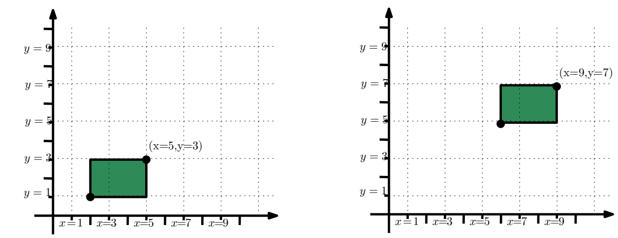
# Lecture 03
# Transformations in 2D
# Short version

We will discuss transformation in 3D, and with full details, later in the course

# Translations (shift) by $(\alpha, \beta)$

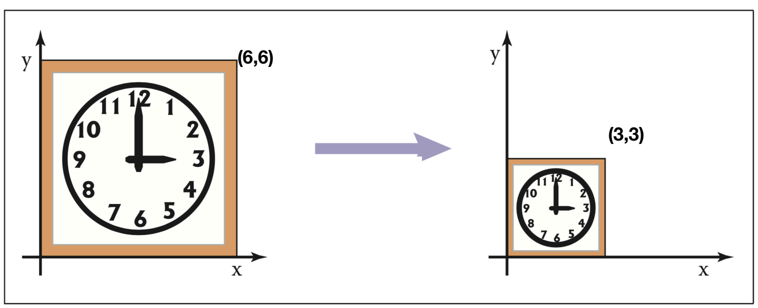
Translation (shift) by $(4, 4)$
$(x, y) \to (x + 4, y + 4)$




- Adding a constant $\alpha$ to the x-coordinate of every point
- Adding a constant $\beta$ to the x-coordinate of every point
- $(x, y) \to (x + \alpha, y + \beta)$

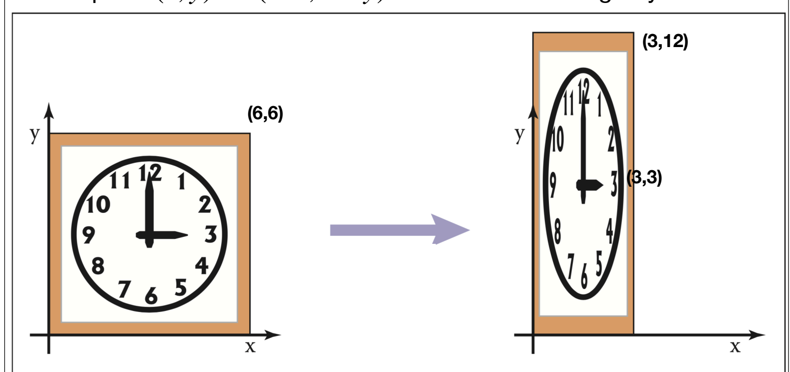
# Scaling

- We can use different constants $(s_x, s_y)$ for the x-axis vs. the y-axis. Then we shift each point *(x,y)* into the point
- $(x, y) \to (s_x \cdot x, \ \ s_y \cdot y)$




# Scaling

- Let s be a constant. If we move each point (x,y) into the point $(x, y) \to (s \cdot x, \ \ s \cdot y)$ we scaled the image by s.



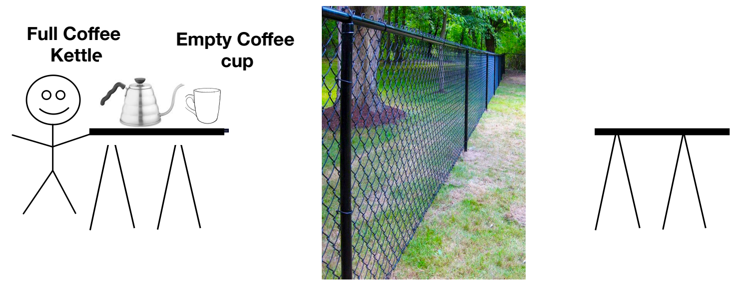
## The mathematician and coffee cup non-funny joke
## Part 1

