Uniform Circular Arrays for Smart Antennas: a Review

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***Abstract*—Over the last decade, wireless technology has snowballed, thereby creating new and improved services at lower costs—also, the need for better coverage, improved capacity, better quality, and more security rises. Thus radio spectrum should be managed in a very efficient way. Smart antennas are capable of use radio spectrum in a much efficient way. So many studies on smart antennas have been done, including uniform linear array and uniform rectangular array. However, not much effort has been dedicated to uniform circular arrays (UCAs). The symmetry that UCA can provide for smart-antennas is unique and can be used in specific applications. It can scan a beam azimuthally through 360° with a bit of change in the radiation pattern. In this paper, the smart antenna has been introduced. Then UCAs has been discussed in details. Then two main issues in the smart antenna (for UCA), estimation of direction-of-arrival and beamforming, have been discussed. Also, the effect of mutual coupling is an issue in every array examined.**

***Index Terms*—Antenna arrays, adaptive arrays, circular arrays, land mobile radio cellular systems, smart antennas, MUSIC, ESPRIT, direction of arrival estimation, LMS algorithm, array signal processing**

I. INTRODUCTION

Spatial processing is the central idea of adaptive antennas or smart-antenna systems. Although it might seem that adaptive antennas have been recently discovered, they date back to World War II with the conventional Bartlett beamformer. It is only of today's advancement in powerful low-cost digital signal processors, general-purpose processors, as well as innovative software-based signal-processing techniques (algorithms) that smart antenna systems have received enormous interest worldwide. Smart antennas can so much capacity improvements in existing wireless infrastructures (especially in dense areas) and use radio spectrum in a much more efficient way. Also, the location finding an aspect of smart antennas we can offer more services to the community (emergencies, law enforcement, etc.) Also, ad hoc networks or WLANs can experience more improvements in their performance by using smart antennas. Due to the beamforming, we may have better SNR so the throughput of the system can increase.

The investigation of smart antennas is primarily rectangular and linear in configuration(geometry). Several techniques and algorithms have been proposed to estimate direction-of-arrival for these configurations, and Also different techniques have been deployed. Nevertheless, a little effort has been dedicated to circular geometry, which has exciting advantages over other geometries due to its symmetric shape. Since a uniform circular array does not have edge elements, directional patterns synthesized with a uniform circular array can be electronically rotated in the plane of the array without significant change in the beam shape.

In this paper, First of all, we introduced smart antennas in general then we investigate uniform circular arrays (UCA) for smart-antenna purposes. Two critical components of smart-antenna technology reviewed here is direction-of-arrival (DOA) estimation and adaptive beamforming. With the former and the aid of a digital signal processor, it is feasible to determine the angles from which sources transmit signals towards an antenna array. With the latter, an antenna radiation pattern beam maximum can be simultaneously placed towards the intended user or Signal of interest (SOl), and, ideally, nulls can be placed towards directions of interfering signals or signals, not of interest (SNRIs). We consider two different methods for DOA estimation, the MUSIC, and the ESPRIT algorithms. For beamforming with uniform circular arrays, we introduced the LMS algorithm, and also we have shown a simulation of one-directional adaptive beamforming to show the power of this method [1], [2] , [6].

II. SMART ANTENNAS

A smart antenna consists of an antenna array combined with digital signal processing in time and space. After the digital signal processor measures the time delays from each antenna array element, it estimates the direction of arrival (DOA) of the signal-of-interest (SOI). It then excites each element (gains and phases of the excitation signal) properly to create a good radiation pattern, which means keeping the pattern's directivity toward the SOI direction and keeping the nulls towards signal-not-of-interest (SNOI) [1], [2].

A. Smart Antennas' Advantages

There are several reasons for the growing interest in smart-antenna systems. The primary one is *increasing capacity*, *increasing range*, *security*, and *locate the users* for emergencies reasons.

1) Increasing Capacity: In dense areas, due to the interference from other users, the signal-to-interference ratio (SIR) is so small. Using smart-antennas, the useful received signal will be increased, and the interference will be lowered so that SIR will be improved [1], [3], [4].

