Stock market price prediction and risk analysis using Geometric Browninan Motion

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Abstract—This project was designed to explore the application of Geometric Brownian Motion (GBM) for the use of stock market prediction and risk analysis. The stock which was the main focus of this project is Apple Inc. (APPL). The GBM model uses the Yahoo Finance API and dataset to simulate future stockprice paths. Using this we can predict risk as well as predict stock prices within a 95% confidence interval.

1. Introduction

Financial analysis is a vital field of research for portfolio managers, investors, and financial institutions. It empowers users to make data-driven decisions about asset allocation, risk management, and investment strategies. Predictions with high accuracy can create methods to identify lucrative opportunities, while also mitigating potential loss.

This report delves into the application of Geometric Brownian Motion (GBM). It is a widely recognized stochastic process which is used in financial modeling to predict stock prices and assess associated risks. GBM is suited to modeling stock prices due to its ability to incorporate randomness and account for the log-normal distribution of prices.

Thi analysis specifically focuses on Apple's stock (AAPL) over the past year. Historical data obtained from Yahoo Finance is used to simulate future price movements.

By leveraging GBM, this study aims to evaluate its predictive accuracy and quantify the risk of negative returns, while providing a comprehensive understanding of the range of possible outcomes. This approach serves to highlight the application of GBM in financial forecasting, while sheddign light on its limitations.

2. Research Questions

1. What is the expected stock price of AAPL after one year?
2. What is the risk of negative returns from AAPL?
3. How accurate is GBM when to comes to stock price prediction?

3. Methods

3.1. Data

The source of data for this project was taken from Yahoo Finance through the yfinance Python library. This dataset contains daily historical data for AAPL over many years. The granularity of this data gets as specific as one minute for the past 7 days, and one hour for intervals greater than that. This dataset contains the stocks opening price, its closing price, its highest prices of the day, its lowest price of the day, and the quantity of shares traded each day as volume. This analysis will be foused on the time period which spans a full year before the present day.

3.2. Geometric Brownian Motion

Geometric Brownian Motion is a stochastic differential equation that can be used to model stock prices. It operates based on the assumption that the percent changes (log returns) of a stock's price will follow a normal distribution and that the stock price follows a log-normal distribution. Due to this, GBM is well suited for capturing the randomness as well as growth trends which are observed in financial markets.

The equation for price modeling is the following.

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Where:

- S_t is the stock price at time t.
- μ is the expected return (drift).
  - Drift represents the average rate of return of a stock over time.
- σ is the volatility.
  - Volatility measures how much a stock's price varies over time.
  - High volatility represents extreme price swings.
- dW_t represents a Wiener process increment.
  - Also known as Brownian motion.
  - This is used to add randomness to the model.
  - Adds randomness at every time step to create a random walk over time.

The discrete approximation for this formula can be represented as the following:

$$S_{t+\delta t} = S_t \cdot exp((\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma \cdot \sqrt{\Delta t} \cdot Z)$$

The discrete approximation is what will be implemented for the purpose of this project.

3.3. Software

This analysis is constructed in Python. It uses yfinance to fetch historical stock data, NumPy for computations and random number generation, and Matplotlib for visualizations and results.

4. Simulation

After conducting the discrete approximation the model is used to simulate 1000 different paths. The figure below show 100 for the sake of visibility.

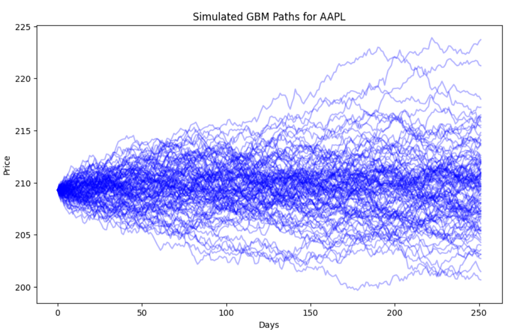

Figure 1. 100 simulated paths with GBM

5. Results

Taking the mean of the final prices of each of these paths gives us our prediction of the final price. The value returned by the model is \$209.68

According to the model, our calculated risk of negative return is 46.8%.

