EE1390
MATRIX PROJECT

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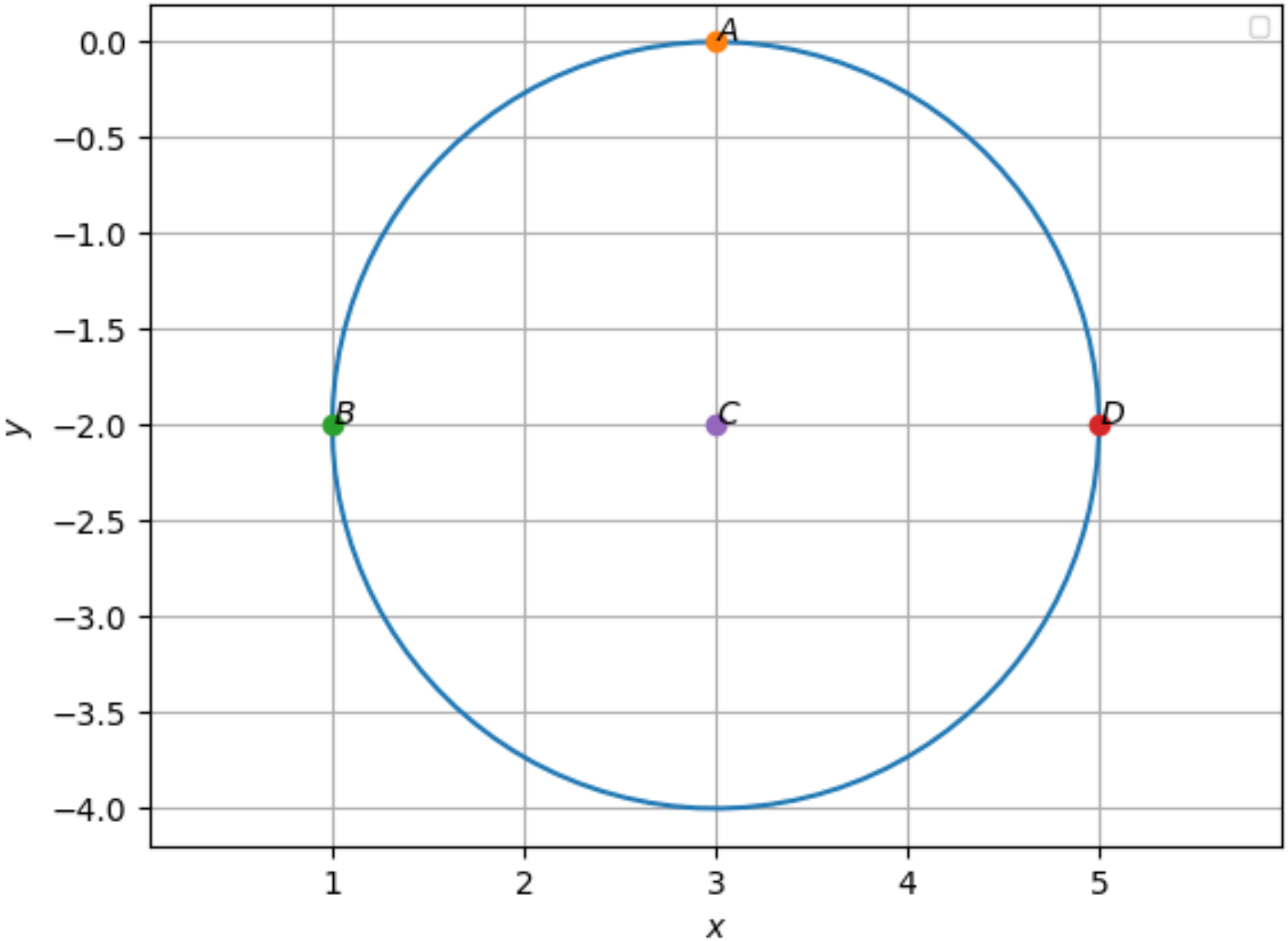
Question 2 from JEE Main 2013

A circle passing through (1,-2) and touching the x-axis at (3,0) also passes through the point

1. (-5,2)
2. (2,-5)
3. (5,-2)
4. (-2,5)

Graph



Solution

We have , the circle passing through $B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

and touches the circle at $A = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

Let the centre of the circle be $C = \begin{bmatrix} 3 \\ k \end{bmatrix}$

Now we know that

$$||C - A|| = ||C - B||$$

let say vector

$$X = ||C - A||$$
$$Y = ||C - B||$$

then

$$X^T X = ||C - A||^2$$

$$Y^T Y = ||C - B||^2$$

So, We have

$$Y^T Y = X^T X$$

$$\begin{bmatrix} 0 & k \end{bmatrix} \begin{bmatrix} 0 \\ k \end{bmatrix} = \begin{bmatrix} 2 & k+2 \end{bmatrix} \begin{bmatrix} 2 \\ k+2 \end{bmatrix}$$

$$k^2 = 4 + (k+2)^2$$

$$k^2 = 4 + k^2 + 4 + 4k$$

$$4k = -8$$

$$k = -2$$

Here we get the centre of a circle $C = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

Here the equation of circle is

$x^T x - 2cx = r^2 - cc^T$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - 2 \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = r^2 - \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Now here we can satisfy each and every point given in options

let take a coordinate (5,-2) for check

$$\begin{bmatrix} 5 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \end{bmatrix} - 2 \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \end{bmatrix} = 2^2 - \begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$29 - 2 * 19 = 4 - 13$

$29 - 38 = -9$

$-9 = -9$

Here, LHS = RHS

hence the point (5,-2) lie on circle

so the correct option is 'C'