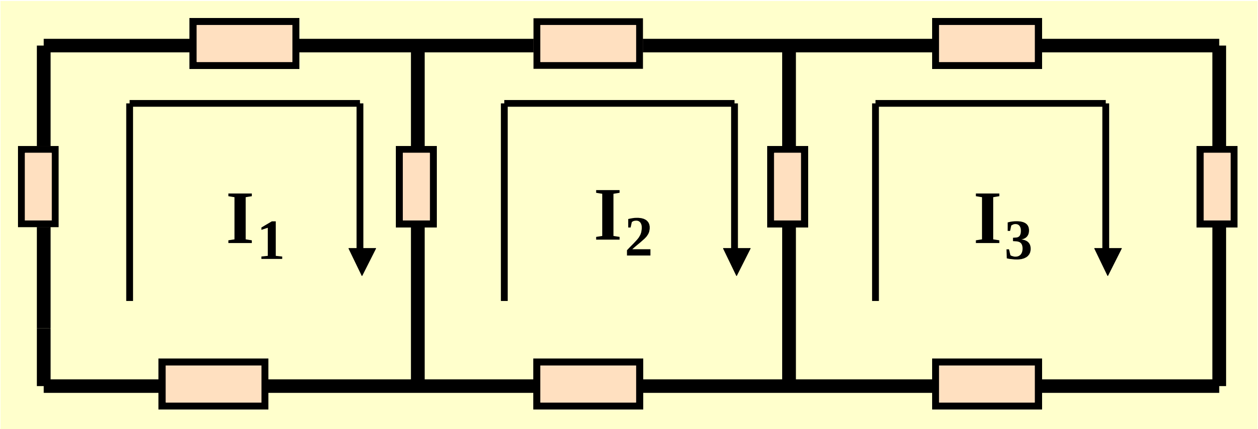
Analysis using KVL to solve for the currents around each closed loop of the network.

Mesh analysis procedure:

1. Assign currents to each closed loop of the network.
2. Apply KVL around each closed loop of the network.
3. Solve the resulting simultaneous linear equation for the loop currents.

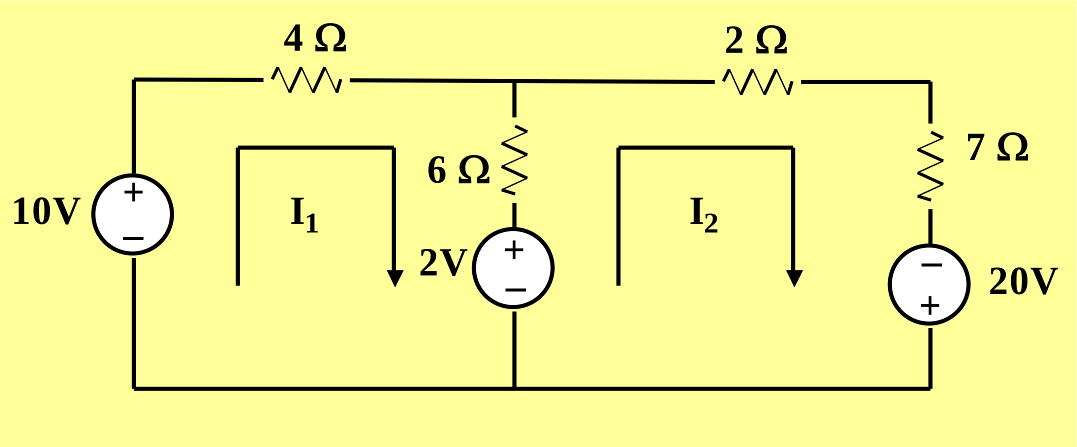
Mesh Analysis: Basic Concepts:

☑ In formulating mesh analysis we assign a mesh current to each mesh.



Mesh Analysis: Example 1

Write the mesh equations and solve for the currents I_1 and I_2.



Circuit for Example 1

Mesh 1 $$4I_1 + 6(I_1 - I_2) = 10 - 2 \quad \text{Eq (1)}$$

Mesh 2 $$6(I_2 - I_1) + 2I_2 + 7I_2 = 2 + 20 \quad \text{Eq (2)}$$

Mesh Analysis: Example 1, continued.

Simplifying Eq (1) and (2) gives,

$$10I_1 - 6I_2 = 8 \qquad \text{Eq (3)}$$

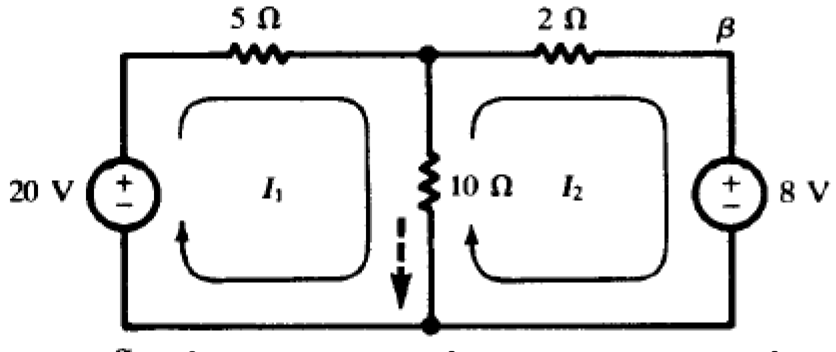
$$-6I_1 + 15I_2 = 22 \qquad \text{Eq (4)}$$

