

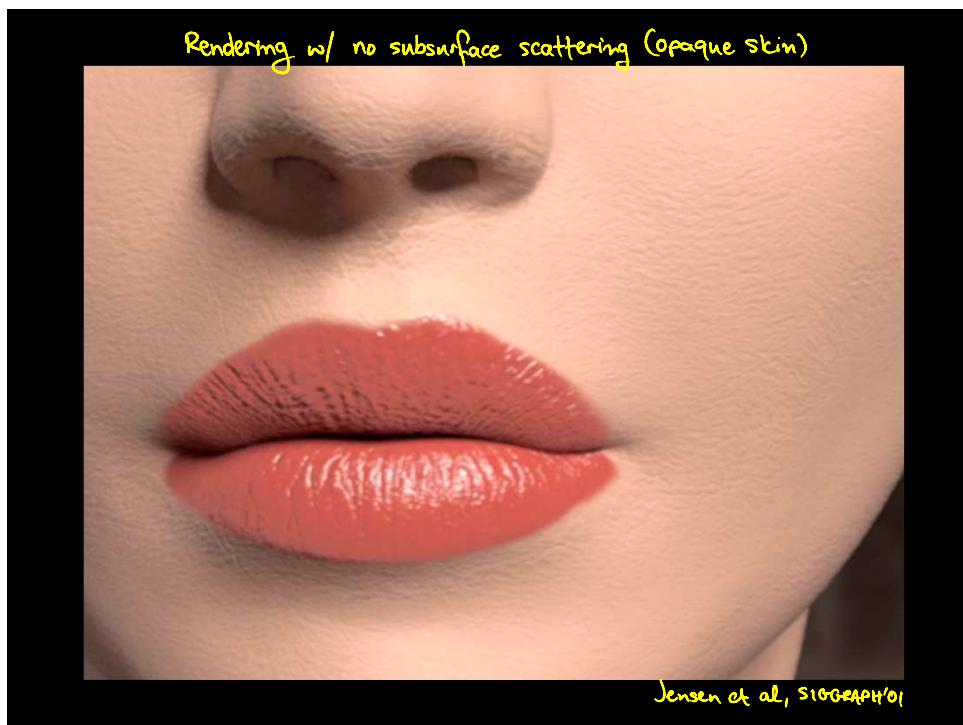
Today's Topics

11. Texture mapping
12. Introduction to ray tracing

Topic 11:

Texture Mapping

- Motivation
- Sources of texture
- Texture coordinates
- Controlling surface appearance with textures
- Texture mapping & scan conversion
- Perspectively-correct texture mapping
- Bump mapping, mip-mapping & env mapping



Texture Mapping: Motivation



Goal: Endow objects with more varied & realistic appearance through complex variations in reflectance

Key features of texture mapping

- efficient to render
- reusable
- can modulate albedo and/or fine geometry



Introduction to Texture Mapping



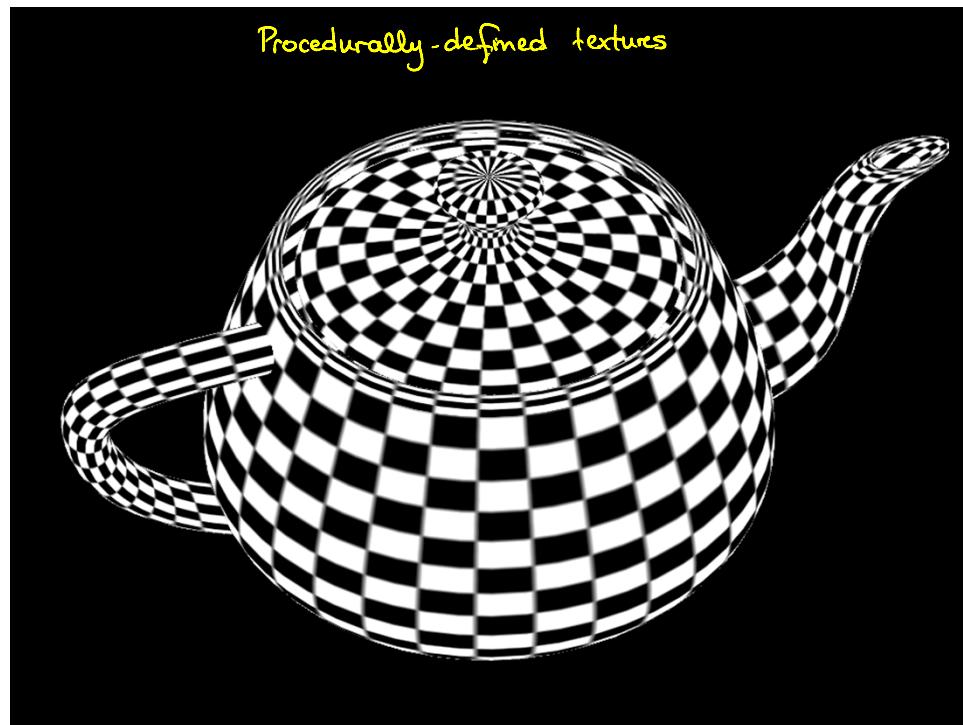
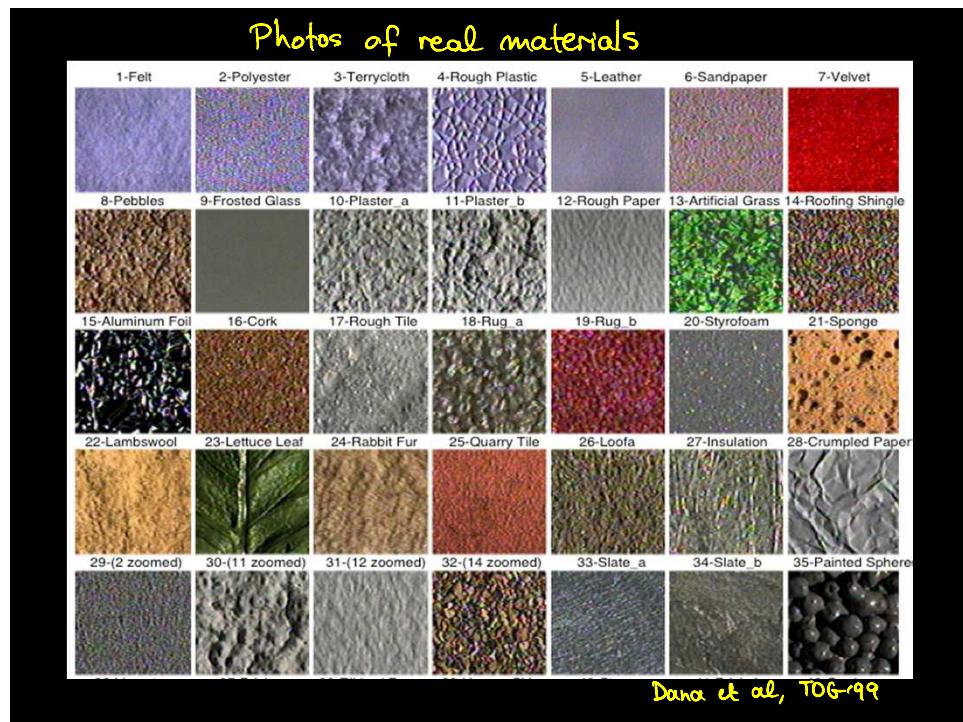
Basic questions:

1. Where do textures come from?
2. How do we map textures onto surfaces?
3. How can textures be used to control appearance?
4. How do we integrate texture mapping and scan conversion?

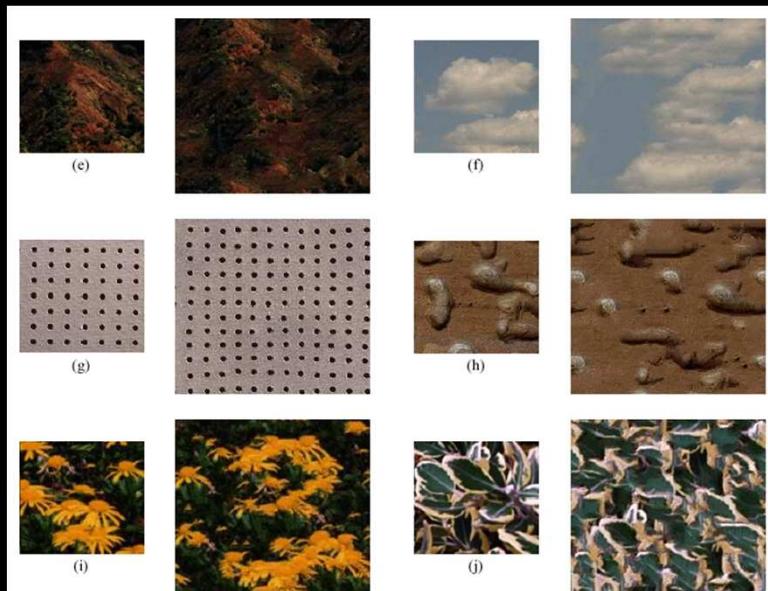
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Texture Synthesis



