CSC 433
Computer Graphics

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Credit: Joshua Levine

Lecture 09
Ray Tracing

Today's Agenda

- Reminders:
  - A02 due!
  - A03 posted!
- Goals for today:
  - Begin our discussing of 3D rendering with Ray Tracing

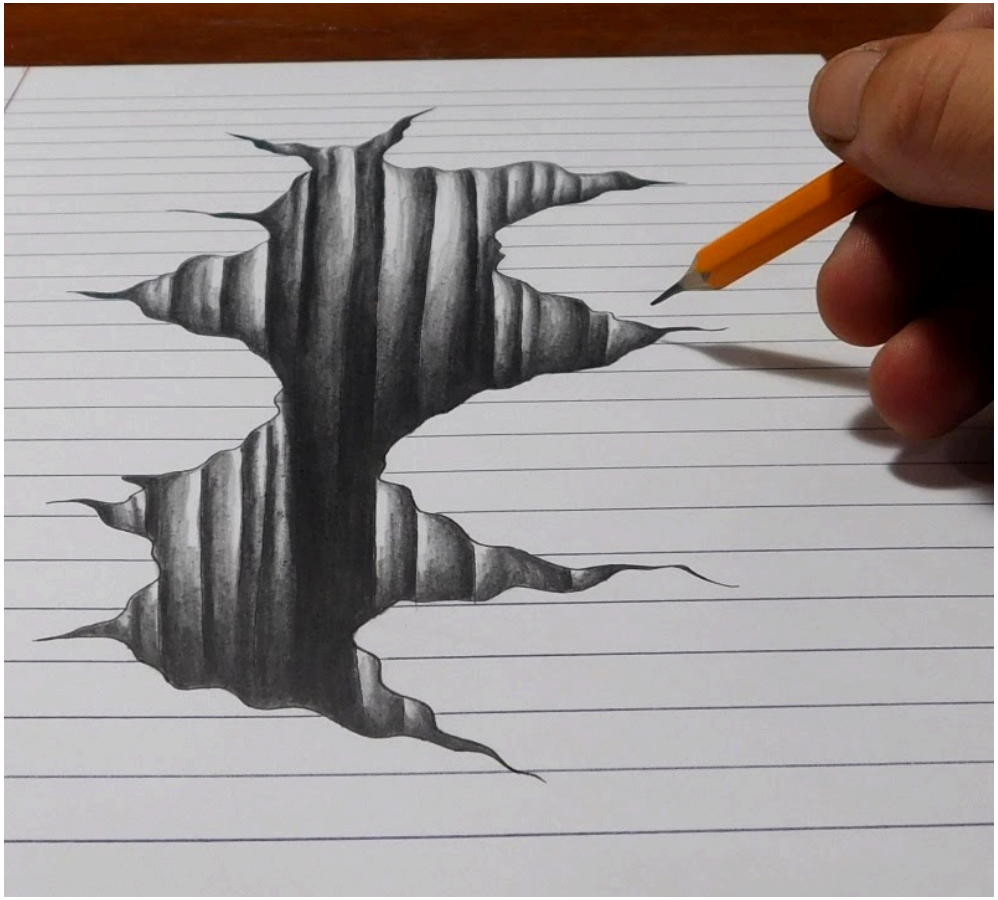
Rendering

What is Rendering?

*"**Rendering** is the task of taking three-dimensional objects and producing a 2D image that shows the objects as viewed from a particular viewpoint"*

Two Ways to Think About How We Make Images

- Drawing



- Photography

Two Ways to Think About Rendering

- Object-Ordered
- Decide, for every object in the scene, its contribution to the image

- Image-Ordered
- Decide, for every pixel in the image, its contribution from every object

Two Ways to Think About Rendering

- Object-Ordered or **Rasterization**

```
for each object {
  for each image pixel {
    if (object affects pixel)
    {
      do something
    }
  }
}
```

- Image-Ordered or **Ray Tracing**

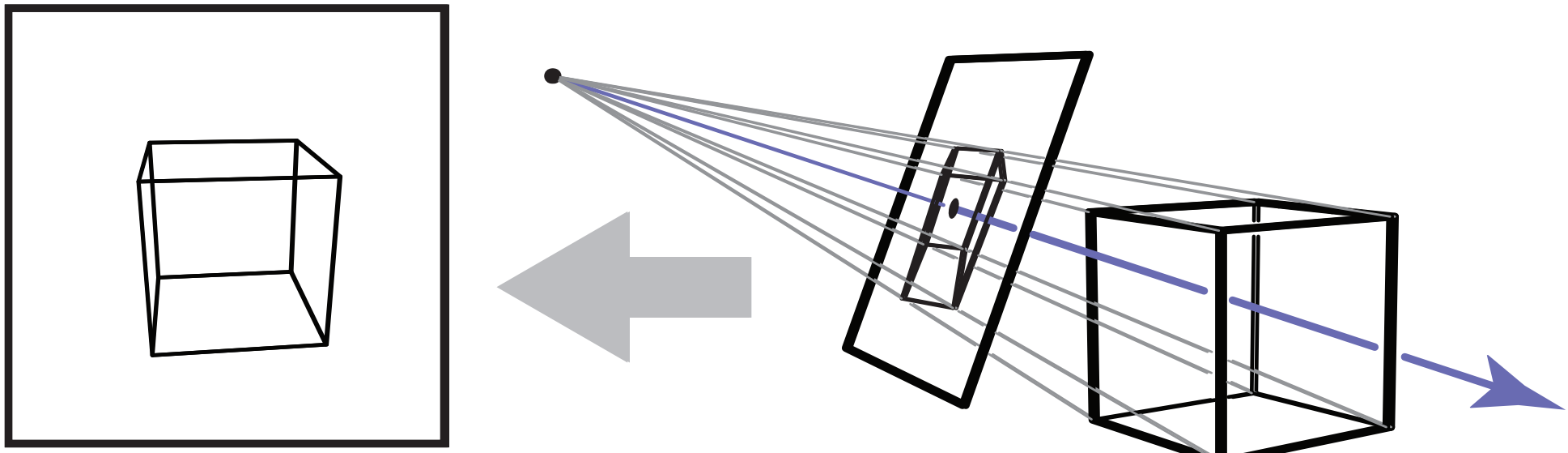
```
for each image pixel {
  for each object {
    if (object affects pixel)
    {
      do something
    }
  }
}
```

TODAY

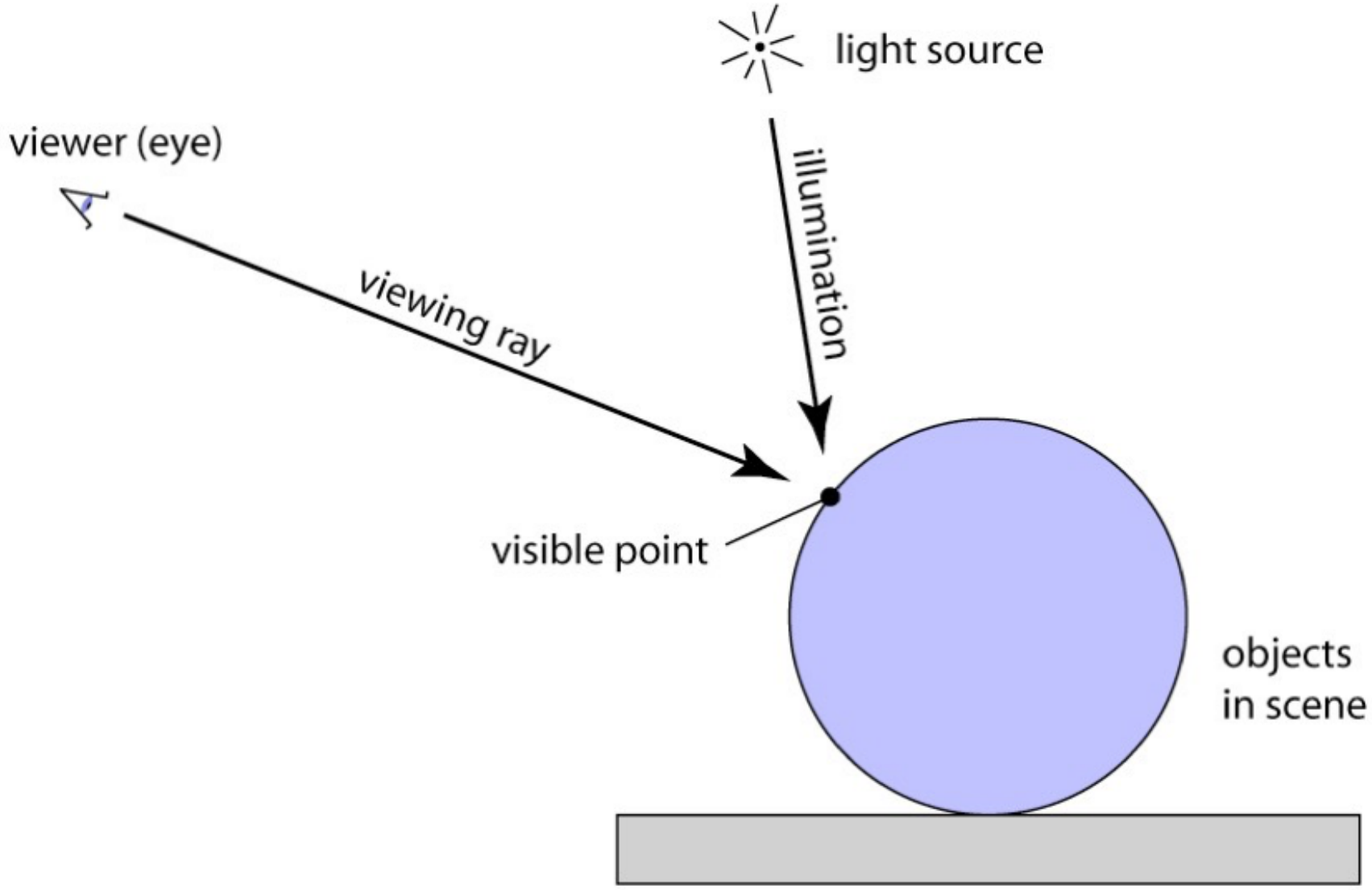
Basics of Ray Tracing

Idea of Ray Tracing

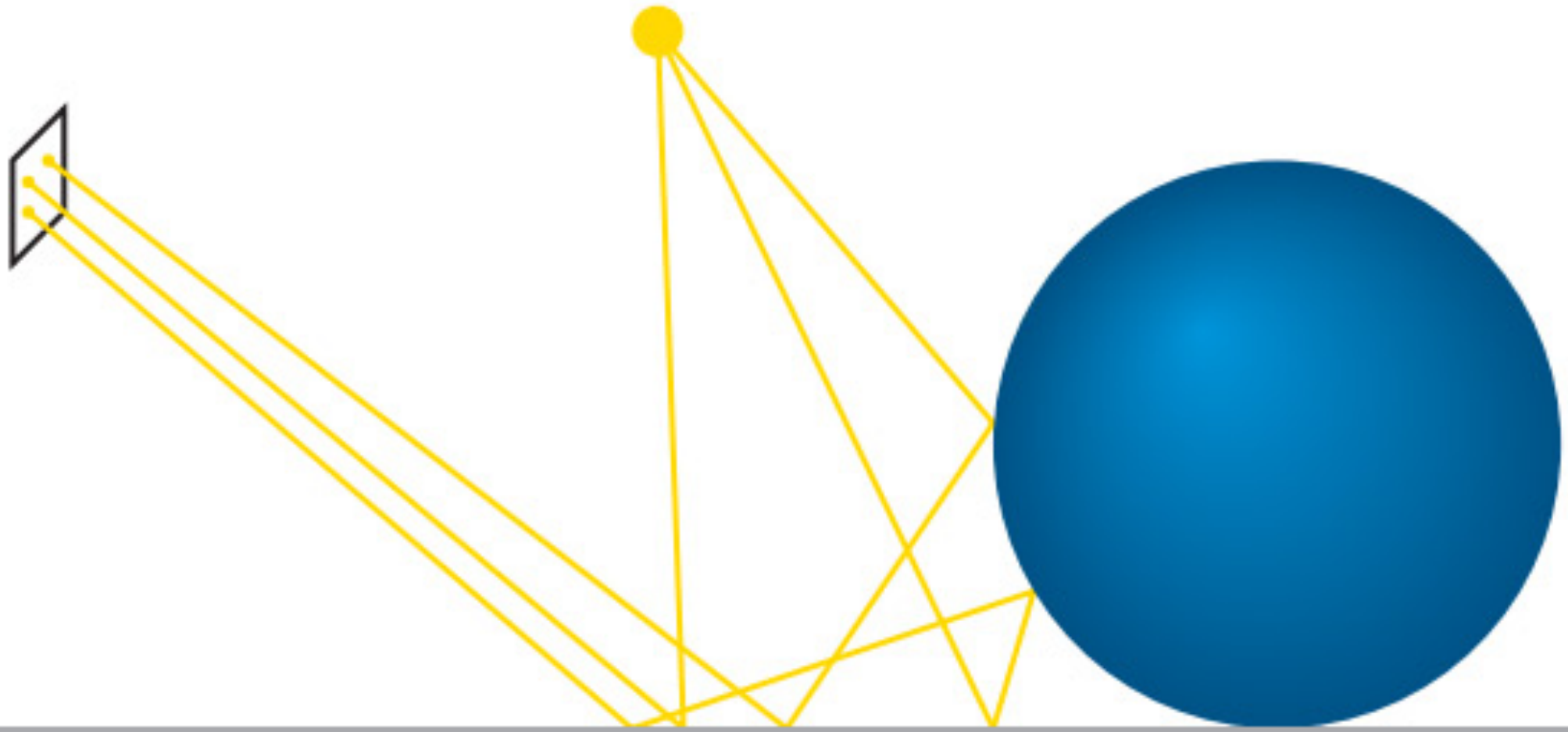
- Ask first, for each pixel: what belongs at that pixel?
- Answer: The set of objects that are visible if we were standing on one side of the image looking into the scene



