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— Computational Quantum Physics —

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Winter term

Home Work (6)

Task 16: (Y-gate operation and single-qubit measurement.)

Suppose, we have the state $|\phi\rangle = \frac{\sqrt{3}}{2}|00\rangle + \frac{1}{2}|11\rangle$ and that a Y-gate is applied to the first qubit. — What are the possible measurement outcomes if both qubits are measured, and what are the respective probabilities for each of these outcomes ??

Task 17: (Average value of various operators.)

Find the average value for the following operators and state:

a) operator $\sigma_x(1)\sigma_z(2)$ for a two-qubit system in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$;

b) operator CNOT for the Bell states $|\Psi^{\pm}\rangle$;

c) operator $(\mathrm{CNOT})_{12}\,(\mathrm{CNOT})_{23}$ for the 3-qubit GHZ and W states.

Task 18: (Measuring a state.)

Suppose, we have the 3-qubit GHZ state $|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and apply a Toffoli gate upon this state, $|\psi_{\mathrm{T}}\rangle := G^{\mathrm{Toffoli}}\,|\psi_{\mathrm{GHZ}}\rangle$. What are the probabilties p_+ and p_- to find the system after a projective measurement in the states $|+++\rangle$ and $|---\rangle$, and where as usual $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$.

Determine (and compare) these probabilities by:

a) calculating the 'scalar products' $p_+ = |\langle +++|\,\psi_{\mathrm{T}}\rangle|^2$ and $p_- = |\langle ---|\,\psi_{\mathrm{T}}\rangle|^2$;

b) by subsequent measurements on all three qubits along the x-axis;

c) by performing a POVM measurement with the POVM elements $E_1 = |+++\rangle\langle +++|$, $E_2 = |---\rangle\langle ---|$ and $E_3 = I - E_1 - E_2$, respectively;

c) by doing a generalized measurement with a proper set of measurement operators, M_i, $i = 1..8$, as given for instance by the projectors of the 3-qubit 'diagonal' basis.

Task 19: (Density operator of a pure state.)

Let ρ denote a density operator. Show that $\mathrm{Tr}(\rho^2) \le 1$ and that $\mathrm{Tr}(\rho^2) = 1$ applies if and only if ρ represents a *pure* state.