Kwatra et al, SIGGRAPH'05

Texture Synthesis

Original Synthesized



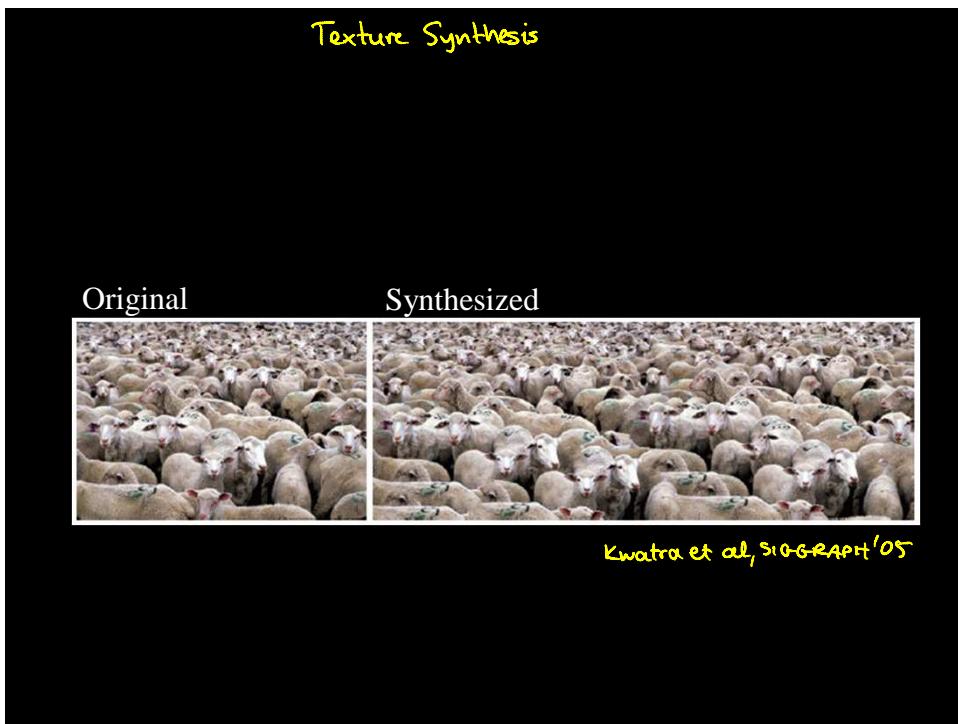
Original



Synthesized



Kwatra et al, SIGGRAPH'05

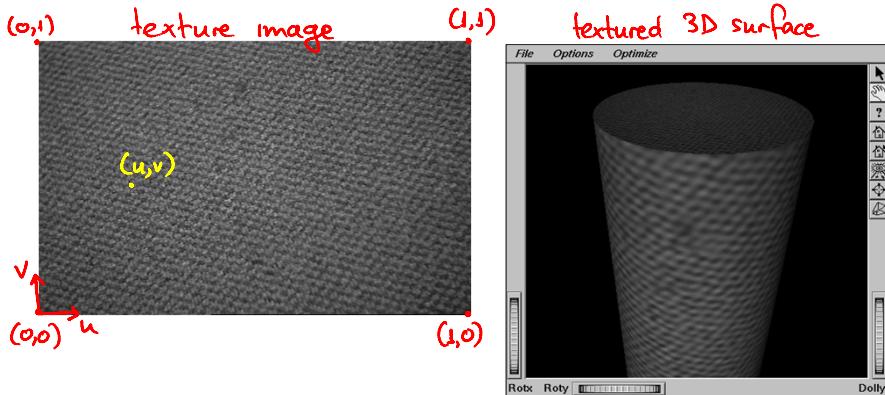


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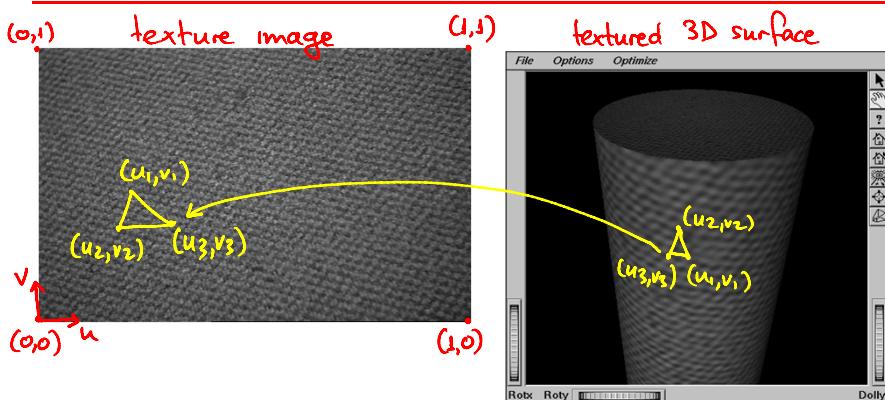
Texture Coordinates



Key ideas

- define a 2D coordinate system for texture image (called texture coordinates)
- define a mapping from triangles to texture coords

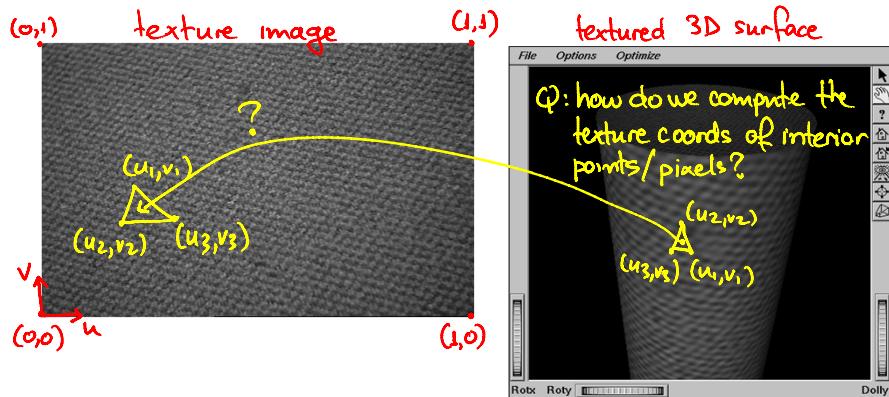
Specifying a Triangle's Texture Coordinates



Two ways to define the mapping:

- ① for each face of mesh, specify the texture coordinates of each vertex of the face
- ② define a continuous mapping from surface pts to texture pts

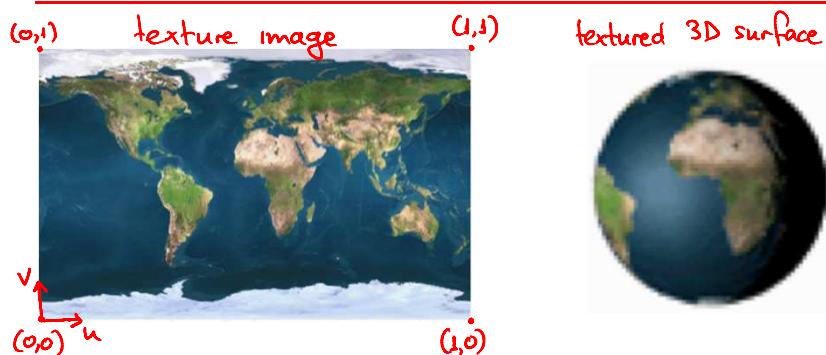
Texture Coords from Triangle Vertices



Two ways to define the mapping:

- ① for each face of mesh, specify the texture coordinates of each vertex of the face
- ② define a continuous mapping from surface pts to texture pts

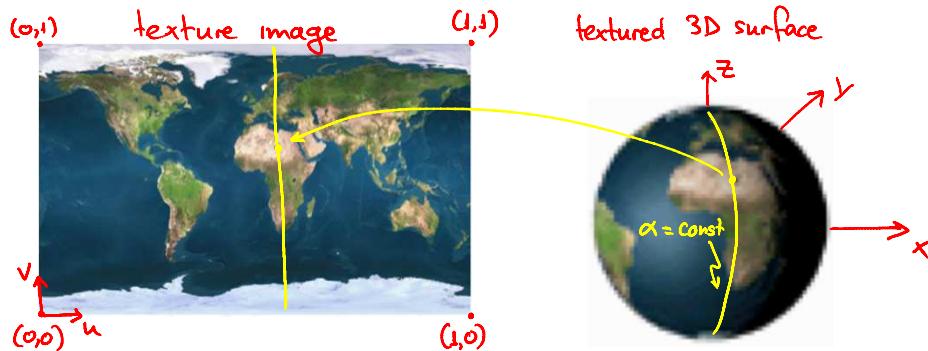
Texture Coords from Surface Parameters



Two ways to define the mapping:

- ① for each face of mesh, specify the texture coordinates of each vertex of the face
- ② define a continuous mapping from surface pts to texture pts

