Prof. Stephan Fritzsche
TPI and HI Jena
— Computational Quantum Physics —

30. October 2014
Winter term

Home Work (1)

Task 1: The determinant $\det(A)$ of a quadratic $n \times n$ matrix $A = (a_{ij})$ can be calculated from Laplace's formula

$$\det(A) \quad = \quad \sum_{k=1}^{n} (-1)^{i+k} \, a_{ik} \, M_{ik} \, ,$$

and where the (so-called) minor M_{ik} is the determinant of the $(n-1) \times (n-1)$ matrix that results from A by removing the i-th row and the k-th column.

a) Write a short program which computes the determinant of a given quadratic matrix by using the formular from above. (Hint: What are the 'building blocks' in order to calculate the determinant recursively ?)

b) Refine this program to support the expansion of the determinant along some given 'row' or 'column'.

c) Calculate the determinant of the matrix

$$A \;=\; \begin{pmatrix} 2 & -3 & 4 & -5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & -8 \end{pmatrix}$$

by taking the expansion along the first and last 'row' as well as the last 'column'.

d) Compare the result with those from a call to some internal or *library* procedure.

e) Calculate the algebraic exact value of the determinant for the matrix

$$B \;=\; \begin{pmatrix} \sqrt{2} & -\sqrt{3} & \sqrt{4} & -\sqrt{5} \\ \sqrt{3} & \sqrt{4} & \sqrt{5} & \sqrt{6} \\ \sqrt{4} & \sqrt{5} & \sqrt{6} & \sqrt{7} \\ \sqrt{5} & \sqrt{6} & \sqrt{7} & -\sqrt{8} \end{pmatrix} .$$

f) Compare the results in single, double and higher precision with the *exact* value.