2) Increasing Range: Using smart antennas provides a much larger range, especially in high noise levels. Smart antennas are more directional than others so that the energy can focus on the intended direction (not wasting in other directions). So the coverage will be improved [1], [3], [4].

3) Security: Smart antennas make connections more secure and solid because the attacker must be position in the same direction as the user or limited directions so the connection's security will be improved [1], [3].

4) Locate the Users: Due to the spatial detection of the smart-antennas, it can determine the user's location in emergencies or other location-based services [1], [3].

B. Smart Antennas' Disadvantages

Smart-Antennas have so many advantages over traditional antennas, but they do suffer from some disadvantages. Their implementation is more complex than the traditional one. Antenna beamforming is computationally intensive. Which means they need an expensive digital signal processor. Overall, it seems that the benefits of smart-antennas overweight over traditional antennas [1], [3].

C. Smart Antenna Systems

Smart-Antenna can refer to different implementations of systems that can focus their directivity toward intended directions while rejecting unwanted interference. These systems can generally be classified as either *Switched-Beam* or *Adaptive Array* [1].

1) Switched-Beam Systems: These systems can choose from many patterns (limited) due to the received signal strength and other conditions to increase SOI gain. However, since the radiation patterns are fixed, the radiation pattern is not that accurate, so the system's performance can decrease due to the decreasing complexity [1].

2) Adaptive Array Systems: Adaptive arrays can create radiation patterns to keep the main lobe of the pattern toward SOI while keeping null towards SNOI. In other words, these systems can customize the radiation pattern for each user Fig. 1 show how it works. This is far superior to the performance of a switched-beam system. Fig. 2 shows a scheme to compare two systems [1].

III. Antenna Design

In the design of smart antennas, the antenna designer should consider the type of antenna elements and array design. Both of them should be designed and chosen due to the application of the antenna.

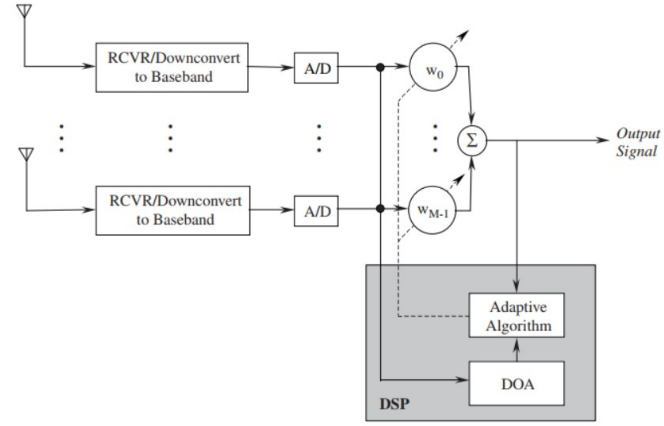

Fig. 1. Functional block diagram of an adaptive array system [1].

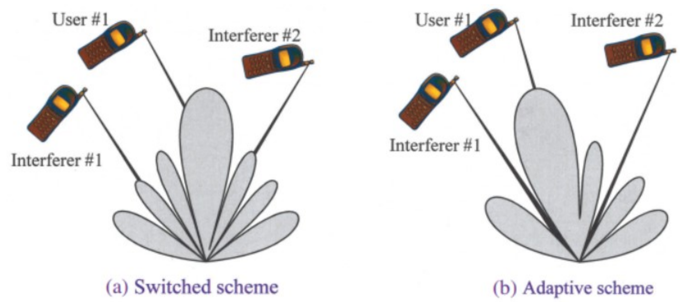

Fig. 2. Comparison of (a) switched-beam scheme, and (b) adaptive array scheme [1].

A. Element Design

Many antenna elements can be selected to form an adaptive array. This includes classic radiators such as dipoles, monopoles, loops, apertures, horns, reflectors, microstrips, and so on. This step of design should be done carefully to suit best for the usage of antenna [1]. One element that meets the requirements of the mobile device is the array of printed elements; patches. There are several analysis methods as well as software packages; one is that of [5].