Example: Mapping Textures on a Sphere



texture coords of $s(\alpha, \beta)$:

$$u = \frac{\alpha}{2\pi}$$
$$v = \frac{\beta}{\pi}$$

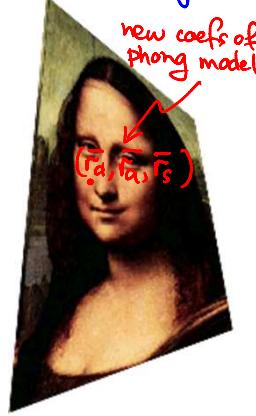
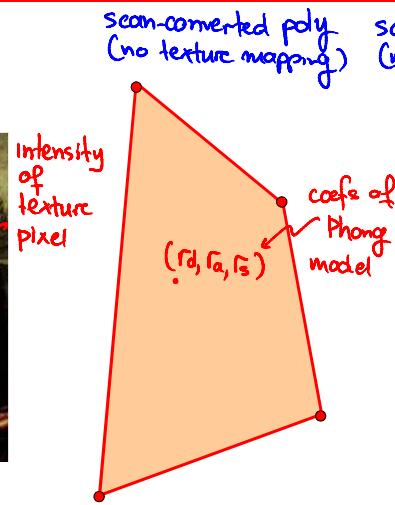
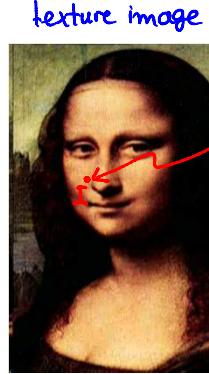
$$s(\alpha, \beta) = \begin{bmatrix} x_0 + r \cos \alpha \sin \beta \\ y_0 + r \sin \alpha \sin \beta \\ z_0 + r \cos \beta \end{bmatrix}$$
$$\alpha \in [0, 2\pi)$$
$$\beta \in [0, \pi)$$

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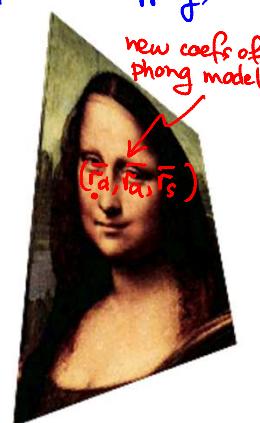
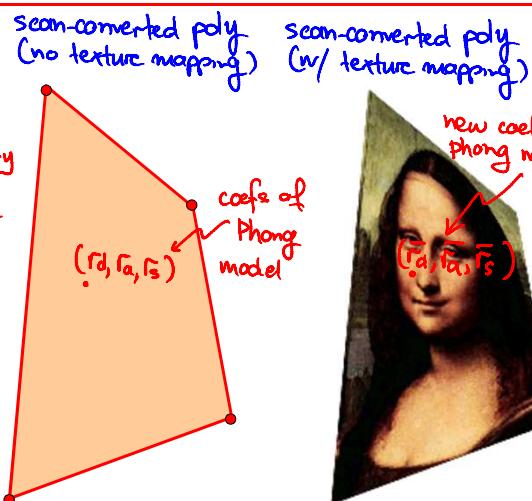
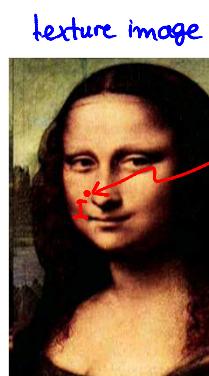
Appearance Control via Texture Mapping



Approach #1 (modulation)

$$\bar{r}_d = I \cdot r_d, \quad \bar{r}_a = I \cdot r_a, \quad \bar{r}_s = I \cdot r_s \quad (\text{assumes } I \text{ normalized to range } [0,1])$$

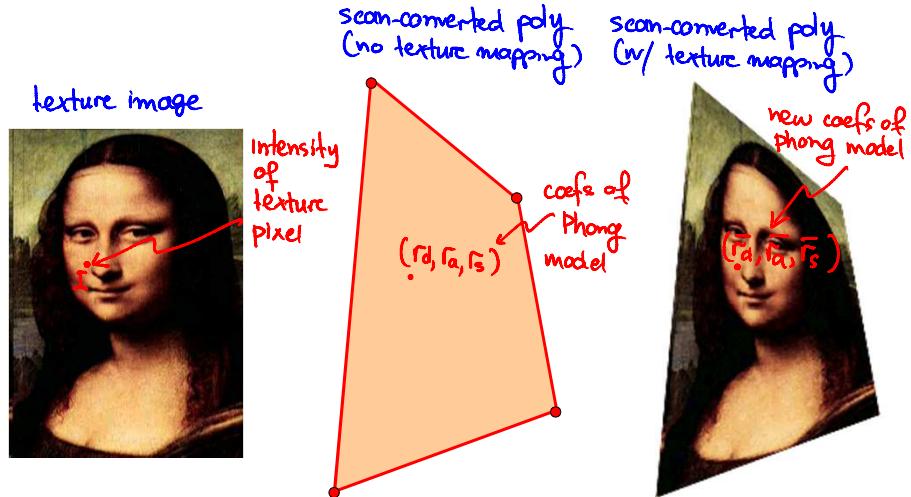
Appearance Control via Texture Mapping



Approach #2 (replacement)

$$\bar{r}_d = I \quad \bar{r}_a = I \quad \bar{r}_s = I$$

Appearance Control via Texture Mapping



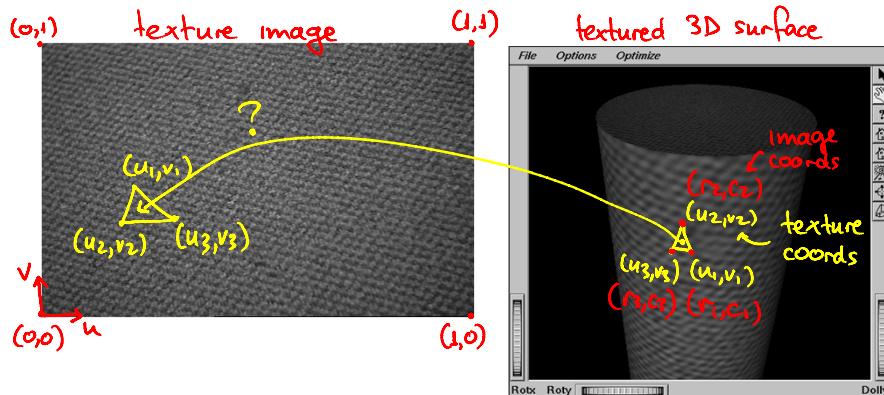
Other options also possible – see `glTexEnvf()` in OpenGL

Topic 11:

Texture Mapping

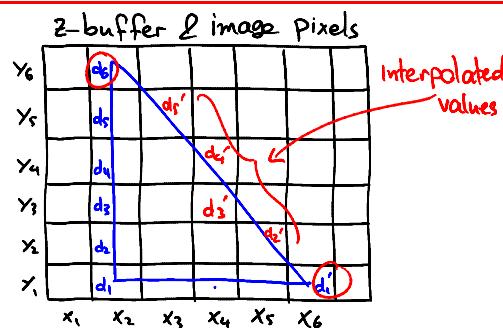
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Texture Coords of Pixels in Triangle's Interior



Q: how do we determine the texture coordinates of interior polygon pixels during scan-conversion?

Reminder: Basic Scan Conversion



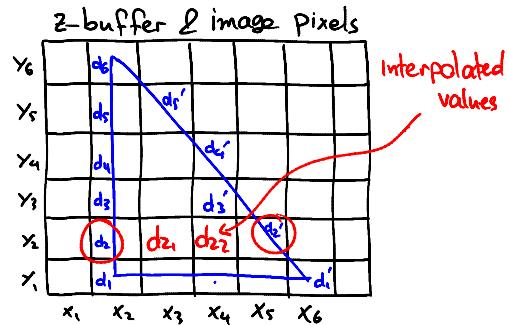
Step a:

for each edge,
interpolate d, L, \dots
between its two
vertices

Step b:

for each scanline
interpolate d, L, \dots
between two edge
pixels

Reminder: Basic Scan Conversion



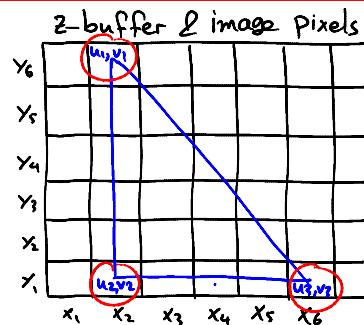
Step a:

for each edge,
interpolate d, L, \dots
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Step b:

for each scanline
interpolate d, L, \dots
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Should we Interpolate Texture Coordinates?

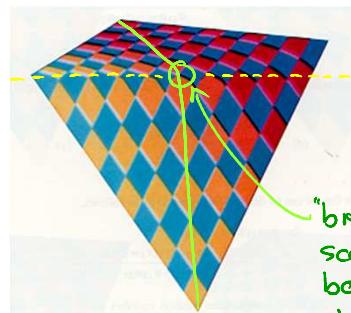


Q: Will this interpolation scheme
also work for texture coordinates?

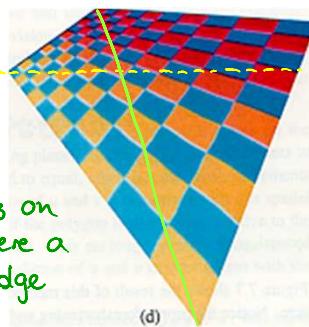
Not very well!

