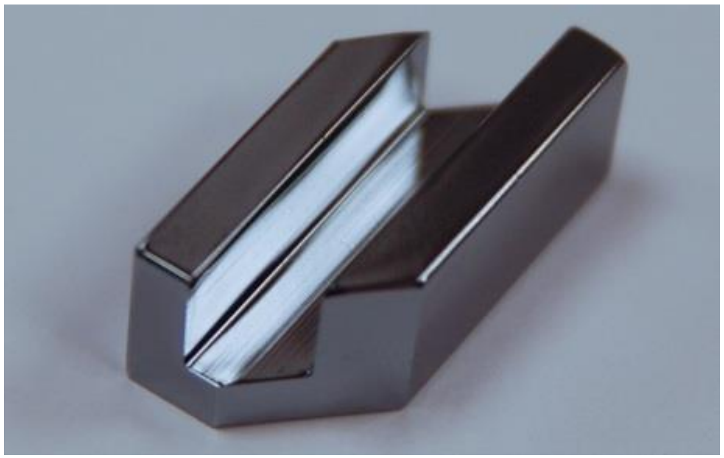
Polarization phenomena in relativistic light-matter interaction

Andrey Surzhykov
Helmholtz Institute Jena

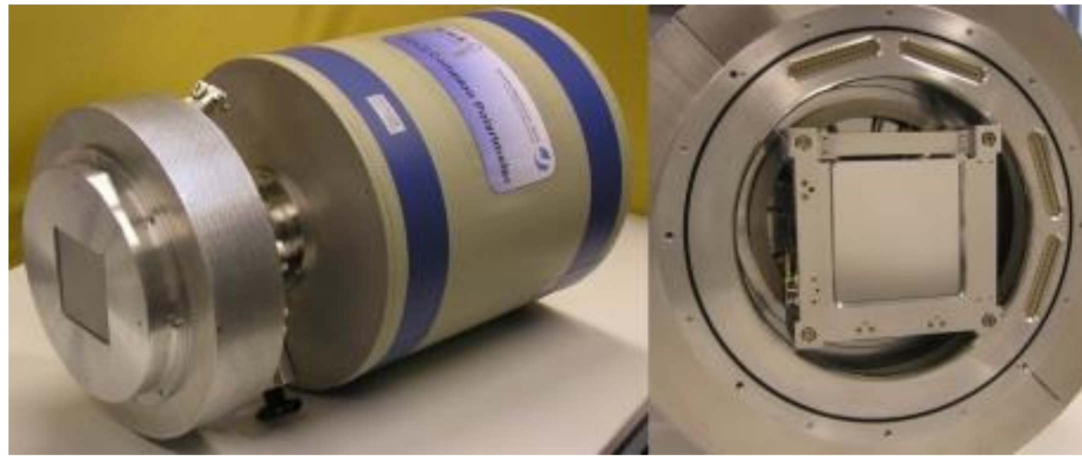
HELMHOLTZ ASSOCIATION

X-ray polarimetry in Jena

High-precision x-ray polarimetry



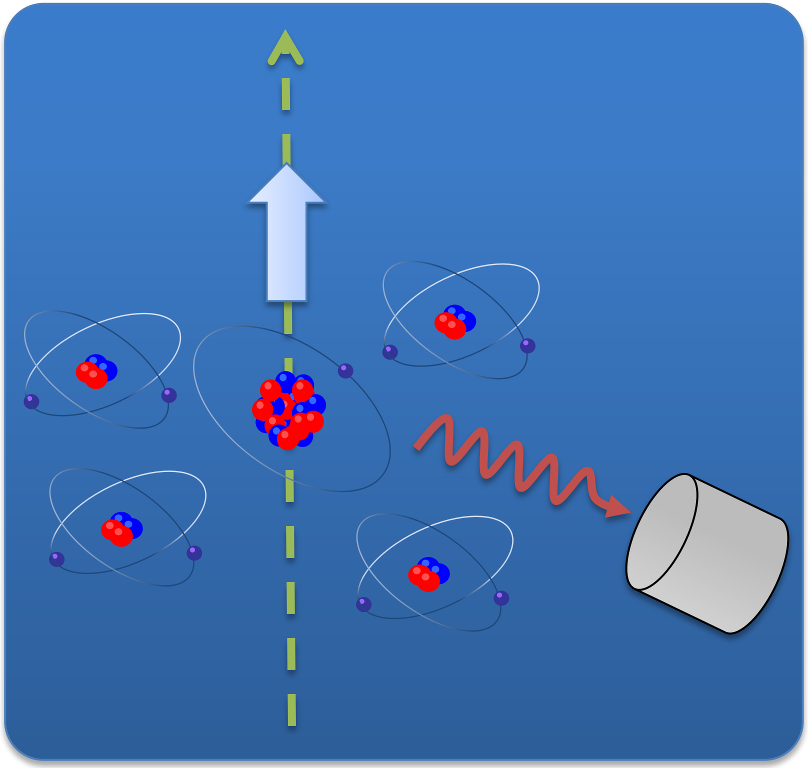
Compton polarimetry of hard x-rays



- What is the physics that we can address by using the novel polarimeters?
  - Non-linear QED processes (e.g. photon-photon scattering)
  - Search for a new physics beyond the Standard Model
  - Structure and dynamics of few-electron systems in strong fields
  - Applications towards nuclear and plasma physics

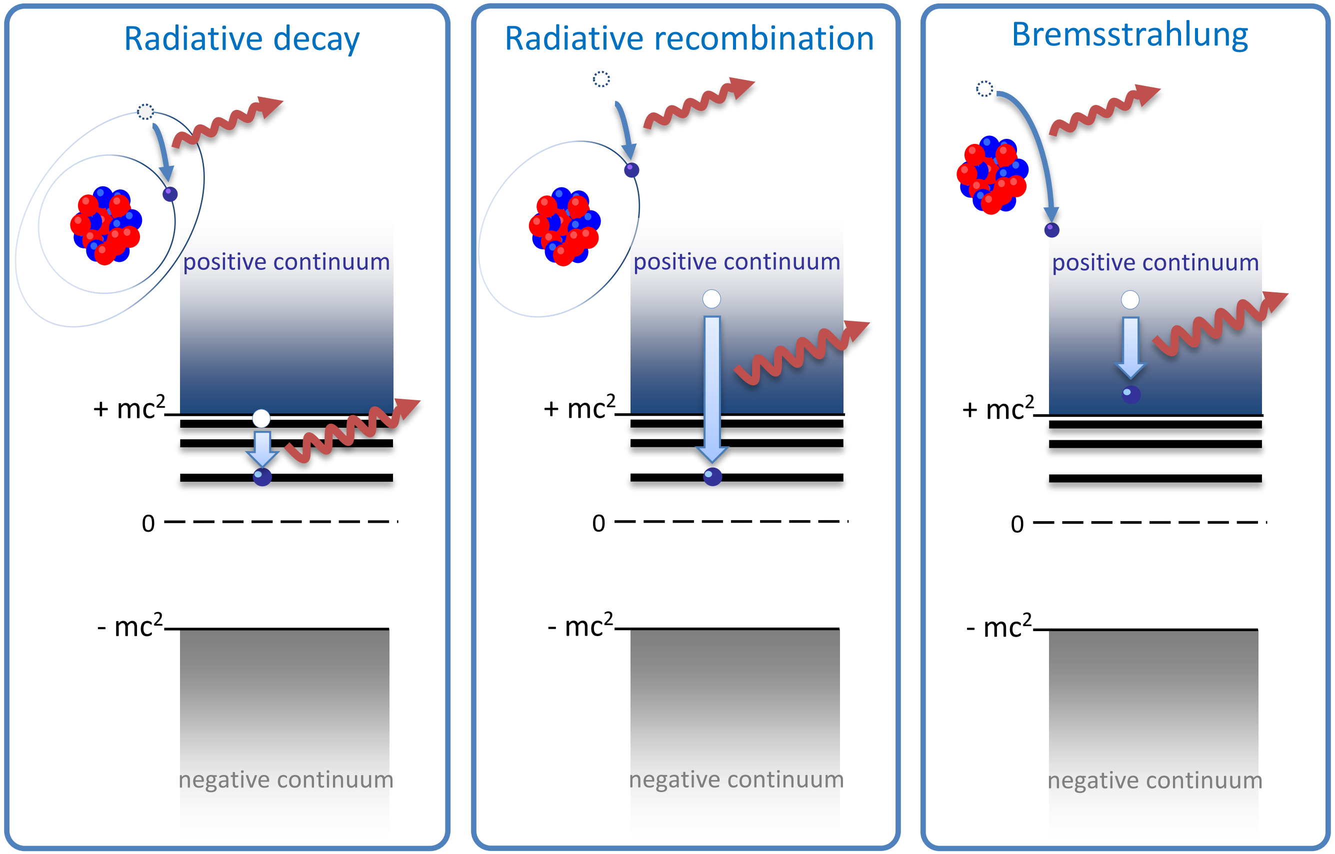
X-ray emission from heavy ions

- In typical experiments fast moving heavy ion collides with electronic or atomic target and x-ray emission is observed.



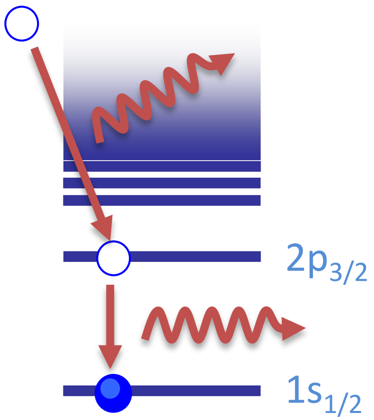
- Which fundamental atomic processes may lead to the emission of x-rays by highly-charged heavy ions?

Electron transitions in Coulomb field



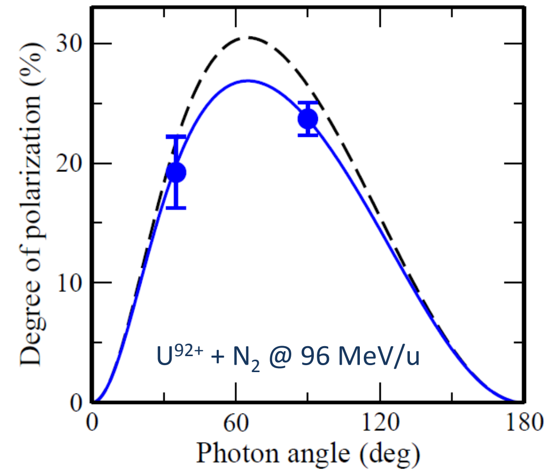
- Bound-state radiative transitions: Multipole-mixing effects
- Radiative recombination: Analysis of parity non-conservation phenomena
- Atomic bremsstrahlung: Polarimetry of electron beams

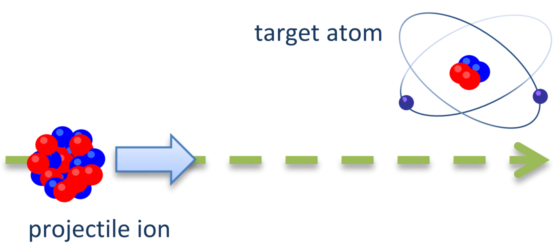
Ly-α_1 emission following radiative electron capture



- A number of studies have been carried out to study angular distribution of Ly-α_1 ($2p_{3/2} \rightarrow 1s_{1/2}$) decay following electron capture.

- Why do we expect at all characteristic emission to be polarized?

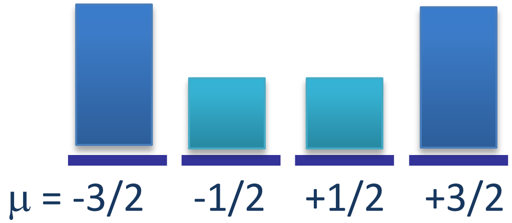




- In ion-atom collision experiments a *preferred* direction for the overall system is defined by the ion direction.

- Residual (excited) ion may appear to be aligned along this direction.

- Example: alignment of $2p_{3/2}$ state: $$A_2 = \frac{\sigma_{\pm 3/2} - \sigma_{\pm 1/2}}{\sigma_{\pm 3/2} + \sigma_{\pm 1/2}}$$



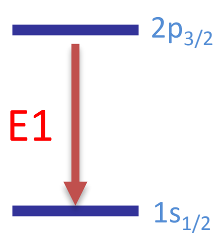
Is alignment the only one parameter that defines the polarization of the decay photons?

