Prof. Stephan Fritzsche
TPI and HI Jena
— Computational Quantum Physics —

27. November 2014
Winter term

Home Work (5)

Task 12: (Measurement of the three-qubit state)

A three-qubit system is in the state

$$|\psi\rangle \;=\; \left(\frac{\sqrt{2}+i}{\sqrt{20}}\,|000\rangle \;+\; \frac{1}{\sqrt{2}}\,|001\rangle \;+\; \frac{1}{\sqrt{10}}\,|011\rangle \;+\; \frac{i}{2}\,|111\rangle \right)$$

a) Is this state normalized ?? What is the probability that the system is found in the state $|000\rangle$ if all 3 qubits are measured ?

b) What is the probability that a measurement on the first qubit only gives 0 ? What is the postmeasurement state of the system in this case ?

Task 13: (Projectors of $\mathbf{v} \cdot \sigma$)

a) Show that the operator $\mathbf{v} \cdot \sigma \;=\; v_x \sigma_x + v_y \sigma_y + v_z \sigma_z$ has eigenvalues ± 1 and that the projectors upon the corresponding eigenspaces are given by $P_\pm \;=\; (I \,\pm\, \mathbf{v} \cdot \sigma)\,/2$.

b) Apply the projector $P_\pm$ to calculate the probability that, for a measurement of the operator $\mathbf{v} \cdot \sigma$, one obtains the result ± 1, if the state prior to the measurement was $|0\rangle$. What is the state of the qubit just after the measurement if the outcome $+1$ was obtained ?

Task 14: (Construct a POVM)

A given source produces a system either in one of two nonorhtogonal states, either $|\psi\rangle$ or $|\phi\rangle$ with scalar product $|\,\langle \psi \,|\, \phi \rangle\,| \;=\; \cos\theta$. — Construct a POVM that help distinguish these states.

Task 15: (Operator product)

Suppose $\hat{A}$ and $\hat{B}$ are commuting Hermitian operators. Prove that $\exp(\hat{A})\,\exp(\hat{B}) \;=\; \exp(\hat{A}\hat{B})$.
Hint: Use the simultaneous diagonalization theorem.