Prof. Stephan Fritzsche
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— Computational Quantum Physics —

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Winter term

Home Work (2)

Task 2:

a) Read the review article "Quantum Computers" by T. D. Ladd *et al.*, Nature 464 (2010) 45-53; DOI: 10.1038/nature08812 [see also folder: Additional-material].

b) Which physical realizations exist for representing a single qubit as well as a sequence of qubits (or quantum registers) ? — What are advantages and weaknesses of these realizations ?

Task 3: Consider the three-qubit state $|\phi\rangle = |+++\rangle$ and the Greenberger-Horne Zeilinger state

$$|\psi\rangle \equiv \left|\mathrm{GHZ}^{(3)}\right\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) .$$

a) Show that these states obey the Cauchy-Schwartz and triangular inequalities.

b) Orthomormalize these states by means of the Gram-Schmidt procedure.

c) Use the Feynman tools to show the same also for the analogue four- and five-qubit states.

d) Calculate the norm of the $U \otimes U$-invariant (two-qubit) mixed Werner state (Werner 1989)

$$\rho_{\mathrm{W}} = p \left|\Psi^{-}\right\rangle \left\langle\Psi^{-}\right| + (1-p)\,\frac{1}{4} I_4$$

for the probabilities $p = 0$, 0.3 and $p = 1$, and where $|\Psi^{-}\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is one of the well-known Bell states.

e) Show the $U \otimes U$ invariance of ρ_W for $U = X, T$ and H.

Task 4: Find the eigenvalues and eigenvectors for the $\pi/8-$gate with the matrix representation

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} .$$