Multipole mixing effects

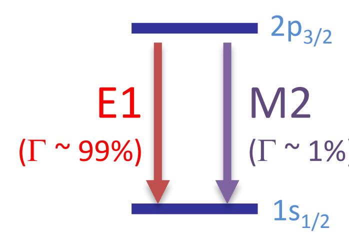
- Beside the alignment, which is defined by the capture process, the structure of the ion also may influence the polarization of emitted photons:

$$P_L(\theta) = \frac{-\frac{3}{2}\,\gamma\,\sin^2\theta}{1+\beta\,P_2(\cos\theta)} \qquad \left\{ \begin{aligned} \beta &\approx \frac{A_2}{2}\left(1+2\sqrt{3}\,\frac{a_{M2}}{a_{E1}}\right) \\ \gamma &\approx \frac{A_2}{2}\left(1-\frac{2}{\sqrt{3}}\frac{a_{M2}}{a_{E1}}\right) \end{aligned} \right.$$

- What is the anisotropy parameters β and γ which defines the Ly-α_1 polarization?
- It reflects both the dynamics and structure of hydrogen-like ions!



Electric dipole approximation



$$\frac{a_{M2}}{a_{E1}} \propto 0.1$$

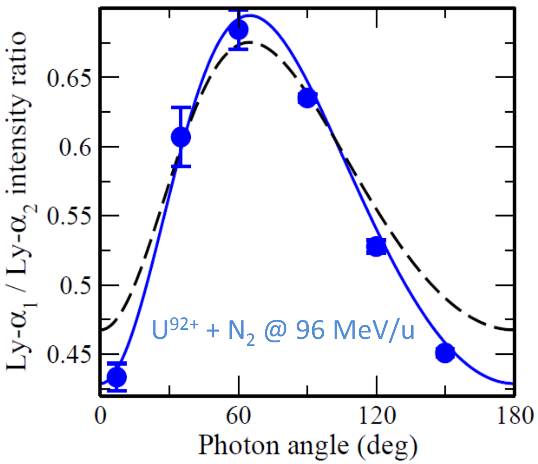
Full treatment

A.Surzhykov, S. Frirzsche, A. Gumberidze, Th. Stöhlker, Phys. Rev. Lett. **88** (2002) 153001

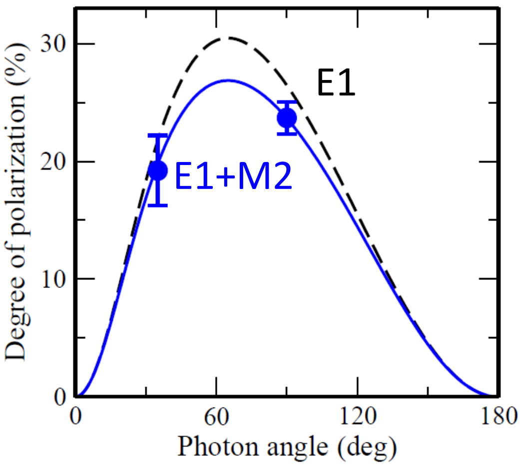
“Decoupling” of structure and dynamics

- Recently we have proposed to perform a simultaneous analysis of the angular distribution and polarization of Ly-α_1 radiation to determine structure and dynamic parameters model-independently.

Angular distribution



Linear polarization



$$W(\theta) \propto 1 + \beta\, P_2(\cos\theta)$$

$$P_L(\theta) = \frac{-\frac{3}{2}\,\gamma\,\sin^2\theta}{1 + \beta\, P_2(\cos\theta)}$$

with

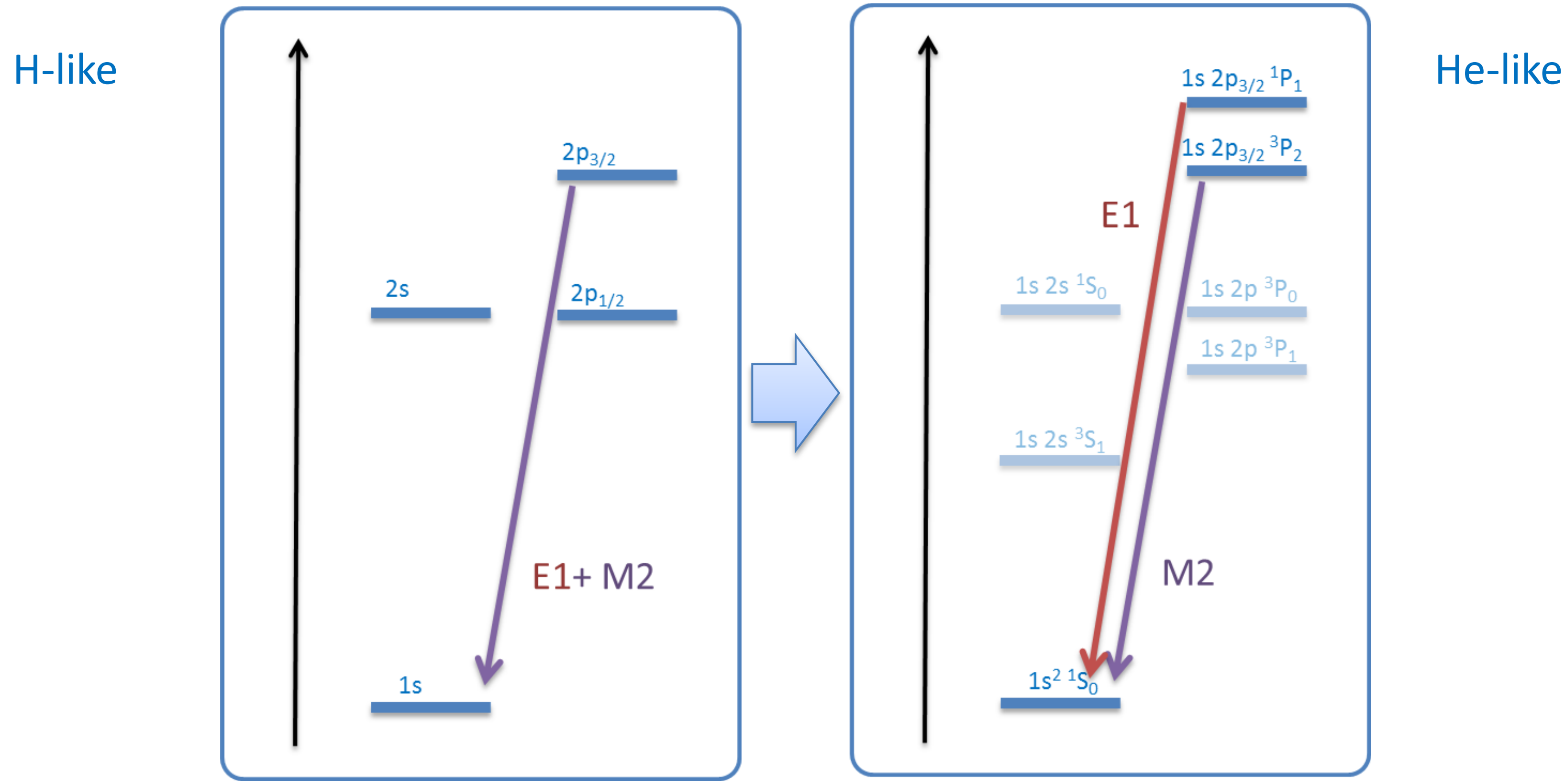
$$\beta \approx \frac{A_2}{2}\left(1 + 2\sqrt{3}\,\frac{a_{M2}}{a_{E1}}\right), \qquad \gamma \approx \frac{A_2}{2}\left(1 - \frac{2}{\sqrt{3}}\,\frac{a_{M2}}{a_{E1}}\right)$$

Alignment parameter $\mathcal{A}_2$		Amplitude ratio a_{M2}:a_{E1}	
Experiment	Theory	Experiment	Theory
-0.451 ± 0.017	-0.457	0.083 ± 0.014	0.0844

Results for:
U^{92+} + N_2 @ 96 MeV/u

G. Weber *et al.*, Phys. Rev. Lett. **105** (2010) 243002

Kα$_1$ decay of helium-like ions



- If one studies the electron capture into 1s 2p$_{3/2}$ state of (finally) helium-like ions with *zero* nuclear spin, one can find:
  - 1s 2p$_{3/2}$ $^1P_1$ decays only via E1 channel
  - 1s 2p$_{3/2}$ $^3P_2$ decays only via M2 channel

No mixing – no information about structure!

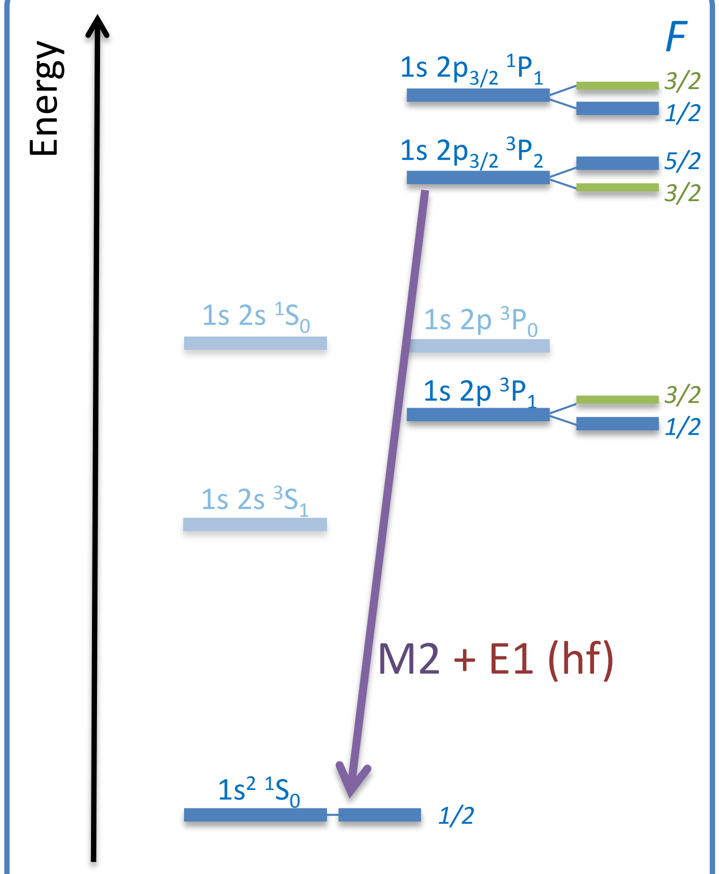
Kα_1 decay of helium-like ions

- Owing to the hyperfine interaction the state $^3P_2$ gets an admixture of $^{1,3}P_1$ levels:

$$|^3P_2\,F\rangle \rightarrow C_{^3P_2}\,|^3P_2\,F\rangle + C_{^1P_1}\,|^1P_1\,F\rangle + C_{^3P_1}\,|^3P_1\,F\rangle$$

- and, hence, can decay not only via magnetic quadrupole (M2) but also electric dipole (E1) channel.

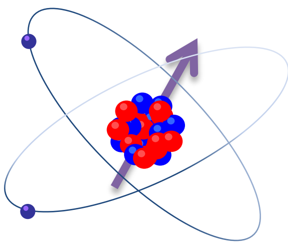
- Interference between the leading M2 and hyperfine-induced E1 may affect polarization properties of the decay radiation and provide information about the nuclear properties!



Example: Ion with nuclear spin I=1/2

- How do nuclear properties enter the polarization?