Key Concepts, in Diagram



Idea: Using Paths of Light to Model Visibility



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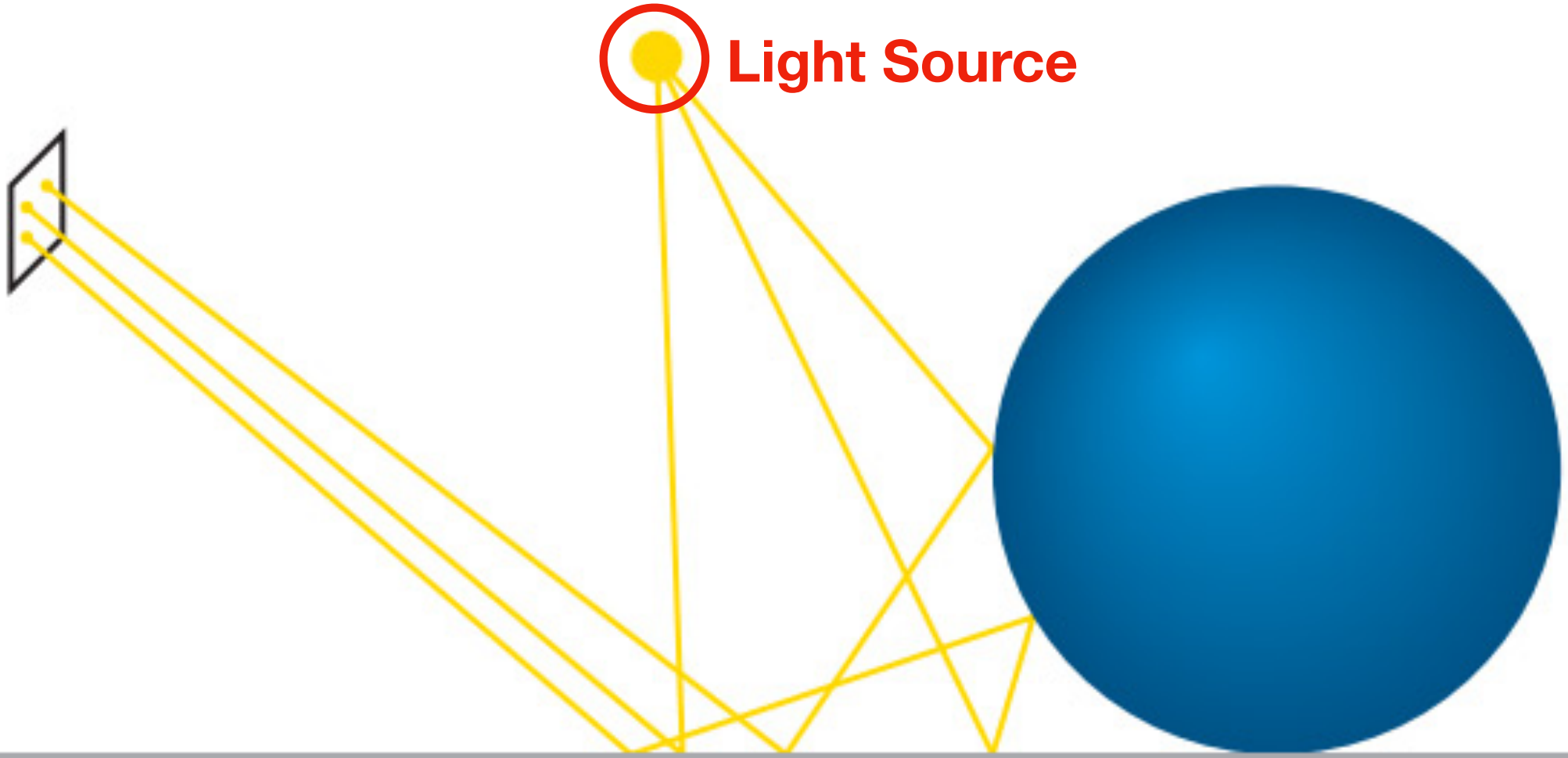
Using Paths of Light to Model Visibility



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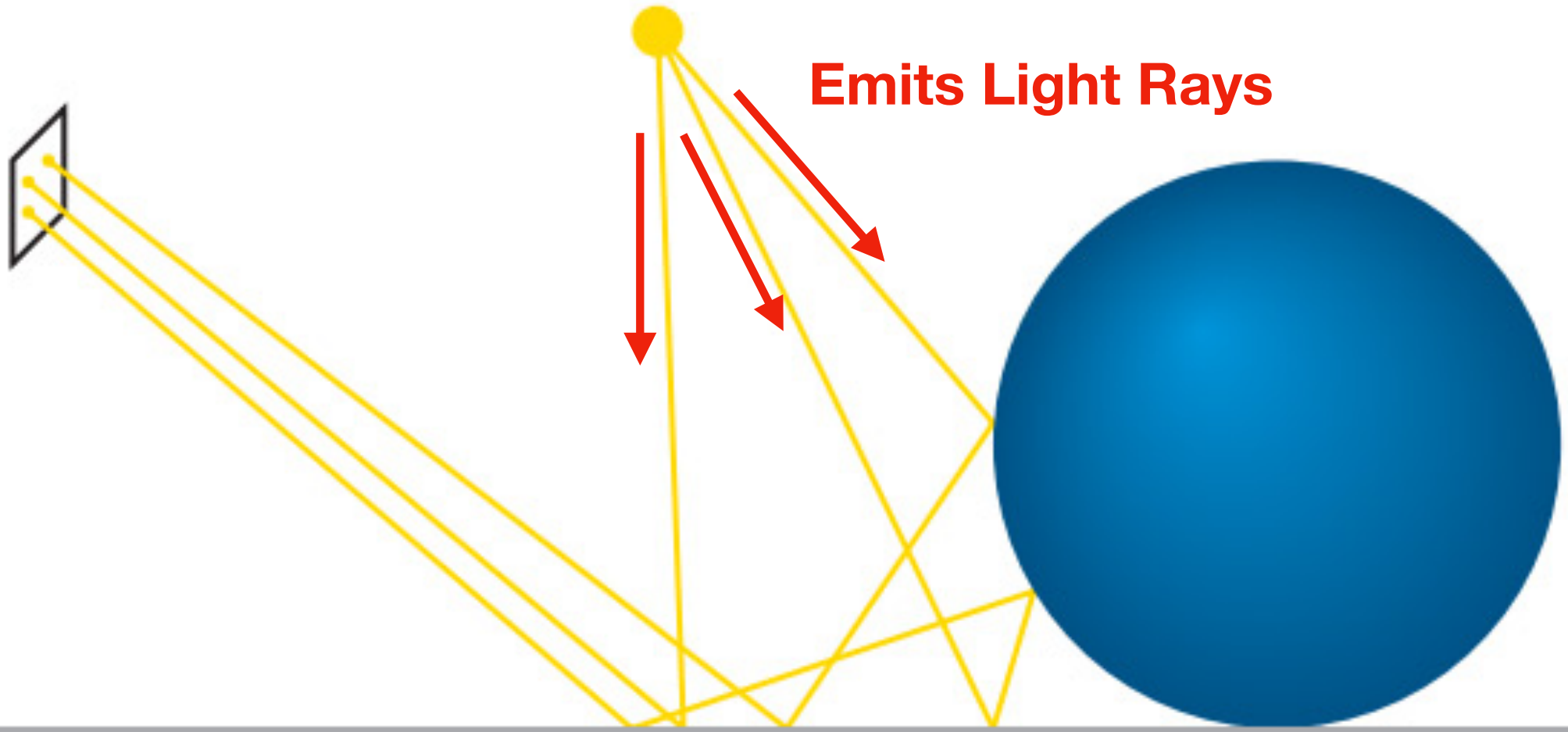
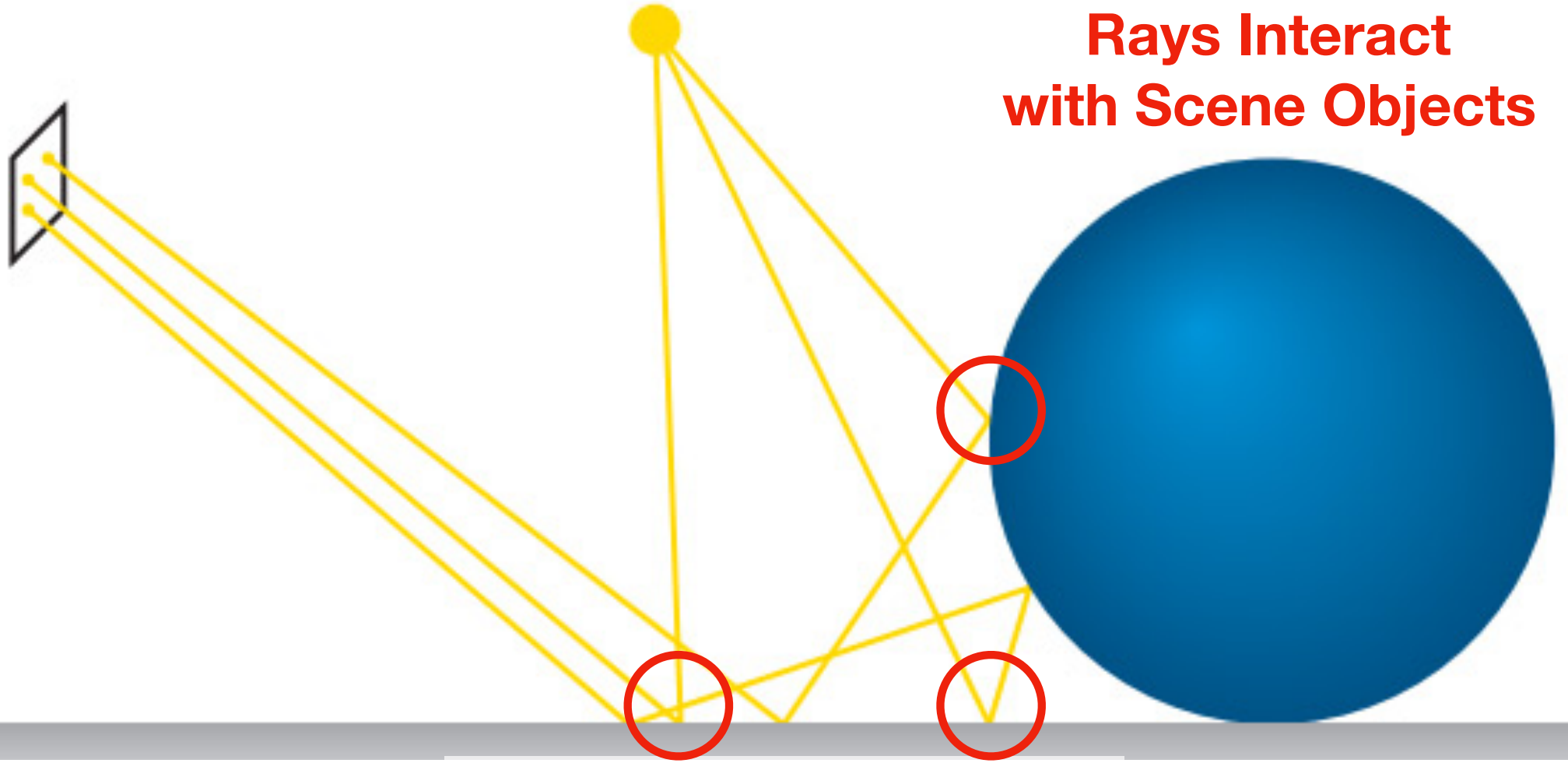
Using Paths of Light to Model Visibility