### Fence


**Solution:**
1. **Walk around the fence,**
2. **fetch coffee kettle,**
3. **walk back pure coffee,**
4. **drink**

## The mathematician and coffee cup non-funny joke
## Part 2

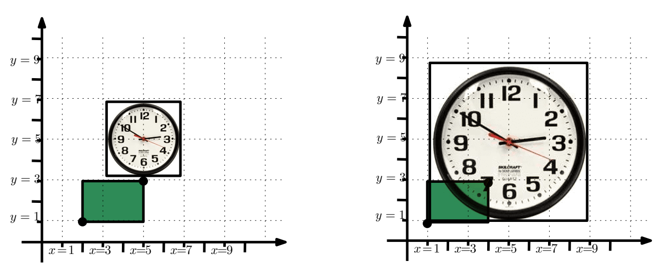
### Fence



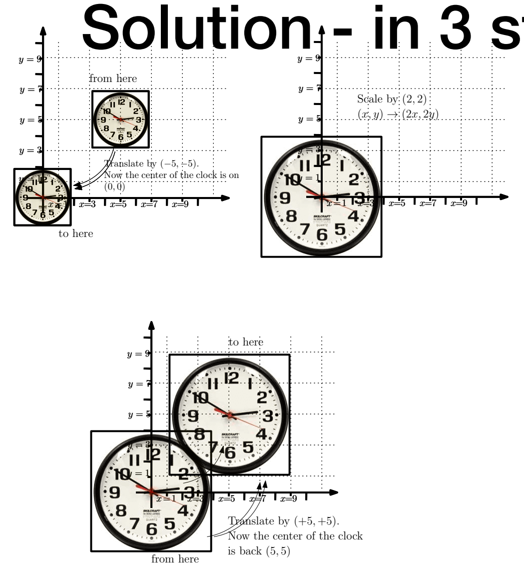

**Solution:**
1. **Bring the coffee Kettle to the other table**
2. **Apply the solution from the previous slide**

# Scale the clock, without changing its center

Problem: scale the clock, but without changing its center and without effecting the green rectangle

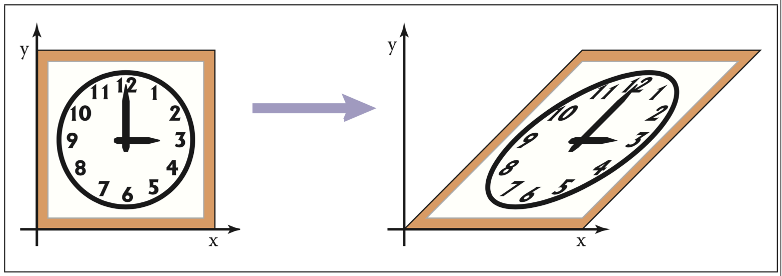


# Solution - in 3 steps



# Shearing

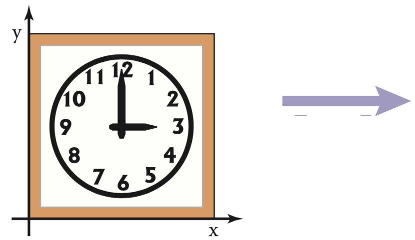
- If we move each point (x,y) into the point
  $(x, y) \rightarrow (x + y,\ y)$ we scaled the image by s.

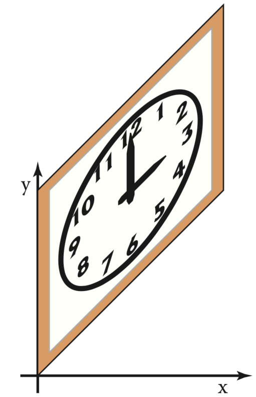



# Shearing

- Vertical shearing shifts each column based on the x value.