B. Array Design

We have to consider some notes to design an array. First of all, The larger an array is the narrower beamwidth it can create, and SOIs and rejecting SNOI will be more accurate. On the other hand, complexity, cost, convergence time for the adaptive algorithm will increase.

Also, the configuration of an array (geometry) is an important design aspect. The usage of antenna should be considered to design a proper smart antenna. Several geometries have been investigated, such as linear arrays, rectangular arrays, and circular arrays.

1) Linear Array: The linear array design is not attractive that much because of its inability to scan 3-D space. The array factor of a linear array of M identical elements with uniform spacing is shown in Fig. 3 . As discussed in class, we have:

$$(AF)_M = \sum_{n=1}^{M/2} w_n \cos\left[(2n-1)\psi_n\right] \tag{1}$$

where

$$\psi_n = \frac{\pi d}{\lambda} \sin\theta \sin\phi + \beta_n \tag{2}$$

In (1) and (2), w_n and β_n represents the amplitude and phase of excitation of the individual elements. By using different techniques, we can change radiation patterns to improve the performance of the system. One of them is Dolph-Tschebyscheff which is discussed in detail in [1].

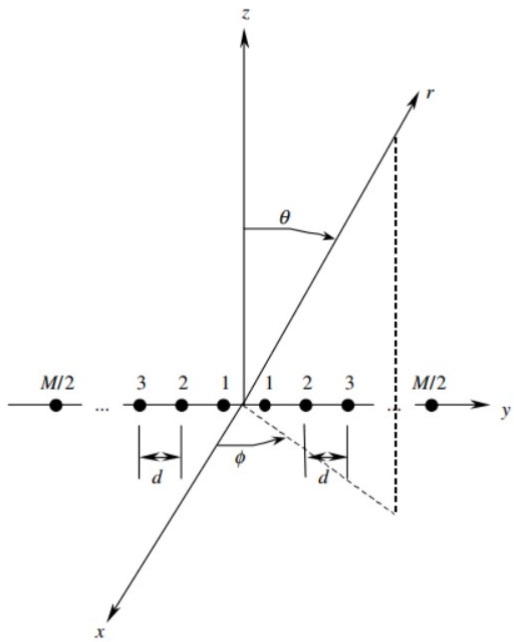

Fig. 3. Linear array with elements along the y-axis [1].

2) Rectangular Array: As discussed in previous sections, the linear array suffers from the inability to scan 3-D space. It is necessary to form the main beam in any direction of θ (elevation) and ϕ (azimuth) in many applications. Planar arrays are more attractive for these applications. one planar array (rectangular) shown in Fig. 4

$$[AF(\theta,\phi)]_{M\times N} = 4 \sum_{m=1}^{M/2} \sum_{n=1}^{N/2} w_{mn} \cos[(2m-1)u] \cos[(2n-1)v] \tag{3}$$

where

$$\begin{aligned} u &= \frac{\pi d_x}{\lambda} \left(\sin\theta\cos\phi - \sin\theta_0\cos\phi_0\right) \\ v &= \frac{\pi d_y}{\lambda} \left(\sin\theta\sin\phi - \sin\theta_0\sin\phi_0\right) \end{aligned} \tag{4}$$

In (3), w_{mn} is the amplitude excitation of individual element.

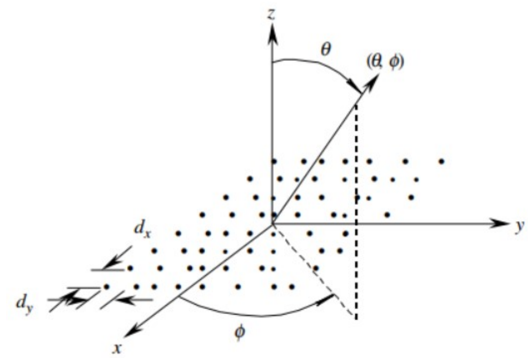

Fig. 4. Planar array with uniformly spaced elements [1].