https://software.intel.com/file/37491

Using Paths of Light to Model Visibility



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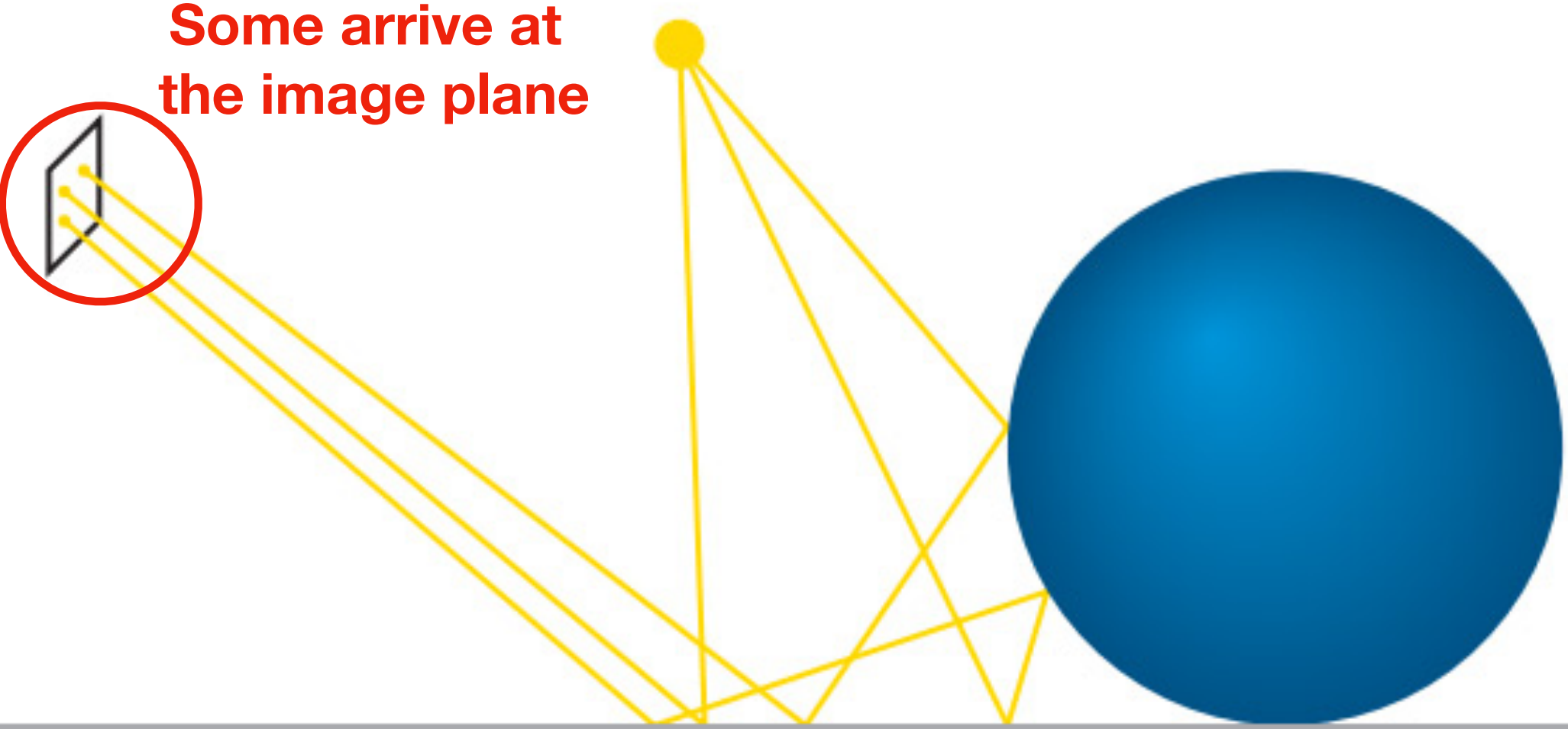
Using Paths of Light to Model Visibility



https://software.intel.com/file/37491

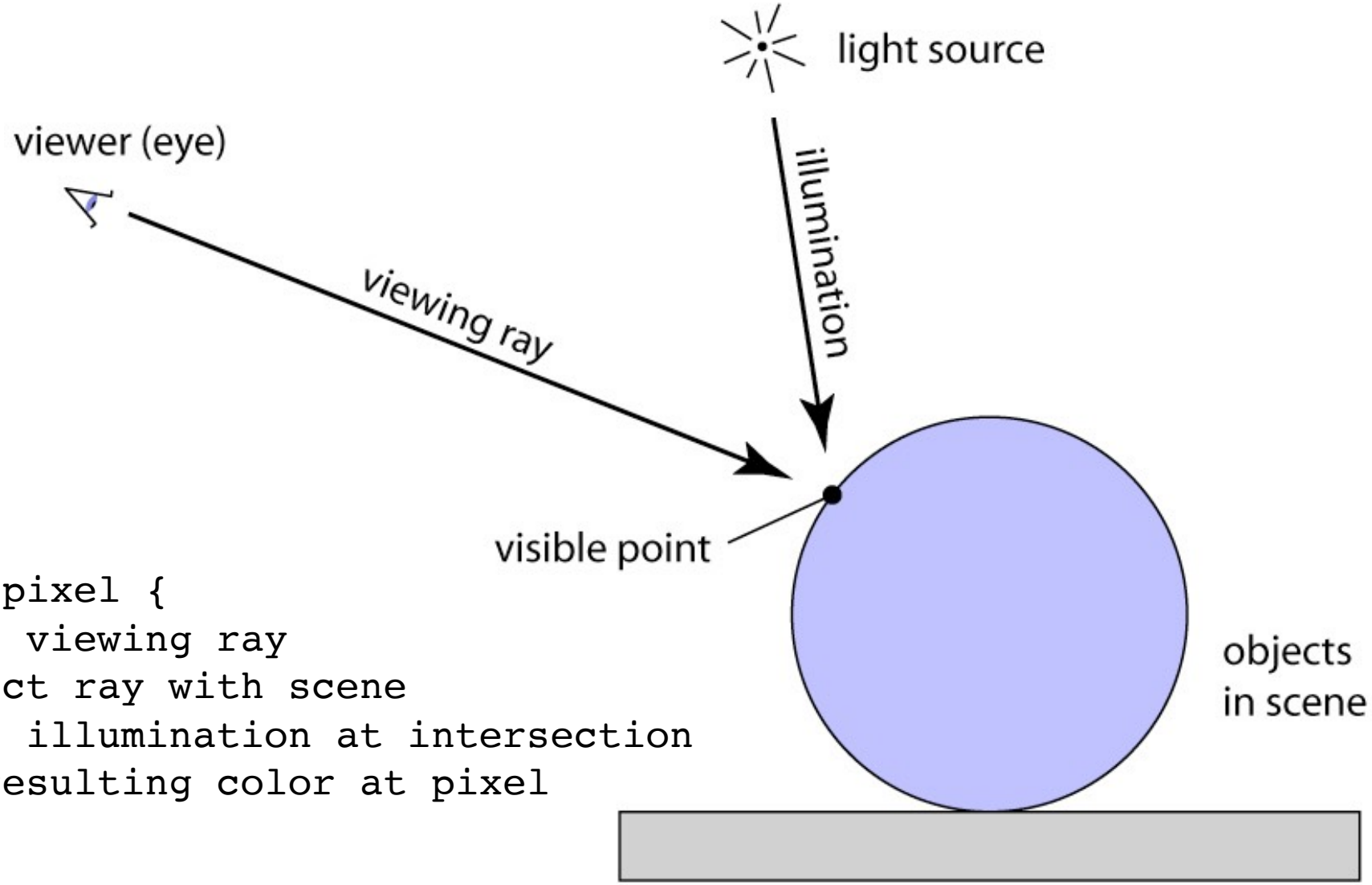
Using Paths of Light to Model Visibility

https://software.intel.com/file/37491

Forwarding vs Backward Tracing

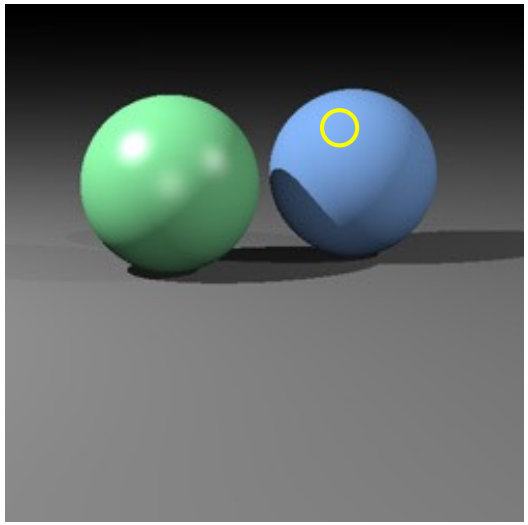
- Idea: Trace rays from light source to image
  - This is slow!
- Better idea: Trace rays **from** image **to** light source

Ray Tracing Algorithm




```
for each pixel {
  compute viewing ray
  intersect ray with scene
  compute illumination at intersection
  store resulting color at pixel
}
```

Ray Tracing Algorithm



Cameras and Perspective

If illumination is uniform and directional-free (ambient light):
for each pixel {
  compute viewing ray
  intersect ray with scene
   copy the color of the object at this point to this pixel.
}

Commonly, we need slightly more involved

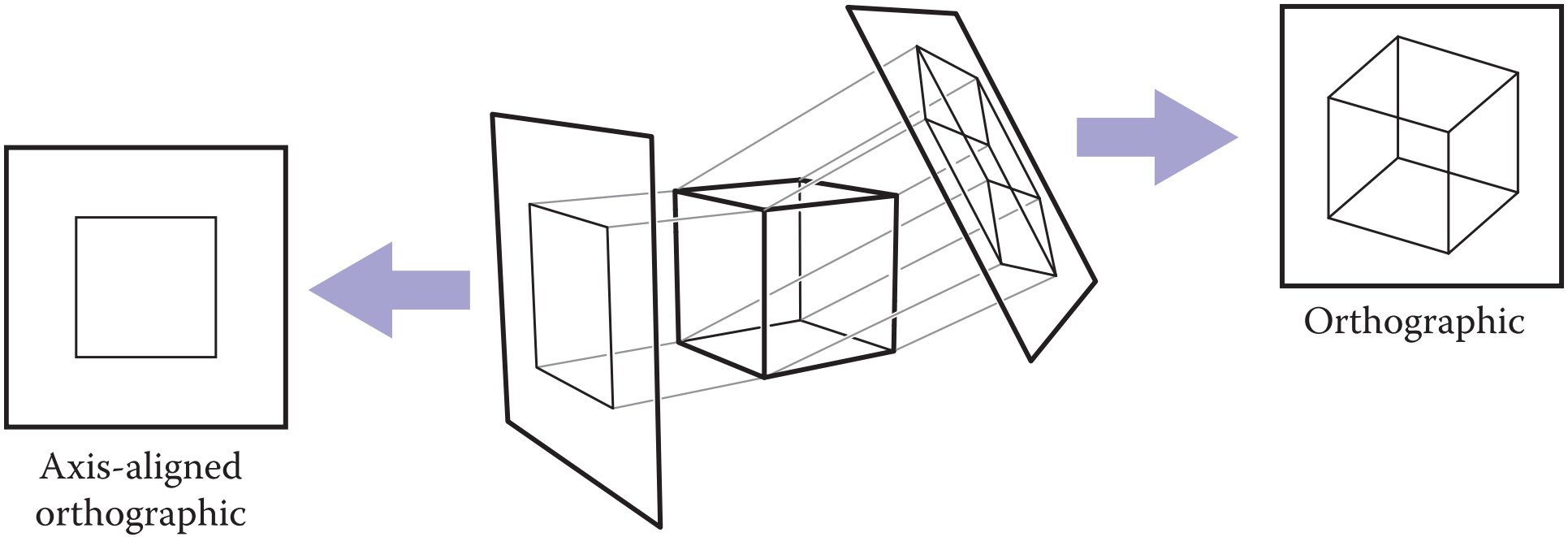
```
for each pixel {
  compute viewing ray
  intersect ray with scene
  compute illumination at intersection
  store resulting color at pixel
}
```

Linear Perspective

- Standard approach is to project objects to an image plane so that straight lines in the scene stay straight lines on the image
- Two approaches:
  - Parallel projection: Results in **orthographic** views
  - Perspective projection: Results in **perspective** views

