Functional programming, Seminar No. 2

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Bindings

The equality sign in Haskall denotes binding:

```
fortyTwo = 42
coolString = "coolString"
```

Local binding with the `let`-keyword:

```
fortyTwo = let number = 43 in number - 1
```

Function definitions

Functions are also defined as bindings:

```
add x y = x + y
userName name = "Username: " ++ name
id x = x
```

The same functions defined via lambda:

```
add = \x y -> x + y
userName = \name -> "Username: " ++ name
id = \x -> x
```

Function application

As in lambda calculus, function application is right associative by default

```
{-
foo x y z = f x y z = ((f x) y) z
-}
```

One may use the dollar infix operator in order to avoid brackets overuse. For example, the following functions have exactly the same behaviour:

```
function  f x y z = f ((x y) z)
function1  f x y z = f $ x y $ z
```

Prefix and infix notation

Any operator or function might be called in prefix and infix:

```
Prelude> map (\x -> x * pi) [1..5]
[3.141592653589793,6.283185307179586,9.42477796076938,12.566370614359172,15.707963267948966]
Prelude> (\x -> x * pi) `map` [1..5]
[3.141592653589793,6.283185307179586,9.42477796076938,12.566370614359172,15.707963267948966]
Prelude> (+) 19 76
95
Prelude> 19 + 76
95
```

One may declare an operator defining its priority and associativity explicitly. Here's an example:

```
(&&) :: Bool -> Bool -> Bool
infixr 3 &&
```

Currying and partial application

Let us recall the function `add` once more:

```
  add x y = x + y
```

Here is an example of a partial application in the following GHCi session

```
add x y = x + y
addFive = add 5
twentyEight = addFive 23
  -- 28
```

Partial application is well-defined since all many-argument functions in Haskell are curried by default.

Immutability and laziness

In Haskell, values are immutable. The GHCi example:

```
Prelude> list = [1,2,3,4]
Prelude> reverse list
[4,3,2,1]
Prelude> list
[1,2,3,4]
Prelude> 10 : list
[10,1,2,3,4]
Prelude> list
[1,2,3,4]
```

Recursion

The straightforward recursion:

```
factorial n = if n == 0 then 1 else n * factorial (n - 1)
```

The factorial function implemented via so-called tail recursion:

```
tailFactorial n = helper 1 n
 where
 helper acc x =
   if x > 1
   then then helper (acc * x) (x - 1)
   else acc
```

Guards

Let us take a look at the factorial implementation via guards:

```
tailFactorial  n = helper 1 n
  where
  helper  acc x | x > 1 = helper (acc * x)  (x - 1)
                | otherwise acc
```

Basic datatypes

The basic datatypes are:

- `Bool`: Boolean values
- `Int`: Bounded integer datatype
- `Integer`: Unbounded integer datatype
- `Char`: Unicode characters
- `()`: Unit value datatype
- If `a` and `b` are types, then `a -> b` is a type
- If `a` and `b` are types, then `(a,b)` is a type
- If `a` is a type, then `[a]` is a type

A type declaration has the following form:

```
term :: type
```

Datatypes and constructors

Let us recall the list of basic data types above and associative constructors with them. A constructor is a term that allows one to obtain a value of the desired type.

`Bool`: Boolean values	`True` and `False`
`Int: Bounded integer datatype`	Integers from -2^{29} to $2^{29}-1$
`Integer`: Unbounded integer datatype	The set of integers
`Char`:	Characters `'0'`, ..., `'9'`, `'a'`, ..., `'z'`, etc
`()`: Unit value datatype:	Just `()`
`a -> b`:	$\lambda x \to m$
`(a,b)`:	if `x :: a` and `y :: b`, then `(x, y) :: (a,b)`
`[a]`, the type of list of elements from a:	the empty list `[]`
`[a]`, the type of list of elements from a:	if `x :: a` and `xs :: [a]`, then `x : xs :: [a]`

Types in GHCi

The command `:t` yields a type of a required expression:

```
Prelude> :t 5
5 :: Num p => p
Prelude> :t (||)
(||) :: Bool -> Bool -> Bool
Prelude> :t [0.5,0.6]
[0.5,0.6] :: Fractional a => [a]
Prelude> :t (\x -> x ++ "guten tag mein ")
(\x -> x ++ "guten tag mein ") :: [Char] -> [Char]
Prelude> :t '/'
'/' :: Char
```

Function declaration with datatypes

Let us recall the examples of function declarations:

```
add x y = x + y
userName name = "Username: " ++ name
id x = x
```

One may annotate these functions with types as follows:

```
add :: Int -> Int -> Int
add x y = x + y

userName :: String -> String
userName name = "Username: " ++ name

id :: Char -> Char
id x = x
```

Note that such calls as `userName 5` or `id ''hello stewart''` cause type errors.

Lists

Let's talk about lists a slightly closely. In Haskell, a list is a homogeneous collection of elements.

