EE3025-Assignment 1

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https://github.com/Alokking/EE3025_Assignment1.git

(5.3) The system with h(n) is defined to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{0.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 Solution

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Given difference equation,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.1}$$

By applying Z-transform we get,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{1.0.2}$$

$$Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{1.0.3}$$

Therefore H(z) is

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{1.0.4}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{1.0.5}$$

$$H(z) = z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{1.0.6}$$

By applying inverse z-transform we get,

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{1.0.7}$$

As the condition for the system to be stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.8}$$

By substituting h(n) we get,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left| \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \right| \tag{1.0.9}$$

$$= \sum_{n=-\infty}^{\infty} \left| (-1)^n \left(\left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \right) \right| \tag{1.0.10}$$

$$= \sum_{n=-\infty}^{\infty} \left| \left[\frac{1}{2}\right]^n u(n) + \left[\frac{1}{2}\right]^{n-2} u(n-2) \right| \tag{1.0.11}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\right]^n u(n) + \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}\right]^{n-2} u(n-2) \tag{1.0.12}$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{2}\right]^n + \sum_{n=2}^{\infty} \left[\frac{1}{2}\right]^{n-2} \tag{1.0.13}$$

$$= \left[\frac{1}{1-\frac{1}{2}}\right] + \left[\frac{1}{1-\frac{1}{2}}\right] \tag{1.0.14}$$

$$= 2 + 2 \tag{1.0.15}$$

$$= 4 < \infty \tag{1.0.16}$$

As the given system satisfies the stability condition.Therefore, the system is stable.

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BIBO Stability: A system is said to be stable, if the bounded input produces the bounded output. Let x(n) is bounded sequence.

$$|x(n)| < M_x \tag{1.0.17}$$

Where M_x is a finite value.From the convolution property,

$$y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k) \tag{1.0.18}$$

$$|y(n)| = \left|\sum_{-\infty}^{\infty} h(k)x(n-k)\right| \tag{1.0.19}$$

$$|y(n)| \leq M_x \sum_{-\infty}^{\infty} |h(k)| \tag{1.0.20}$$

Because all x(k) less than M_x.
As M_x is finite, for $|y(n)|$ to be finite

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \tag{1.0.21}$$

$$\text{then}\, |y(n)| \leq M_y < \infty \tag{1.0.22}$$

Hence, we can say that the output is bounded if the impulse response is absolutely summable.

Given-

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.0.23}$$

$$y(n) = 0 \text{ for } y < 0 \tag{1.0.24}$$

and the given bounded input is

$$x(n) = \{1, 1, 2, 4, 3, 1\} \tag{1.0.25}$$

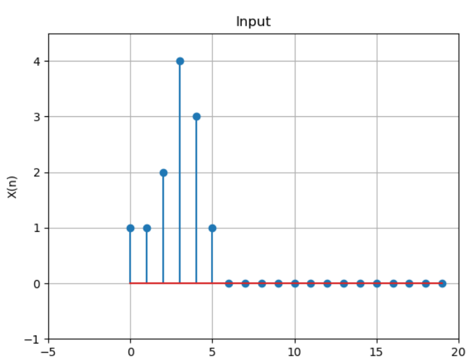

Fig. 0: Input

and the output we get,

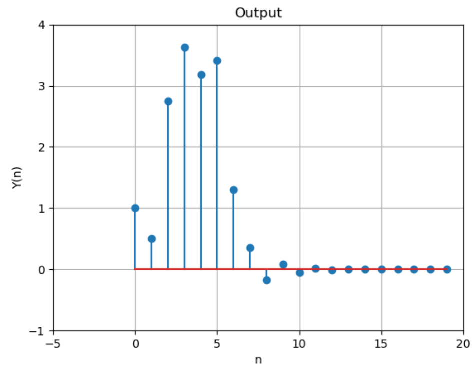

Fig. 0: Bounded output

Here we can see that we are getting bounded output for the bounded input.Hence, the given system is stable.