3) Circular Array: In the literature for adaptive antennas, not as much attention has been devoted to circular configurations despite their ability to offer some advantages. A noticeable advantage results from the azimuthal symmetry of the Uniform Circular Array (UCA) geometry. Because a UCA does not have edge elements, directional patterns synthesized by this geometry can be electronically scanned in the azimuthal plane without a significant change in beam shape.

The angular position of the nth element of the array is given by

$$\phi_n = 2\pi\left(\frac{n}{N}\right), n = 1, 2, \ldots, N \tag{5}$$

The unit vector $\mathbf{a}_r$ from the origin is represented in Cartesian coordinates by

$$\hat{\mathbf{a}}_r = \hat{\mathbf{a}}_x \sin\theta\cos\phi + \hat{\mathbf{a}}_y \sin\theta\sin\phi + \hat{\mathbf{a}}_z \cos\theta \tag{6}$$

The unit vector $\mathbf{a}_{pn}$ from the origin to the nth element of the array is written as

$$\hat{\mathbf{a}}_{pn} = \hat{\mathbf{a}}_x \cos\phi_n + \hat{\mathbf{a}}_y \sin\phi_n, n = 1, 2, \ldots, N \tag{7}$$

The vector $\Delta\mathbf{r}_n$ represents the differential distance by which the planar wavefront reaches the nth element of the array relative to the origin, and it is given by

$$\Delta\mathbf{r}_n = \hat{\mathbf{a}}_r a \cos\psi_n \tag{8}$$

Since the wavefront is incoming and not radiating outwards, ψ_n can be expressed as

$$\begin{aligned} \cos\psi_n &= -\,\hat{\mathbf{a}}_r \cdot \hat{a}_{pn} \\ &= -\left(\hat{\mathbf{a}}_x \sin\theta\cos\phi + \hat{\mathbf{a}}_y \sin\theta\sin\phi + \hat{\mathbf{a}}_z \cos\theta\right) \\ &\quad \cdot \left(\hat{\mathbf{a}}_x \cos\phi_n + \hat{\mathbf{a}}_y \sin\phi_n\right) \\ &= -\left(\sin\theta\cos\phi\cos\phi_n + \sin\theta\sin\phi\sin\phi_n\right) \\ &= -\sin\theta\left(\cos\phi\cos\phi_n + \sin\theta\sin\phi_n\right) \\ &= -\sin\theta\cos\left(\phi - \phi_n\right), \quad n = 1, 2, \ldots, N \end{aligned} \tag{9}$$

Therefore we have

$$\Delta\mathbf{r}_n = -\hat{\mathbf{a}}_r a \sin\theta\cos\left(\phi - \phi_n\right), \quad n = 1, 2, \ldots, N \tag{10}$$

Furthermore, assuming that the wavefront passes through the origin at time $t = 0$, it impinges on the nth element of the array at the relative time of

$$\tau_n = -\frac{a}{c} \sin\theta \cos\left(\phi - \phi_n\right), n = 1, 2, \ldots, N \qquad (11)$$

where c is the speed of light in free space. Positive time delay indicates that the wavefront impinges on the nth element after it passes through the origin, whereas negative time delay indicates that the wavefront impinges on the nth element before it arrives at the origin. Moreover, based on Equation (9), the element-space circular array steering vector is given by

$$\mathbf{a}(\boldsymbol{\theta}) = \mathbf{a}(\zeta, \phi) = \left[e^{\zeta \cos(\phi-\phi)}, e^{\varsigma \cos(\phi-\phi_2)}, \ldots, e^{\varsigma \cos(\phi-\phi_N)}\right]^T \qquad (12)$$

where the elevation dependence is through the parameter $\varsigma = ka \sin\theta$, and the vector $\boldsymbol{\theta} = (\varsigma, \phi)$ is used to represent source arrival directions [1], [6].

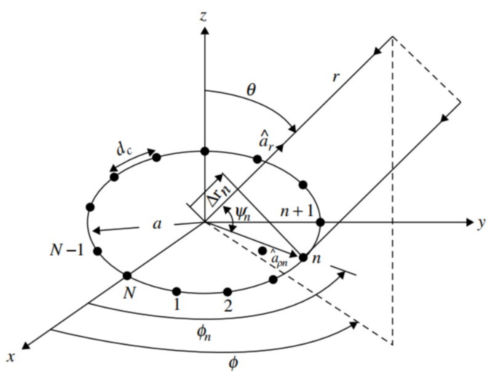

Fig. 5. Geometry of uniform circular array of N elements [6].