Theoretical background



- Hamiltonian of helium-like ion with non-zero nuclear spin:

$$\hat{H} = \hat{H}_0 + \hat{H}_{hf}$$

$$\hat{H}_0 = \sum_{i=1,2} \hat{h}_i + V(\boldsymbol{r}_1, \boldsymbol{r}_2)$$

Magnetic dipole hyperfine operator

$$\hat{H}_{hf} = \sum_{\lambda} (-1)^{\lambda} M_{-\lambda}^{(1)} T_{\lambda}^{(1)}$$

- We can find eigenfunctions of the Hamiltonian $\hat{H}$ by making expansion:

$$|\alpha\ F\ M_F\rangle = \sum_{\beta\ J} C_{\beta J} \sum_{M_I M_J} \langle I\ M_I\ J\ M_J | F\ M_F\rangle\, |I M_I\rangle |J M_J\rangle$$

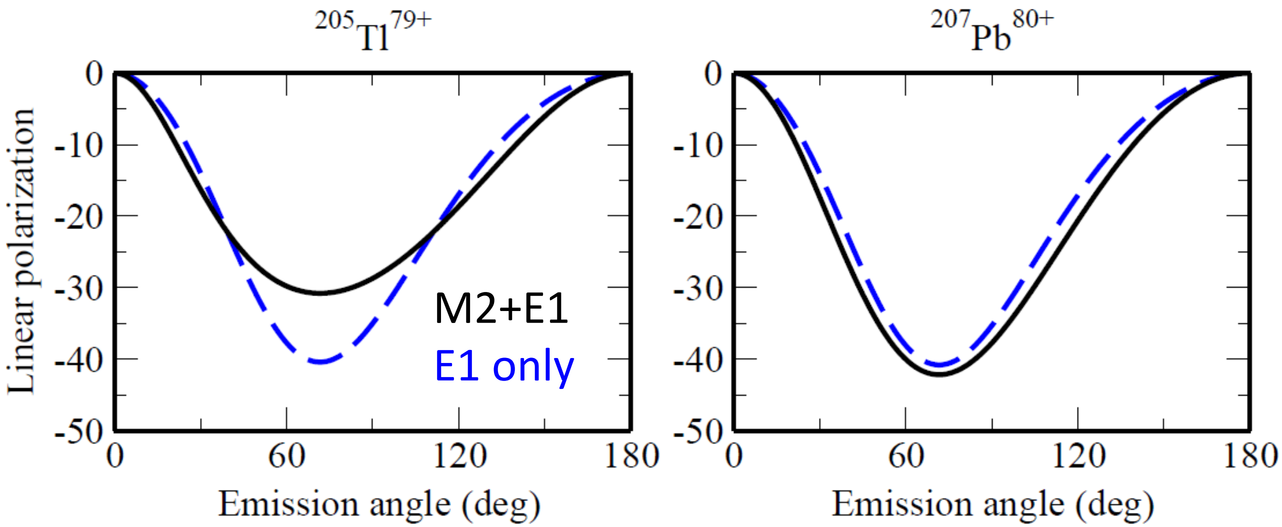
- Expansion coefficients $C_{\beta J}$ can be then found by diagonalization of Hamiltonian matrix.
- In order to perform such a diagonalization, one needs first to evaluate matrix elements of the magnetic dipole hyperfine operator:

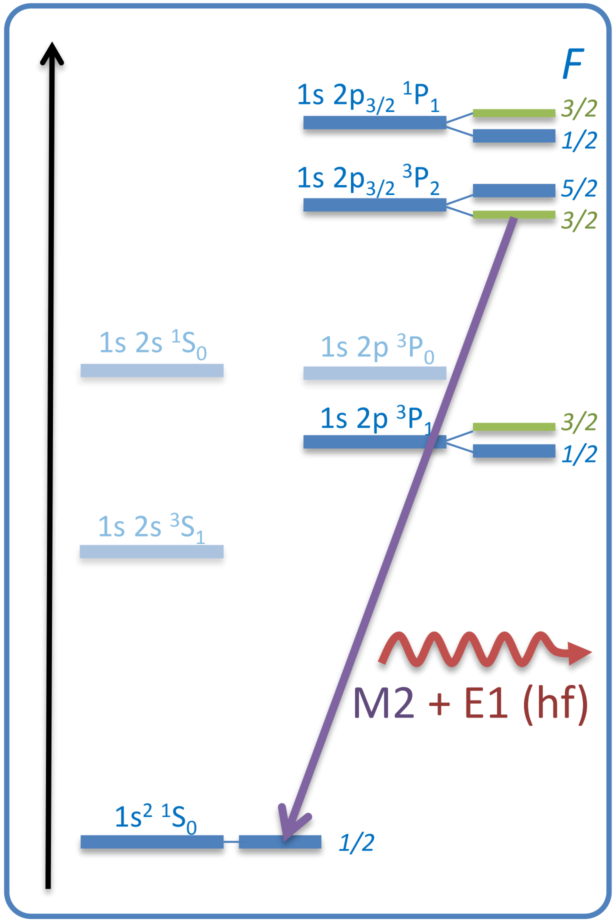
$$\langle \alpha\ F\ M_F | \hat{H}_{hf} | \alpha'\ F\ M_F\rangle \propto \mu_I = g_I\ I\ \mu_N$$

Nuclear magnetic moment

Hyperfine-resolved transitions

- We studied the linear polarization of F = 3/2 → F = 1/2 transition for two helium-like ions:
  - $^{205}Tl^{79+}$, $I = 1/2$, $\mu_I = 1.64\ \mu_N$
  - $^{207}Pb^{80+}$, $I = 1/2$, $\mu_I = 0.59\ \mu_N$



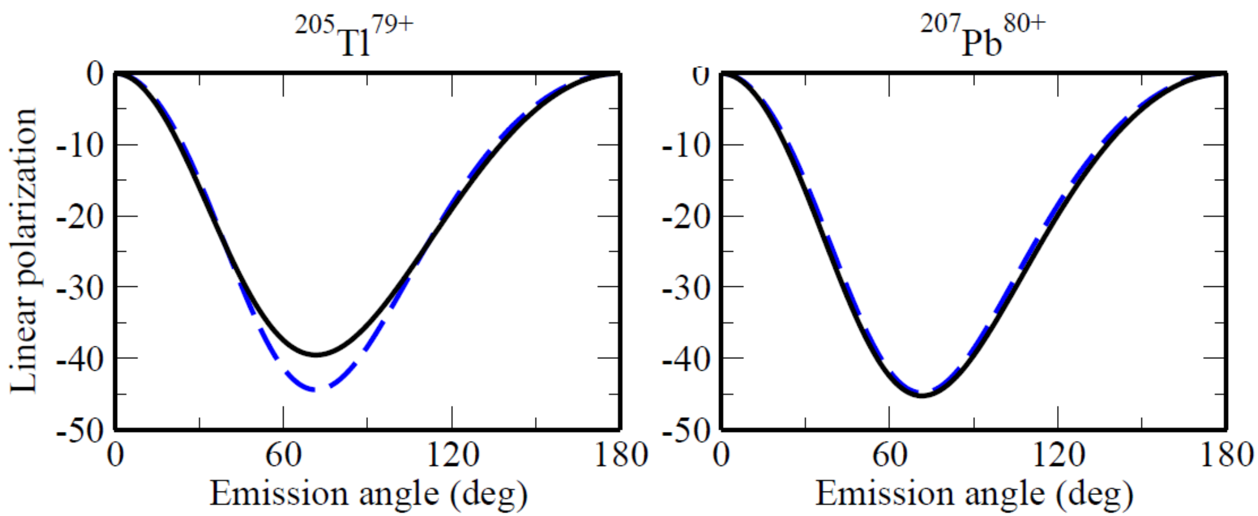


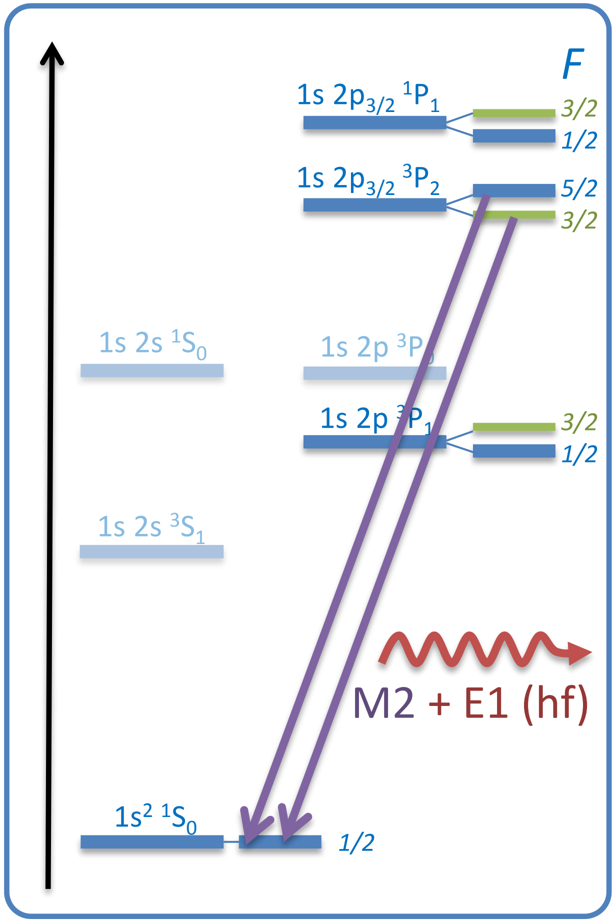
$$P_L(\theta) = \frac{-\frac{3}{2}\,\gamma\,\sin^2\theta}{1+\beta\,P_2(\cos\theta)} \qquad \beta \approx 1 - 2\sqrt{3}\,\frac{a_{E1}}{a_{M2}} \qquad \gamma \approx 1 + \frac{2}{\sqrt{3}}\frac{a_{E1}}{a_{M2}}$$

A.Surzhykov, Y. Litvinov, Th. Stöhlker, and S. Frirzsche, Phys. Rev. A (2013) submitted

Fine-structure transitions

- We studied the linear polarization of $1s\,2p_{3/2}\ {}^3P_2 \rightarrow 1s^2\ {}^1S_0$ transition for two helium-like ions:
  - ${}^{205}Tl^{79+}$, $I = 1/2$, $\mu_I = 1.64\ \mu_N$
  - ${}^{207}Pb^{80+}$, $I = 1/2$, $\mu_I = 0.59\ \mu_N$

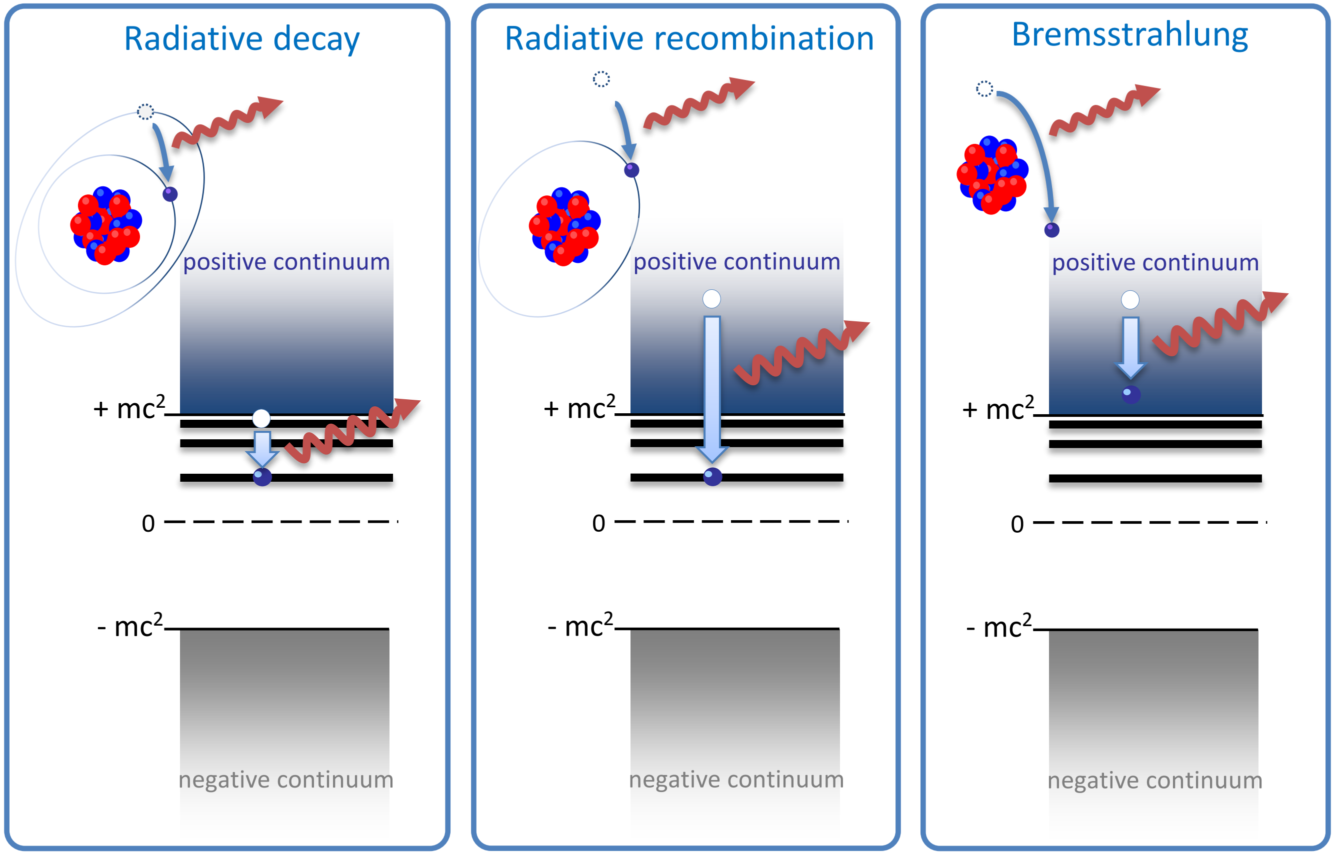




- Even after averaging over the hyperfine transitions one can observe the effect of M2-E1(hf) mixing!

A.Surzhykov, Y. Litvinov, Th. Stöhlker, and S. Frirzsche, Phys. Rev. A (2013) submitted

Electron transitions in Coulomb field



- Bound-state radiative transitions: Multipole-mixing effects
- Radiative recombination: Analysis of parity non-conservation phenomena
- Atomic bremsstrahlung: Polarimetry of electron beams

Parity violation studies

Gravitational	Weak	Electromagnetic	Strong	
	(Electroweak)		Fundamental	Residual
Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
10^{-41}	0.8	1	25	Not applicable to quarks
10^{-41}	10^{-4}	1	60	
10^{-36}	10^{-7}	1	Not applicable to hadrons	20

- Standard Model suggests the unified description of the electromagnetism and the weak interaction.
- Note that electromagnetic interaction preserves parity while weak interaction – not!