Artifacts Caused by Naive Scan Conversion

Scan-converted polygon



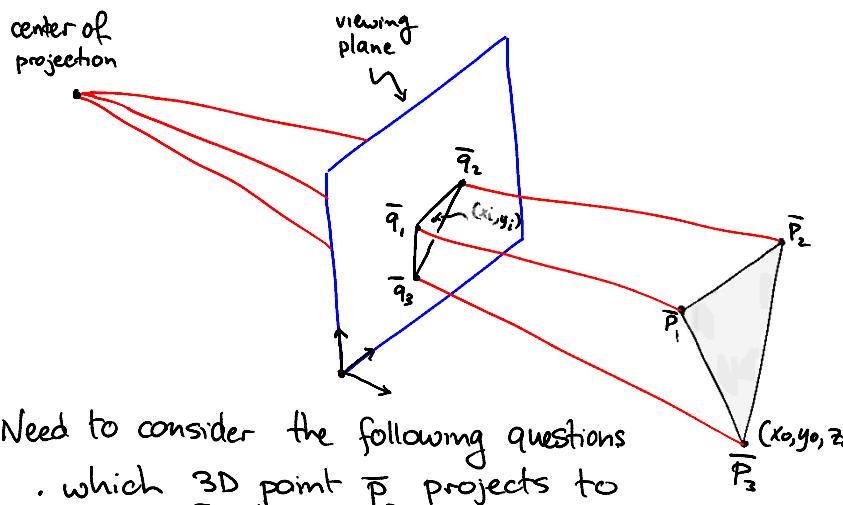
Correct appearance



"break" occurs on
scanline where a
bounding edge
changes

(d)
Wolberg, 1991

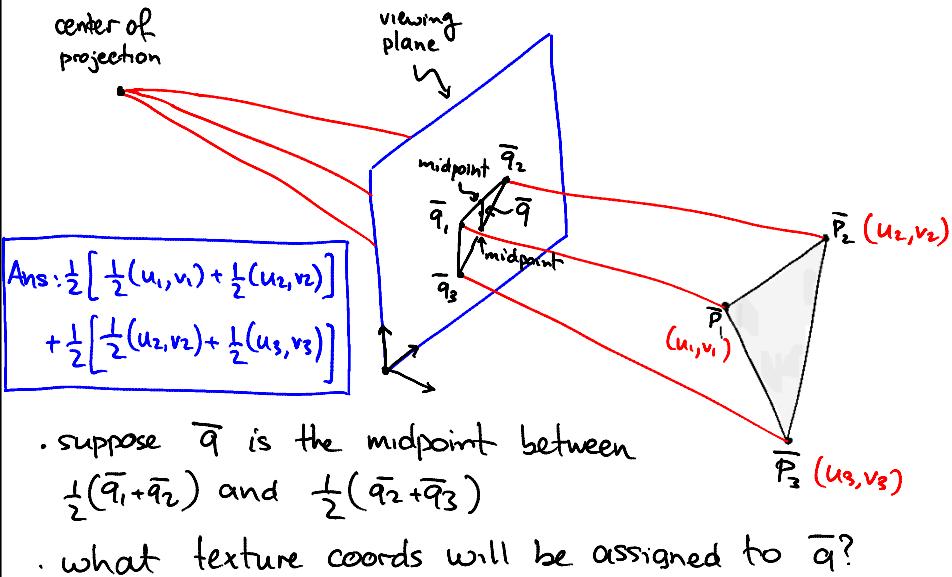
Why Do These Artifacts Occur?



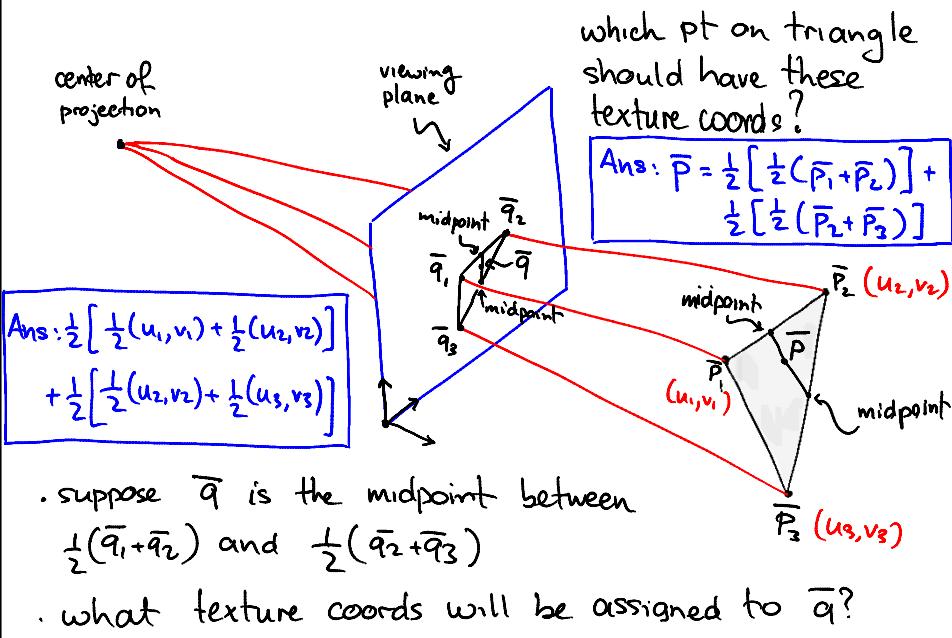
Need to consider the following questions

- which 3D point \bar{p} projects to pixel $\bar{q} = (x_i, y_i)$?
- what are the texture coords of point \bar{p} ?

Why Do These Artifacts Occur?



Why Do These Artifacts Occur?



Why Do These Artifacts Occur?

The diagram shows a 3D scene with a viewing plane. A point \bar{P} on the viewing plane is projected from a point P on a triangle in 3D space. The triangle has vertices $\bar{P}_1, \bar{P}_2, \bar{P}_3$ with texture coordinates $(u_1, v_1), (u_2, v_2), (u_3, v_3)$. The projection of P onto the viewing plane is labeled \bar{q} . The diagram highlights several midpoints: midpoints between \bar{q}_1, \bar{q}_2 and \bar{q}_1, \bar{q}_3 , and midpoints between \bar{P}_1, \bar{P}_2 and \bar{P}_1, \bar{P}_3 . Handwritten text asks, "which pt on triangle should have these texture coords?" and provides the answer:

$$\text{Ans: } \bar{P} = \frac{1}{2} \left[\frac{1}{2} (\bar{P}_1 + \bar{P}_2) \right] + \frac{1}{2} \left[\frac{1}{2} (\bar{P}_2 + \bar{P}_3) \right]$$

Below the diagram, two points are listed:

$$\text{Ans: } \frac{1}{2} \left[\frac{1}{2} (u_1, v_1) + \frac{1}{2} (u_2, v_2) \right] + \frac{1}{2} \left[\frac{1}{2} (u_2, v_2) + \frac{1}{2} (u_3, v_3) \right]$$

$$+ \frac{1}{2} \left[\frac{1}{2} (u_2, v_2) + \frac{1}{2} (u_3, v_3) \right]$$

- suppose \bar{q} is the midpoint between $\frac{1}{2}(\bar{q}_1 + \bar{q}_2)$ and $\frac{1}{2}(\bar{q}_2 + \bar{q}_3)$
- problem: \bar{P} will not project to \bar{q} in general

Why Do These Artifacts Occur?

The diagram is identical to the one above, showing a 3D scene with a viewing plane. A point \bar{P} on the viewing plane is projected from a point P on a triangle in 3D space. The triangle has vertices $\bar{P}_1, \bar{P}_2, \bar{P}_3$ with texture coordinates $(u_1, v_1), (u_2, v_2), (u_3, v_3)$. The projection of P onto the viewing plane is labeled \bar{q} . The diagram highlights several midpoints: midpoints between \bar{q}_1, \bar{q}_2 and \bar{q}_1, \bar{q}_3 , and midpoints between \bar{P}_1, \bar{P}_2 and \bar{P}_1, \bar{P}_3 . Handwritten text asks, "which pt on triangle should have these texture coords?" and provides the answer:

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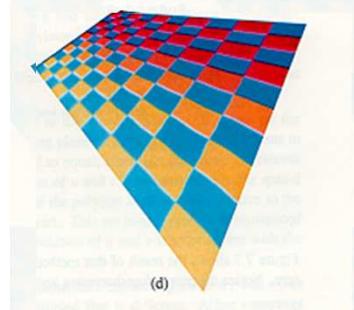
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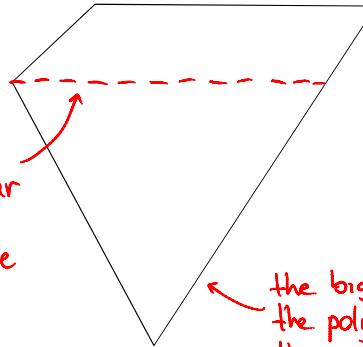
$$+ \frac{1}{2} \left[\frac{1}{2} (u_2, v_2) + \frac{1}{2} (u_3, v_3) \right]$$

- perspective projection does NOT preserve ratios!
- problem: \bar{P} will not project to \bar{q} in general

