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— Advanced Quantum Theory —

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Winter Term

Home Work (5)

Task 1: Molecule (3 Points)

In this task, we consider a diatomic molecule that is aligned along the x-axis, where the two identical atoms are placed at $x = \pm a$. This molecule is described by the potential

$$V(\boldsymbol{r}) = v(\boldsymbol{r} + \boldsymbol{a}) + v(\boldsymbol{r} - \boldsymbol{a}) \qquad \boldsymbol{a} = a\boldsymbol{e}_x$$

In an experiment, this molecule is probed by a beam of thermal neutrons with energy 0.1 eV that comes in along the z-axis.

What is the interatomic distance, when the first minimum of the cross section within the $x - z$ plane ($\varphi = 0$) is observed at $\vartheta = 9°$, where φ is the angle to the positive x-axis.

Hint: You do not need to know the potential of the atoms, express the cross section in terms of a single atom's potential $v(\boldsymbol{r})$. The neutron mass is $m_n c^2 = 940\,\text{MeV}$ and $\hbar c = 197\,\text{MeV fm}$

Task 2: Angular Dependence of the Differential Cross Section (3 Points)

In this task, we want to analyze the angular dependence of the differential cross section, which is a common tool to experimentally gain information about the potential.

a) Plot the angular dependence ϑ of the differential cross section for s-wave and p-wave scattering, then include the interference between s and p-wave scattering. For this analysis we want to use the arbitrarily chosen scattering phases $\delta_\ell = \pi/2$.

b) Use Born's approximation, to plot the angular dependence of the differential cross section for the square well and the Yukawa potential. Repeat this for at least two different values of the product kb (for the Yukawa potential, we have of course $b = 1/\kappa$ as the characteristic length scale).

Task 3: Scattering from a crystal lattice (3 Points)

Let us consider the elastic scattering of an incident particle with momentum $\mathbf{k}$ by a crystal lattice with sites

$$\mathbf{R}_n = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3\,.$$

Here $n = (n_1, n_2, n_3)$ is a multi-index, n_i are integers, and $\mathbf{a}_i$ denotes the lattice vectors. The wave function of a particle scattered by a lattice site n is given by

$$\psi^{(\text{out})}(\mathbf{r}) = \mathrm{e}^{i\mathbf{k}\cdot\mathbf{R}_n} \frac{\mathrm{e}^{ik|\mathbf{r}-\mathbf{R}_n|}}{|\mathbf{r}-\mathbf{R}_n|}\,.$$

a) Show that for $r \gg R_n$ one can write

$$\psi^{(\text{out})}(\mathbf{r}) \approx \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{R}_n} \frac{\mathrm{e}^{ikr}}{r} ,$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is the momentum transfer, and $\mathbf{k}'$ refers to the momentum of the scattered particle.
b) The scattering amplitude F for the full crystal is the sum of scattering amplitudes for each lattice site n

$$F = \sum_n \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{R}_n} .$$

Show the condition on the momentum transfer $\mathbf{q}$ that maximizes the scattering amplitude F.