- Besides high–energy experiments at colliders and linear accelerators worldwide, precision electroweak measurements in **atomic physics** attract currently much attention since they allow to explore low–energy regime.

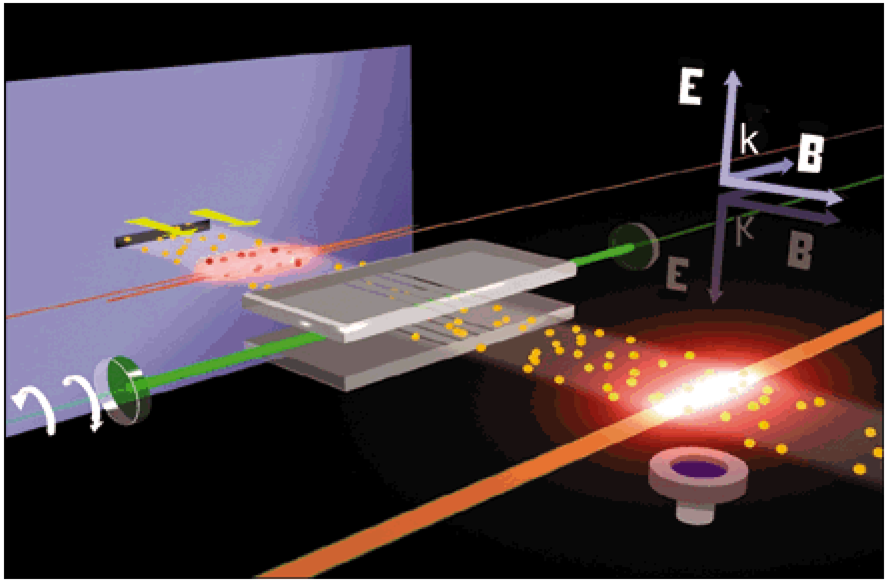
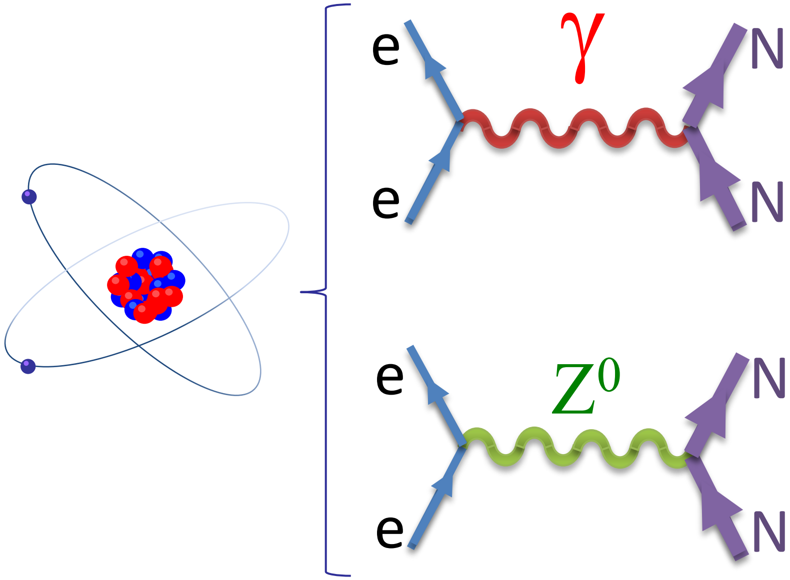
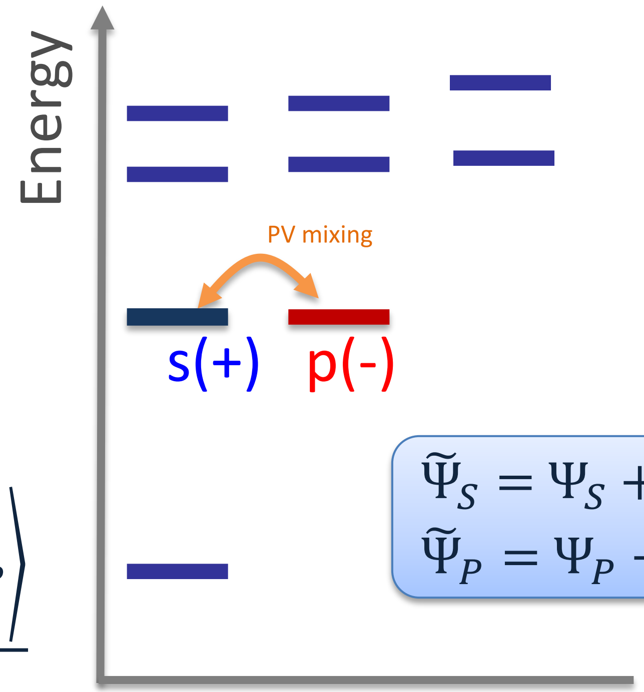


Figure drawn by T. Andrews, University of Colorado

Atomic parity violation



- Exchange of neutral Z boson between nucleus and electrons leads to the mixing of atomic levels with different parities.



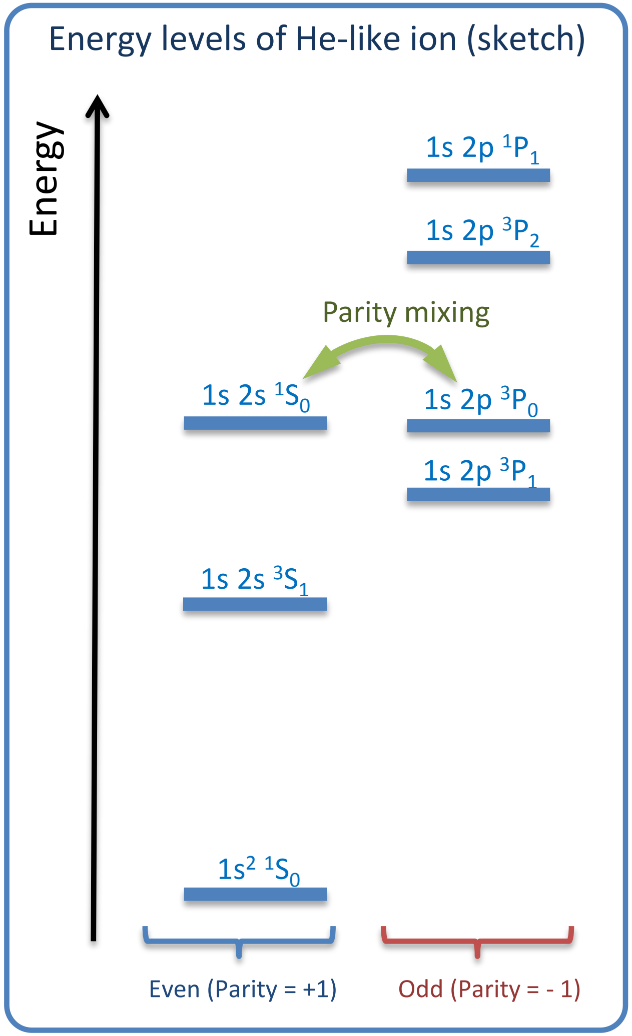
$$\tilde{\Psi}_S = \Psi_S + i\eta\Psi_P$$
$$\tilde{\Psi}_P = \Psi_P + i\eta\Psi_S$$

- Mixing coefficient for the states with opposite parities:

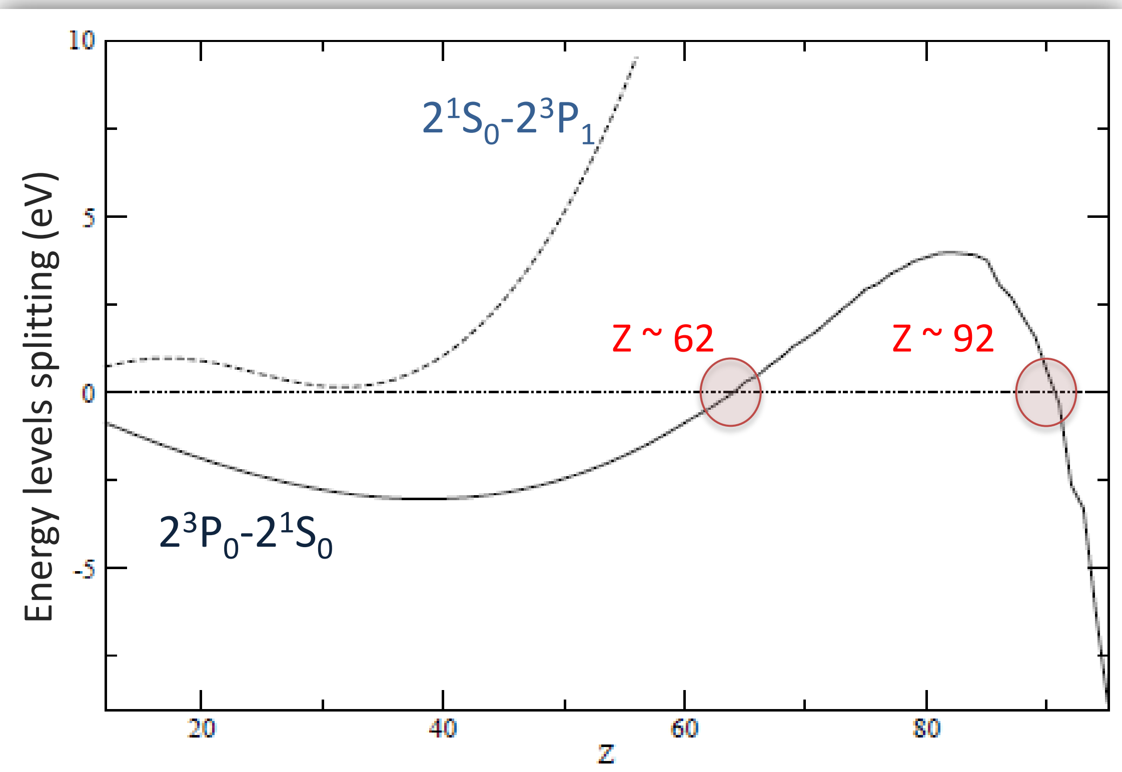
$$\eta = \frac{\left\langle \Psi_S \left| \frac{G_F}{\sqrt{2}} \left(-\frac{Q_W}{2}\gamma_5 + \frac{\kappa}{I}\boldsymbol{\alpha}\cdot\boldsymbol{I} \right) \rho(\boldsymbol{r}) \right| \Psi_P \right\rangle}{\boxed{E_S - E_P} - i\,\Gamma/2}$$

Energy splitting should be small!

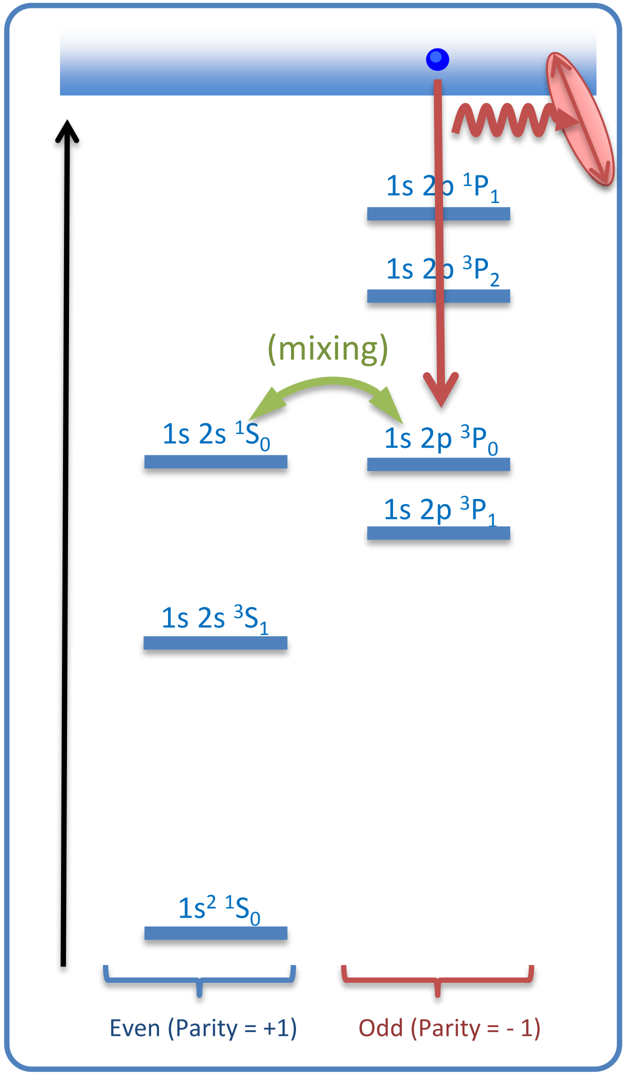
PV studies with helium-like ions



- Helium-like ions provide a unique tool for studying parity violation phenomena in atomic systems:
  - Simple systems (just two electrons)
  - Large electron-nucleus overlap
  - Small 2^1S_0-2^3P_0 energy splitting



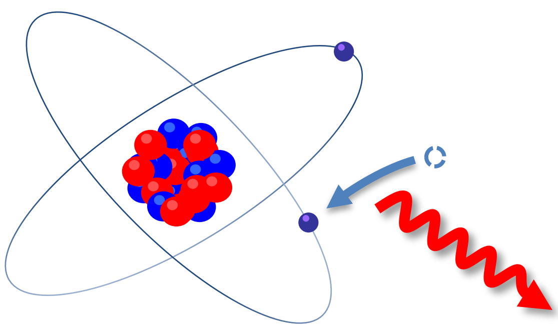
Free-bound electron transitions



- A number of theoretical proposals have been made recently to employ bound-free ionic transitions for the PV studies.
- One of the very promising probe–processes which may be efficiently used to measure the parity violation effects is the **radiative recombination** (RR) of electrons with initially hydrogen–like (finally helium–like) ions.

