Prof. Stephan Fritzsche
TPI and HI Jena
— Computational Quantum Physics —

3. Januar 2015
Winter term

Home Work (9)

Task 28: (Reading.)

Read the article "Measurement-based quantum computation" by J. Briegel *et al.*, Nature Physics 5 (2009) 19; DOI: 10.1038/NPHYS1157.

Task 29: (Single-qubit gates.)

a) Prove that an arbitrary single-qubit unitary operation can be written in the form

$$U \;=\; \exp(i\alpha)\, R_{\mathbf{n}}(\theta)$$

where α and θ are real parameters, and $\mathbf{n}$ is a real three-dimensional unit vector. Here, $R_{\mathbf{n}}(\theta)$ denotes a rotation of angle θ with regard to the axis $\mathbf{n}$.

b) Find the values of α, θ and $\mathbf{n}$ for the following gates:

$$\sigma_x \;=\; \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad H \;=\; \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad S \;=\; \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}.$$

Task 30: (Determine the circuit from a given gate.)

The matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

corresponds to a simple two-qubit circuit that involves only one 'controlled-something' gate. Determine this circuit.

Task 31: (Bell states and the parity bit.)

The Bell states can be written compactly as

$$\beta_{xy} \;=\; \frac{|0y\rangle \;+\; (-1)^x \;|1\bar{y}\rangle}{\sqrt{2}}$$

where $\bar{y}$ means 'not' y. x is often called the phase bit und y the parity bit.

Explore and discuss how the operator $Z \otimes Z$ acts upon the Bell states.