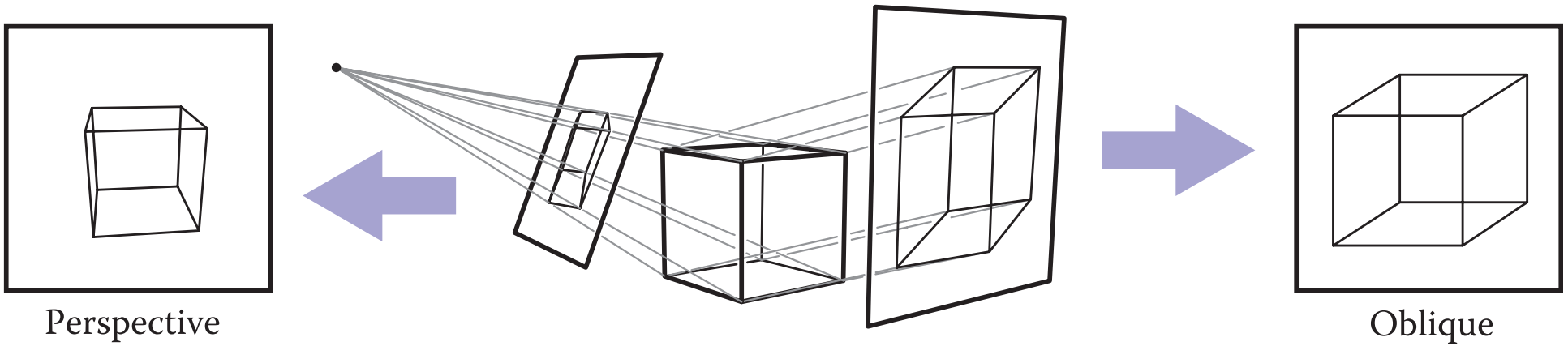
Orthographic Views

- Points in 3D are moved along parallel lines to the image plane.
- Resulting view determined solely by choice of projection direction and orientation/position of image plane



Perspective Views

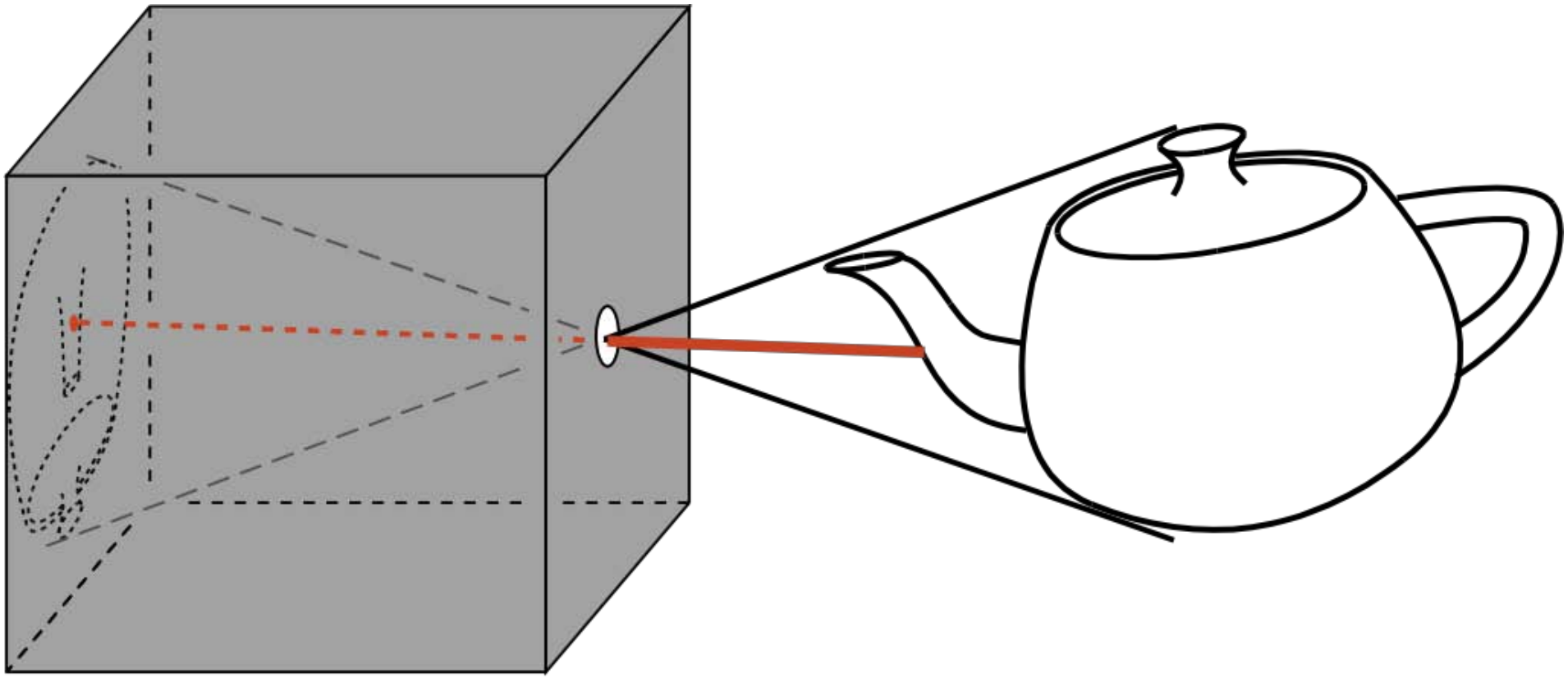
- But, objects that are further away should look smaller!
- Instead, we can project objects through a single viewpoint and record where they hit the plane.



Pinhole Cameras

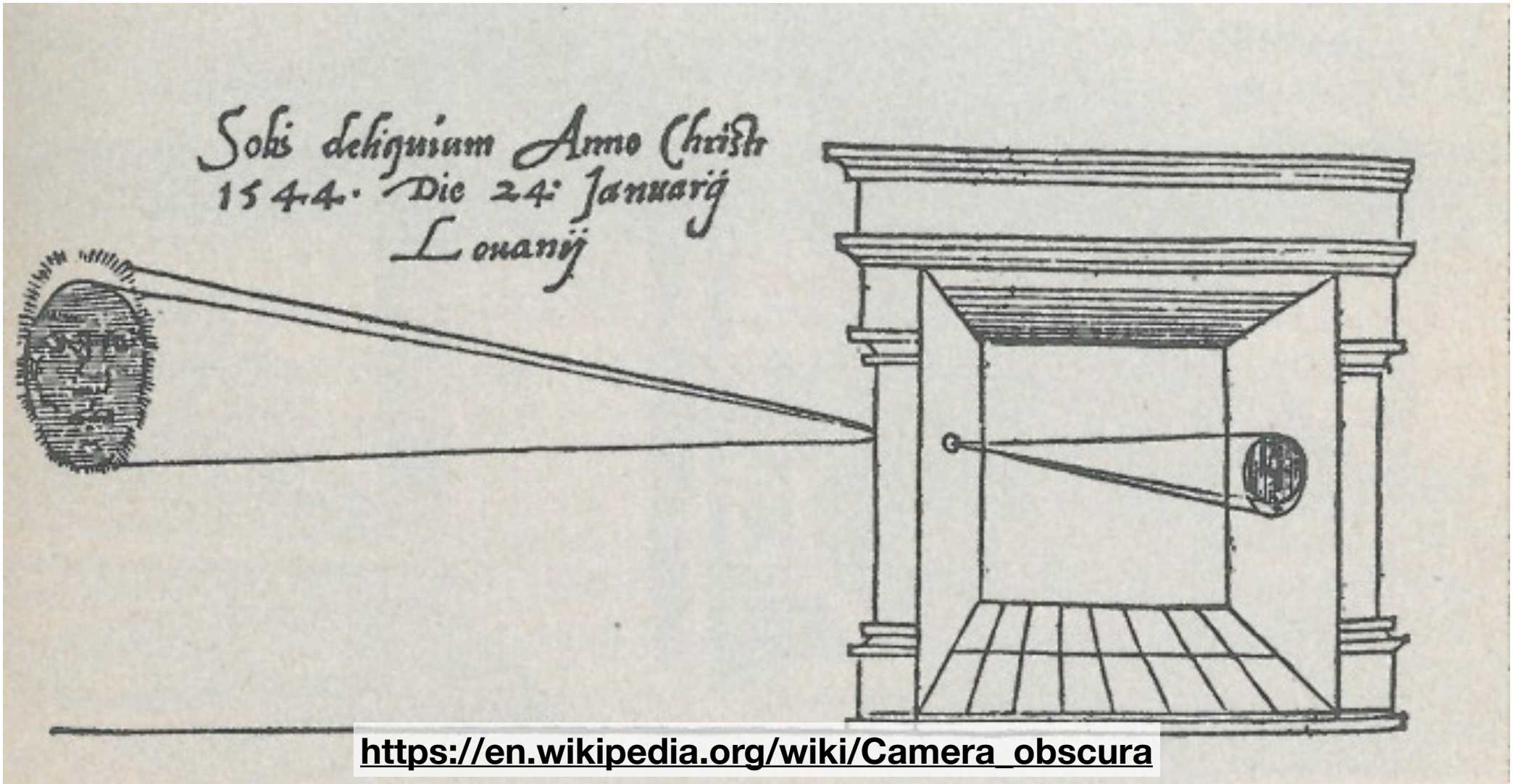
- Idea: Consider a box with a tiny hole. All light that passes through this hole will hit the opposite side

- Produced image inverts



Camera Obscura

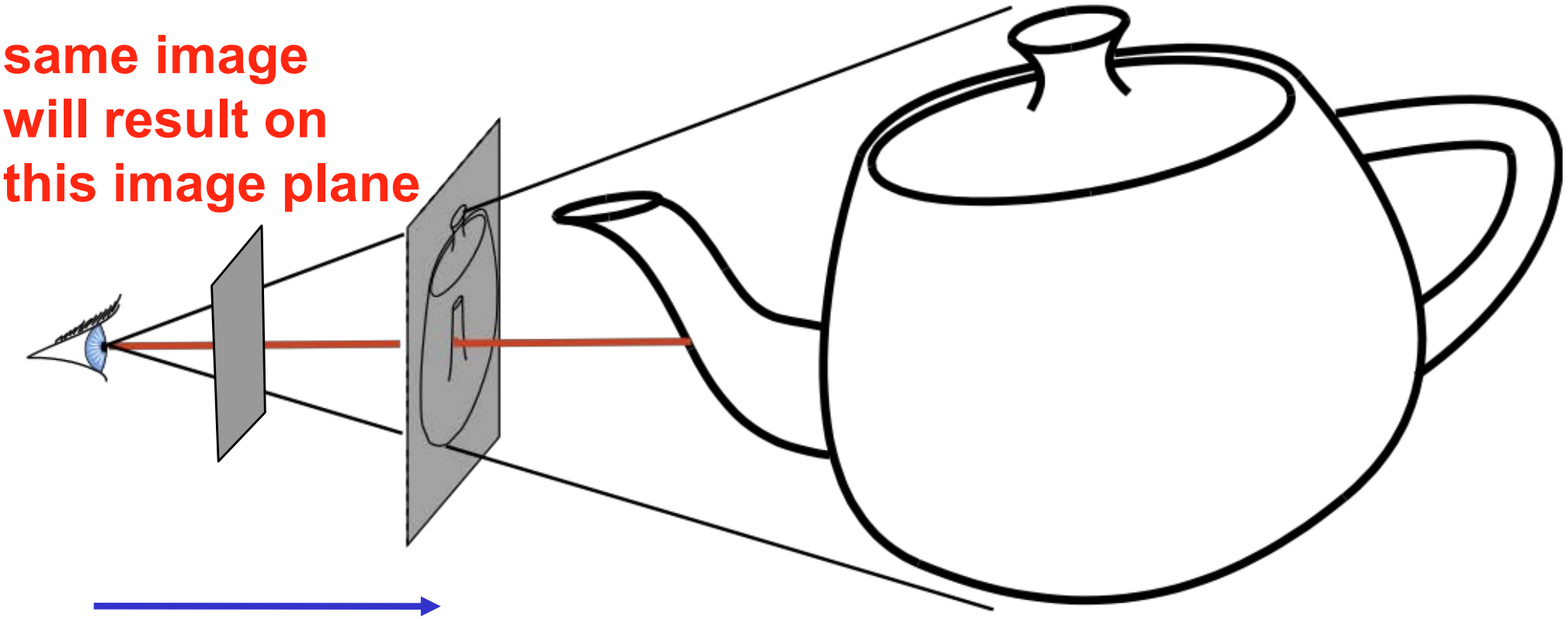
- Gemma Frisius, 16th century



https://en.wikipedia.org/wiki/Camera_obscura

Simplified Pinhole Cameras

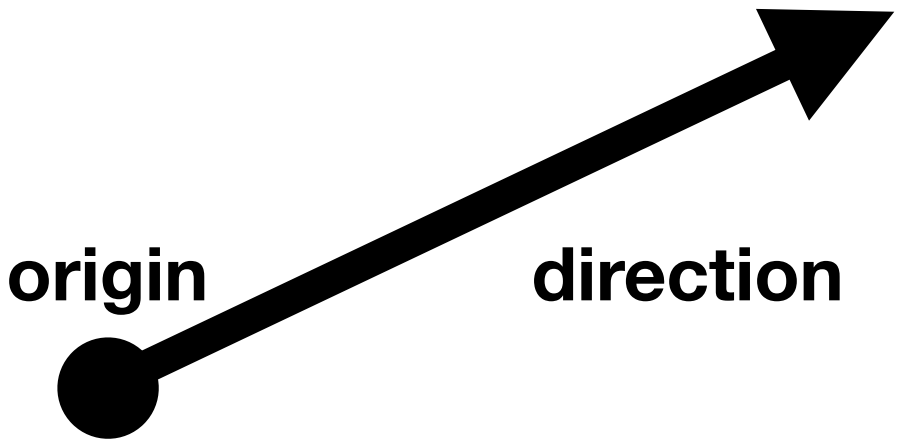
- Instead, we can place the eye at the pinhole and consider the eye-image pyramid (sometimes called **view frustum**)