IV. Smart Antenna Beam forming with UCA

In smart antennas, knowledge and data from the environment are gained and applied via algorithms processed by a digital signal processor (DSP). The goal of DSP is to:

1) DOA: Estimate the direction of arrival (DOA) of all impinging signals.

2) Adaptive Beam forming: Create the appropriate weights to ideally steer the maximum radiation of the antenna pattern toward the SOI and place nulls toward the SNOI.

3) Mutual Coupling: Moreover, in some cases, deal with mutual coupling problems.

V. DOA Estimation with UCA

In this section, we investigate some algorithms for estimation DOA with UCA.

A. Real Beam-Space MUSIC for UCA

In other configurations of an array, most of the DOA algorithms used are the MUltiple SIgnal Classification (MUSIC) algorithm, which utilizes the fact that signal vectors are orthogonal to the noise subspace [6]. MUSIC offers numerous advantages over the element-space operation, including reduced computation, since subspace estimates are obtained via real-valued Eigen decompositions and enhanced performance in correlated-source scenarios due to the attendant forward-backward averaging effect [7]. An exciting version of the MUSIC algorithm is the Real Beam-space MUSIC algorithm for uniform circular arrays (UCA-RB-MUSIC). The complete development of the algorithm can be found in [7].

TABLE I
The geometries used to test the UCA-RB-MUSIC algorithm.

	Case 1	Case 2
Number of elements	$N = 8$	$N = 10$
Radius of the UCA	$a = 0.6\lambda$	$a = 0.75\lambda$
Number of impinging sources	$K = 3$	$K = 4$
SNR per equal-power source	10 dB	10 dB
Number of collected samples	3000	4000

TABLE II
The directions of arrival of the signals used to test the UCA-RB-MUSIC algorithm.

Case 1	Case 2
$\theta_1 = 30°, \phi_1 = 120°$	$\theta_1 = 30°, \phi_1 = 300°$
$\theta_2 = 45°, \phi_2 = 210°$	$\theta_2 = 45°, \phi_2 = 150°$
$\theta_3 = 60°, \phi_3 = 300°$	$\theta_3 = 60°, \phi_3 = 240°$
	$\theta_4 = 75°, \phi_4 = 60°$

1) Simulation Results of Direction Finding with the UCA-RB-MUSIC Algorithm: To test UCA-RB-MUSIC Algorithm two cases have been investigated, which shown in Table I and II. Results in Fig. 6 and Fig. 7 shows that this algorithm works properly.

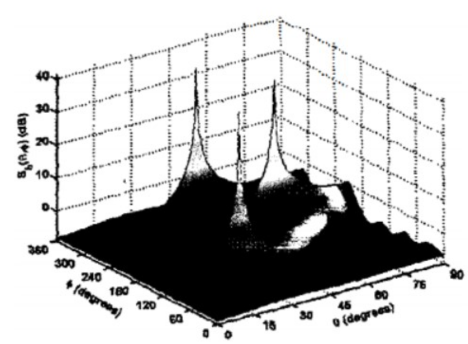

Fig. 6. The UCA-RB-MUSIC spectrum for Case 1 of Table 1. [6].

2) Comparison of Classical MUSIC with the UCA-RB-MUSIC Algorithm: The classical MUSIC algorithm is a high-resolution viral algorithm applicable to arbitrary but known configuration and response. An exciting investigation compares the classical MUSIC algorithm with the UCA-RB MUSIC algorithm, developed exclusively for uniform circular arrays.