Can we see the parity mixing effects when observing linear polarization of the x-rays emitted in the electron capture into 1s $2p_{1/2}$ $^3P_0$ state of helium-like ions?

QED treatment of the electron capture



initial state: H-like ion + electron — final state: He-like ion + photon

$$\hat{\rho}_i = \underbrace{\hat{\rho}_e \otimes \hat{\rho}_{ion}} \quad \Rightarrow \quad \hat{\rho}_f = \underbrace{\hat{R}\,\hat{\rho}_i\,\hat{R}^+}$$

$$\langle k\lambda|\hat{\rho}_\gamma|k\lambda'\rangle = \sum_{m_a m_a'} \langle \alpha_a\, j_a\, m_a|\hat{\rho}_{ion}|\alpha_a\, j_a\, m_a'\rangle\, \mathcal{M}_{m_a\lambda}\, \mathcal{M}_{m_a'\lambda'}^{\;*}$$

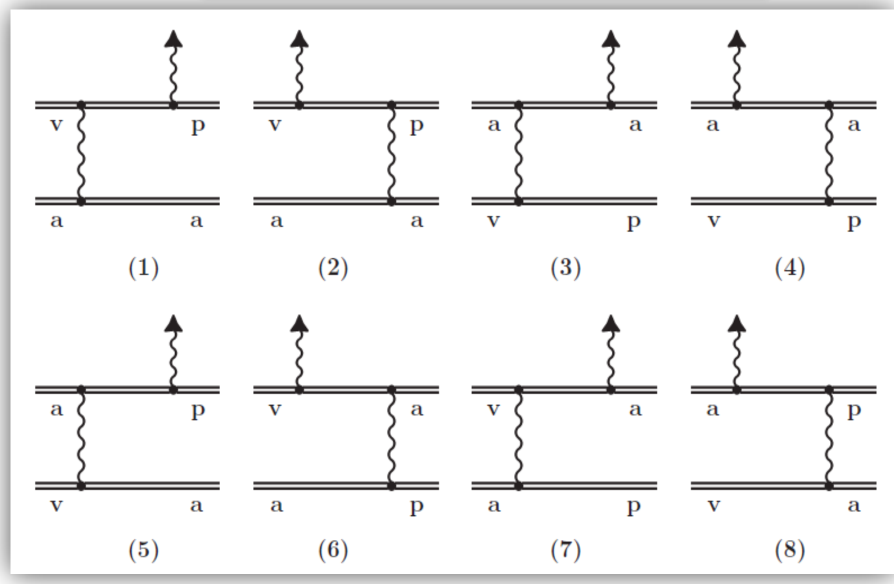
$$\mathcal{M}_{m_a\lambda} \propto \mathcal{M}^{(0)} + \sum_{i=1}^{8} \mathcal{M}^{(1,i)}$$

- Transition amplitudes account for the interelectronic interactions:

$$\mathcal{M}^{(1,8)} = -\sum_n \frac{\langle a|\hat{R}|n\rangle\langle vn|\hat{I}(\epsilon_a - \epsilon_v)|ap\rangle}{\epsilon_a - \epsilon_v + \epsilon - \epsilon_n}$$

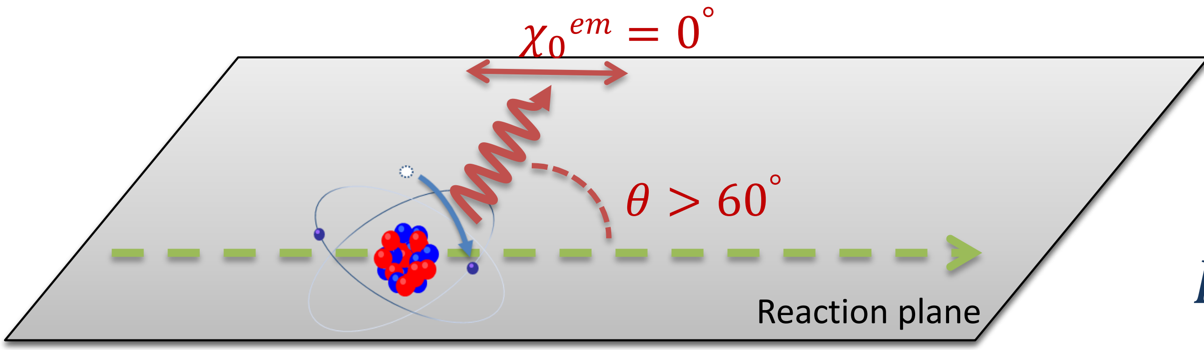
- And are evaluated with the help of relativistic Green's function:

$$G_E(\boldsymbol{r},\boldsymbol{r}') = \sum_n \frac{|\psi_n(\boldsymbol{r})\rangle\langle\psi_n(\boldsymbol{r}')|}{\epsilon_n - \epsilon}$$



A. Surzhykov, A. Artemyev, and V. Yerokhin, PRA **83** (2011) 062710

Orientation of the L-REC linear polarization



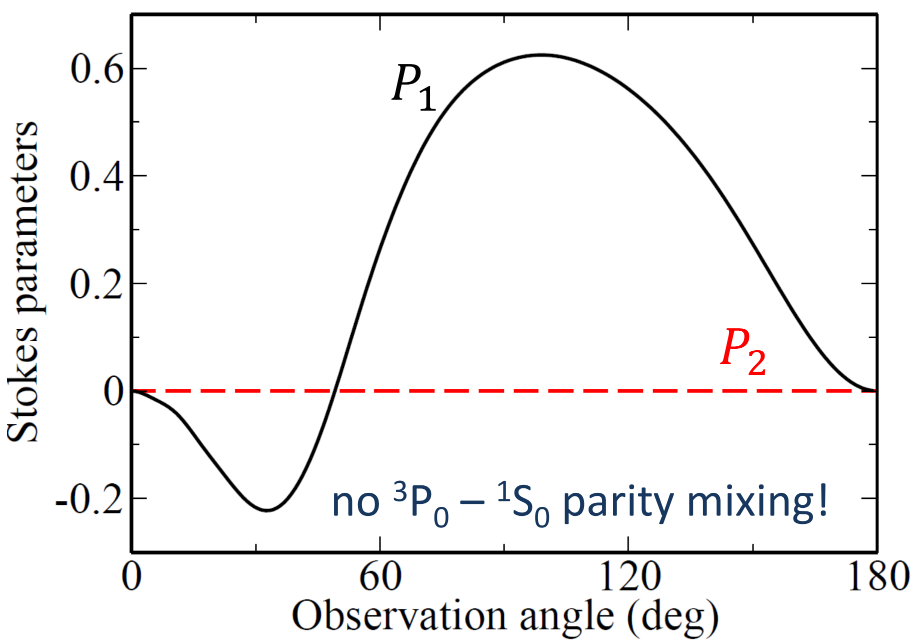
- Linear polarization of emitted light is usually described in terms of Stokes parameters:

$$P_1 = \frac{I_0 - I_{90}}{I_0 + I_{90}} \qquad P_2 = \frac{I_{45} - I_{135}}{I_{45} + I_{135}}$$

- Or in terms of polarization ellipse parameters:

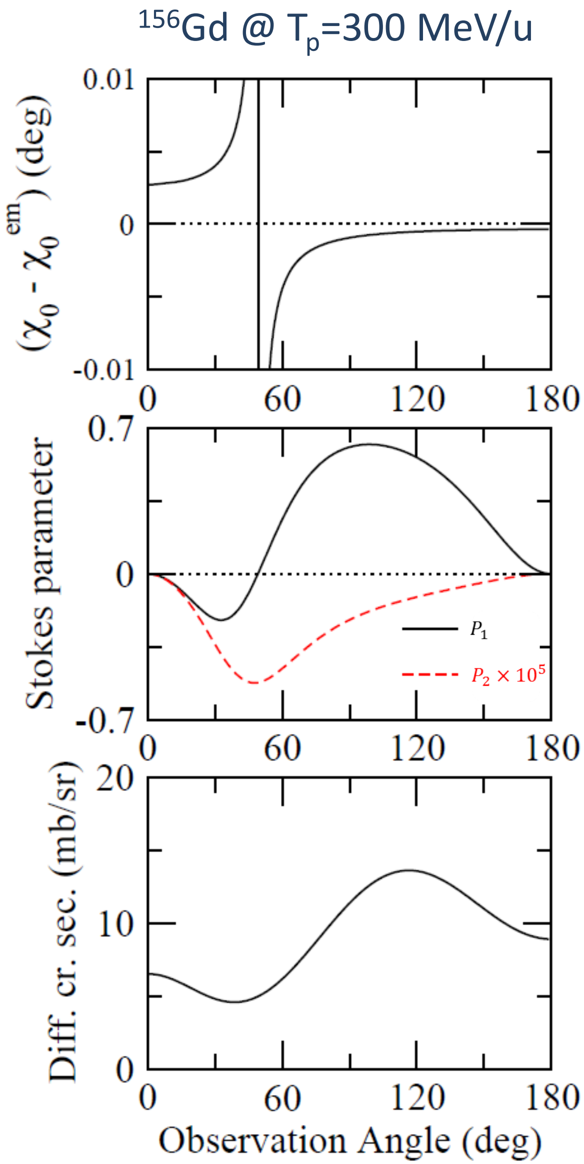
$$P_L = \sqrt{P_1^{\,2} + P_2^{\,2}} \qquad \boxed{\tan \chi_0 = \frac{P_2}{P_1}}$$

^{156}Gd @ T_p=300 MeV/u, REC into 2^3P_0 state



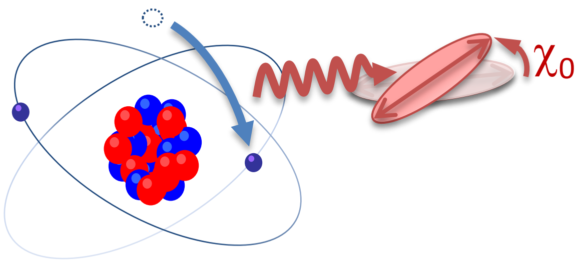
If no parity mixing occur, the rotation angle of the polarization ellipse would be exactly 0 or 90 degrees depending on the observation angle and ion energy!