Defining Rays

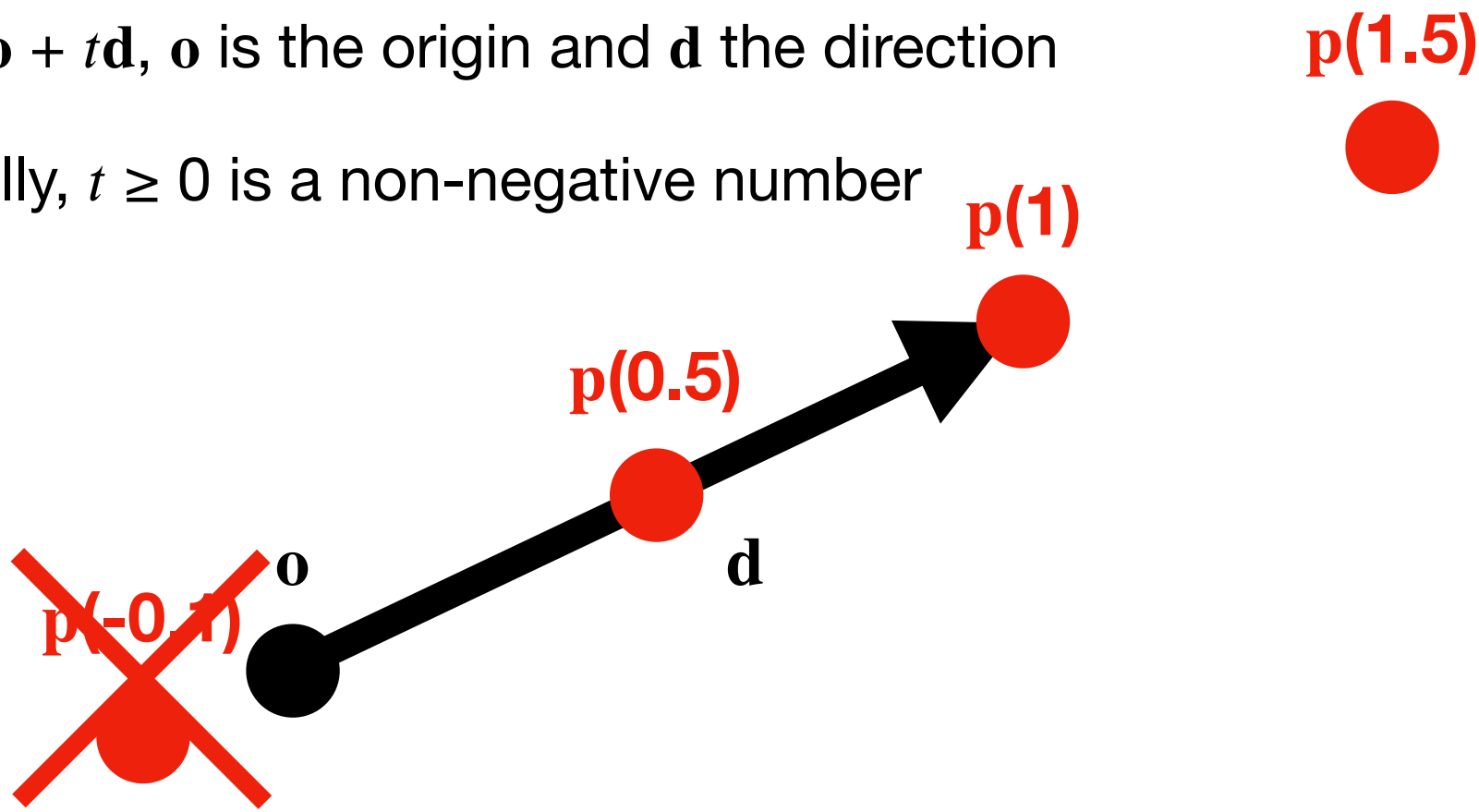
Mathematical Description of a Ray

- Two components:
  - An **origin**, or a position that the ray starts from
  - A **direction**, or a vector pointing in the direction the ray travels
    - Not necessarily unit length, but it's sometimes helpful to think of these as normalized

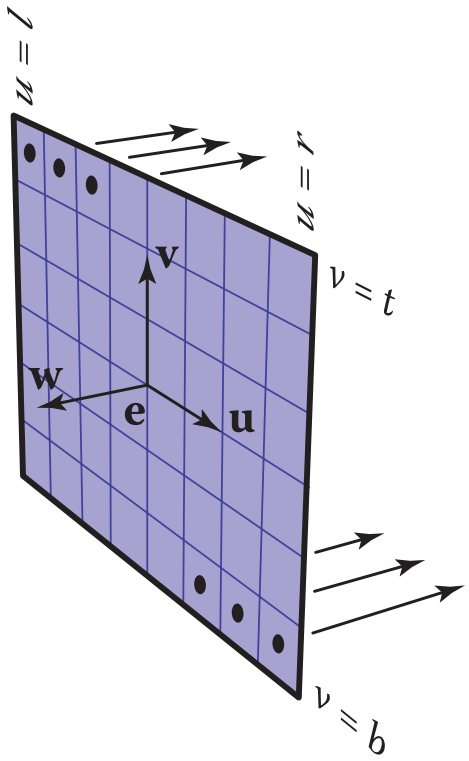


Mathematical Description of a Ray

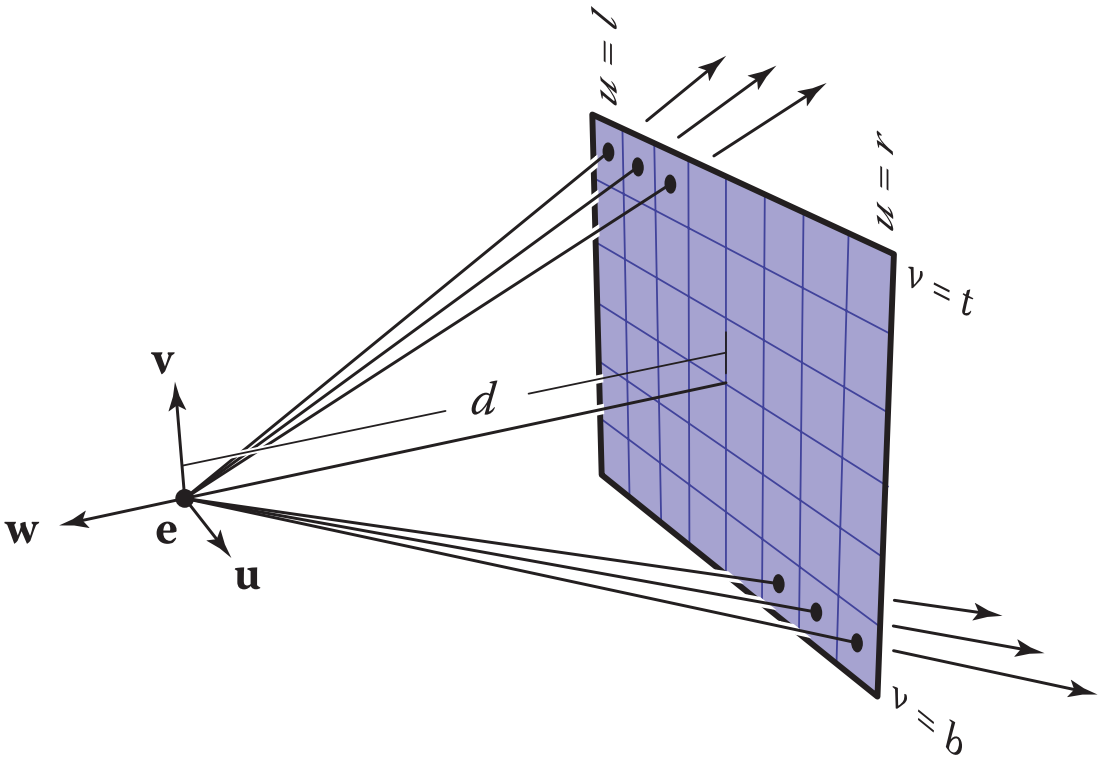
- Rays define a family of points, $\mathbf{p}(t)$, using a **parametric** definition
- $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$, $\mathbf{o}$ is the origin and $\mathbf{d}$ the direction
- Typically, $t \geq 0$ is a non-negative number



Orthographic vs. Perspective Rays



Parallel projection
same direction, different origins



Perspective projection
same origin, different directions

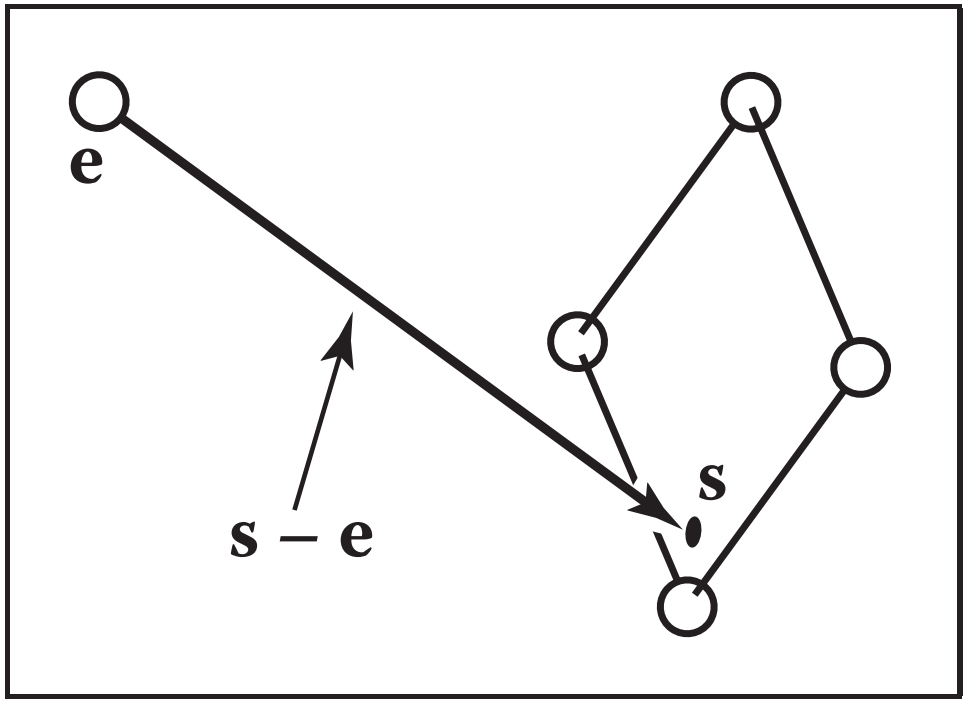
Defining o and d in Perspective Projection

- Given a viewpoint, $\mathbf{e}$, and a position on the image plane, $\mathbf{s}$

  $\mathbf{o} = \mathbf{e}$

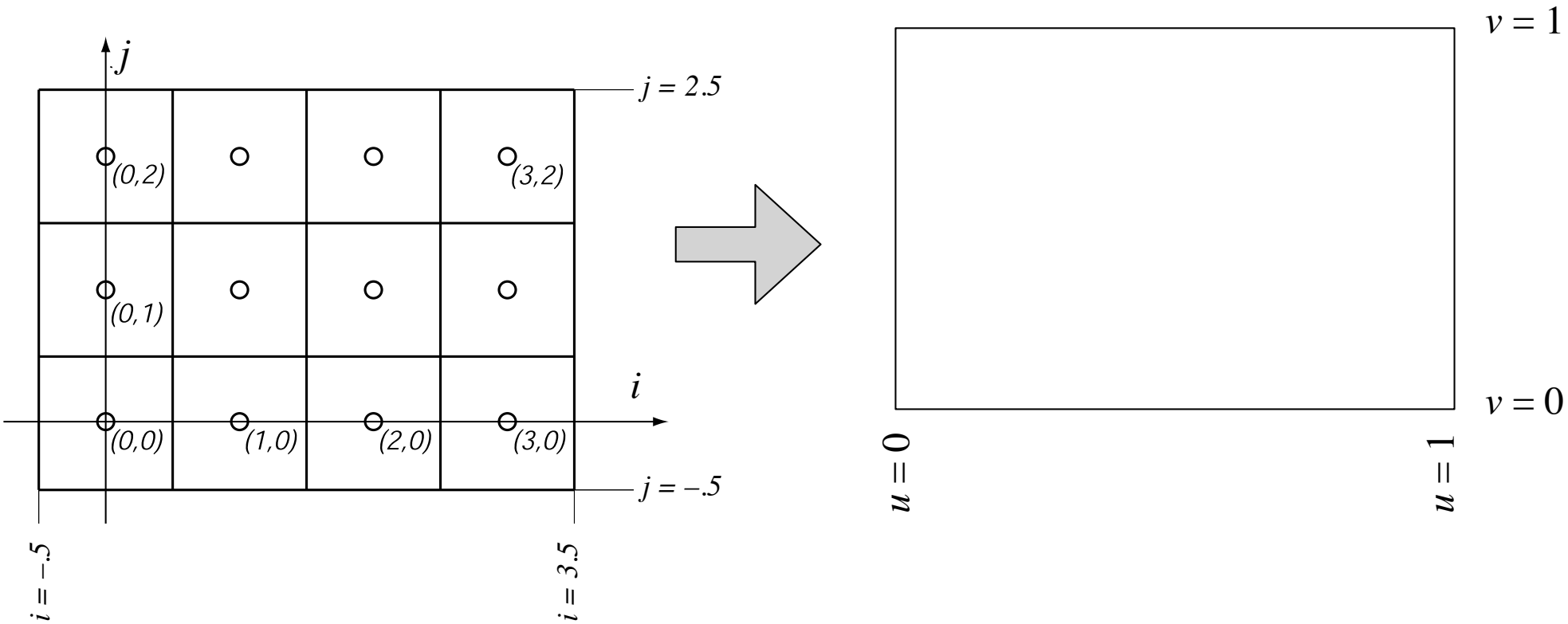
  $\mathbf{d} = \mathbf{s} - \mathbf{e}$