Reducing Artifacts of Naive Scan Conversion



break
will occur
on this
scanline



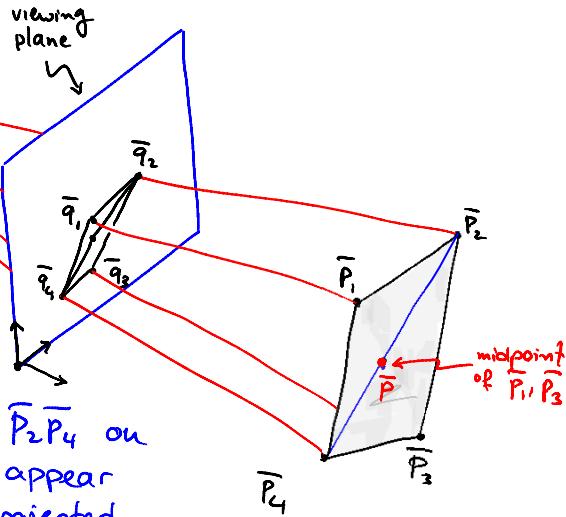
the bigger
the polygon,
the more
noticeable
the artifacts

Reducing Artifacts of Naive Scan Conversion

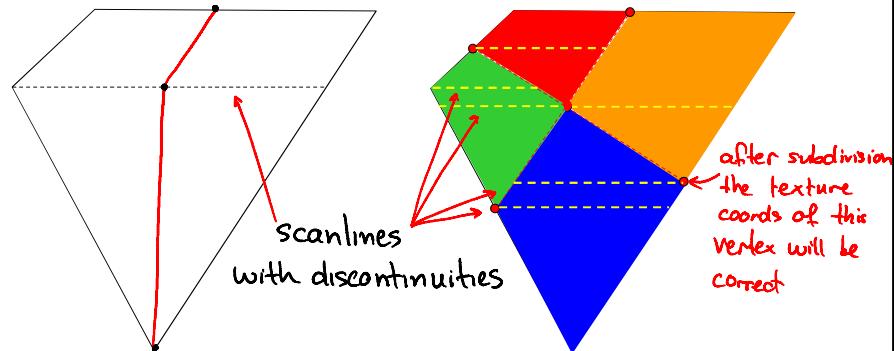
center of
projection

viewing
plane

The straight line $\overrightarrow{P_2 P_4}$ on
the polygon will appear
“broken” in the projected
triangle

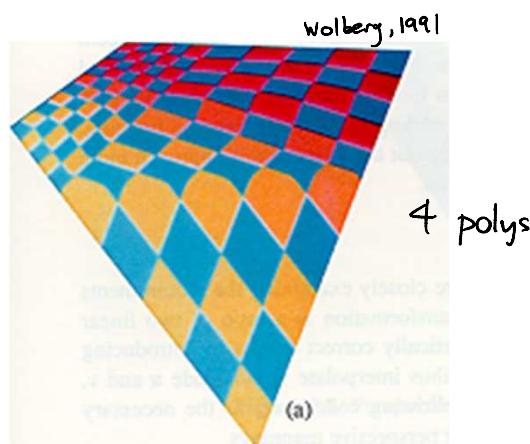


Reducing Artifacts of Naive Scan Conversion



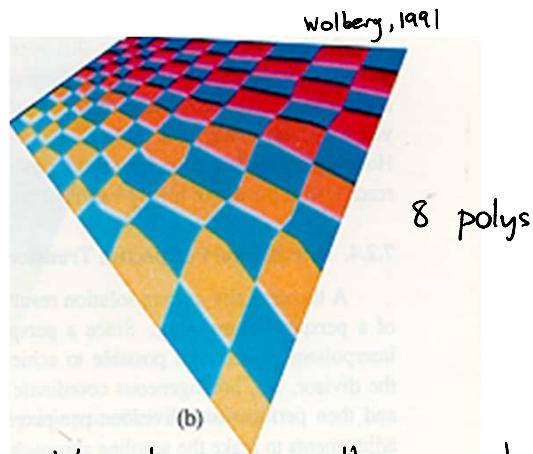
It is possible to reduce the magnitude of distortions by subdividing the polygon

Reducing Artifacts by Polygon Subdivision



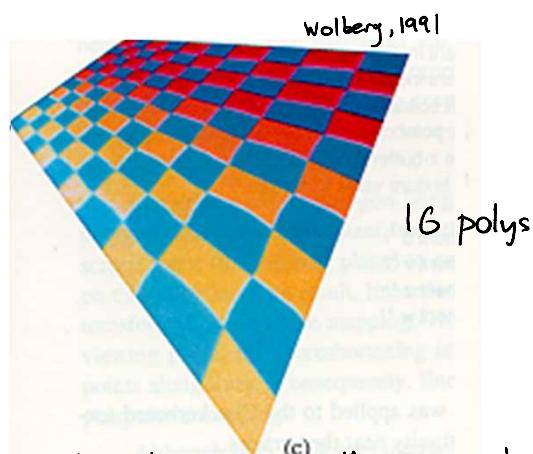
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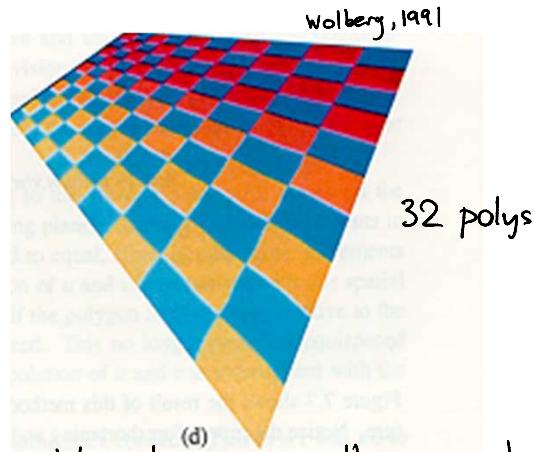
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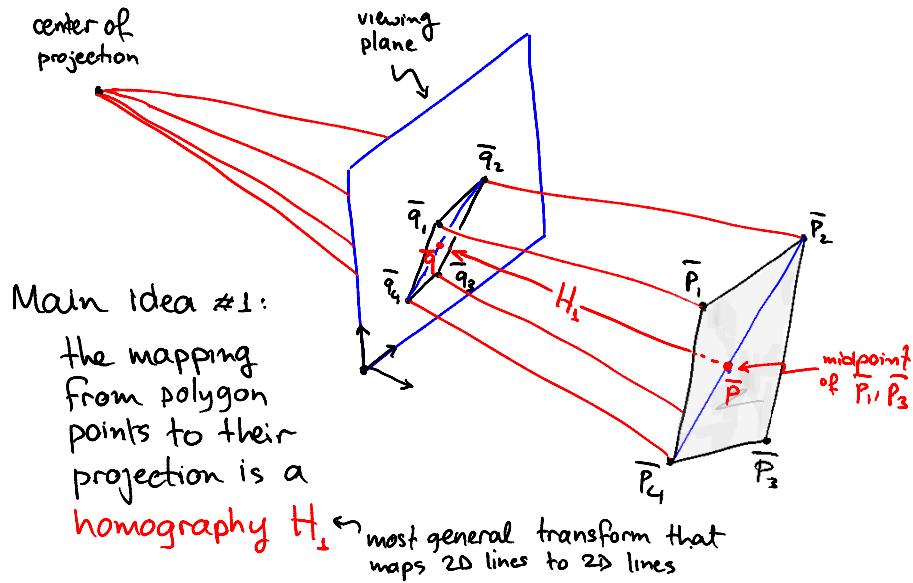
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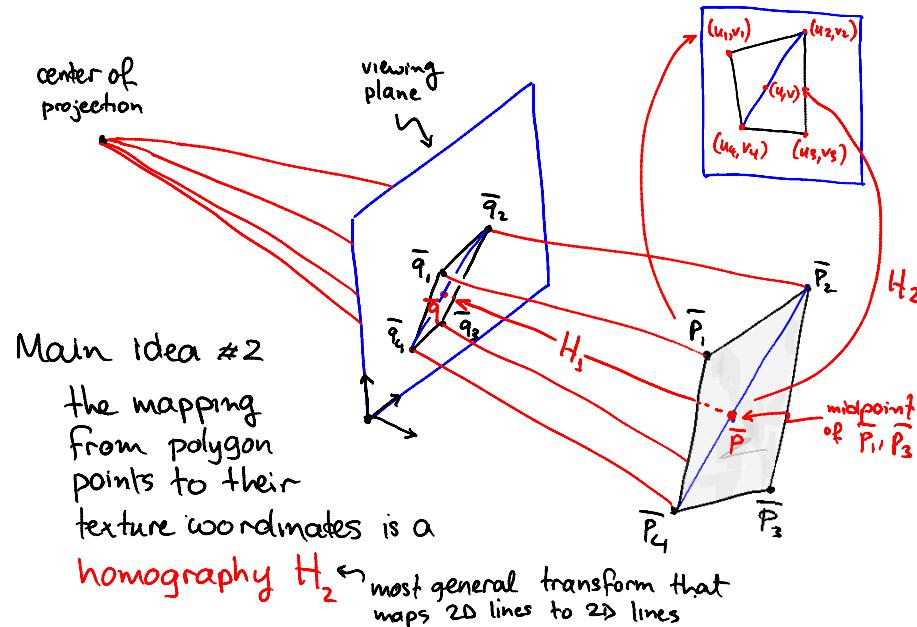
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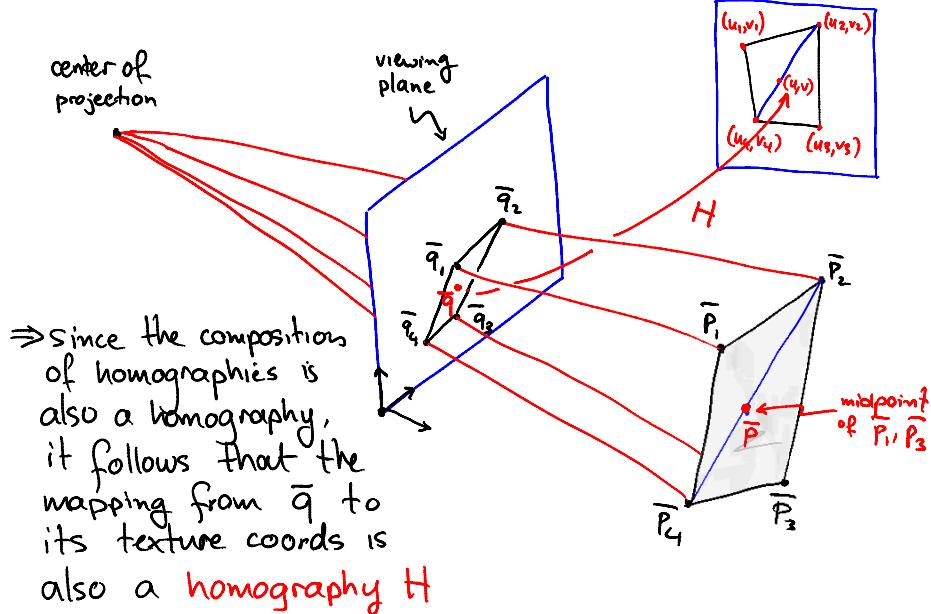
Perspectively-Correct Texture Mapping



