Prof. Stephan Fritzsche
TPI and HI Jena
— Computational Quantum Physics —

8. December 2014
Winter term

Home Work Topics (Exam)

This course aims for learning and dealing with — more or less — simple computations on many-particle quantum systems and, especially, multi-qubit systems. For the final home work (exam) of this course, please, select and individually work out in good detail one of the following tasks. We shall agree about your selection latest until January, 15th, 2015, on the basis 'first comes, first served'. You will have to submit (and present) your results until the last lecture week of this term, however not later than February, 28th, 2105, please.

To solve these tasks, work out a (Maple, Mathematica, ...) worksheet or other write-up, in which you shall include all the necessary (sub-) procedures of your code as well as the individual steps of computation. Please, follow the style of either the FEYNMAN write-up in Comp. Phys. Commun. **185** (2014) 1697, section 4 or the worksheet Feynman-examples-CQP-2015-maple18.mw, inlcuding some proper explanations about your logic and how to follow your reasoning. If possible, please present this worksheet to all of us and demonstrate of how it works in detail.

In the following, we shall refer to the following pre-defined states, the

- Bell states: $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$ and $|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$;

- diagonal Bell states, i.e. the mixture of the two-qubit Bell states:
$$\begin{aligned}
\rho &= p_I \left|\Phi^+\right\rangle\left\langle\Phi^+\right| + p_x \left|\Psi^+\right\rangle\left\langle\Psi^+\right| + p_y \left|\Psi^-\right\rangle\left\langle\Psi^-\right| + p_z \left|\Phi^-\right\rangle\left\langle\Phi^-\right| \\
&= \frac{1}{2}\begin{pmatrix} p_I + p_z & 0 & 0 & p_I - p_z \\ 0 & p_x + p_y & p_x - p_y & 0 \\ 0 & p_x - p_y & p_x + p_y & 0 \\ p_I - p_z & 0 & 0 & p_I + p_z \end{pmatrix},
\end{aligned}$$
and where the (classical) weights p_i have to fulfil $\sum_i p_i = 1$;

- n-qubit GHZ states: $|GHZ\rangle_n = \frac{1}{\sqrt{2}}\left(|00...0\rangle + |11..1\rangle\right)$;

- n-qubit W states: $|W\rangle_n = \frac{1}{\sqrt{n}}\left(|00...01\rangle + |00...10\rangle + ... + |10...00\rangle\right)$;

- n-qubit symmetric Dicke states with e excitations (Mandel and Wolf 1995):
$$|n, e\rangle = \begin{pmatrix} N \\ e \end{pmatrix}^{1/2} \sum_i P_i\left(|1_1, 1_2, ..., 1_e, 0_{e+1}, ..., 0_N\rangle\right),$$
and where the $\{P_i\}$ refers to the set of all distinct permutations of the qubits. The state $|n, 1\rangle$ is the same as the n-qubit W state. You can access these states also by mean of the FEYNMAN tools.

The references given below are provided also in the folder Additional-material (with the prefix d-).

Task (Multi-qubit Stokes parameter):

The Stokes parameter of a given (mixed) state provide a simple means to support the tomographic reconstruction of a given density matrix, i.e. a quantum system in a given state.

a) Evaluate and compare graphically the multi-qubit Stokes parameter of the Bell states $|\Psi^{\pm}\rangle$, the diagonal Bell states with selected weights;

b) Calculate and discuss the same for the 3- and 4-qubit W and GHZ-states;

c) Calculate the Stokes parameter also for the 3-qubit symmetric Dicke states.

Reference: J.B. Altepeter *et al.*, Adv. Atomic Molecular and Optical Physics **52** (2005) 105.

Task (Measurement statistics in projective measurements):

For some quantum system in a given state, the probability for outcome m of a measurement can be calculated by following the postulate about quantum measurements. In real or computational 'experiments', in contrast, of course only one of all possible outcomes is measured at a given time, and the measurements have to be repeated on a quite large number of individually prepared systems in order to determine the probabilties.

Show and discuss how such a measurement statistics approaches the quantum-mechanical probabilities for a sufficiently large number of measurements and, especially,

a) for Pauli measurements on 3- and 4-qubit GHZ and W states;

b) for Bell-state measurements on the 3-qubit GHZ and Dicke states;

Please, make a proper selection of the possible measurements, the qubits to be measured as well as a graphical comparison with regard to the number of measurements and the calculated probabilities.

Task (Local operations on entangled states):

The GHZ and W states are known to belong to (topologically) different classes of entangled states that cannot be transformend into each other by applying just local (single-qubit) operations. Apply one or several single-qubit gates in order to:

a) attempt minimizing the trace distance and the Hilbert-Schmidt distance between the 3-qubit GHZ and W state;

b) do the same but in terms of the (so-called) fidelity.

c) Moreover, is it possible (and, if yes, how) to transform the 3-qubit Dicke states by single-qubit gates into a GHZ state ?

Task (Quantum data compression):

Recently, Rozema *et al.* (2014) discussed and realized a protocol in which an ensemble of quantum bits (qubits) can in principle be perfectly suppressed into exponentially fewer qubits. Implement and discuss such a protocol that help compress the information of a 3-qubit (product) state into 2 qubits. Try to extend this scheme towards a larger number of qubits, say, $n = 4, 5, 6, ...$

Reference: L. A. Rozema *et al.*, Phys. Rev. Lett. **113** (2014) 160504.

Task (Entanglement witness):

An entanglement *witness* is a functional of the density operator that help distinguish between the entangled and separable states of a system. Such entanglement witnesses can be either linear or nonlinear functionals of the density matrix. Explore and compare such entanglement witnesses for the two-qubit Bell and diagonal Bell states. Moreover, show how a device-independent entanglement witness (DIEW) can be calculated for two- and three-qubit states. Evaluate and discuss this witness especially for the 3-qubit GHZ, W and Dicke states.

References: J.T. Barreiro *et al.*, Nature Physics **9** (2013) 539; J. Dai *et al.*, Phys. Rev. Lett. **113** (2014) 170402.

Task (Entanglement purification):

Entanglement purification describes some protocol(s) in quantum information processing that help to distil highly entangled states from less entangled states. These protocols are typically based on controlled-NOT (CNOT) or similar quantum gates. — Explore, implement and discuss one or two entanglement purification protocols for two-qubit states, provided you have available (any number of) identical copies of the system. Apply this protocol especially to the diagonal Bell state with different weigths and compare the distance of the purified state with the four Bell states (for instance, in terms of the fidelity).

Task (Quantum teleportation):

Explain and demonstrate how teleportation can be used to transfer some (unknown) one- and two-qubit quantum state from Alice to Bob. Implement and demonstrate such a teleportation for the Bell and diagonal Bell states. Moreover, how can such a scheme be extended towards (entangled) 3- and multi-qubit systems ?

Reference: M. Fuwa *et al.*, Phys. Rev. Lett. **113** (2014) 223602.

Task (One-way quantum computations):

So-called *one-way* or *measurement-based quantum computing* refers to a method in which an entangled resource state is first prepared, and where then single-qubit measurements are performed with forward control in order to 'steer' a system into a desired state. Often, the resource state is either a cluster or graph state. — Show and explain step-wise how a (given) general single-qubit [SU/(2)] operation can be realized by some subsequent measurements of a five-qubit chain state.

Reference: R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86** (2001) 5188.