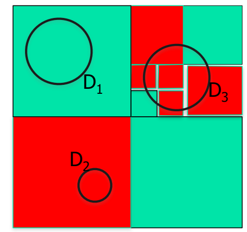
Quadtrees and R-trees

A data simple data structure for geometric objects(e.g. points, houses, an image, 3D scene)

Support efficiently a very wide variety of queries.

Hierarchical Partition of the scene

QuadTrees



Assume we are given a red/green picture defined a $2^h \times 2^h$ grid. E.g. pixels.
Each pixel is either **green** or **red.**

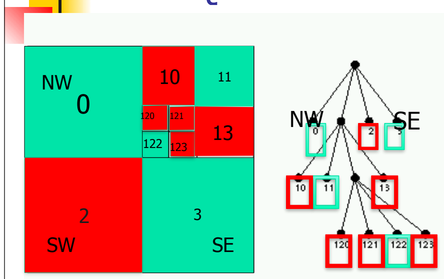
(more general and interesting examples – soon)

Need to represent the shape "compactly"

Need a data structure that could answers multiple types of queries. For example:

1. For a given point q, is q red or green ?
2. For a given query disk D, are there any green points in D ?
3. How many green points are there in D ?
4. Etc etc

QuadTrees



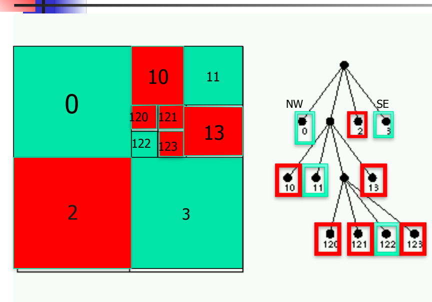
- Assume we are given a red/green picture defined on a $2^h \times 2^h$ grid of **pixels**.
- Each pixel has as a unique color (Green or Red)
- **Every node $v \in T$ is associated with a geometric region $R(v)$**

Alg <u>constructQT for a shape S</u>.

- **input** – a node $v \in T$, and a shape S.
- **Output** – a Quadtree T_v representing the shape of S *within* $R(v)$).

- If S is fully green in $R(v)$, or S is fully red in $R(v)$ – then
- v is a leaf, labeled Green or Red. Return ;
- Otherwise, divide $R(v)$ into 4 equal-sized quadrants, corresponding to nodes v.*NW,* v.*NE,* v.*SW,* v.*SE.*
- Call <u>constructQT</u> recursively for each quadrant.

QuadTrees



Consider a picture stored on an $2^h \times 2^h$ grid. Each pixel is either red or green.

We can represent the shape "compactly" using a QT.

Height – at most h.

Point location operation – given a point q, is it black or white

– takes time O(h)

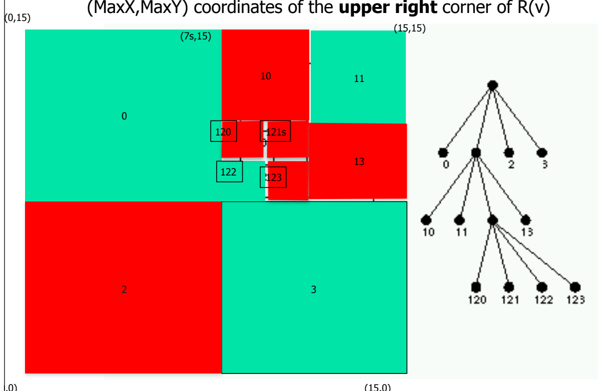
- could it be much smaller ?

Many other operations are very simple to implement.

