Prof. Stephan Fritzsche
TPI and HI Jena
— Advanced Quantum Theory —

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Winter Term

Homework (1)

Task 1: Eigenfunctions

If $\hat{H}$ is an operator, the equation $\hat{H}\phi(x) = E\phi(x)$ indicates that $\phi(x)$ is an eigenfunction of $\hat{H}$, and E is the corresponding eigenvalue. In addition to statisfying this eigenvalue equation the function $\phi(\boldsymbol{r})$ must also be well-behaved in order to be an acceptable eigenfunction. By well-behaved we mean that $\phi(x)$ is single valued, finite, and continuous and that the first derivative of $\phi(x)$ is continuous. Which of the following functions are well-behaved? For those functions that are not well-behaved, indicate the reason.

(a) $\phi = x$, $x \geq 0$, and $\phi = 0$ otherwise
(b) $\phi = x^2$
(c) $\phi = e^{-|x|}$
(d) $\phi = e^{-x}$
(e) $\phi = \cos x$
(f) $\phi = \sin |x|$
(g) $\phi = e^{-x^2}$
(h) $\phi = 1-x^2$, $-1 \leq x \leq 1$, $\phi = 0$ otherwise.

Task 2: Wave-function normalization

A wave function ψ at a given time (say $t = 0$) can be written in the form $\psi(\boldsymbol{r}) = \sum_n c_n \phi_n(\boldsymbol{r})$, where $\{\phi_n\}$ are eigenfunctions of an arbitrary operator. Prove $\sum_n |c_n|^2 = 1$.

Task 3: Probability of a measurement

A certain system is in a state that is described by the wave function ψ at a specific time (say $t = 0$). According to quantum mechanics a measurement of the physical quantity O can only give one of the eigenvalues of the operator $\hat{O}$ that is associated with O. What is the probability of obtaining the eigenvalue O_n in a given measurement?
Hint: Expand ψ in the eigenfunctions of $\hat{O}$.

Task 4: Commutators

The commutator of two quantum mechanical operators $\hat{A}$ and $\hat{B}$ is defined by $\left[\hat{A}, \hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A}$. Calculate the commutator $[\hat{x}, \hat{p}_x]$ of the position and momentum (in one-dimensional systems) by using the definitions of these operators: $\hat{x} = x$ and $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$.