Radiative electron capture into 2^3P_0 state



- For the capture of unpolarized electrons by unpolarized ions and measure tilt angle of the REC polarization:

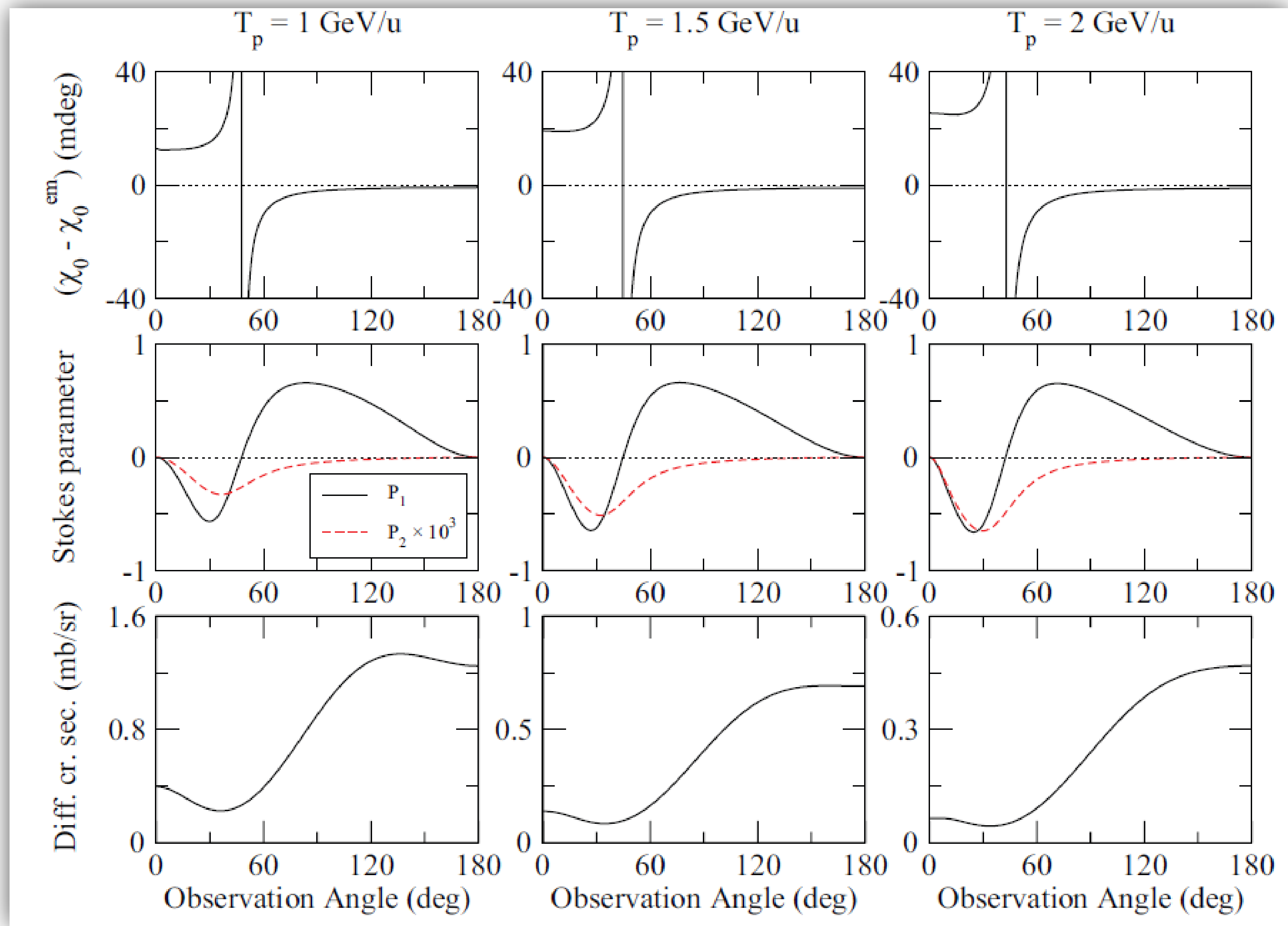
$$\boxed{\chi_0 - \chi_0{}^{\mathrm{em}} = \eta \cdot \mathcal{F}(\theta, T_p) + \mathcal{O}(\eta^3)}$$



- The PNC-induced tilt angle increases with the projectile energy!

J. Gunst *et al.*, PRA **87** (2013) 032714

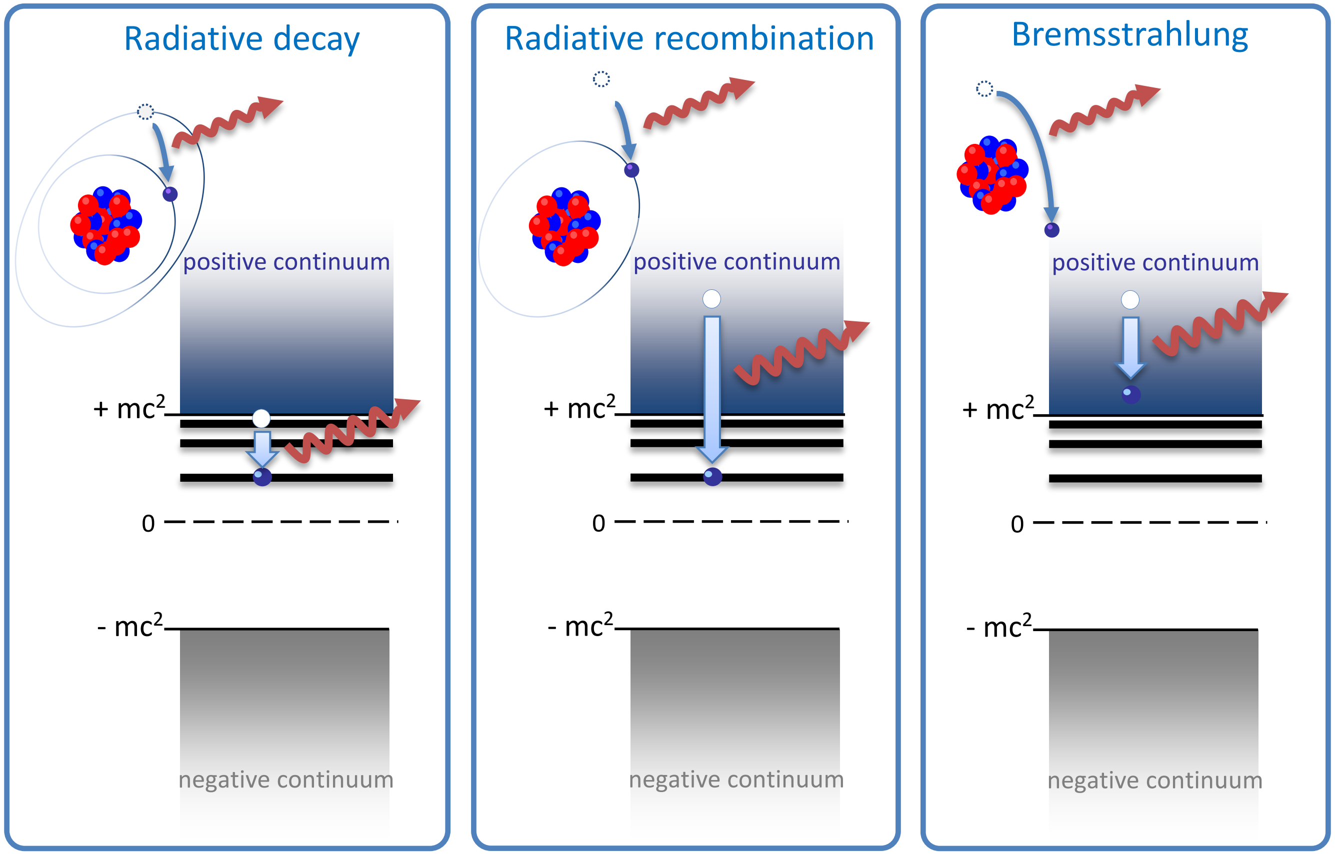
Radiative electron capture into 2^3P_0 state



- The PNC-induced tilt angle may increase to 0.03 deg.
- But very high collision energies of few GeV/u are required!

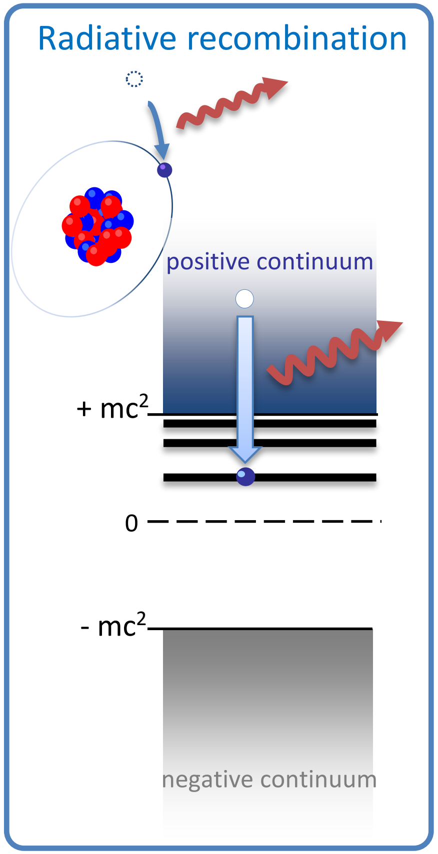
J. Gunst *et al.*, PRA **87** (2013) 032714

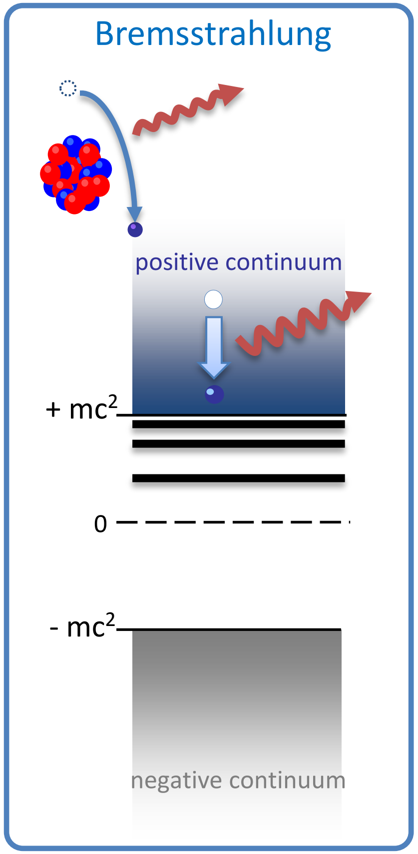
Electron transitions in Coulomb field



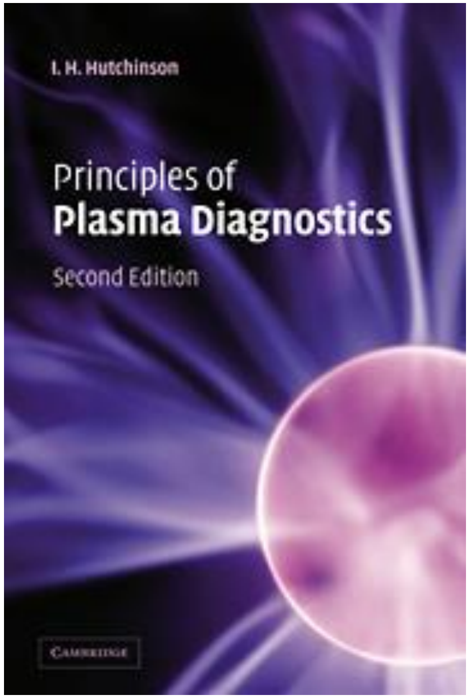
- Bound-state radiative transitions: Multipole-mixing effects
- Radiative recombination: Analysis of parity non-conservation phenomena
- Atomic bremsstrahlung: Polarimetry of electron beams

Atomic bremsstrahlung





- Bremsstrahlung attracts much current attention:
  - Studies in the short-wavelength limit
  - Applications towards electron polarimetry
  - Highly-ionized plasma diagnostics

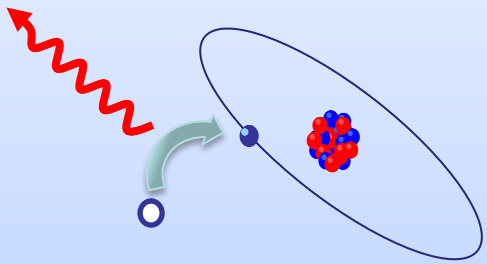


S. Tashenov et al., Phys. Rev. Lett. **107**, 173201 (2011)
R. Märtin *et al.*, Phys. Rev. Lett. **108,** 264801 (2012)

Bremsstrahlung vs. recombination

- From the theoretical viewpoint, analysis of bremsstrahlung is more complicated task if compared to the radiative recombination.

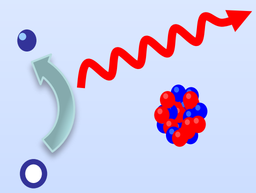
Radiative recombination
(free-bound electron transition)



$$M_{if} = \int \psi_b{}^{+}(\boldsymbol{r})\, \boldsymbol{\alpha}\, \boldsymbol{\varepsilon}\, e^{i\boldsymbol{kr}}\;\; \psi_i(\boldsymbol{r})\, d\boldsymbol{r}$$

After capture electron is in well-defined state (with given parity and angular momentum).