$$I_1 = 2.2105 \text{ A}$$

$$I_2 = 2.3509 \text{ A}$$

Mesh Analysis: Example 2

- Obtain the current in each branch of the network using the mesh current method



- The currents I_1 and I_2 are chosen as shown on the circuit diagram. Applying KVL around the left loop, starting at point α

$$-20 + 5I_1 + 10(I_1 - I_2) = 0$$

Mesh Analysis

- Now around the right loop, starting at point β,

$$8 + 10(I_2 - I_1) + 2I_2 = 0$$

- Rearranging terms in both the equations

$$15I_1 - 10I_2 = 20$$
$$-10I_1 + 12I_2 = -8$$

- Solving simultaneously we get I_1 =2A and I_2= 1A
- The current in the center branch, shown dotted, is I_1 - I_2 = 1 A.

Node Analysis

- Node voltage method, one principal nodes is selected as the reference and equations based on KCL are written at the other principal nodes
- At each other principal nodes, a voltage is assigned, where it is understood that this is a voltage with respect to the reference node
- These voltages are the unknowns and, when determined by a suitable method

Nodal Analysis

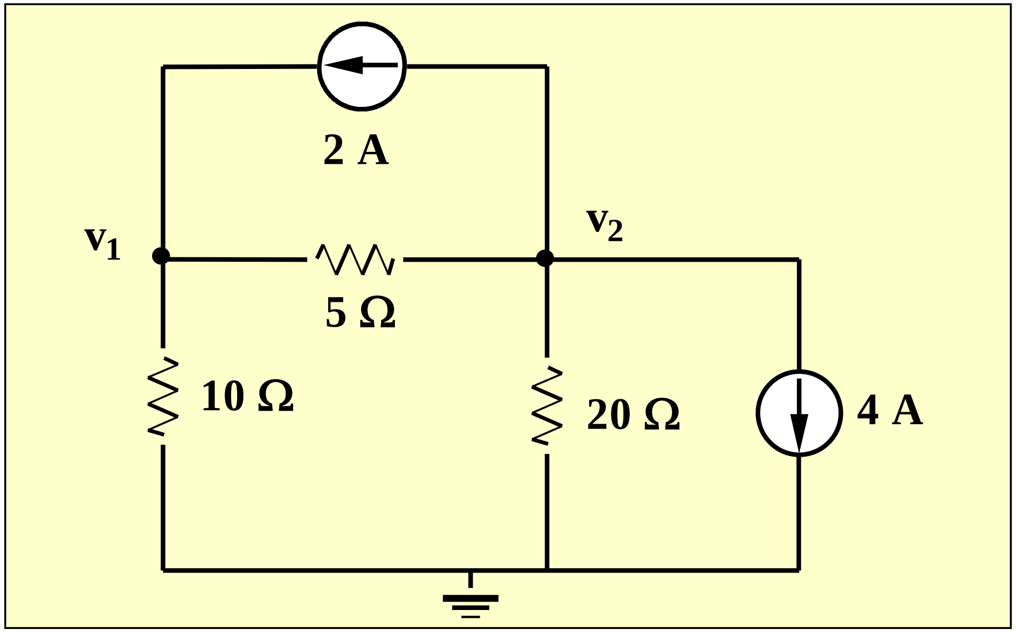
Analysis using KCL to solve for voltages at each common node of the network

Nodal analysis procedure:

1. Determine the number of common nodes and reference node within the network.
2. Apply KCL at each of the common nodes in the network
3. Solve the resulting simultaneous linear equation for the nodal voltages.

Circuit Analysis

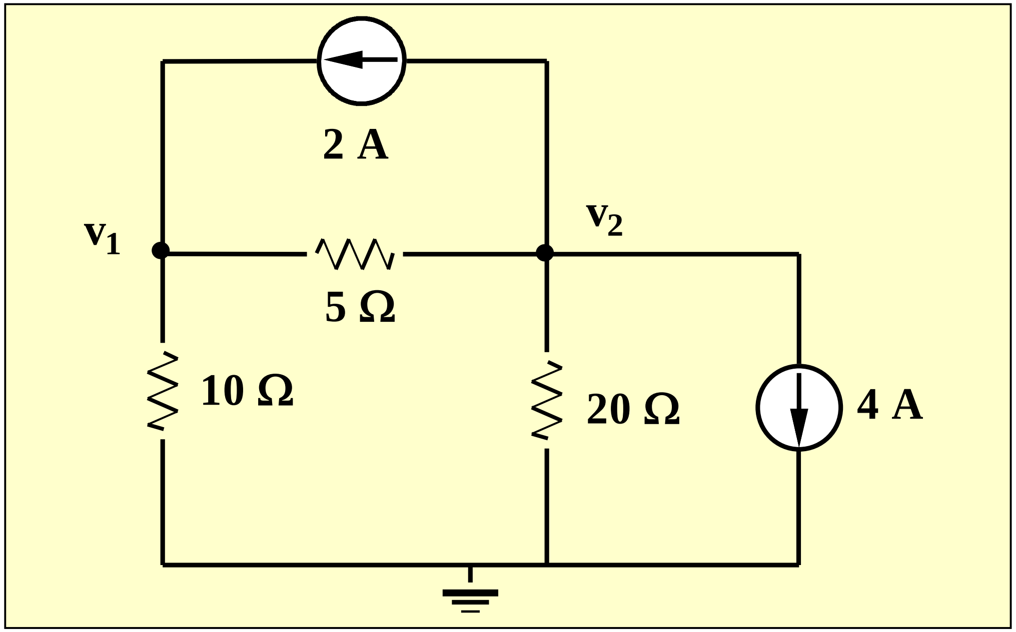
Nodal Analysis:



