

Homework (6)

Task 1: Rabi Oscillations

In the previous task, we found that the exact (Rabi) solution to the two-level evolution equations is

$$|c_2(t)|^2 = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2 \left(\frac{\sqrt{\Omega^2 + \Delta^2}}{2} t \right).$$

In the lecture, we also found the perturbative solution

$$|c_2(t)|^2 \approx \frac{\Omega^2}{\Delta^2} \sin^2 \left(\frac{\Delta}{2} t \right),$$

where $\Delta = \omega - \omega_0$ is the detuning. Show that both solutions agree in the limit of small Rabi frequencies.

Task 2: Hyperfine Structure

a) Show that the magnetic dipole interaction described by the interaction Hamiltonian

$$H_{\text{HFS}} = A \mathbf{I} \cdot \mathbf{J},$$

leads to the interval rule

$$E_F - E_{F-1} = AF.$$

b) How can the previous result be used to assign quantum numbers to experimental results?

Task 3: Natural Line Shape

Show that the line shape due to spontaneous emission is given by a Lorentzian.