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— Theoretical Atomic Physics —

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Winter Term

Homework (5)

Task 1: Evolution Equations for a Two-Level Atom

The interaction Hamiltonian for an electric field in the dipole approximation is given by

$$H_I = e\boldsymbol{r} \cdot \boldsymbol{E}_0 \cos(\omega t) \ .$$

Prove that the ansatz for a two-level system

$$|\Psi(t)\rangle = c_1(t)\,|1(t)\rangle + c_2(t)\,|2(t)\rangle$$

leads to the evolution equations shown in the lecture:

$$i\dot{c}_1 = \Omega \cos(\omega t) \exp(-i\omega_0 t)\, c_2 \ ,$$
$$i\dot{c}_2 = \Omega^* \cos(\omega t) \exp(i\omega_0 t)\, c_1 \ .$$

Task 2: Rabi Oscillations

a) Solve the two-level equations derived in the previous task to obtain $c_1(t)$ and $c_2(t)$. Use the initial conditions $c_1(0) = 1$ and $c_2(0) = 0$, and apply the rotating wave approximation, to find

$$|c_2(\tau)|^2 = \frac{\Omega^2}{W^2} \sin^2\left(\frac{W\tau}{2}\right)$$

as given in the lecture.

b) Take the general solution of the two-state evolution equations to show that the action of a resonant ($\omega = \omega_0$) pulse of length τ on a two-state system in an arbitrary superposition can be described by the matrix

$$M = \begin{pmatrix} \cos(\phi/2) & -i\sin(\phi/2) \\ -i\sin(\phi/2) & \cos(\phi/2) \end{pmatrix} ,$$

where the phase shift is given by $\phi = \Omega\tau$.

c) Calculate how a π-pulse acts on an arbitrary state vector.

d) Consider two consecutive π pulses acting on $|1\rangle$

e) Show how a $\pi/2$-pulse acts on $|1\rangle$.

f) What happens, if two $\pi/2$-pulses act on $|1\rangle$? What happens, if between the pulses, the state $|2\rangle$ is shifted by an angle ϕ?