Prof. Stephan Fritzsche | 30. Januar 2016
TPI and HI Jena | Winter Term
— Advanced Quantum Theory —

# Home Work (14)

**Task 1:** Pure vs. Mixed States (2 Points)

Consider the following density matrices:

$$\textbf{a)}\quad \rho = \begin{pmatrix} 3/4 & 0 \\ 0 & 1/4 \end{pmatrix} \qquad \textbf{b)}\quad \rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \qquad \textbf{c)}\quad \rho = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}$$

$$\textbf{d)}\quad \rho = \begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix} \qquad \textbf{e)}\quad \rho = \begin{pmatrix} 1/2 & \frac{1-i}{2\sqrt{2}} \\ \frac{1+i}{2\sqrt{2}} & 1/2 \end{pmatrix}$$

Which of these density operators represent pure and which one mixed states? If the state is pure, determine the state vector, and find an ensemble representation otherwise.

**Task 2:** Density Operator of a Pure State (2 Points)

Let $\rho$ denote a density operator. Show that $\mathrm{Tr}\left(\rho^2\right) \leq 1$ and that $\mathrm{Tr}\left(\rho^2\right) = 1$ applies if and only if $\rho$ represents a pure state.

**Task 3:** Density Matrix of a Spin-1/2 particle (2 Points)

Let a spin-1/2 particle be in the spin state

$$|\psi\rangle = \sum_{\mu=\pm 1/2} a_\mu \, |\chi_\mu\rangle \; .$$

**a)** Find the density matrix which describes the spin state of this particle.
**b)** Determine the polarization vector $\boldsymbol{P} = \langle \psi \,|\, \boldsymbol{\sigma} \,|\, \psi \rangle$ in terms of the coefficients $a_\mu$.