Perspectively-Correct Texture Mapping



Perspectively-Correct Texture Mapping



Perspectively-Correct Texture Mapping

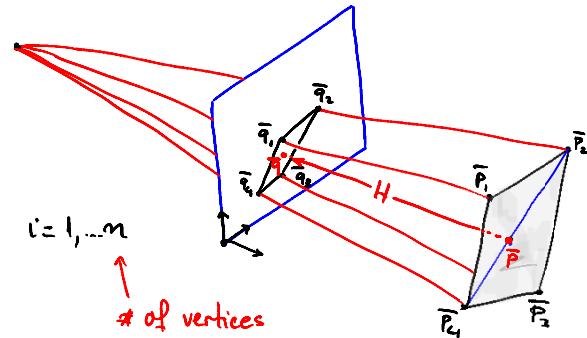
- ① Compute the homography H such that

$$H \bar{q}_i \approx \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} \quad i=1, \dots, n$$

projection of
ith vertex in
homogeneous 2D
coords

texture coords
of ith vertex
express as
homogeneous
vectors

of vertices



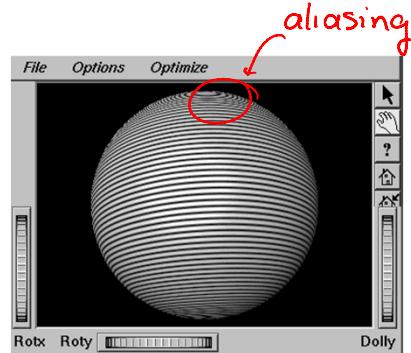
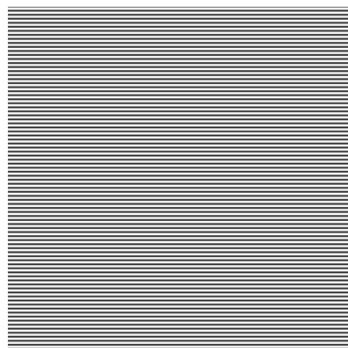
- ② Use H to compute the texture coords of a pixel $\bar{q} = (x_i, y_i, 1)$ in homogeneous coords

Topic 11:

Texture Mapping

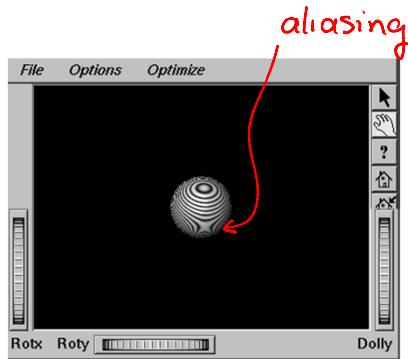
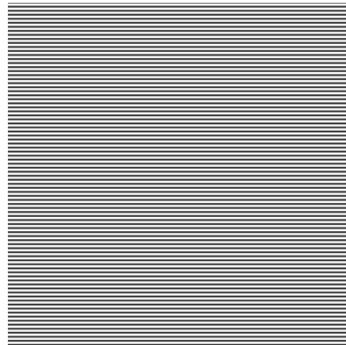
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Aliasing During Texture Mapping



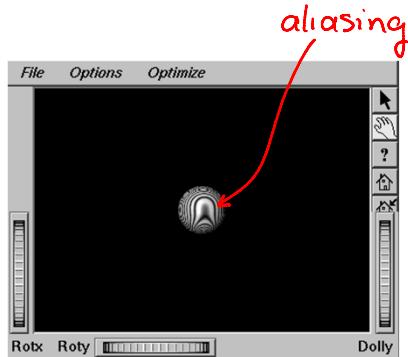
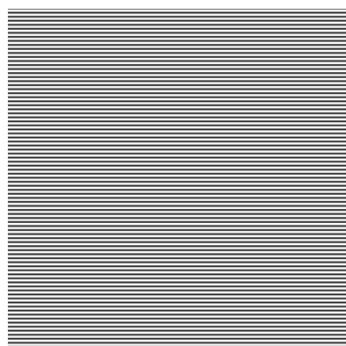
Texture mapping can produce significant aliasing artifacts when the texture images contain rapid variations (aka "high frequencies")

Aliasing During Texture Mapping



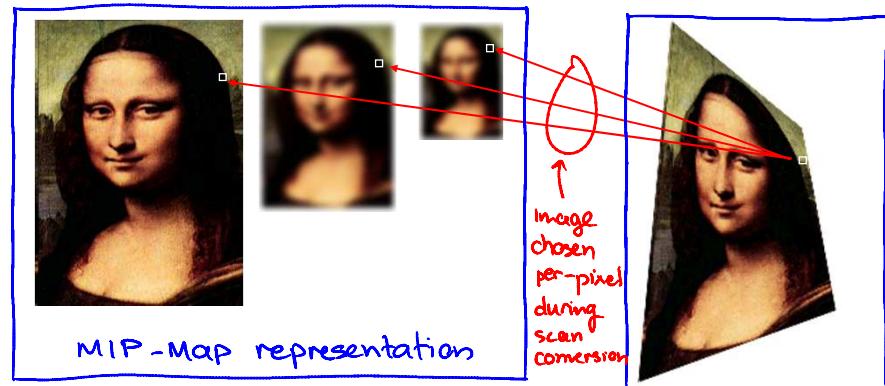
Texture mapping can produce significant aliasing artifacts when the texture images contain rapid variations (aka "high frequencies")

Aliasing During Texture Mapping



Texture mapping can produce significant aliasing artifacts when the texture images contain rapid variations (aka "high frequencies")

MIP-Mapping: Basic Idea



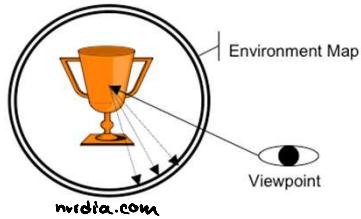
Solution: Use high-resolution texture for rendering objects that are close & low-res (i.e. blurred) texture when they are far away

We can use textures that perturb normals instead of colors or reflectances



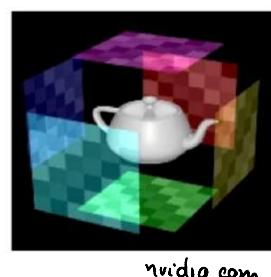
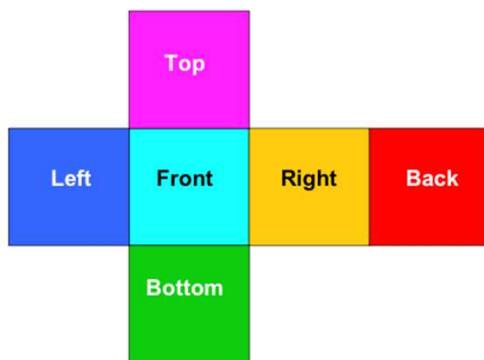
2D Image Bump Mapping Using a 24-bit Bitmap