After this comparison (detail in [6]) we can conclude that the classical MUSIC is great in any cases but UCA-RB-

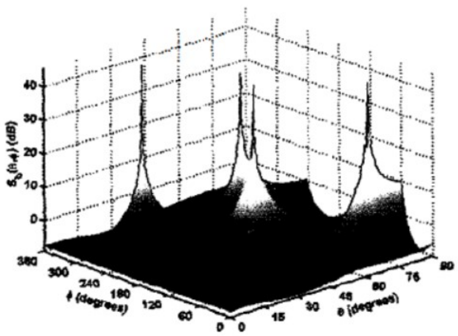

Fig. 7. The UCA-RB-MUSIC spectrum for Case 2 of Table 1 [6].

MUSIC was very poor in some cases. This is a significant disadvantage of the UCA-RB-MUSIC algorithm, since it can identify the directions of not more than N/2 sources with an N-element uniform circular array with excellent performance, compared to the classical MUSIC algorithm, which can resolve the directions of arrival from up to N sources with very adequate performance [6].

B. The UCA-ESPRIT Algorithm

We have seen UCA-RB-MUSIC and classical MUSIC algorithms benefit, but these two algorithms are intensive in computational aspects. However, the UCA-ESPRIT, which can be found in [7] is a powerful and simple algorithm to compute. It should be noted that UCA-ESPRIT is totally different from the classical algorithm.

1) Simulation Results of Direction Finding with the UCA-ESPRIT Algorithm: Simulation result of this algorithm can be found in Table III. As it can be seen this algorithm work properly until $SNR = -20$ which is a good performance.

TABLE III
THE DIRECTIONS OF ARRIVAL OF THE SIGNALS USED TO EVALUATE THE PERFORMANCE OF THE UCA-ESPIRT ALGORITHM [6].

Exact	
$\theta_1 = 15°$	$\phi = 150°$
$\theta_2 = 30°$	$\phi_2 = 210°$
$\theta_3 = 45°$	$\phi_3 = 330°$
SNR per equal-power source: $+10$ dB	
$\theta_1 = 15.0010°$, std $= 0.0276°$	$\phi_1 = 150.0066°$, std. $= 0.1387°$
$\theta_2 = 30.0159°$, std $= 0.0390°$	$\phi_2 = 209.9967°$, std. $= 0.0528°$
$\theta_3 = 45.2591°$, std. $= 0.0558°$	$\phi_3 = 329.9947°$, std $= 0.0465°$
SNR per equal-power source: -10 dB	
$\theta_1 = 14.9395°$, std. $= 0.3275°$	$\phi_1 = 149.8370°$, std. $= 1.9409°$
$\theta_2 = 30.0713°$, std. $= 0.5320°$	$\phi_2 = 210.1875°$, std. $= 0.8380°$
$\theta_3 = 45.2124°$, std. $= 0.7346°$	$\phi_3 = 330.0246°$, std. $= 0.7250°$
SNR per equal-power source: -20 dB	
$\theta_1 = 15.2311°$, std. $= 4.4905°$	$\phi_1 = 143.1731°$, std. $= 28.8028°$
$\theta_2 = 27.4981°$, std. $= 5.2720°$	$\phi_2 = 209.1216°$, std. $= 12.1141°$
$\theta_3 = 42.7270°$, std. $= 6.3526°$	$\phi_3 = 329.2504°$, std. $= 11.0682°$

VI. ADAPTIVE BEAMFORMING WITH UCA

The information supplied by the DOA algorithm is processed by means of an adaptive algorithm to ideally steer the maximum radiation of the antenna pattern toward the SOI and place nulls in the pattern toward the SNOIs by adjusting amplitude and phases of elements of UCA.

There are different techniques to perform adaptive-beamforming. One of the simplest is the least mean squares (LMS) algorithm. An injected pilot signal simulates a received signal from the desired look direction. This allows the array to be trained so that its directional pattern has a main lobe in the previously specified look direction. At the same time, the signal-processing unit can reject any incident interference with an angle of propagation different from that of the desired signal. By minimizing the mean square error between an a priori known training sequence and the signal received at the receiver, the complex excitation weights are updated at each iteration until stability is achieved. The entire process leads to a maximum of the antenna directivity pattern towards the direction of the desired signal, and nulls are formed at the interference angles. *It does not need the DOA information* but instead uses the reference signal, or training sequence, to adjust the magnitudes and phases of each weight to match the time delays created by the impinging signals into the array [1], [6].