Storing the **range** R(v) of a node

Each node v is associated with a range R(v) – a square. The node v stores (in addition to other info) 4 values

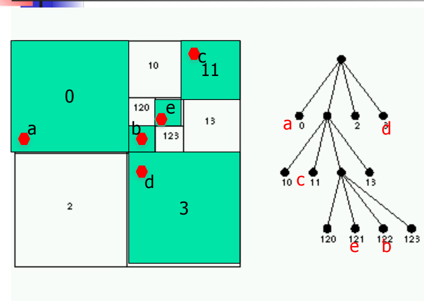
(MinX,MinY) – coordinates of the **lower left** corner of R(v)
(MaxX,MaxY) coordinates of the **upper right** corner of R(v)



QuadTree for a set of points

Now consider a set of points (red) but on a $2^h \times 2^h$ grid.

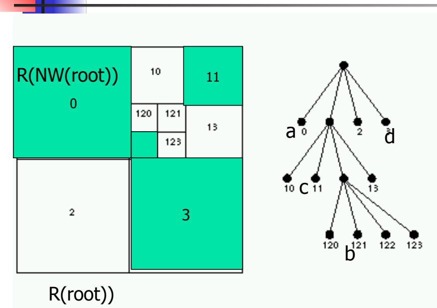
Splitting policy: Split until each quadrant contains ≤1 point.



Build a similar QT, but we stop splitting a quadrant when it contain ≤1 point (or some other small constant)
Point location operation – given a point q, is it black or white
– takes time O(h) (and less in practice)

Many other splitting polices are very simple to implement.
(eg. A leaf could contain contains ≤17 points)

Regions of nodes



In general, every node v is associated with a region R(v) in the plane

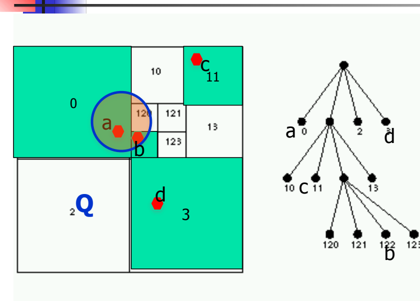
R(root) is the whole region

The smallest area of R(v) is a single pixel.

Let NW(v) denote the North West child of v. (similarly NE, SW, SE)

R(v) = is the union of
R(NW(v)), R(NE(v)) R(SW(v)), R(SE(v))

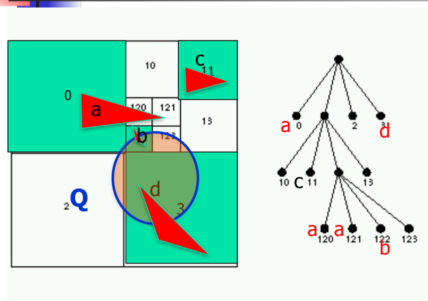
QuadTrees for a set of points



Report(Q,v)
// Q – a query disk
/*report all the points in stored at the subtree rooted at v, which are also inside Q. */

1. If v is NULL – **return**.
2. If R(v) is disjoint from Q – **return**
3. If R(v) is fully contained in Q – report all points in the subtree rooted at v.
4. If v is a leaf – check each point in R(v) if inside Q
5. Else
   - Report(Q, NW(v))
   - Report(Q, NE(v))
   - Report(Q, SW(v))
   - Report(Q, SE(v))

QuadTrees for shape



Input: Set S of triangles $S=\{t_1 \ldots t_n\}$

Splitting policy: Split quadrant if it intersects more than 1 triangle of S.

Note – a triangle might be stored in multiple leaves. Some leaves might store no triangles.

Finding all triangles inside a query region Q – essentially same Report Report(Q,v) as before (minor modifications)

Inserting a new segment

insert(segment s ,node v) {
// Inserting a segment s into an existing node v of QT
// v might or might not be a leaf
If v is NULL - Error
If R(v) is disjoint from x – Return. Else
If v is not a leaf, then for each child **u** of **v**, call **insert(s,u)**;
Else // v is a leaf
Add **s** to **v.SegmentsList**
If number of segments in v.SegmentsList too long (e.g. >3) Call **Split(v)**
}