Environment Mapping



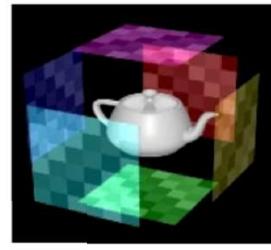
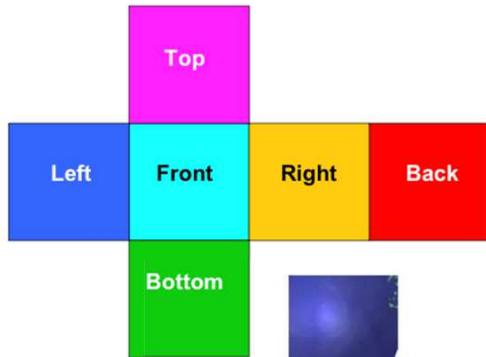
- Place the center of projection inside a sphere or cube
- Texture-map the sphere/cube INTERIOR with a photo
- Rendered images now correspond to views of the environment where photo was taken
- Particularly effective technique for rendering reflective objects

Environment Mapping



To create a full 360-degree environment map, we must texture all 6 interior faces of the cube

Environment Mapping



nvidia.com

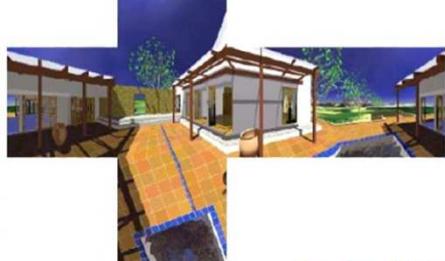
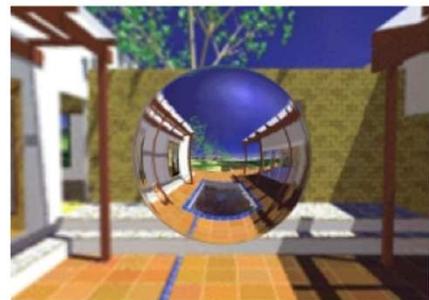


← texture map

nvidia.com

Environment Mapping

environment mapping
result



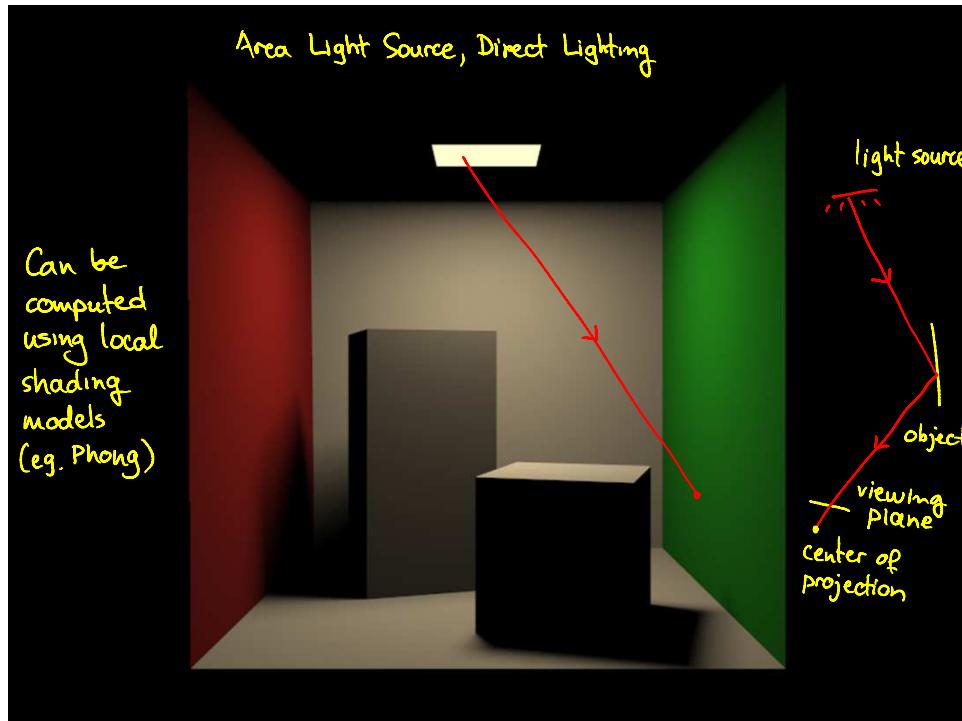
← texture map

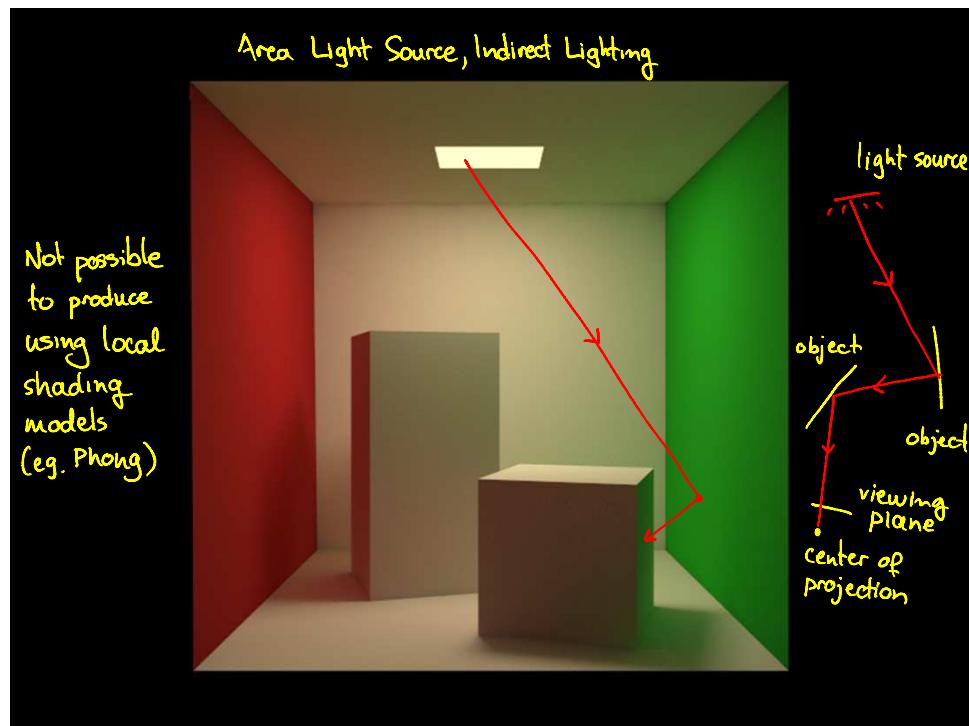
Image from slides by

Topic 12:

Basic Ray Tracing

- Introduction to ray tracing
- Computing rays
- Computing intersections
 - ray-triangle
 - ray-polygon
 - ray-quadratic
 - the scene signature
- Computing normals
- Evaluating shading model
- Spawning rays
- Incorporating transmission
 - refraction
 - ray-spawning & refraction

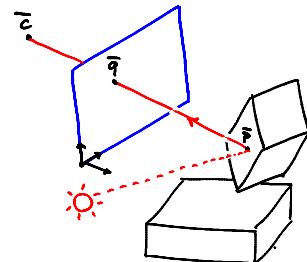




Rasterization vs. Ray Tracing

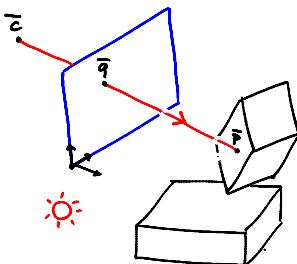
Rasterization:

- Project geometry onto image
- Compute pixel color using local shading model



Ray tracing:

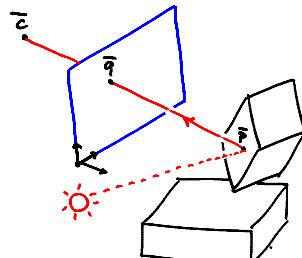
- Project pixels (aka "image samples") backwards onto scene
- Compute pixel color at \bar{q} by estimating light reaching \bar{p} directly or indirectly



Ray Tracing: Basic Idea

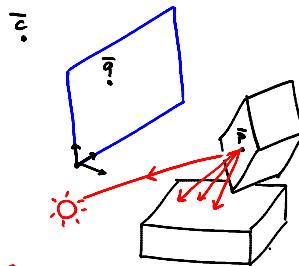
Rasterization:

- Project geometry onto image
- Compute pixel color using local shading model

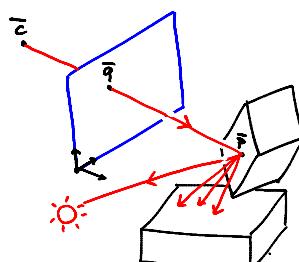
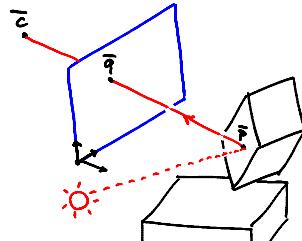
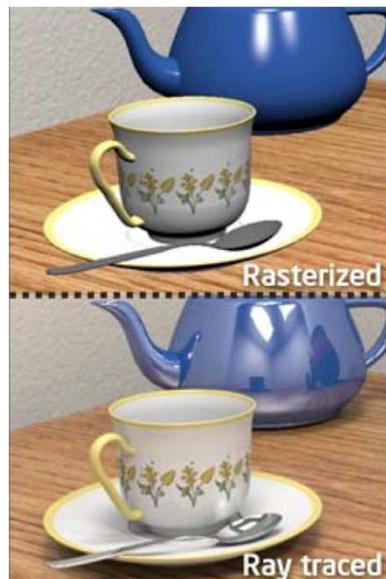