A. LMS Method

MSE based cost function can be written as

$$\begin{aligned} J_{\mathrm{MSE}}\left(E\left[\varepsilon_k^2\right]\right) &= E\left[\left(d_k - \mathbf{w}^H \mathbf{x}_k\right)^2\right] \\ &= E\left[d_k^2 - 2 d_k \mathbf{w}^H \mathbf{x}_k + \mathbf{w}^H \mathbf{x}_k \mathbf{x}_k^H \mathbf{w}\right] \\ &= d_k^2 - 2\mathbf{w}^H E\left[d_k \mathbf{x}_k\right] + \mathbf{w}^H E\left[\mathbf{x}_k \mathbf{x}_k^H\right] \mathbf{w} \end{aligned} \tag{13}$$

where d_k is desired signal. w_k is weights needs to found. Moreover, it is an approximation of the steepest descent method using an estimator of the gradient instead of the actual value of the gradient, since computation of the actual value of the gradient is impossible because it would require knowledge of the incoming signals a priori. Therefore, at each iteration in the adaptive process, the estimate of the gradient is of the form

$$\widehat{\nabla}[J(\mathbf{w})]_k = \begin{bmatrix} \frac{\partial J(\mathbf{w})}{\partial w_0} \\ \vdots \\ \frac{\partial J(\mathbf{w})}{\partial w_L} \end{bmatrix} \tag{14}$$

where J is the cost function that has to minimized. Hence, according to the method of steepest descent, the iterative equation that updates the weights at each iteration is

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \widehat{\nabla}[J(\mathbf{w})]_k \tag{15}$$

where μ is the step size. In order to assure convergence of the weights, w_k, the step size μ is bounded by the condition [1].

$$0 < \mu < \frac{1}{\lambda_{\max}} \tag{16}$$

Other adaptive algorithms that may apply to UCAs are the recursive least square (RLS), the conjugate gradient (GC), the constant modulus (CM), and the decision-directed (DD) algorithms [6].

TABLE IV
THE DIRECTIONS OF ARRIVAL OF THE SIGNALS USED TO STUDY OUE-DIMENSIONAL ADAPTIVE BEAMFORMING WITH UNIFORM CIRCULAR ARRAYS EMPLOYING THE LEAST-MEAN-SQUARES ALGORITHM.

	Case 1
Signal of interest	$\theta_1 = 45°, \phi_1 = 0°$
Signals not of interest	$\theta_2 = 15°, \phi_2 = 0°$ $\theta_3 = 75°, \phi_3 = 0°$

If the signal environment is stationary, the weights are easily computed by solving the normal equations. However, in practice, the signal environment is dynamic or time varying, and therefore, the weights need to be computed with adaptive methods.

B. Simulation of One-Dimensional Adaptive Beamforming

In Fig. 8 to Fig. 11 we can see an example one-dimensional adaptive beamforming. We can see the information of ideal radiation pattern in Table IV. The result in Fig. 9 shows that adaptive beamforming works fine.

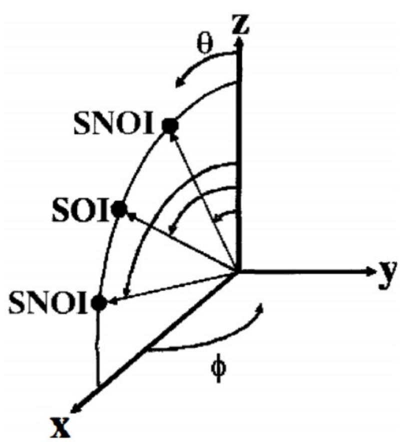

Fig. 8. One-dimensional adaptive beamforming along elevation for fixed azimuth.