- And thus $\mathbf{p}(t) = \mathbf{e} + t(\mathbf{s} - \mathbf{e})$



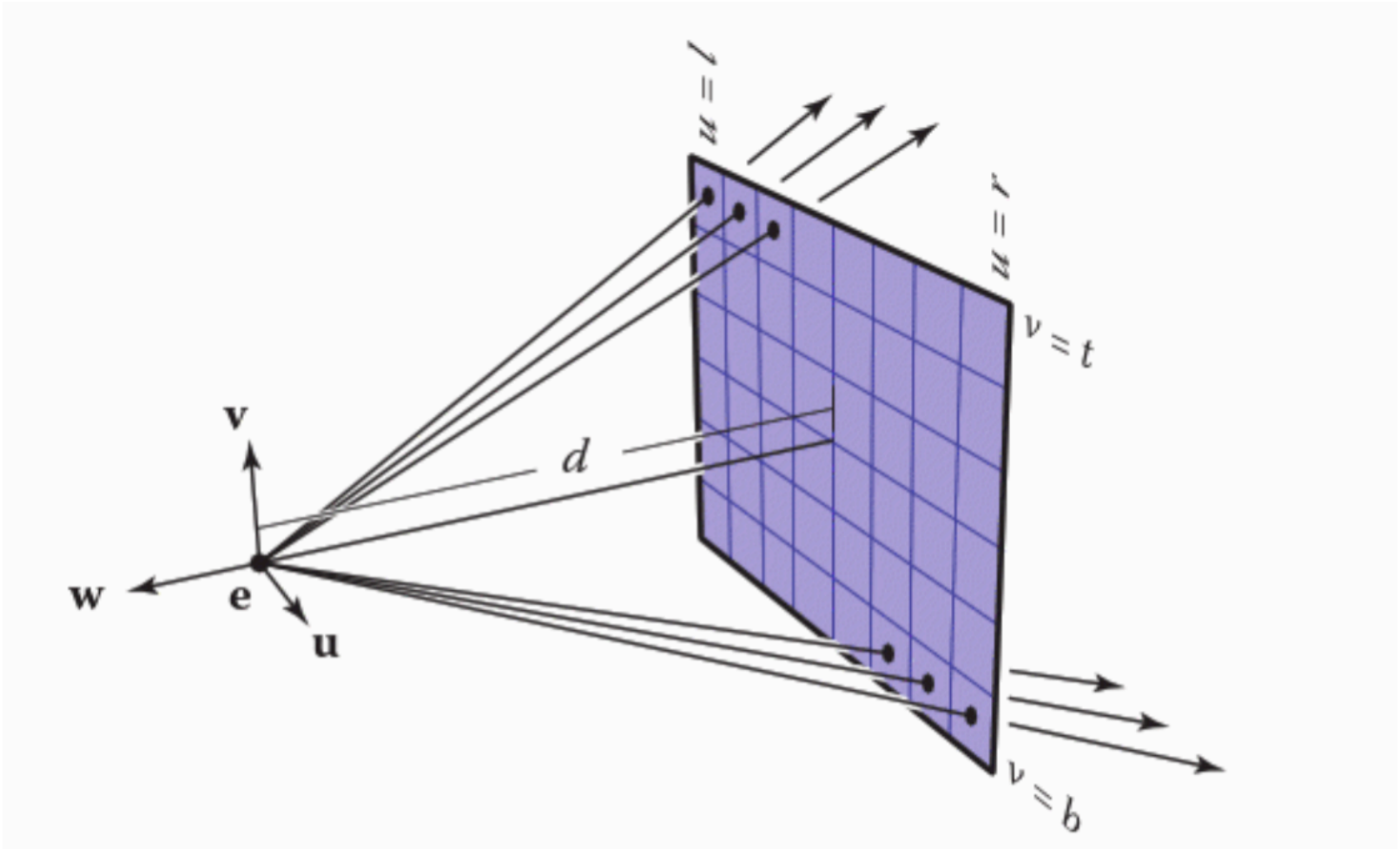
Pixel-to-Image Mapping

- Exactly where are pixels located? Must convert from pixel coordinates (i,j) to positions in 3D space (u,v,w)
- What should w be?



$$u = (i + 0.5)/n_x$$

$$v = (j + 0.5)/n_y$$

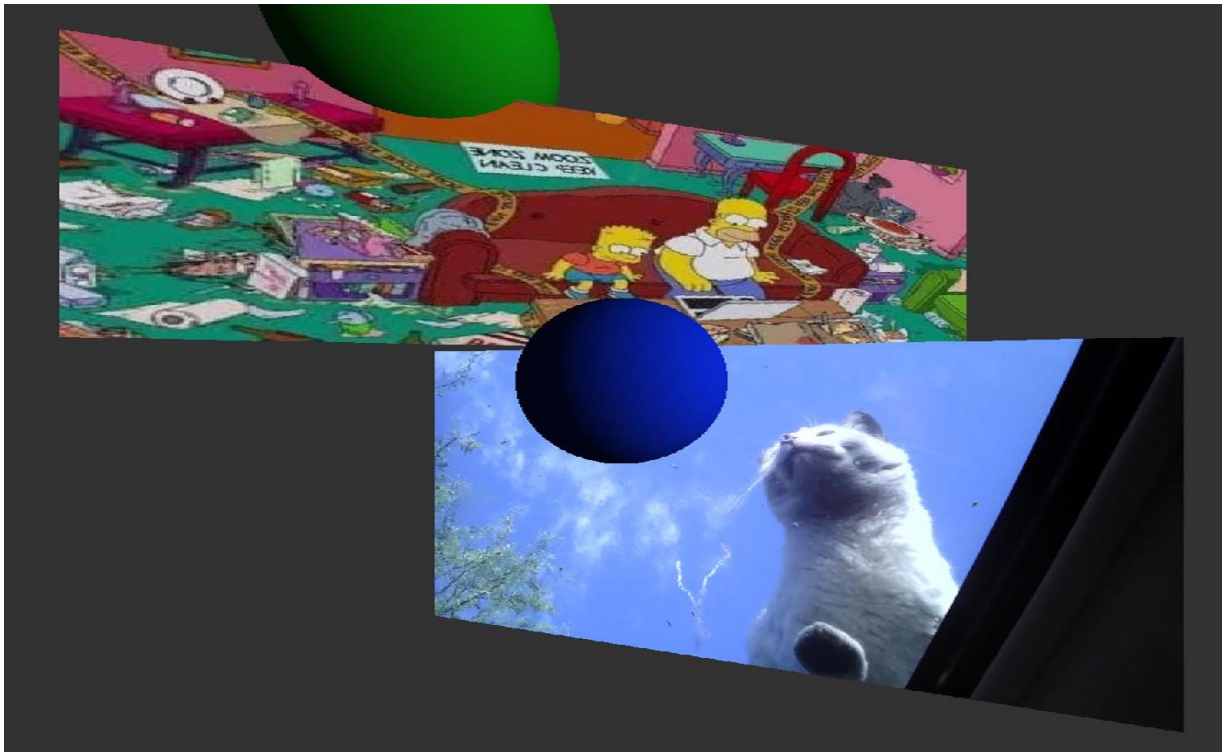

u = l
u = r
v = t
v = b
v
d
w
e
u

Camera Components

- Definition of an image plane
    - Both in terms of pixel resolution AND position in 3D space or more frequently in **field of view** and/or **distance**
- Viewpoint
- View direction
- Up vector

Assignment 3. Balls and Billboards

Input: JSON file describing locations of billboards and spheres.
Images placed on the billboards.
Output: scene showing what a viewer could see



In our trivia corner today: Line-Scan cameras

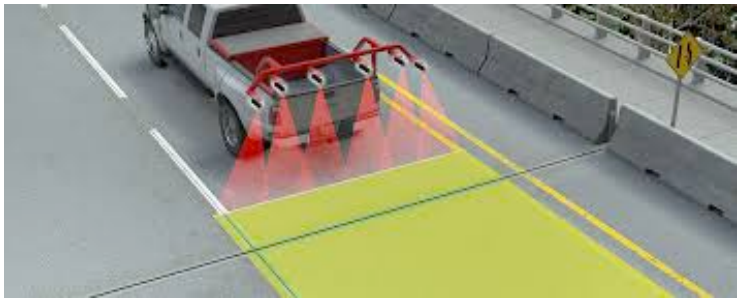
$1300

Model No.	MV-CL020-40GC
Type	2k pixels line scan GigE camera
Pixel Size	7 µm
Horizontal Frequency	1 Hz to 51 kHz

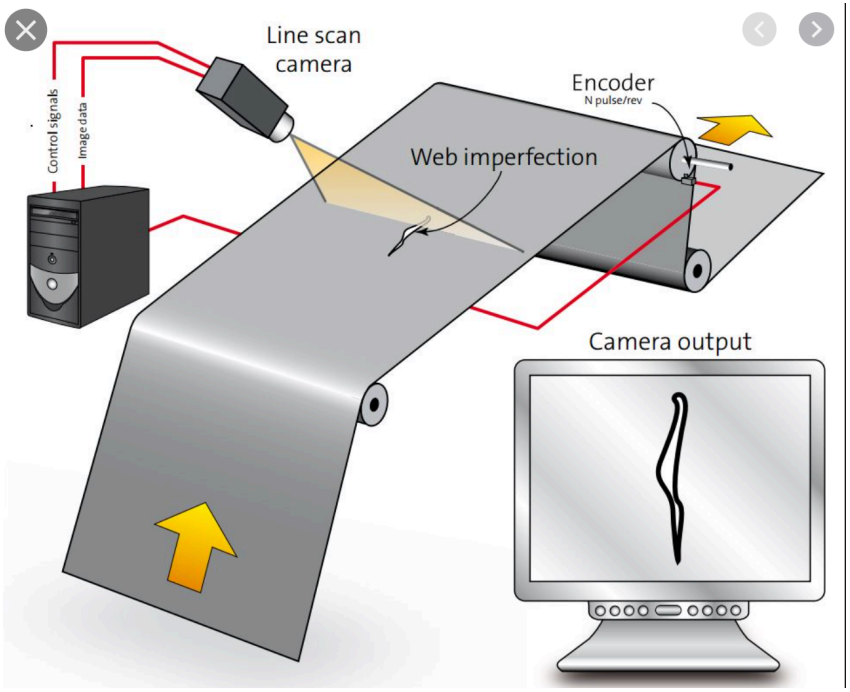
Takes 51000 images /second
But each image is 1D