Ray tracing:

- Project pixels (aka "image samples") backwards onto scene
- Compute pixel color at \bar{q} by estimating light reaching \bar{p} directly or indirectly
done by recursively casting rays from \bar{p} to possible incident directions

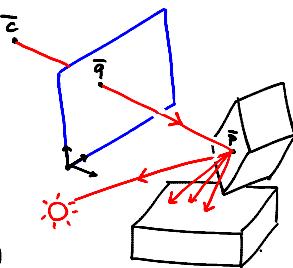
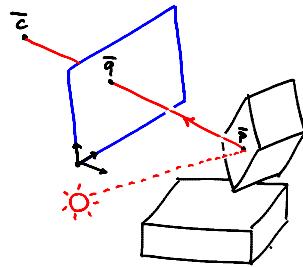


Ray Tracing: Basic Idea



Ray Tracing: Advantages

- highly customizable
(“plug-ins” for reflectance models, ray sampling functions)
- can model shadows, arbitrary reflections (e.g. mirrors), refractions, indirect illumination, sub-surface scattering, ...
- parallelizable
- allows trading off speed for accuracy (through #cast rays)
-

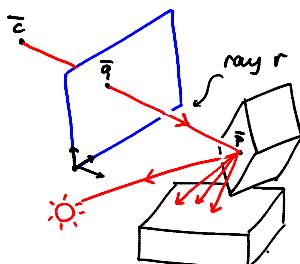


Ray Tracing: Basic Algorithm

Basic loop:

for each pixel \bar{q}
① cast ray r through \bar{q}
② find 1st intersection of \bar{q} with scene (i.e. point \bar{p})
③ estimate amount of light reaching \bar{p}

④ estimate amount of light travelling from \bar{p} to \bar{q} along ray r



Ray Tracing: Basic Algorithm

Basic loop:

for each pixel \bar{q}

① cast ray r through \bar{q}

② find 1st intersection of \bar{q} with scene (i.e. point \bar{P})

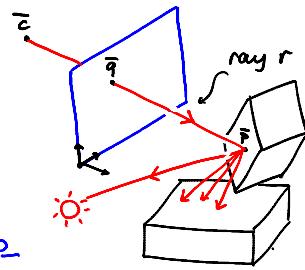
③ estimate amount of light reaching \bar{P}

a. "spawn" rays r_1, r_2, \dots, r_k from \bar{P} in various directions

b. if ray r_i hits a light source, estimate light travelling along r_i and stop

c. else apply loop recursively to ray r_i

④ estimate amount of light travelling from \bar{P} to \bar{q} along ray r



Ray tracing in the movies



Christensen et al, 2006

www.povray.org/community/hof



"Main street (blue)"



"The Cool Cows"

www.povray.org/community/hof



"The Dark Side of Trees"



"Capriccio" F. Obukhov et al, 2003

Online Ray Tracing Competitions



MARCO LUCINI 1997

www.intc.org/stills

380K triangles, 104 lights, full global illumination in real time



SIGGRAPH'05 Course by Slusallek et al



Computational Issues in Basic Ray Tracing

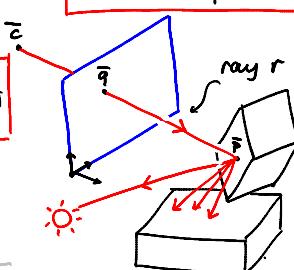
Basic loop:

for each pixel \bar{q}

defining
the ray r

computing ray-scene
intersections

- ① cast ray r through \bar{q}
- ② find 1st intersection of \bar{q} with scene (i.e. point \bar{P})
- ③ estimate amount of light reaching \bar{P}
 - a. "spawn" rays r_1, r_2, \dots, r_k from \bar{P} in various directions
 - b. if ray r_i hits a light source, estimate light travelling along r_i and stop
 - c. else apply loop recursively to ray r_i
- ④ estimate amount of light travelling from \bar{P} to \bar{q} along ray r
evaluating reflectance model at \bar{P}





Ray Tracing: Basic Algorithm

Basic loop:

for each pixel \bar{q}

① cast ray r through \bar{q}

② find 1st intersection of \bar{q} with scene (i.e. point \bar{p})

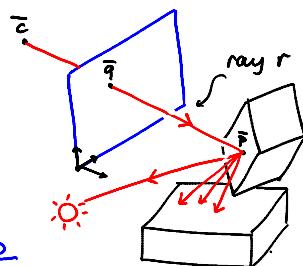
③ estimate amount of light reaching \bar{p}

a. "spawn" rays r_1, r_2, \dots, r_k from \bar{p} in various directions

b. if ray r_i hits a light source, estimate light travelling along r_i and stop

c. else apply loop recursively to ray r_i

④ estimate amount of light travelling from \bar{p} to \bar{q} along ray r



Topic 12:

Basic Ray Tracing

- Introduction to ray tracing
- Computing rays
 - Computing intersections
 - ray-triangle
 - ray-polygon
 - ray-quadratic
 - the scene signature
 - Computing normals
 - Evaluating shading model
 - Spawning rays
 - Incorporating transmission
 - refraction
 - ray-spawning & refraction

Computing the Ray Through a Pixel

Basic loop:

for each pixel \bar{q}

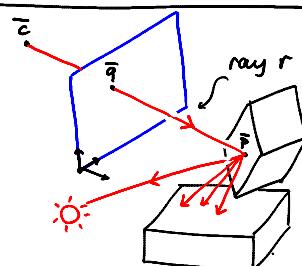
defining
the ray r

- ① cast ray r through \bar{q}
- ② find 1st intersection of \bar{q} with scene (i.e. point \bar{p})
- ③ estimate amount of light reaching \bar{p}

- ④ estimate amount of light travelling from \bar{p} to \bar{q} along ray r

Compute ray r given

- discrete pixel position
- world-to-camera matrix M_{wc}
- camera-to-canonical view matrix M_{cv}



Computing the Ray Through a Pixel: Main Idea

Idea: ray through \bar{q} contains the points

$$\bar{c} + \lambda(\bar{q}_w - \bar{c}) \quad \lambda \in \mathbb{R}$$

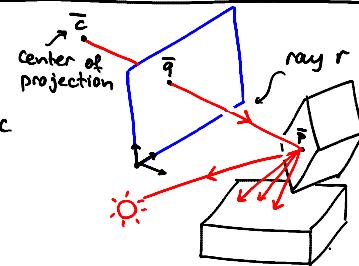
where \bar{q}_w are the world coordinates

of pixel \bar{q}

↑ how do we compute
 \bar{q}_w ?

Compute ray r given

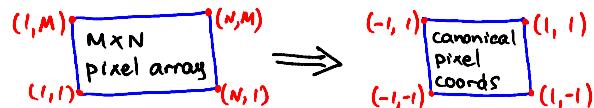
- discrete pixel position
- world-to-camera matrix M_{wc}
- camera-to-canonical view matrix M_{cv}



Computing the Ray Through a Pixel: Steps

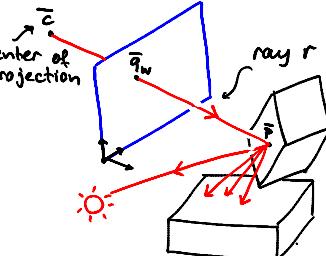
Computing homogeneous coords of \bar{q}_w :

- Convert discrete pixel coordinates (row & column number) to canonical coordinates (x_i, y_i) that lie in the range $[-1, 1]$



Compute ray r given

- discrete pixel position
- world-to-camera matrix M_{wc}
- camera-to-canonical view matrix M_{cv}



Computing the Ray Through a Pixel: Steps

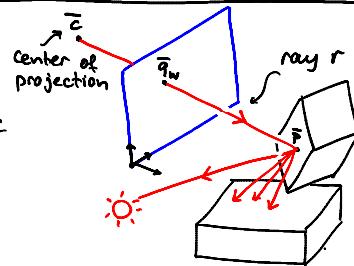
Computing homogeneous coords of \bar{q}_w :

- ① Compute canonical view coordinates (x_i, y_i)
- ② Compute homogeneous 3D canonical view coordinates of \bar{q} :

$$\bar{q}_v = \begin{bmatrix} x_i \\ y_i \\ 0 \\ 1 \end{bmatrix}$$

Compute ray r given

- discrete pixel position
- world-to-camera matrix M_{wc}
- camera-to-canonical view matrix M_{cv}



Computing the Ray Through a Pixel: Steps

Computing homogeneous coords of \bar{q}_w :

- ① Compute canonical view coordinates (x_i, y_i)
- ② Compute homogeneous 3D canonical view coordinates, \bar{q}_v .
- ③ Convert to world coordinates:

$$\bar{q}_w = M_{wc}^{-1} \cdot M_{cv}^{-1} \bar{q}_v$$

Compute ray r given

- discrete pixel position
- world-to-camera matrix M_{wc}
- camera-to-canonical view matrix M_{cv}

