EE3025-Assignment 1

ALOK SAROJ - EE18BTECH11003

https://github.com/Alokking/EE3025_Assignment1.git

(5.3) The system with h(n) is defined to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{0.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 Solution

¢

Given difference equation,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.1}$$

By applying Z-transform we get,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{1.0.2}$$

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{1.0.3}$$

Therefore H(z) is

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{1.0.4}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{1.0.5}$$

$$H(z) = z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{1.0.6}$$

By applying inverse z-transform we get,

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{1.0.7}$$

As the condition for the system to be stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.8}$$

By substituting h(n) we get,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left| \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \right| \tag{1.0.9}$$

$$= \sum_{n=-\infty}^{\infty} \left| (-1)^n \left(\left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \right) \right| \tag{1.0.10}$$

$$= \sum_{n=-\infty}^{\infty} \left| \left[\frac{1}{2}\right]^n u(n) + \left[\frac{1}{2}\right]^{n-2} u(n-2) \right| \tag{1.0.11}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\right]^n u(n) + \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\right]^{n-2} u(n-2) \tag{1.0.12}$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{2}\right]^n + \sum_{n=2}^{\infty} \left[\frac{1}{2}\right]^{n-2} \tag{1.0.13}$$

$$= \left[\frac{1}{1-\frac{1}{2}}\right] + \left[\frac{1}{1-\frac{1}{2}}\right] \tag{1.0.14}$$

$$= 2 + 2 \tag{1.0.15}$$

$$= 4 < \infty \tag{1.0.16}$$

As the given system satisfies the stability condition.Therefore, the system is stable.