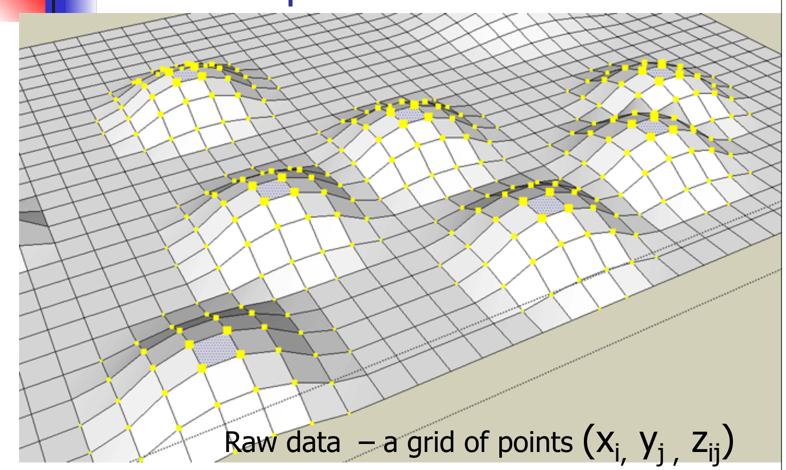
Split(v){
// Assumption – v is a leaf, but has too many segments in its list
// Create 4 children for v (make sure they know which region they cover.)
For each child u of v
For each segment **s** in **v.SegmentsList** Call **insert(s, u)**
Empty **v.SegmentsList**
}

Material from this slide is optional for CSs345

Raw data – a grid of points (x_i, y_j, z_{ij})

For every grid point i,j

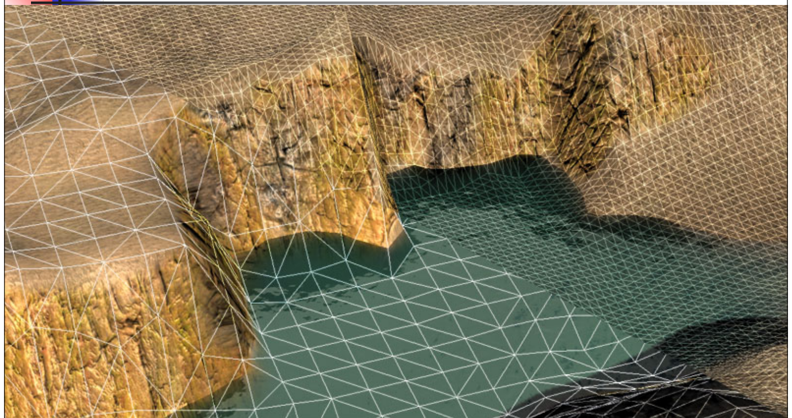
Terrain representations



Raw data – a grid of points (x_i, y_j, z_{ij})

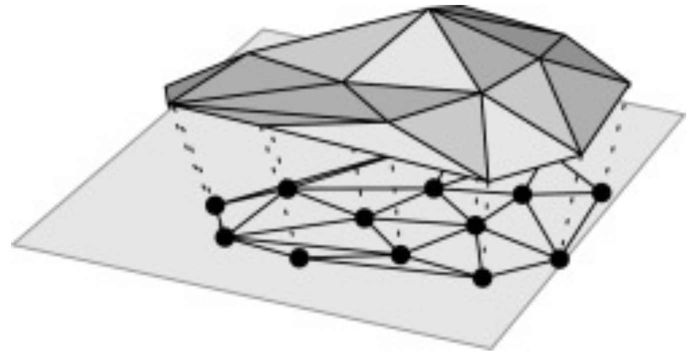
For every grid point i,j

Triangulated terrain (TIN – Triangulated irregular network)



Each triangle approximately fits the surface below it

How to find good triangulation ?



