Prof. Stephan Fritzsche
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— Computational Quantum Physics —

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Home Work (7)

Task 20: (Pure vs. mixed states.)

There are given the following density matrices:

$$a)\ \rho = \begin{pmatrix} 3/4 & 0 \\ 0 & 1/4 \end{pmatrix}, \qquad b)\ \rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix},$$

$$c)\ \rho = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}, \qquad d)\ \rho = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix},$$

$$e)\ \rho = \begin{pmatrix} 1/2 & \frac{1-i}{2\sqrt{2}} \\ \frac{1+i}{2\sqrt{2}} & 1/2 \end{pmatrix}.$$

Which of these density operators represent pure and which one mixed states ? — If the state is pure, then determine the state vector, and find an ensemble representation otherwise.

Task 21: (Mixed state density operators.)

Proof that $\mathrm{Tr}(\rho^2) < 1$ for mixed state !

Task 22: (Stern-Gerlach filter.)

A beam of electrons, prepared as a 50:50 statistical mixture with spin projections $\mu_z = \pm 1/2$ (along the $z-$axis), is sent through a Stern-Gerlach filter which admits only particles with spin projection $+1/2$ either on the $x-$axis, $y-$axis or $z-$axis.

Determine the probabilities that an electron will penetrate the corresponding filters. — Compare and discuss these results. How can one distinguish the pure and mixed electron states by using the Stern-Gerlach technique ?

Task 23: (Density matrix of spin-1/2 particle.)

Let a spin-1/2 particle be in the spin state

$$|\psi\rangle = \sum_{\mu=\pm 1/2} a_\mu\, |\chi_\mu\rangle\ .$$

a) Find the density matrix which describe the spin state of this particle.

b) Find the polarization vector $\mathbf{P} = \langle \psi | \sigma | \psi \rangle$ in terms of the coefficients a_μ.

MERRY CHRISTMAS AND A HAPPY NEW YEAR 2015 !