Bremsstrahlung
(free-free electron transition)



$$M_{if} = \int \psi_f{}^{+}(\boldsymbol{r})\, \boldsymbol{\alpha}\, \boldsymbol{\varepsilon}\, e^{i\boldsymbol{kr}}\;\; \psi_i(\boldsymbol{r})\, d\boldsymbol{r}$$

After capture electron is in continuum state which still can be represented as an expansion of spherical waves.

$$\psi_b(\boldsymbol{r}) = \begin{pmatrix} g_{n\kappa}(r)\, \Omega_{\kappa\mu}(\hat{\boldsymbol{n}}) \\ i f_{n\kappa}(r)\, \Omega_{\kappa\mu}(\hat{\boldsymbol{n}}) \end{pmatrix}$$

Bound state has well-define symmetry!

$$\psi_f(\boldsymbol{r}) = \sum_{\kappa\mu} C_{\kappa\mu} \begin{pmatrix} g_{n\kappa}(r)\, \Omega_{\kappa\mu}(\hat{\boldsymbol{n}}) \\ i f_{n\kappa}(r)\, \Omega_{\kappa\mu}(\hat{\boldsymbol{n}}) \end{pmatrix}$$

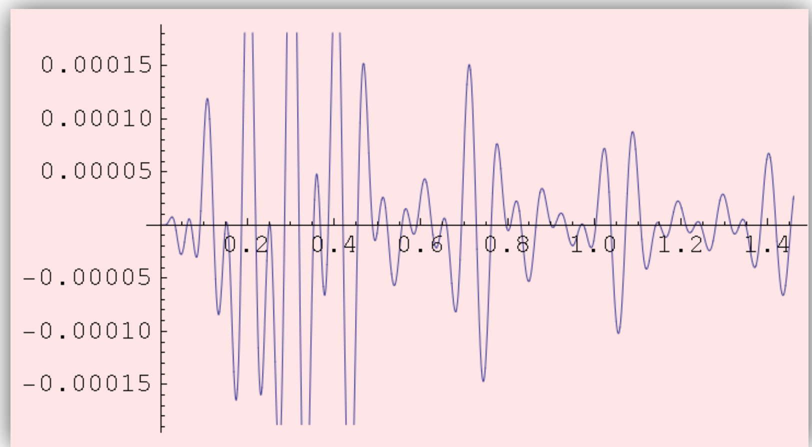
Continuum state is superposition of spherical waves!

Free-free radial integrals

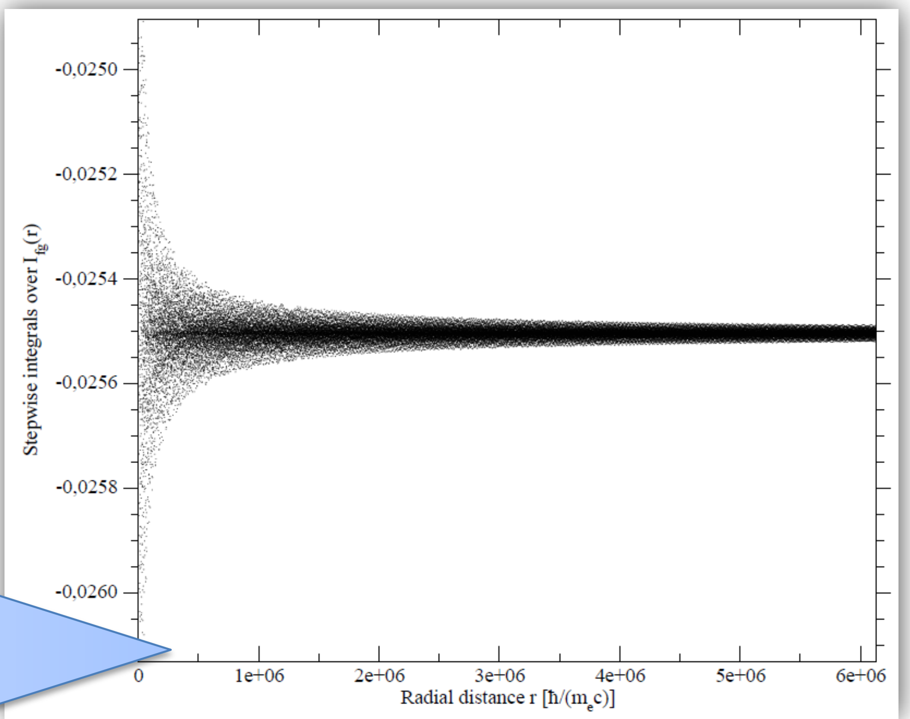
- After making use of angular momentum algebra we may express bremsstrahlung amplitude as a (infinite but convergent) sum of radial free-free integrals:

$$M_{ib} = \int \psi_f^{\,+}(\boldsymbol{r})\, \boldsymbol{\alpha}\, \boldsymbol{\varepsilon}\, e^{i\boldsymbol{kr}}\; \psi_i(\boldsymbol{r})\, d\boldsymbol{r} = \sum_{n_i n_f L} C_{n_i n_f L} \int_0^{\infty} f_{n_i}(r)\, j_L(qr) g_{n_f}(r)\, r^2\, dr$$

Large and small components of Dirac's wavefunctions

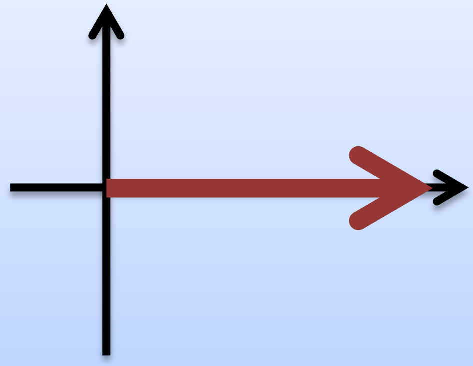


Example of integrand $I(r) = f_{n_i}(r)\, j_L(qr) g_{n_f}(r)\, r^2$

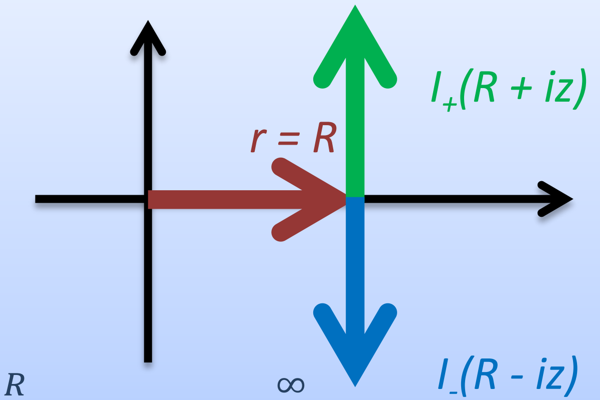


Direct integration is extremely time consuming and can not be used in calculations!

Complex contour rotation



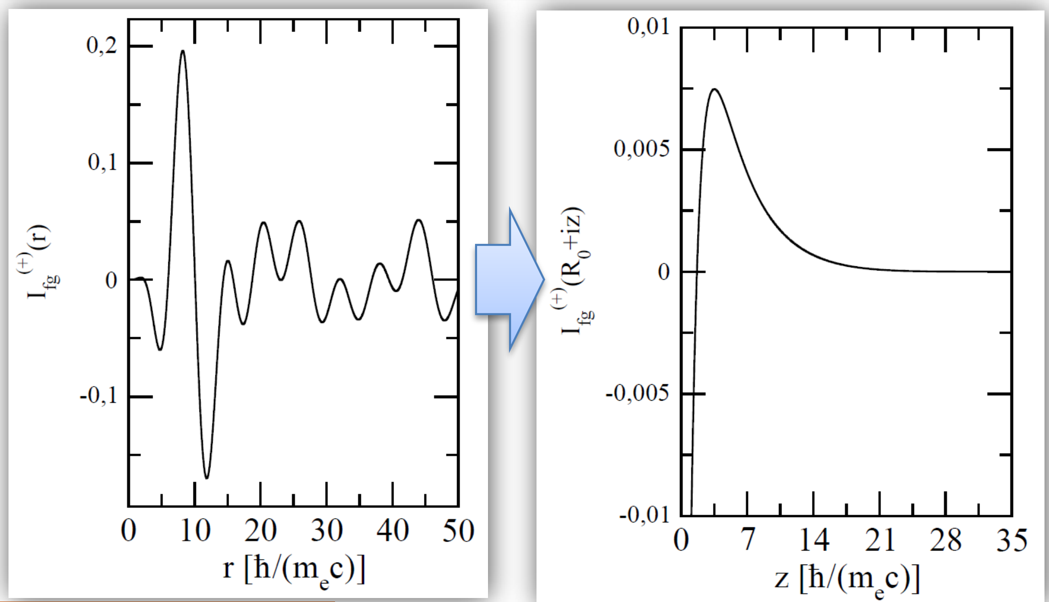
$$\mathfrak{I} = \int_0^\infty I(r)\,dr$$



$$\mathfrak{I} = \int_0^R I(r)\,dr + i\int_0^\infty dz[I_+(R+iz) - I_-(R+iz)]$$

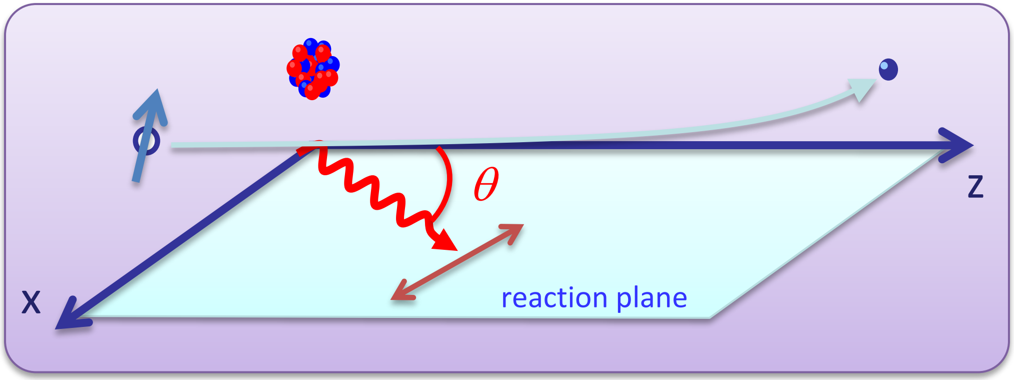
- The method is based on the representation of radial wavefunctions as sum of Whittaker functions:

$$f_n(r), g_n(r) \sim a_n\, W_{-\alpha,\beta}(-2ipr) + b_n\, W_{\alpha,\beta}(2ipr)$$



V. Yerokhin and A.Surzhykov, Phys. Rev. A **82**, 062702 (2010).

Polarization correlations: Theory



- How polarization of bremsstrahlung photons is affected if incident electrons are themselves polarized?
- We may use density matrix theory!

initial state: ion + electron

final state: ion + electron + photon

$$\hat{\rho}_i = \overbrace{\hat{\rho}_e \otimes \hat{\rho}_{ion}} \quad \Rightarrow \quad \hat{\rho}_f = \overbrace{\hat{R}\,\hat{\rho}_i\,\hat{R}^+}$$

$$\langle k\lambda|\hat{\rho}_\gamma|k\lambda'\rangle = \sum_{m_s m_s'} \langle p_e m_s|\hat{\rho}_{ion}|p_e\, m_s'\rangle\, M_{if}(m_s)\, M_{if}{}^*(m_s')$$

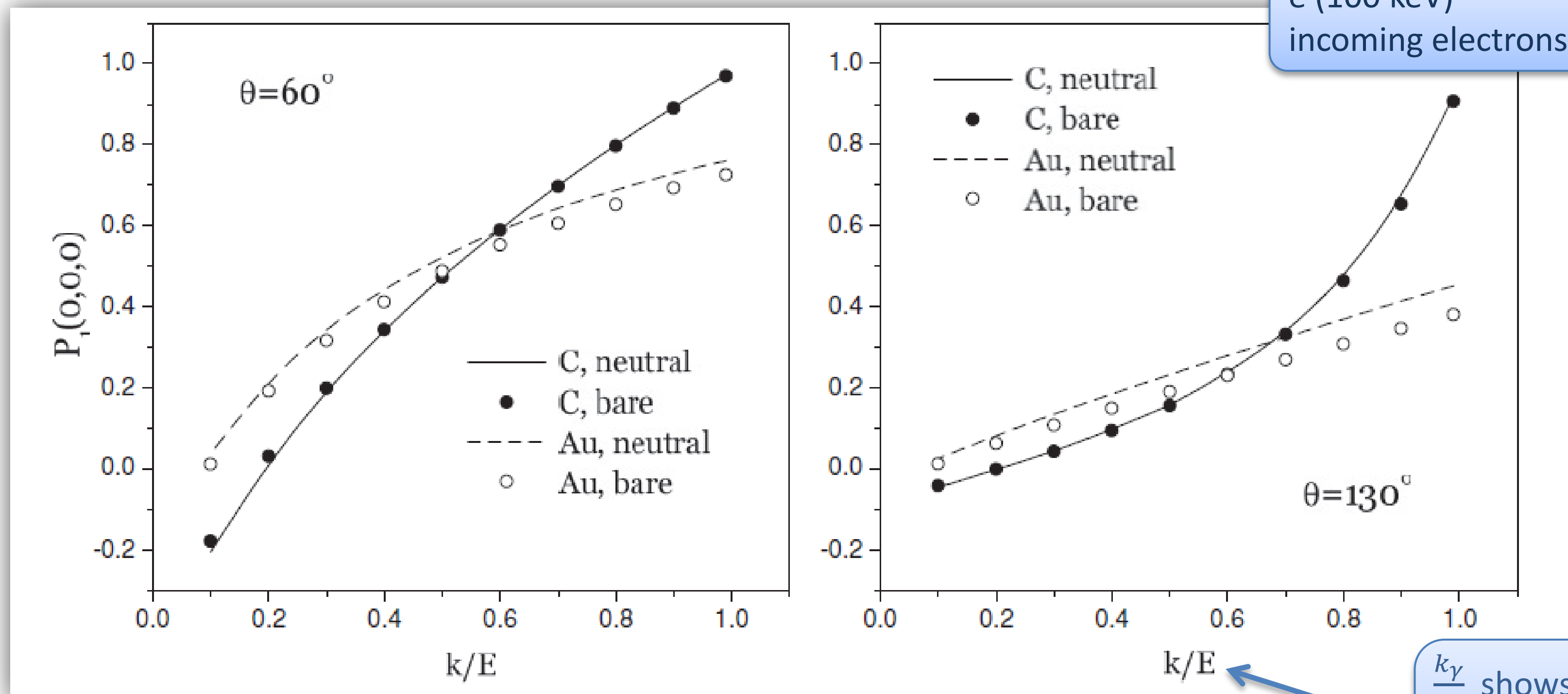
- Density matrix (being 2x2 matrix) is parameterized by three Stokes parameters, two of which describe linear polarization of emitted light:

$$P_1 = \frac{I_0 - I_{90}}{I_0 + I_{90}}, \quad P_2 = \frac{I_{45} - I_{135}}{I_{45} + I_{135}}$$