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(credit SCALGO)

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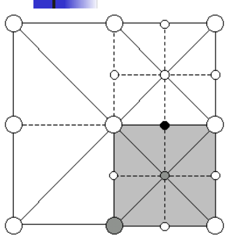
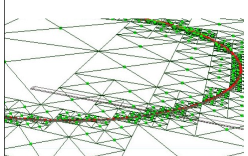


- Input – a very large set of points $S=\{ (x_i , y_j, z_{ij}) \}$.
- z_{ij} is the elevation at point (x_i , y_i)
- Want to create a surface, consists of triangles, where each triangle interpolates the data points underneath it.
- Idea: Build a QT T for the 2D points.
- (if want triangles: Each quadrant is split into 2 triangles)
- Assign to each vertex the height of the terrain above it.
- The approximated elevation of the terrain at any point is the linear interpolation of its elevated vertices.

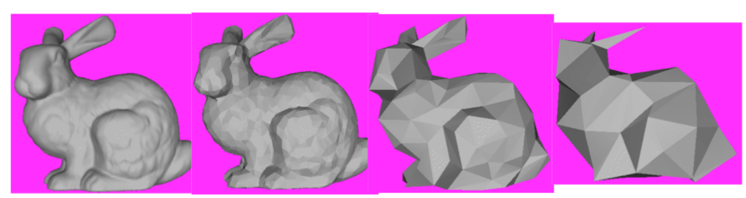
QT Split Policy: Splitting a quadrant into 4 sub-quadrants:



- split a node v if for some date point $(x_i , y_i) \in R(v)$, the elevation of z_{ij} is too far from the the corresponding triangle. If not, leave v as a leaf.
- That is, (x_i ,y_j , z_{ij}) it is too far from the interpolated elevation.
- Note: A quadrant might contain a huge number of points, but they behave smoothly. E.g. all a the sloop of a mountain, but this slope is more or less linear.

Level Of Details

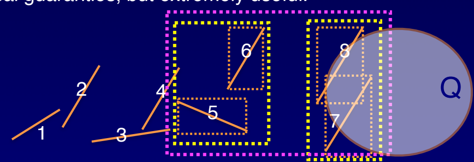
- Idea – the same object is stored several times, but with a different level of details
- Coarser representations for distant objects
- Decision which level to use is accepted `on the fly'
  (eg in graphics applications, if we are far away from a terrain, we could tolerate usually large error)



69,451 polys | 2,502 polys | 251 polys | 76 polys

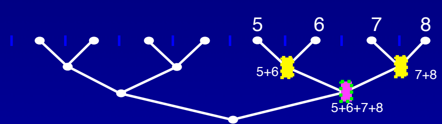
R-trees

- Input: A set S of shapes (segments in this example)
- Prepare a tree that could assists finding the segments intersecting a query region.
- Fewer theoretical guaranties, but extremely useful.



- We compute for each segment its bounding bounding box (rectangle).
- These are the leaves of *T*
- Match pairs of bounding boxes. For example 1-2, 3-4, 5-6, 7-8. For each such pair, compute their bounding boxes. Each node in level 2 is such a box.
- Match these bounding boxes. These are the nodes of level 3.
- In general for every node *v*, $BB(v) = BB\big(BB(v.right) \bigcup BB(v.left)\big)$

Once a query region Q is given we determine whether it intersect BB(root)
If not, we are done. If yes, check recursively if Q intersect BB(v.left) and BB(v.right)



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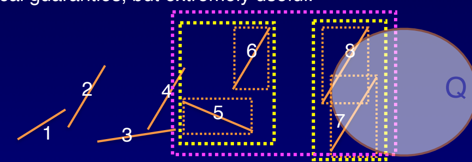
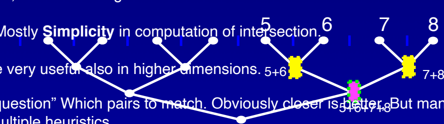
- Problem in reporting: Many "false alarms" : BBs that intersect Q while their segments don't.
- Should we use axis parallel BB instead of something that could "snag" better? For example, rotated rectangles ?
- Answer: Mostly **Simplicity** in computation of intersection.
- R-trees are very useful also in higher dimensions.
- Other big question" Which pairs to match. Obviously closer is better, But many variants Multiple heuristics





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