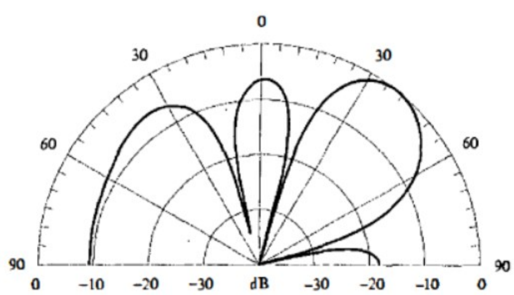

Fig. 9. The radiation pattcm with respect to θ for fixed ϕ (Case 2 of Table 4) resulting from one-dimensional adaptive beamforming along elevation for fixed azimuth

More detail on comparison and simulation of one-dimensional and two-dimensional adaptive beamforming can be found in [6].

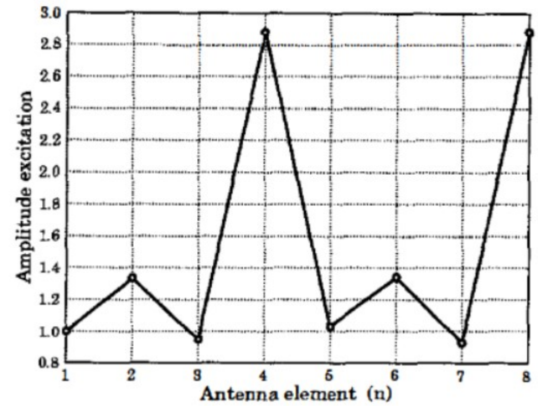

Fig. 10. One-dimensional adaptive beamforming along elevation for fixed azimuth.

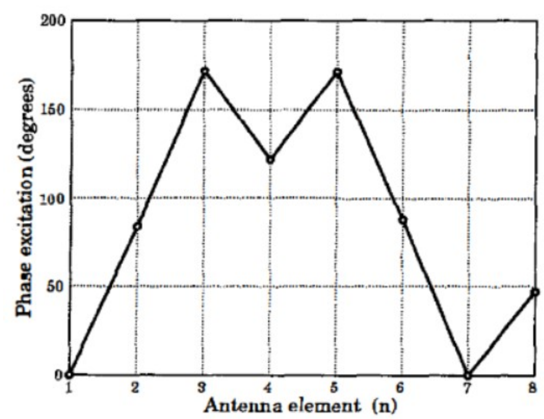

Fig. 11. One-dimensional adaptive beamforming along elevation for fixed azimuth.

VII. MUTUAL COUPLING

The mutual coupling between antenna elements affects the antenna parameters like terminal impedances, reflection coefficients, antenna array performance in terms of radiation characteristics, output signal-to-interference noise ratio (SINR), and radar cross-section (RCS). This coupling effect is also known to directly or indirectly influence the steady-state and transient response, the resolution capability, interference rejection, and direction-of-arrival (DOA) estimation competence of the array. Researchers have proposed several techniques and designs for optimal phased array performance in a given signal environment, counteracting the coupling effect. MOre detail can be found in [8].

VIII. CONCLUSION

In this paper, a brief on smart antennas has been proposed. Rectangular and linear arrays have been investigated briefly to propose an overview of smart antennas. The Uniform circular array has been investigated in detail. Two significant issues (estimation of DOA and adaptive beamforming) in smart antennas have been discussed, and mutual coupling has been introduced. For DOA estimation, two primary methods have been investigated. In the first approach, two versions of the MUSIC algorithm have been discussed; classical MUSIC and UCA-RB-MUSIC, but classical ones have better performance than modified one.

The other approach for direction-of-arrival estimation examined in this paper was the UCA-ESPRIT algorithm. Within the simulation, results were impressively precise compared to the exact directions of arrival, even for scenarios where the noise power was dominant over the incoming signals. This approach, due to the computational aspects, can be favored over MUSIC approaches.

For beamforming issues, the uniform circular array was shown to have excellent beamforming capabilities in directing the maximum towards the direction of the signal of interest and deep nulls towards the directions of the signals not of interest. LMS method has been introduced briefly, which is an efficient method for beamforming (finding excitation amplitude and phase for each element in the array). Also, one-directional applications of this method have been simulated, which shows an accurate result for beamforming. Also, the mutual coupling issue has been introduced, and some papers have been proposed to deal with this problem in UCAs.

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