Other line scan cameras could reach 12000 pixels per scan

Why are they used

- Road monitoring
- 

Better handling noise,
optical distortions
Better dynamic range

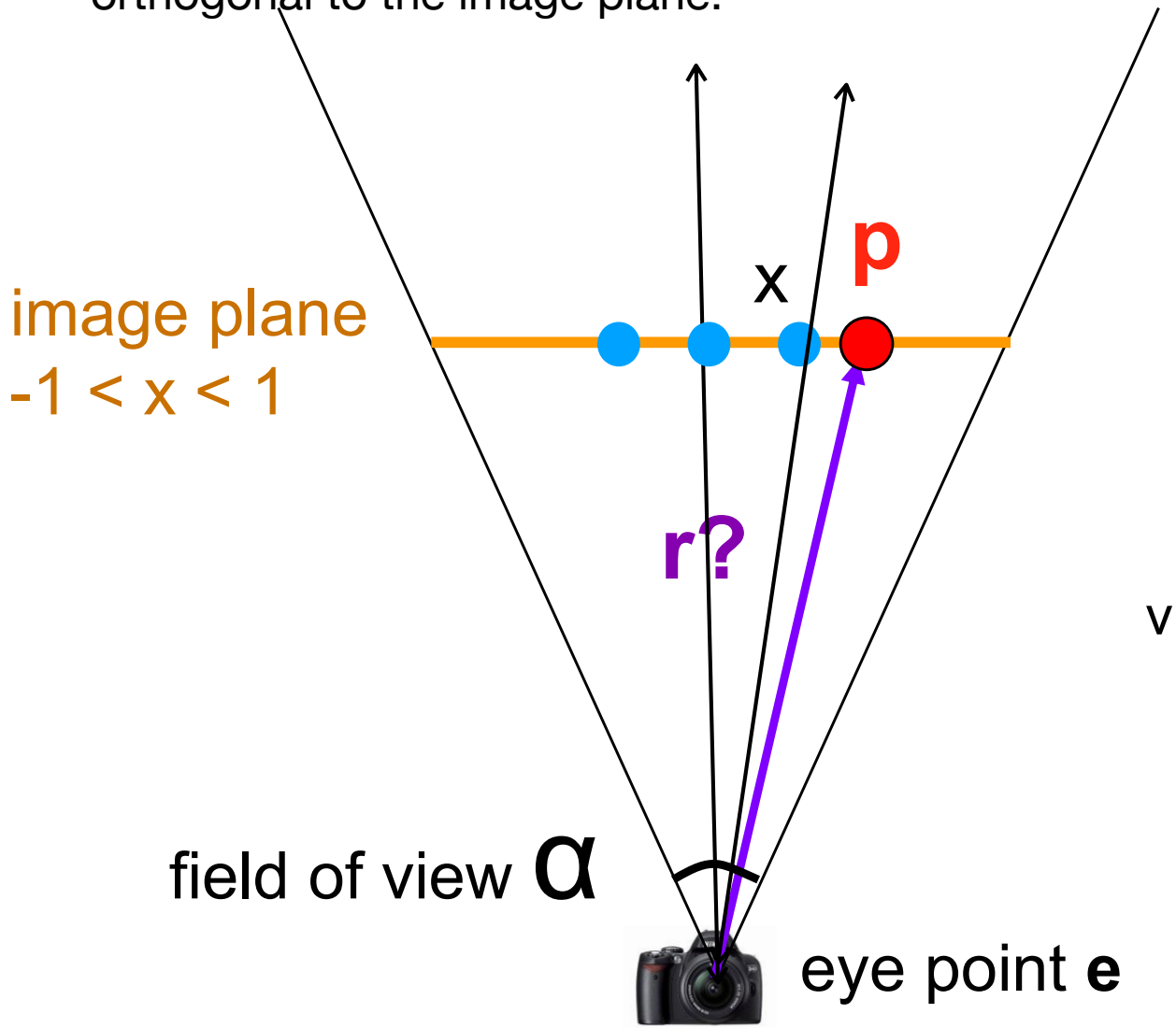


Having the line-scan cameras as a excuse,

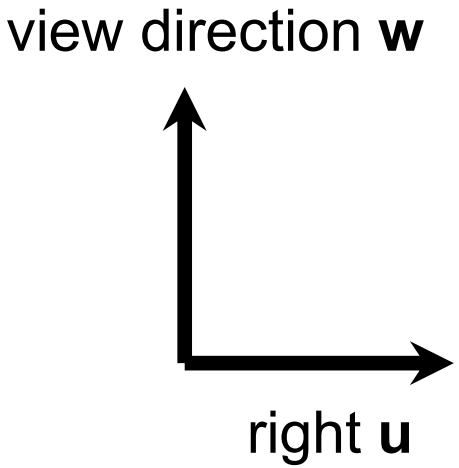
Lets study ray-tracing in the 2D world

Ray Generation in 2D

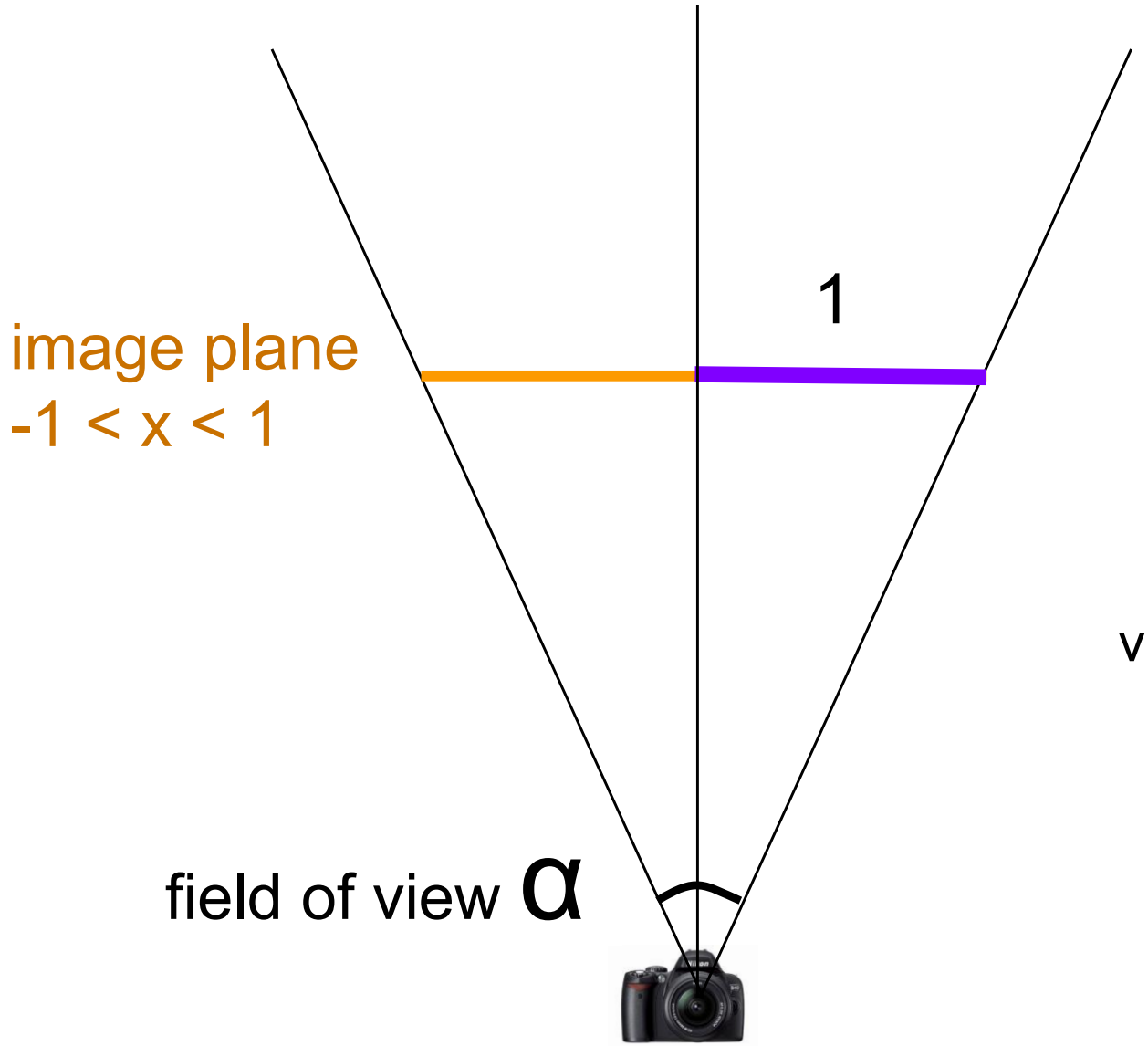
- The image plane (should actually call it ``image line")
- Our algorithm will assign a color to each pixel, by tracing a ray through the pixel and check the color of the object it hits
- User determined the location **e** of the camera, the direction **w** through, and orthogonal to the image plane.



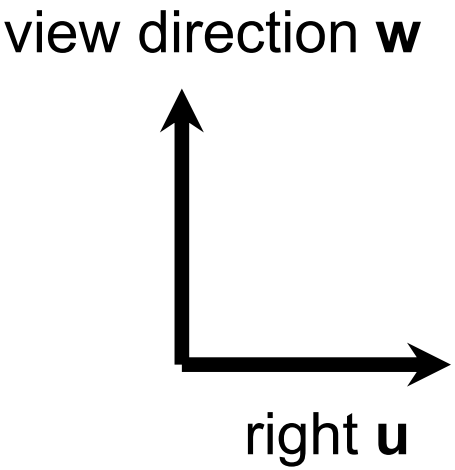
p is point on image plane at coordinate x, we want to know the direction of the ray **r**



Ray Generation in 2D

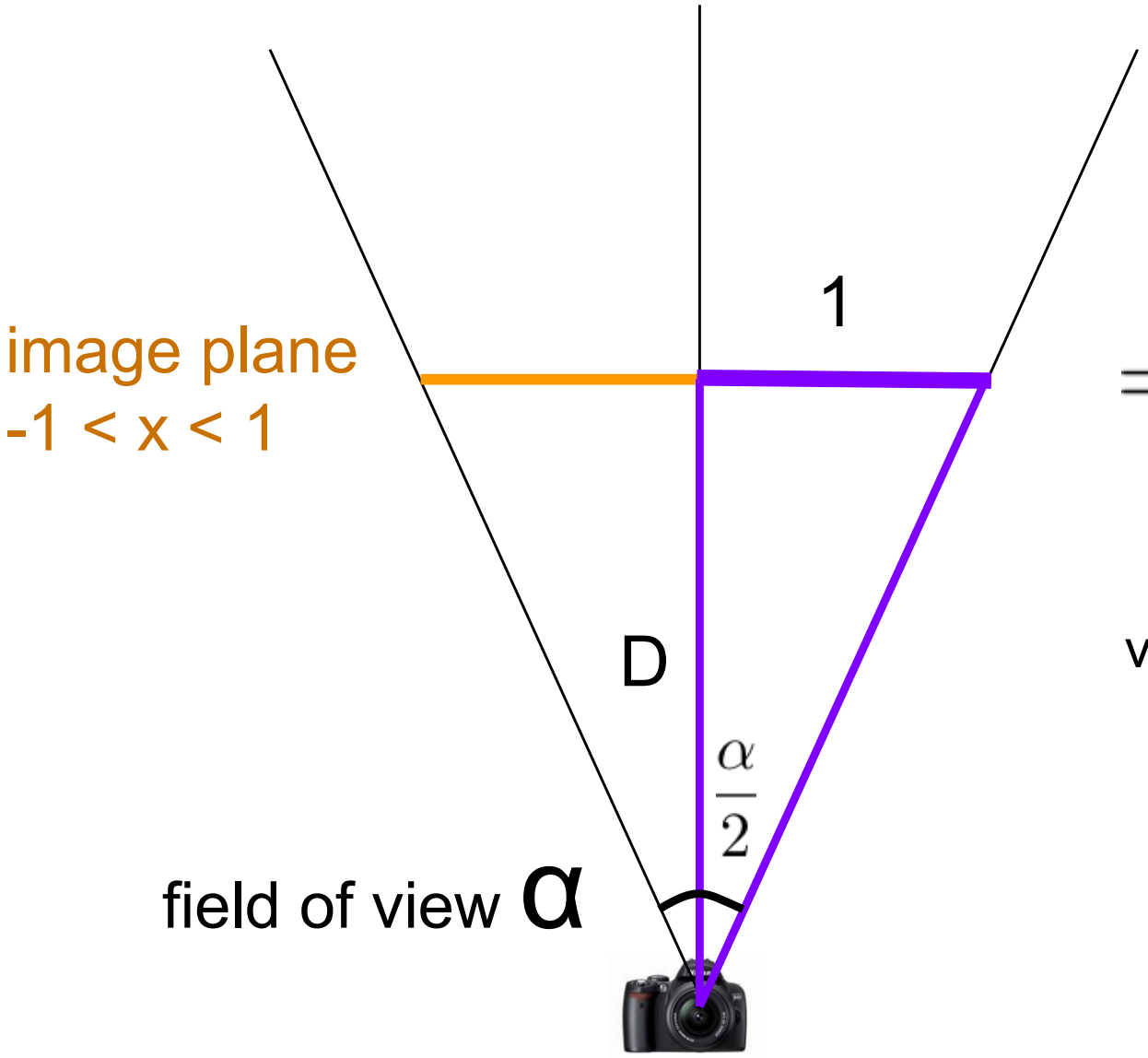


What is the distance D to the screen so that the normalized coordinates go to 1?



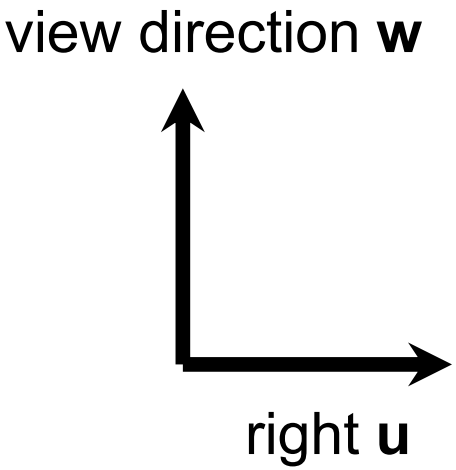
The user specifies the field of view angle `fov_angle`.