For the given circuit
Find V_1 and V_2.

Circuit Analysis

Nodal Analysis:



At v_1:

$$\frac{V_1}{10}+\frac{V_1-V_2}{5}=2 \qquad \textbf{Eq 1}$$

At v_2:

$$\frac{V_2-V_1}{5}+\frac{V_2}{20}=-6 \qquad \textbf{Eq 2}$$

Circuit Analysis

Nodal Analysis: Clearing Equations;

From Eq 1:

$$V_1 + 2V_1 - 2V_2 = 20$$

or

$$3V_1 - 2V_2 = 20 \qquad \text{Eq 3}$$

From Eq 2:

$$4V_2 - 4V_1 + V_2 = -120$$

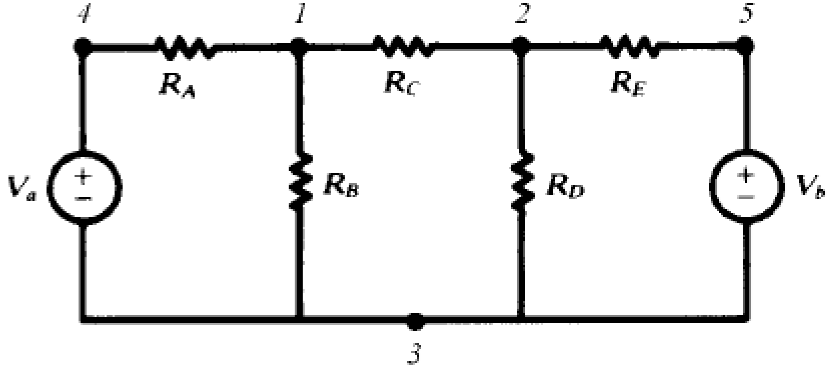
or

$$-4V_1 + 5V_2 = -120 \qquad \text{Eq 4}$$

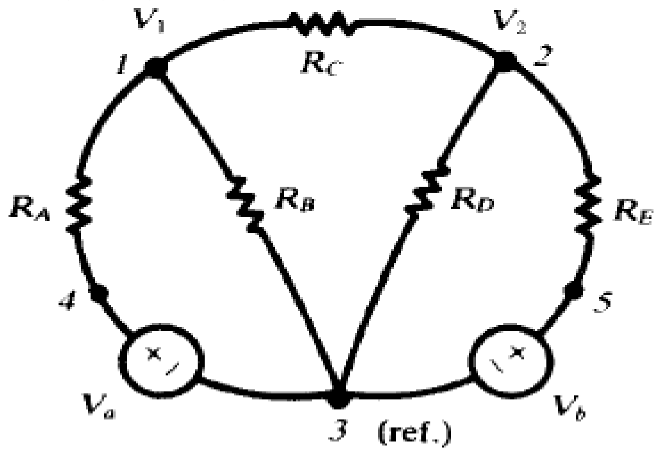
Solution: $V_1 = -20$ V, $V_2 = -40$ V

Node Analysis

- Example, The network shown in Fig contains five nodes, where 4 and 5 are simple nodes and 1, 2, and 3 are principal nodes



- This can be redrawn as shown



Node Analysis

- The network is redrawn and node 3 selected as the reference for voltages V1 and V2. KCL requires that the total current out of node 1 be zero

$$\frac{V_1 - V_a}{R_A} + \frac{V_1}{R_B} + \frac{V_1 - V_2}{R_C} = 0$$

- Similarly, the total current out of node 2 must be zero

$$\frac{V_2 - V_1}{R_C} + \frac{V_2}{R_D} + \frac{V_2 - V_b}{R_E} = 0$$

Node Analysis

- Put two equations for V_1 and V_2 in matrix form and solve

$$\begin{bmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_a/R_A \\ V_b/R_E \end{bmatrix}$$

Summary

- Source transformation simplifies the analysis of some circuits
- Voltage source in series with a resistor is transformed into a current source in parallel with a resistor and vice versa
- Mesh analysis is a systematic technique to evaluate all voltages and currents in a circuit based on Kirchoff's Voltage Law and Ohm's Law.
- Nodal Analysis is a step-by-step approach to solve circuits, It is based on Kirchoff's Current Law