$$(x, y) \rightarrow (x,\ x + y)$$

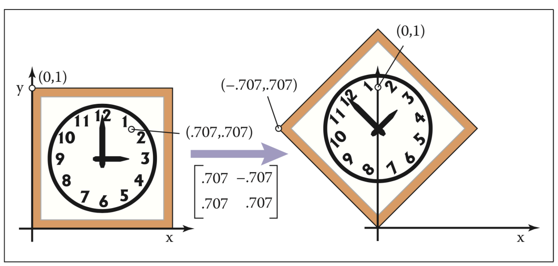


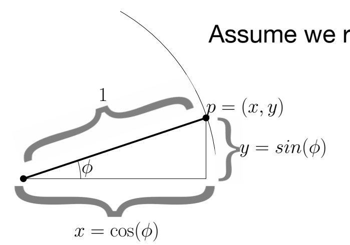

# Rotation

- Rotate counterclockwise by an angle $\phi$ about the origin.

$$(x, y) \rightarrow (\underbrace{x\cos\phi - y\sin\phi}_{\textbf{New } x},\ \underbrace{x\sin\phi + y\cos\phi}_{\textbf{New } y})$$

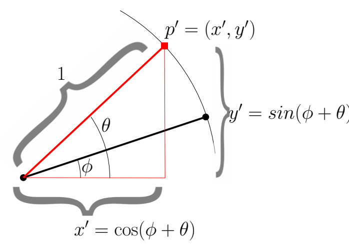



Assume we rotate $p$ by an angle $\theta$ CCW



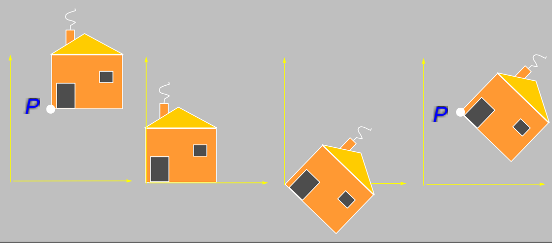

$$\cos(\phi + \theta) =$$

$$\underbrace{\cos(\phi)}_{=x}\cos(\theta) - \underbrace{\sin(\phi)}_{=y}\sin(\theta)$$



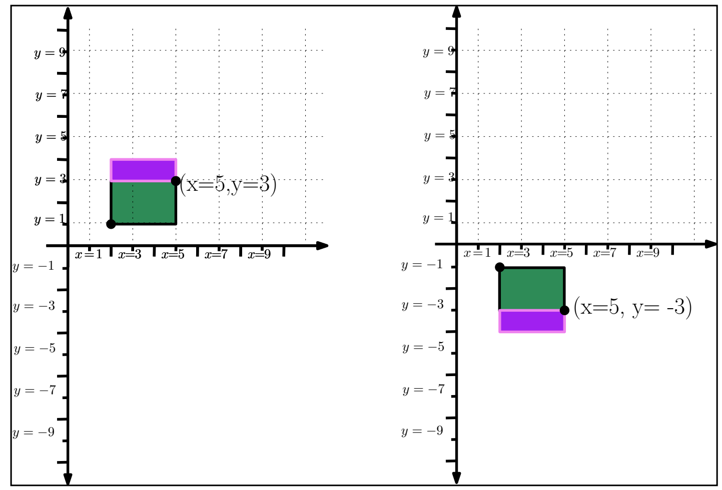
## Transformation Composition

- What operation rotates by $\theta$ around $P = (p_x, p_y)$ ?
  - Translate $P$ to origin
  - Rotate around origin by $\theta$
  - Translate back



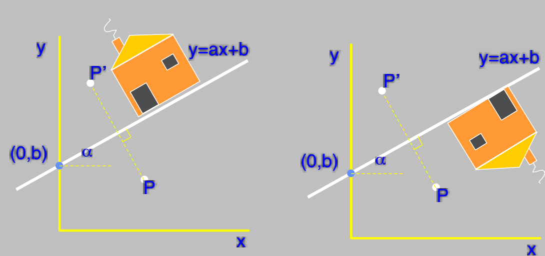



## Reflection on the x-axes: $(x, y) \rightarrow (x, -y)$




## Arbitrary Reflection - promo
## We will get back to it later in the semester

$\alpha = \tan^{-1}(a)$




Shift by (0,-b)
Rotate by $-\alpha$
Reflect through x
Rotate by $\alpha$
Shift by (0,b)

Very scarrrrry….
Unless we represent transformation by matrices
And then it is trivial.