We can take a look at the distribution of our models final

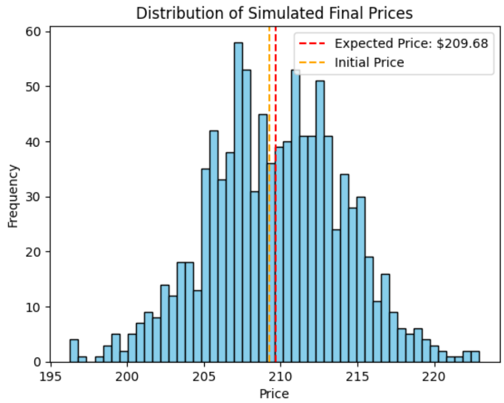

Figure 2. Distributions of simulated final prices with inital and expected price

prices to get a better understanding of our simulation results while comparing the years initial price, to our obtained expected price:

Based on this distribution, we can find a 95% confidence interval, which estimates that the price will be between $200.77 and $218.21.

5.1. Discussion of Results

The expected final price was $209.68, and our 95% confidence interval ranged from $200.77 to $218.21. The actual price was $208.37 after 1 year, which fell within the predicted range. The expected value was had a percent error of 0.629%. While this is a very accurate result. This is likely due to chance, as the model will underestimate events which are rare, but impactful. Market behavior is also more volatile than a constant sigma assumes.

6. Conclusions and Limitations

6.1. Conclusions

While Geometric Brownian Motion provides a computationally efficient framework for modeling stock prices, its limitations highlight the need for caution when applying it to real-world scenarios on its own. It is best suited for scenarios where the assumptions of constant drift and volatility are reasonable approximations.GBM also functions best when the focus is on long-term trends rather than short-term market dynamics.

6.2. Limitations

While GBM can be a powerful tool when it comes to stock price modeling, its limitations make it a tool best used in conjunction with others. GBM lacks the ability to capture sudden jumps in the market. Due to unexpected events such as earnings reports, regulatory changes, or geopolitical tensions, jumps in the market are inevitable. GBM operates with a continuous nature which fails to capture these abrupt shocks to the market. GBM within this model assumes that volatility is constant. However, this is not entirely realistic, as volatility can potentially change greatly over time. This limits the models applicability due to the dynamic nature of market conditions. This models exclusion of interest rates, transaction costs, and liquidity also create some unreliability, as true cost and risk may not be fully represented. Overall the model, while remaining a very valuable tool, is a bit too simplistic and does not account for the complex behaviors such as momentum, market cycles, and mean reversion seen in the market.

References

[1] Hull, J. C. *Options, Futures, and Other Derivatives*. 10th ed., Pearson, 2017. (Referenced for the mathematical formulation and assumptions of Geometric Brownian Motion.)

[2] Shreve, S. E. *Stochastic Calculus for Finance I: The Binomial Asset Pricing Model*. Springer, 2004. (Referenced for the theoretical background on stochastic processes and their application to financial modeling.)

[3] Fama, E. F. "Efficient Capital Markets: A Review of Theory and Empirical Work." *The Journal of Finance*, vol. 25, no. 2, 1970, pp. 383–417. (Referenced for the assumption of no arbitrage and efficient markets.)

[4] Merton, R. C. "Option Pricing When Underlying Stock Returns Are Discontinuous." *Journal of Financial Economics*, vol. 3, no. 1–2, 1976, pp. 125–144. (Referenced for the limitations of GBM, including its inability to model jumps and time-varying volatility.)

[5] Yahoo Finance. "Yahoo Finance Historical Data." Available at: https://finance.yahoo.com. (Used to retrieve historical stock data for analysis.)