

EE3025-Assignment 1

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Python codes

https://github.com/Alokking/ee3025_As1/tree/main/codes

Latex-tikz codes

https://github.com/Alokking/EE3025_Assignment1.git

(5.3) The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (0.0.1)$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

1 SOLUTION

¶

Given difference equation,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \quad (1.0.1)$$

By applying Z-transform we get,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \quad (1.0.2)$$

$$Y(z) = \frac{2(z^2 + 1)}{z(2z + 1)}X(z) \quad (1.0.3)$$

Therefore $H(z)$ is

$$H(z) = \frac{2(z^2 + 1)}{z(2z + 1)} \quad (1.0.4)$$

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \quad (1.0.5)$$

$$H(z) = z^{-1} \left[\frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}} \right] \quad (1.0.6)$$

By applying inverse z-transform we get,

$$h(n) = \left[\frac{-1}{2} \right]^n u(n) + \left[\frac{-1}{2} \right]^{n-2} u(n-2) \quad (1.0.7)$$

As the condition for the system to be stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \quad (1.0.8)$$

By substituting $h(n)$ we get,

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} \left| \left[\frac{-1}{2} \right]^n u(n) + \left[\frac{-1}{2} \right]^{n-2} u(n-2) \right| \quad (1.0.9)$$

$$= \sum_{n=-\infty}^{\infty} \left| (-1)^n \left(\left[\frac{-1}{2} \right]^n u(n) + \left[\frac{-1}{2} \right]^{n-2} u(n-2) \right) \right| \quad (1.0.10)$$

$$= \sum_{n=-\infty}^{\infty} \left| \left[\frac{1}{2} \right]^n u(n) + \left[\frac{1}{2} \right]^{n-2} u(n-2) \right| \quad (1.0.11)$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{1}{2} \right]^n u(n) + \sum_{n=-\infty}^{\infty} \left[\frac{1}{2} \right]^{n-2} u(n-2) \quad (1.0.12)$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{2} \right]^n + \sum_{n=2}^{\infty} \left[\frac{1}{2} \right]^{n-2} \quad (1.0.13)$$

$$= \left[\frac{1}{1 - \frac{1}{2}} \right] + \left[\frac{1}{1 - \frac{1}{2}} \right] \quad (1.0.14)$$

$$= 2 + 2 \quad (1.0.15)$$

$$= 4 < \infty \quad (1.0.16)$$

As the given system satisfies the stability condition. Therefore, the system is stable.

¶

BIBO Stability: A system is said to be stable, if the bounded input produces the bounded output.

Let $x(n)$ is bounded sequence.

$$|x(n)| < M_x \quad (1.0.17)$$

Where M_x is a finite value. From the convolution property,

$$y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k) \quad (1.0.18)$$

$$|y(n)| = \left| \sum_{-\infty}^{\infty} h(k)x(n-k) \right| \quad (1.0.19)$$

$$|y(n)| \leq M_x \sum_{-\infty}^{\infty} |h(k)| \quad (1.0.20)$$

Because all $x(k)$ less than M_x .

As M_x is finite, for $|y(n)|$ to be finite

$$\sum_{-\infty}^{\infty} |h(n)| < \infty \quad (1.0.21)$$

$$\text{then } |y(n)| \leq M_y < \infty \quad (1.0.22)$$

Hence, we can say that the output is bounded if the impulse response is absolutely summable.

Given-

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \quad (1.0.23)$$

$$y(n) = 0 \text{ for } y < 0 \quad (1.0.24)$$

and the given bounded input is

$$x(n) = \{1, 1, 2, 4, 3, 1\} \quad (1.0.25)$$

and the output we get,

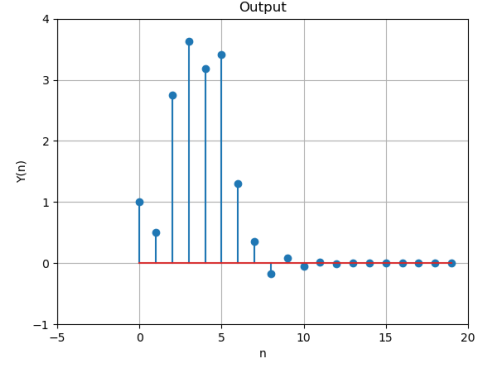


Fig. 0: Bounded output

Here we can see that we are getting bounded output for the bounded input. Hence, the given system is stable.

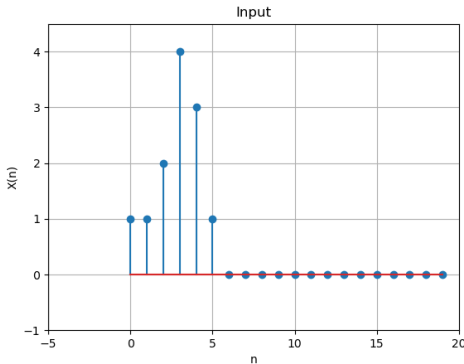


Fig. 0: Input