Bremsstrahlung polarization

- For the scattering of unpolarized electrons the second Stokes parameter P_2 is *zero*.

e (100 keV)
incoming electrons are unpolarized

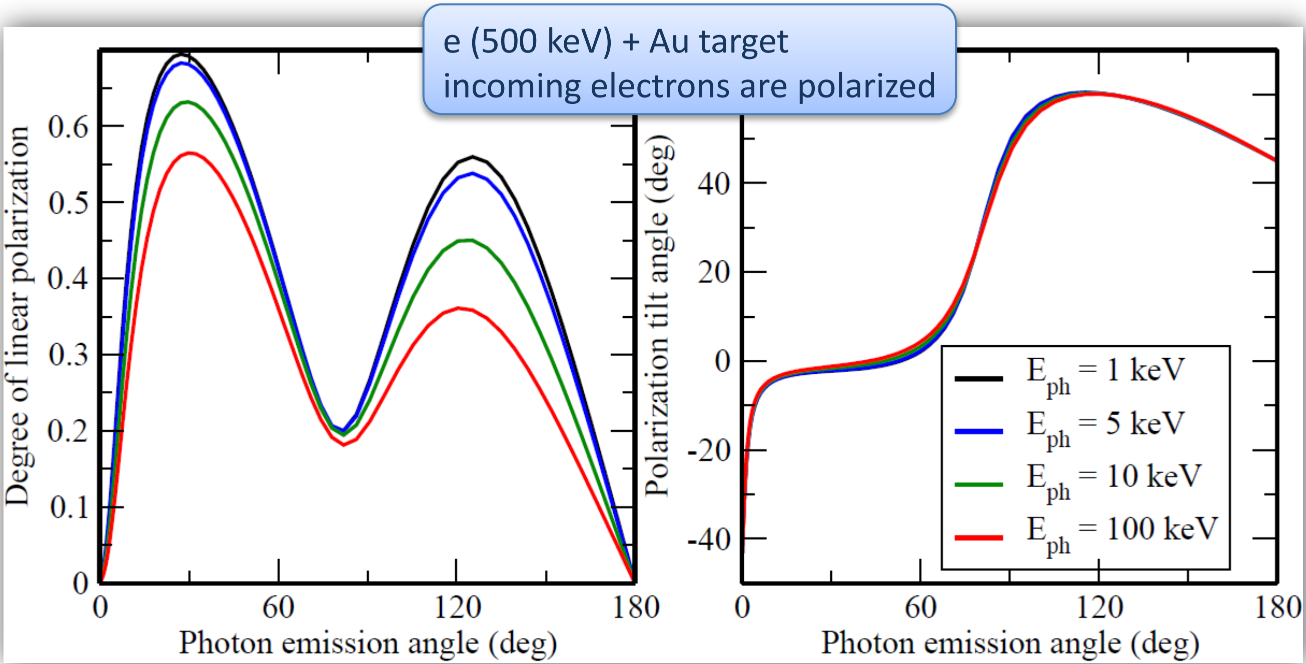


$\frac{k_\gamma}{E_i}$ shows which portion of electron energy is taken away by the photon!

- In order to describe electron-atom bremsstrahlung, we have used the screening approximation based on the Dirac-Fock calculations.

V. Yerokhin and A.Surzhykov, Phys. Rev. A **82**, 062702 (2010).

Bremsstrahlung polarization correlations



Stokes parameters translate into the experimental observables P_L and χ, i.e., the degree of linear polarization and the polarization rotation angle, by the following relations

$$P_L = \sqrt{{P_1}^2 + {P_2}^2}$$

$$\tan\chi = \frac{P_1}{P_2}$$

- Based on the analytical analysis of the electron wavefunctions we found:

$$P_1 \cong \frac{k_\gamma}{E_i}\, f_1(Z,\theta) + \cdots,\ P_2 \cong P_e \frac{k_\gamma}{E_i}\, g_1(Z,\theta) + \ldots$$

- Which implies for the experimentally observable parameters:

$$P_L = \frac{k_\gamma}{E_i} \cdot F_L\,(P_e, Z, \theta), \qquad \tan 2\chi = P_e \cdot F_\chi(Z,\theta)$$

Bremsstrahlung polarization correlations

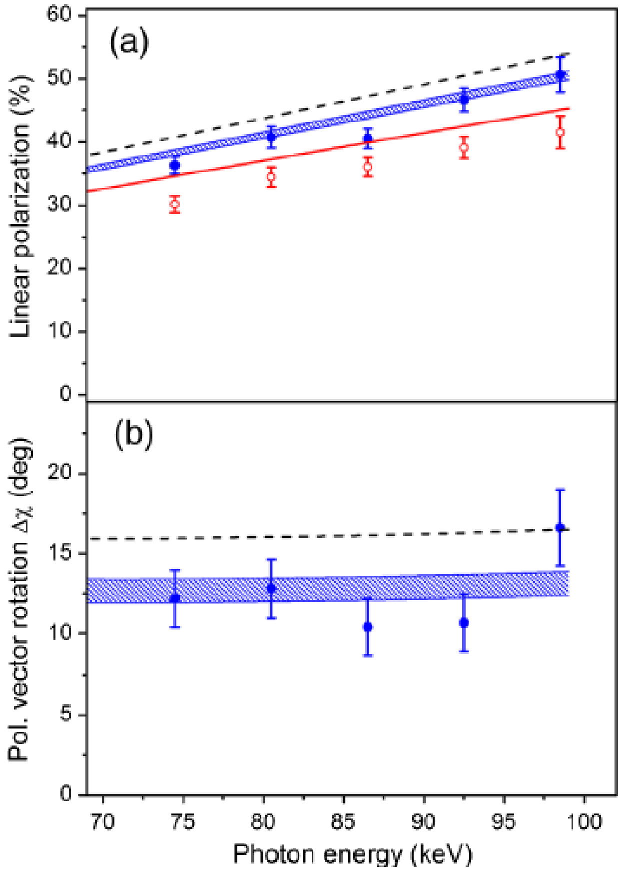


FIG. 3 (color online). (a) Degree of linear polarization of bremsstrahlung arising from transversal polarized (filled symbols) and unpolarized electrons (open symbols) in comparison to theory (shaded area, solid and dashed lines), see text for details. (b) Rotation angle of the bremsstrahlung polarization-axis with respect to the reaction plane.

e (100 keV) + Au target
incoming electrons are polarized

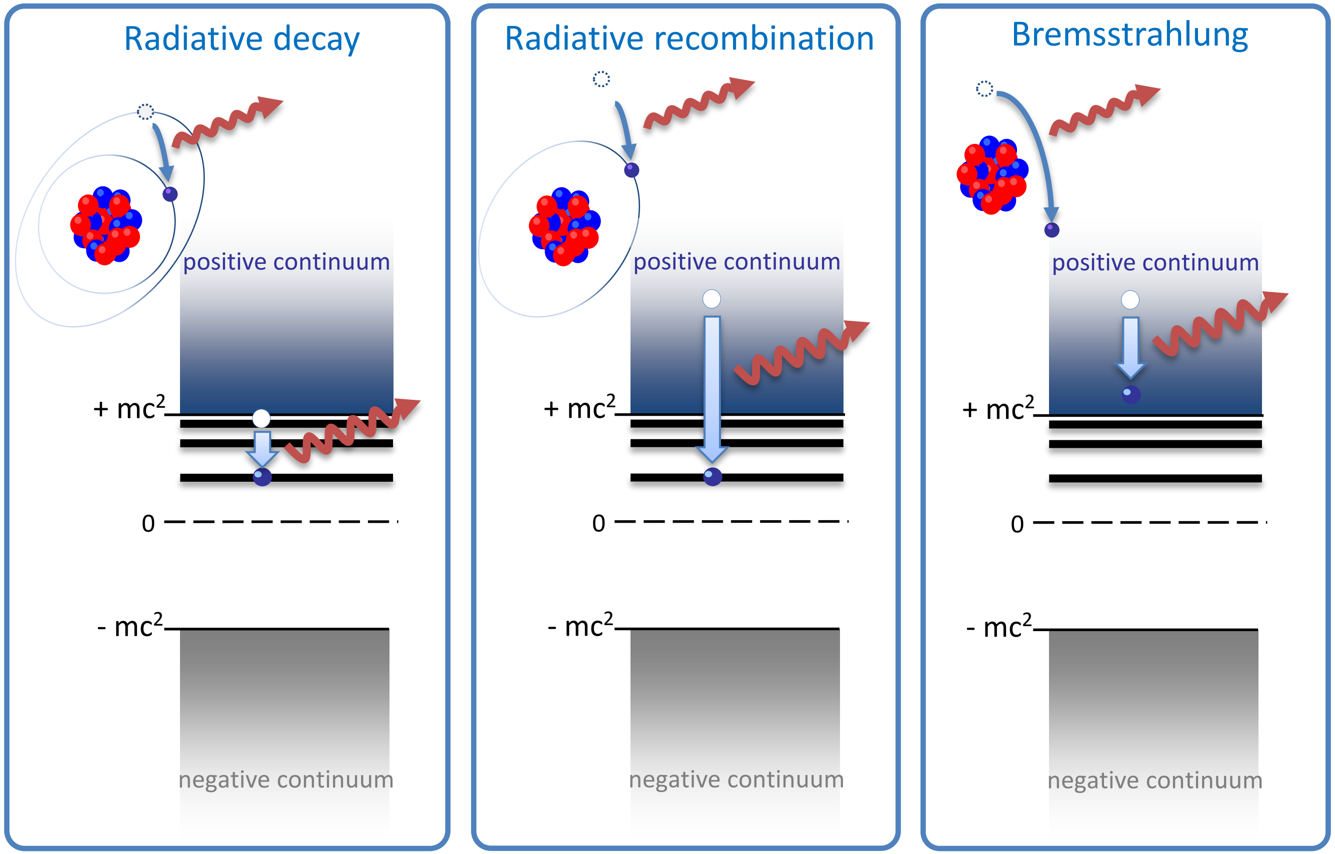
- Scaling of the polarization parameters:

$$P_L = \frac{k_\gamma}{E_i} \cdot F_L\,(P_e, Z, \theta), \quad \tan 2\chi = P_e \cdot F_\chi(Z,\theta)$$

- The polarization tilt angle can be used as a probe of the electron polarization!

R. Märtin *et al.*, Phys. Rev. Lett. **108,** 264801 (2012)

Electron transitions in Coulomb field



- **Bound-state radiative transitions:** Multipole-mixing effects
- **Radiative recombination:** Analysis of parity non-conservation phenomena
- **Atomic bremsstrahlung:** Polarimetry of electron beams

Many thanks to

Anton Artemyev
Stephan Fritzsche
Renate Märtin
Thomas Stöhlker
Günter Weber
Helmholtz Institute Jena

Jonas Gunst
Stanislav Tashenov
University of Heidelberg

Alexandre Gumberidze
Yuri Litvinov
GSI Helmholtz centre

Anna Maiorova
Vladimir Shabaev
St. Petersburg State University

Vladimir Yerokhin
St. Petersburg Technical University

Guenter Plunien
Andrey Volotka
TU Dresden

Paul Indelicato
ENS, Paris

Financial support

HELMHOLTZ
ASSOCIATION

Thank you very much for your attention!