Prof. Stephan Fritzsche
TPI and HI Jena
— Computational Quantum Physics —

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Winter term

Home Work (4)

Task 9: (Pauli matrices)

a) Find the eigenvalues, eigenvectors and the diagonal representation of the Pauli matrices.

b) Show that the Pauli matrices are hermitian and unitary.

c) Show that any 2×2 hermitian matrix A can always be expressed in terms of the unitary matrix I and the three Pauli matrices as:

$$A = c_0 I + c_1 \sigma_x + c_2 \sigma_y + c_3 \sigma_z \,,$$

where c_0, c_1, c_2 and c_3 are real numbers.

Task 10: (Distributed gates)

Suppose we have the Bell state $|\psi\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$ and the three operators

$$A = X \otimes Z, \qquad B = I \otimes X \otimes Z \otimes I, \qquad D = I \otimes C_{\text{not}}$$

with the CNOT gate $C_{\text{not}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

Determine the following states:

a) $A\,|\psi\rangle$

b) $B\,|0+10\rangle$

c) $B\,(|\psi\rangle \otimes |\psi\rangle)$

d) $D\,|000\rangle$

e) $D\,|+++\rangle$

f) $(D \otimes X)\,(|\psi\rangle \otimes |\psi\rangle)$

Task 11: (Two hermitian operators $A,\ B$) Suppose that A and B are hermitian. Show that $A \otimes B$ is then also hermitian.