Prof. Stephan Fritzsche
TPI and HI Jena
— Advanced Quantum Theory —

23. Januar 2016
Winter Term

Home Work (13)

Task 1: Properties of Clebsch-Gordan coefficients (2 Points)

Use the properties of Clebsch-Gordan coefficients to show that the following coefficients vanish

a) $\langle 2\,1,\ 1\,0|2\ -1\rangle$

b) $\langle 0\,0,\ 1\,0|3\,0\rangle$

c) $\langle 1\,0,\ 1\,0|1\,0\rangle$

Task 2: Two spin-1/2 particles (2 Points)

Use the rule for adding angular momentum with Clebsch-Gordan coefficients to construct the coupled spin states of two spin$-1/2$ particles.

Task 3: Landau-Yang theorem (4 Points)

Prove the Landau-Yang theorem which says that the vector particle with momentum $J = 1$ can not decay into two photons (photon's spin is 1).

Hint: The spatial part of wave-function of two photons should be symmetric.

Task 4: Probability in a Rotated Coordinate System (2 Points)

Assume a particle with angular momentum $\ell = 2$ and projection $m = 1$ on the z-axis of the coordinate system Σ. Let Σ' be a coordinate system that is rotated by $\vartheta = 60°$ with respect to Σ. Find the probability $P(m')$ that the particle has projection m' on the z'-axis.

Hint: You might need:

$d^2_{m,m'}$	2	1	0	-1	-2
2	$\frac{(1+\cos(\beta))^2}{4}$	$-\frac{\sin(\beta)(1+\cos(\beta))}{2}$	$\frac{1}{2}\sqrt{\frac{3}{2}}\sin(\beta)^2$	$-\frac{\sin(\beta)(1-\cos(\beta))}{2}$	$\frac{(1-\cos(\beta))^2}{4}$
1	$\frac{\sin(\beta)(1+\cos(\beta))}{2}$	$\frac{2\cos(\beta)^2+\cos(\beta)-1}{2}$	$-\sqrt{\frac{3}{2}}\sin(\beta)\cos(\beta)$	$-\frac{2\cos(\beta)^2-\cos(\beta)-1}{2}$	$-\frac{\sin(\beta)(1-\cos(\beta))}{2}$
0	$\frac{1}{2}\sqrt{\frac{3}{2}}\sin(\beta)^2$	$\sqrt{\frac{3}{2}}\sin(\beta)\cos(\beta)$	$\frac{3\cos(\beta)^2-1}{2}$	$-\sqrt{\frac{3}{2}}\sin(\beta)\cos(\beta)$	$\frac{1}{2}\sqrt{\frac{3}{2}}\sin(\beta)^2$
-1	$\frac{\sin(\beta)(1-\cos(\beta))}{2}$	$-\frac{2\cos(\beta)^2-\cos(\beta)-1}{2}$	$\sqrt{\frac{3}{2}}\sin(\beta)\cos(\beta)$	$\frac{2\cos(\beta)^2+\cos(\beta)-1}{2}$	$-\frac{\sin(\beta)(1+\cos(\beta))}{2}$
-2	$\frac{(1-\cos(\beta))^2}{4}$	$\frac{\sin(\beta)(1-\cos(\beta))}{2}$	$\frac{1}{2}\sqrt{\frac{3}{2}}\sin(\beta)^2$	$\frac{\sin(\beta)(1+\cos(\beta))}{2}$	$\frac{(1+\cos(\beta))^2}{4}$