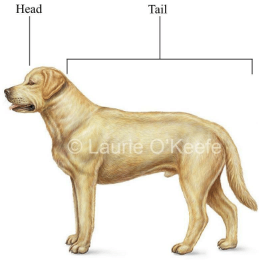
```
empty :: [Int]
empty = []

ten  :: [Int]
ten  = [10]

tenEleven  :: [Int]
tenEleven  = 11 : ten

tenElevenTwelve :: [Int]
tenElevenTwelve = 12 : tenEleven
-- 12 : (11 : [])
```

Lists. Ranges

```
oneToFive :: [Int]
oneToFive = [1..5]

oneToSevenOdd :: [Int]
oneToSevenOdd = [1,3..7]

nat :: [Int]
nat [0,1..]

evens :: [Int]
evens = [0,2,4..]
```

Lists. Heads and Tails

```
Prelude> tail [1..5]
[2,3,4,5]
Prelude> head [1..5]
1
Prelude> head []
*** Exception: Prelude.head: empty list
Prelude> tail []
*** Exception: Prelude.tail: empty list
```



Dogs according to Haskell

Other helpful list functions

```
Prelude> drop 3 [1..5]
[4,5]
Prelude> take 4 [1..10]
[1,2,3,4]
Prelude> replicate 3 "dratuti"
["dratuti","dratuti","dratuti"]
Prelude> zip [1,2,3] "abc"
[(1,'a'),(2,'b'),(3,'c')]
Prelude> unzip [(1,'a'),(2,'b'),(3,'c')]
([1,2,3],"abc")
Prelude> words "Anna Daniel \t\n\n LeonidYakubovich"
["Anna","Daniel","LeonidYakubovich"]
Prelude> [34..72] !! 7
41
```

List compeherension

```
Prelude> [ (i, j) | i <- [1..10], j <- [2..12], j - i < 4, j + i > 20 ]
[(9,12),(10,11),(10,12)]
Prelude> take 10 [ (i,j) | i <- [1..], j <- [1..i-1], gcd i j == 1 ]
[(2,1),(3,1),(3,2),(4,1),(4,3),(5,1),(5,2),(5,3),(5,4),(6,1)]
Prelude> [ c | c <- "the picture of dorian grey", c < 'o']
"he ice f dian ge"
Prelude> [ c | c <- "the picture of dorian grey", c < 'o', fromEnum c < 104 ]
"e ce f da ge"
```

Higher order functions

Function is a first-class object and one may pass any function as an argument:

```
inc, dec :: Int -> Int
inc x = x + 1
dec x = x - 1

changeTwiceBy :: (Int -> Int) -> Int -> Int
changeTwiceBy operation value = operation (operation value)

seven :: Int
seven = changeTwiceBy inc 5

three :: Int
three = changeTwiceBy dec 5
```

Case-expressions

Case-expressions allows one to perform case analysis within an observed function.

```
getFont :: Int -> String
getFont n =
  case n of
    0 -> "PLAIN"
    1 -> "BOLD"
    2 -> "ITALIC"
    _ -> "UNKNOWN"
```

Theoretical Flashback. Reduction strategies

Here we recall some of relatable definition from lambda calculus:

1. A term M is called *weakly normalisable* (WN), if there exists some halting reduction path that starts from M
2. A term M is called *strongly normalisable* (SN), if any reduction path that starts from M terminates

It is clear, that SN implies WN, not vice versa. In other words, there exists a term, that has an infinite reduction path, but it has a finite reduction path.

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$(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta$
$(\lambda y.[x := (\lambda z.z)])((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta$
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$(\lambda z.z)[y := (\lambda x.xx)(\lambda x.xx)] \rightarrow_\beta$
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From the other hand:

$(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta$
$(\lambda xy.x)(\lambda z.z)(xx)(x := [\lambda x.xx]) \rightarrow_\beta$
$(\lambda xy.x)(\lambda z.z)((\lambda x.xx)(\lambda x.xx)) \rightarrow_\beta \dots$
spasiti pamagiti pajalusta ya tak bolshe ni magu (((((((9(99(

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- Can we distinguish all possible ways of term reduction?

In a matter of fact, we need to distinguish all possible ways of application reduction, so far as we have no other options in the remaining cases:

1. If x is a variable, then x is already in normal form
2. If a term has the form $\lambda x.M$, then we reduce M

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Theorem

Let M be a term such that M has a normal form M', then M might be reduced to M' via normal order

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- The Haskell reduction has a call-by-value strategy. Informally, such a stragety is called *lazy*. Laziness denotes that Haskell doesn't compute a value if it's not needed at the moment
- Call-by-name reduction reduces reducible terms to the bitter end, but it's not always optimal, unfortunately

Haskell reduction

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We evalutate $(1+2)$ twice, even if we know that $1+2=3$ a priori. I sincerely hope that you have no doubts about it. The question of optimality is still relevant.

The notion of a weak head normal form

In Haskell, reduction evalutates a term to a weak head normal form, where the outermost must be either constructor or lambda. Here are example: WHNFs from the left and non-WHNFs from the right

```haskell
78

2 : [1,2]

'p' : ("ri" ++ "vet")

[1, 1 + 2, 1 + 3]

("hel" ++ "lo", "world")

\x -> (x + 2) + 2

\xs -> zip xs [1, 3+2]
```

```haskell
1 + 665

(\x -> x ++ "guten tag mein herr ") "Heinrich"

length [1..145]

(\f g x -> f (g x)) $ (\x y -> y)
```

Pure functions and side-effects

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- A side-effect function is a function that may yield different value passing the same arguments. Mathematically, such a function is not function at all.
- Haskell functions are (mostly) pure ones, but Haskell isn't confluent as a version of lambda calculus

The failure of confluece

Let us consider the following quite simple example. In Haskell one has a function called `seq`. According to Hackage, "The value of `seq a b` is bottom if a is bottom, and otherwise equal to b." The listing below demostrates the failure of confluence:

```
seq :: a -> b -> b
seq _|_ _ = _|_
seq _ b  = b

dno = undefined

seq dno 14        == dno
seq (dno . id) 14 == 14
```

Finally

On this seminar, we

- got acquinted with the basic Haskell syntax and basic data types
- learnt the underlying aspects of Haskell semantics
- discussed pure functions and the example of the confluence failure

On the next seminar, we

- start to learn polymorphism and its advantages
- introduce typeclasses
- study the first examples of crucially important examples of type classes