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— Computational Quantum Physics —

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Home Work (3)

Task 5: (Unitary transform of hermitian operator)

Suppose A is a hermitian operator, U denotes a unitary operator and $A' = UAU^+$. Then show that:

a) A' is hermitian

b) The eigenvalues of A' are the same as of A.

Task 6: (Single-qubit rotations)

Let $\mathbf{v} = (v_1, v_2, v_3)$ be any real, three-dimensional unit vector and θ a real number. Prove that

$$\exp(i\theta\, \mathbf{v} \cdot \sigma) = \cos\theta\, I + i \sin\theta\, \mathbf{v} \cdot \sigma \,,$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ is the vector of Pauli matrices.

Task 7: (Hadamard operator)

For a single qubit, the Hadamard operator is usually written in terms of the Pauli matrices:

$$H = \frac{1}{\sqrt{2}} (\sigma_x + \sigma_z) = \frac{1}{\sqrt{2}} (X + Z) \,.$$

a) Determine how this operator acts on the vectors $|0\rangle$ and $|1\rangle$ in the 2−dimensional vector space $\mathbb{C}^2$.

b) Express the Hadamard operator in the outer product notation (i.e. in terms of the operators $|0\rangle\langle 0|\,,\ |0\rangle\langle 1|\,,\ \ldots$).

c) Find the Hadamard operator on n qubits, $H^{\otimes n} = \underbrace{H \otimes H \otimes ... \otimes H}_{n}$ in the outer product notation.

$$H = \frac{1}{\sqrt{2}} (\sigma_x + \sigma_z) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Task 8: Prove that the entangled Bell state $|\psi\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$ cannot be represented in the form $|\psi\rangle = |a\rangle\,|b\rangle$ where $|a\rangle$ and $|b\rangle$ are single-qubit states.