Prof. Stephan Fritzsche
TPI and HI Jena
— Advanced Quantum Theory —

2. Januar 2017
Winter Term

Home Work (10)

Task 1: Permutation Operator for Spin One-Half Fermions (2 Points)

Show that the operator

$$P_{12} = \frac{1}{2}\sum_{\mu=0}^{3} \sigma_\mu \otimes \sigma_\mu \,, \qquad \text{with} \qquad (\sigma_\mu) = (\mathbf{1}_2, \boldsymbol{\sigma})$$

swaps the two spin variables, i.e. $P_{12}\Psi\left(m_{s_1}, m_{s_2}\right) = \Psi\left(m_{s_2}, m_{s_1}\right)$, when applied to the basis states

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} , \ldots$$

Task 2: Interacting Fermions (2 Points)

Consider two identical interacting spin one-half particles in the potential

$$V(x_1, x_2) = \frac{1}{2}\left(x_1^2 + x_2^2\right) + \frac{\lambda}{2}\left(x_1 - x_2\right)^2 ,$$

where λ is a positive coupling constant. For brevity, we have set $\hbar = m = \omega = 1$

a) Decouple the two oscillators by introducing center-of-mass coordinates $R = \frac{1}{\sqrt{2}}\left(x_1 + x_2\right)$ and $r = \frac{1}{\sqrt{2}}\left(x_1 - x_2\right)$.

b) Find the ground state energy E_0 and the corresponding wave function $\Psi_0(x_1, x_2)$, including the spin part.