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— Theoretical Atomic Physics —

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Winter Term

Homework (1)

Task 1: Gravitational Force in the Hydrogen Atom

When treating atomic systems, the gravitational force is usually neglected. Apply perturbation theory to estimate the energy shift of the 1s state in hydrogen, when the gravitational interaction between the electron and the nucleus is taken into account.

Task 2: Creation of Holes in the Dirac Sea

How much energy is required to create an electron-position pair in the field of a calcium nucleus ($Z = 20$), if the electron is captured into the $1s$ ground state of the ion.
Hint: Use the non-relativistic formula for the bound-state energies, $E_n = -Z^2/2n^2$[a.u.].

Task 3: Electron Capture by a Proton

We consider the capture of a free electron by a proton to form hydrogen in one of its $2p_{1/2}$ or $2p_{3/2}$ excited levels.
a) Estimate the ratio for populating the $2p_{1/2}$ and $2p_{3/2}$ levels in this capture process.
b) Estimate the same for populating the $3d_{3/2}$ and $3d_{5/2}$ levels.

Task 4: Muonic Hydrogen

The muon is a fermion, much like an electron, but with mass $m_\mu = 105.7\,\mathrm{MeV/c^2}$ and same charge. In a hydrogen atom, the muon can be captured and therefore replace the electron, to obtain muonic hydrogen.
a) What is the ground state binding energy of the muon?
b) What chemical element does muonic lithium, where only one electron is replaced by a muon, resemble most?

Task 5: Transition energies in hydrogen-like uranium

Calculate the relativistic energies of the characteristic $\mathrm{Ly}-\alpha_1$ ($2p_{3/2} \to 1s_{1/2}$) and $\mathrm{Ly}-\alpha_2$ ($2p_{1/2} \to 1s_{1/2}$) radiation in hydrogen-like U^{91+} ions. Compare these transition energies with the corresponding nonrelativistic results from the Schrödinger equation.