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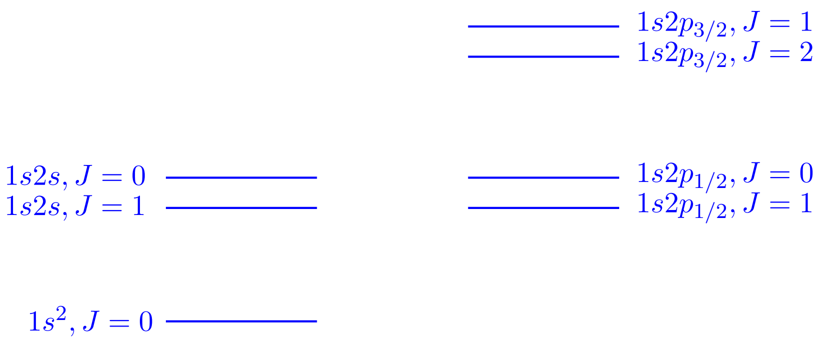
— Theoretical Atomic Physics —

15. Dezember 2015
Winter Term

Homework (4)

Task 1: Selection Rules for a Helium-Like System

Identify the possible one-photon transitions in the given helium-like system.



Task 2: Stark Quenching of the Hydrogen $2S_{1/2}$ State

The hydrogen $2S_{1/2}$ can, due to the selection rules, not decay to the ground state via a one-photon transitions (when spin is neglected). Therefore, it decays via a two-photon process, so that its lifetimes is long compared to the dipole-allowed $2P_{1/2} - 1S_{1/2}$ transition. We already saw that an external electric field E leads to mixing between the $2s$ and $2p$ states. Therefore, as the $2S_{1/2}$ mixes with the $2P_{1/2}$ state in the presence of an electric field, the transition to the ground state becomes allowed, such that the lifetime becomes smaller as the electric field strength is increased. We also need to consider the Lamb shift of $\delta = 1.058\,\mathrm{GHz}$ that increases the energy of the $2S_{1/2}$ state and therefore lifts the degeneracy with the $2P_{1/2}$ state.

a) Argue that the Hamiltonian describing the mixing is given by

$$H = \begin{pmatrix} 2\pi\delta\hbar & eE \left\langle 2P_{1/2} \,\middle|\, z \,\middle|\, 2S_{1/2} \right\rangle \\ eE \left\langle 2P_{1/2} \,\middle|\, z \,\middle|\, 2S_{1/2} \right\rangle & -i\hbar A/2 \end{pmatrix},$$

where A is the transition rate $2P_{1/2} \to 1S_{1/2}$, calculated in the previous homework.

b) Evaluate the matrix elements, and find the probability of finding a state initially prepared as $2S_{1/2}, M_J = 1/2$ in the $2P_{1/2}, M_J = 1/2$ state. Use the result to calculate the decay rate of the $2S_{1/2}$ state to the ground state

$$A_{2S\to 1S} = A_{2P\to 1S} \cdot \frac{3e^2E^2a_0^2}{\hbar^2\left((2\pi\delta)^2 + A^2/4\right)}$$

Here you can assume, that the energy of the states does not change, as we are considering small electric field strengths.

c) Calculate the lifetime for an electric field of $E = 100\mathrm{V/m}$