`We need to calculate the distance D to the image plane`



$$\tan\frac{\alpha}{2} = \frac{1}{D}$$

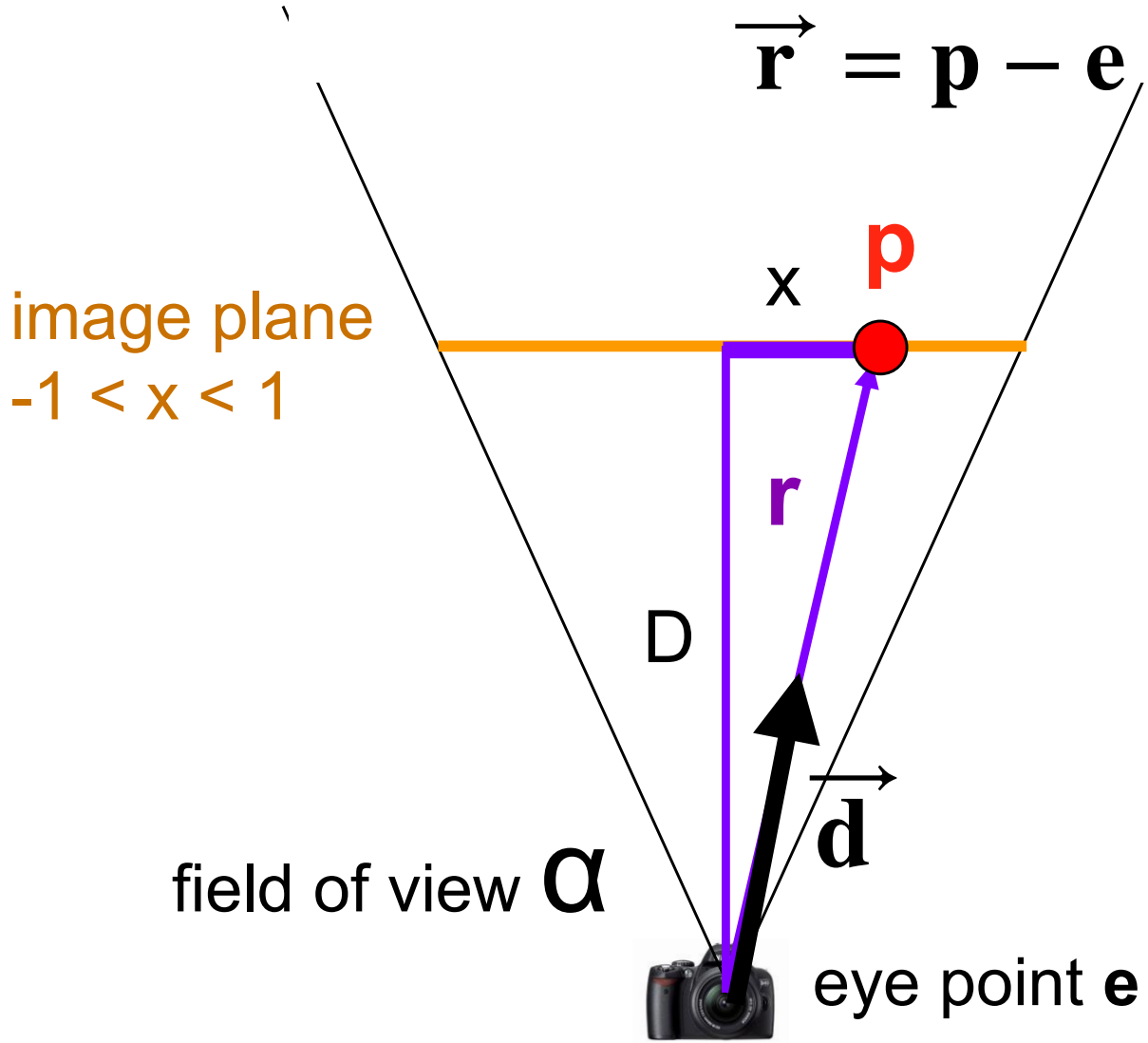
$$\Rightarrow D = \frac{1}{\tan(\alpha/2)}$$



Ray Generation in 2D

$$\vec{\mathbf{p}} = \mathbf{e} + x\vec{\mathbf{u}} + D\vec{\mathbf{w}}$$

$$\vec{\mathbf{r}} = \mathbf{p} - \mathbf{e}$$

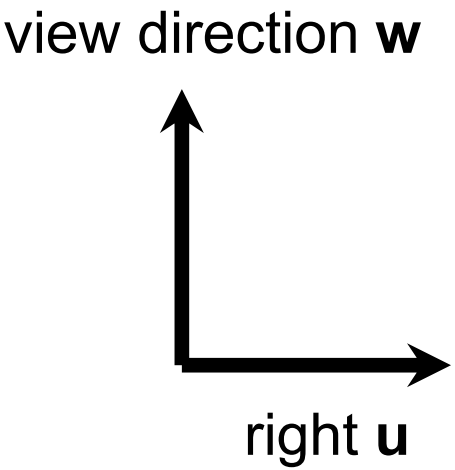


D already computed

Lets normalized $\vec{\mathbf{r}}$

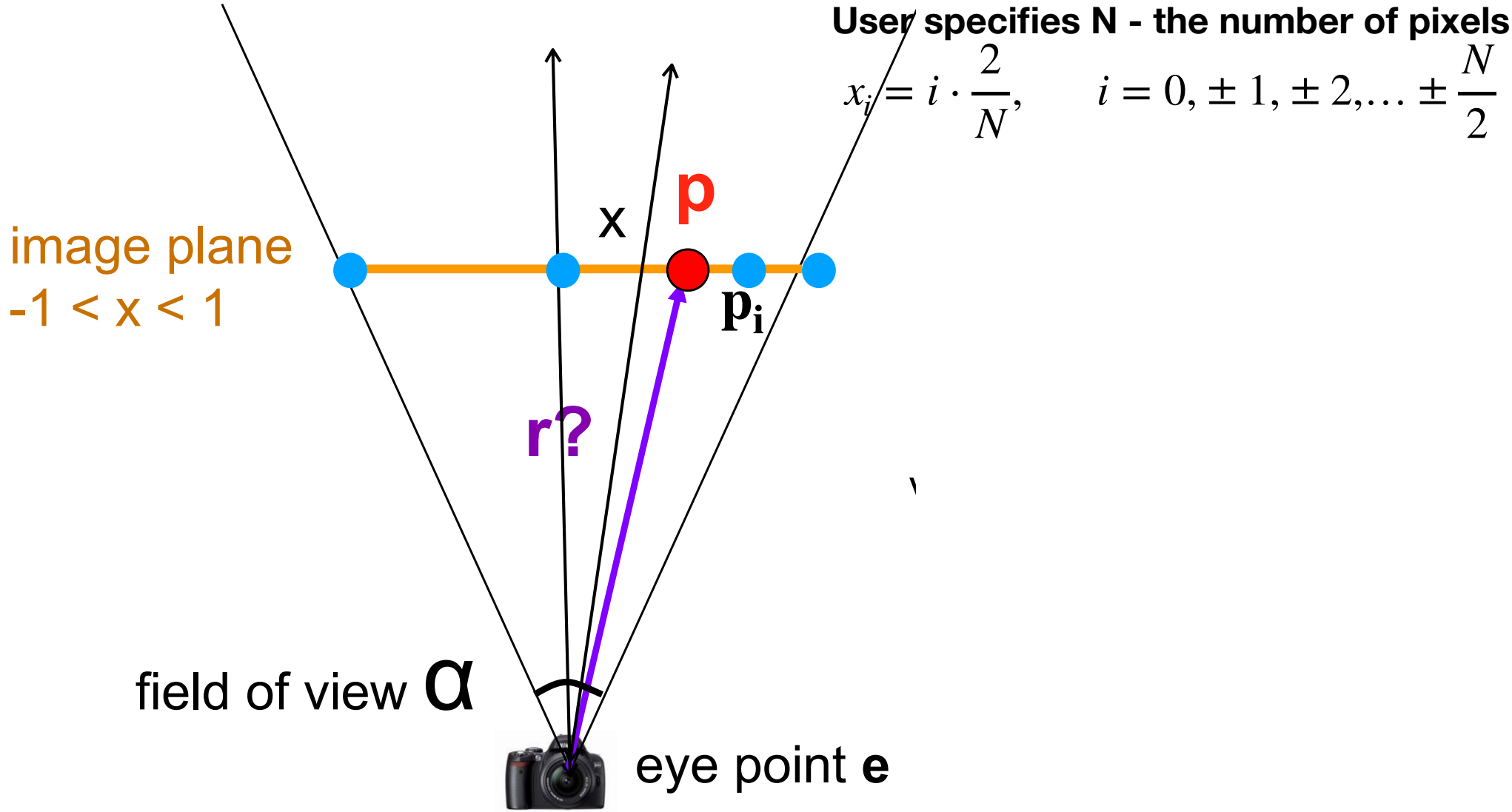
$\vec{\mathbf{d}} = \vec{\mathbf{r}} / |\vec{\mathbf{r}}|$

The ray is $\vec{\mathbf{P}}(\mathbf{t}) = \mathbf{e} + t\vec{\mathbf{r}}$



Calculating all the rays

$$\vec{\mathbf{p}}_\mathbf{i} = \mathbf{e} + x_i\vec{\mathbf{u}} + D\vec{\mathbf{w}}$$



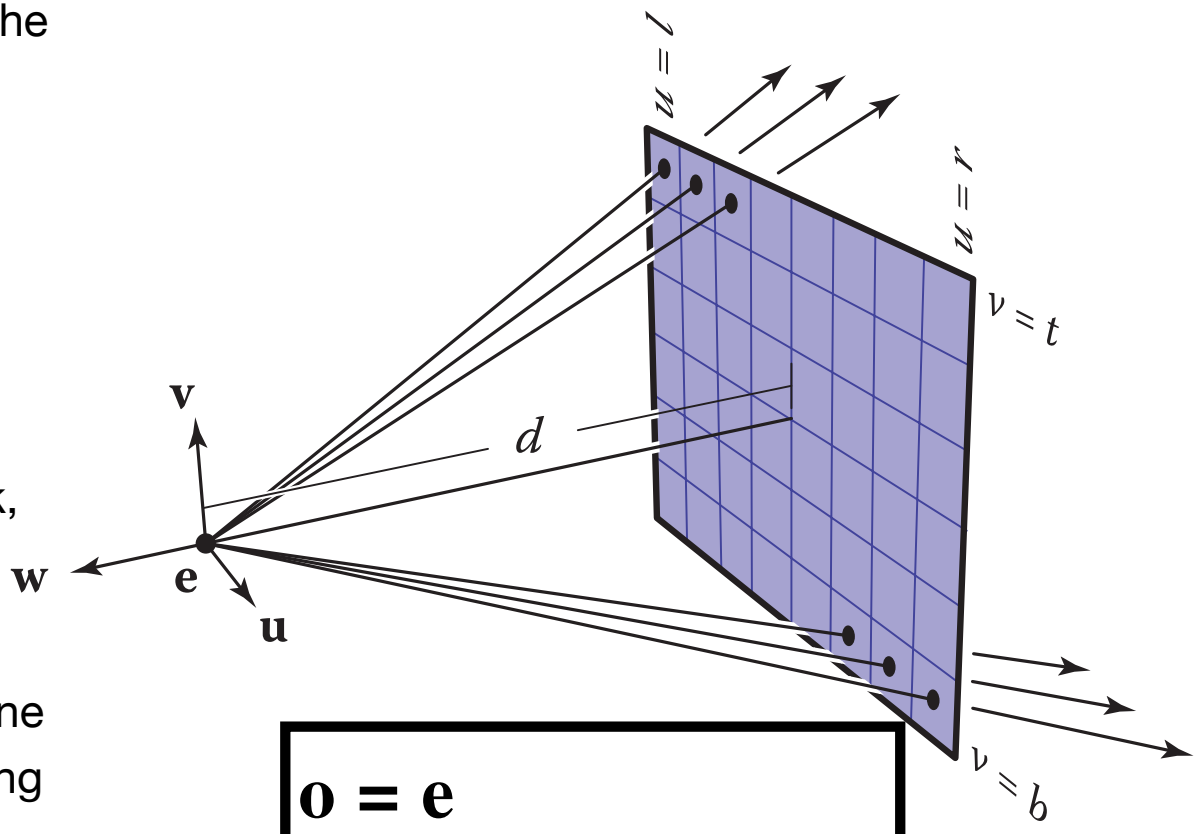
From 2D to 3D

- Moving from 2D to 3D is essentially the same thing once you can define the positions of pixels in uvw space

from now on, we use *d* to denote distance to image plane

- Following the convention of the book, **w** is the negated viewpoint vector

- The up vector, **v**, can be used to define a local coordinate space by computing **u**



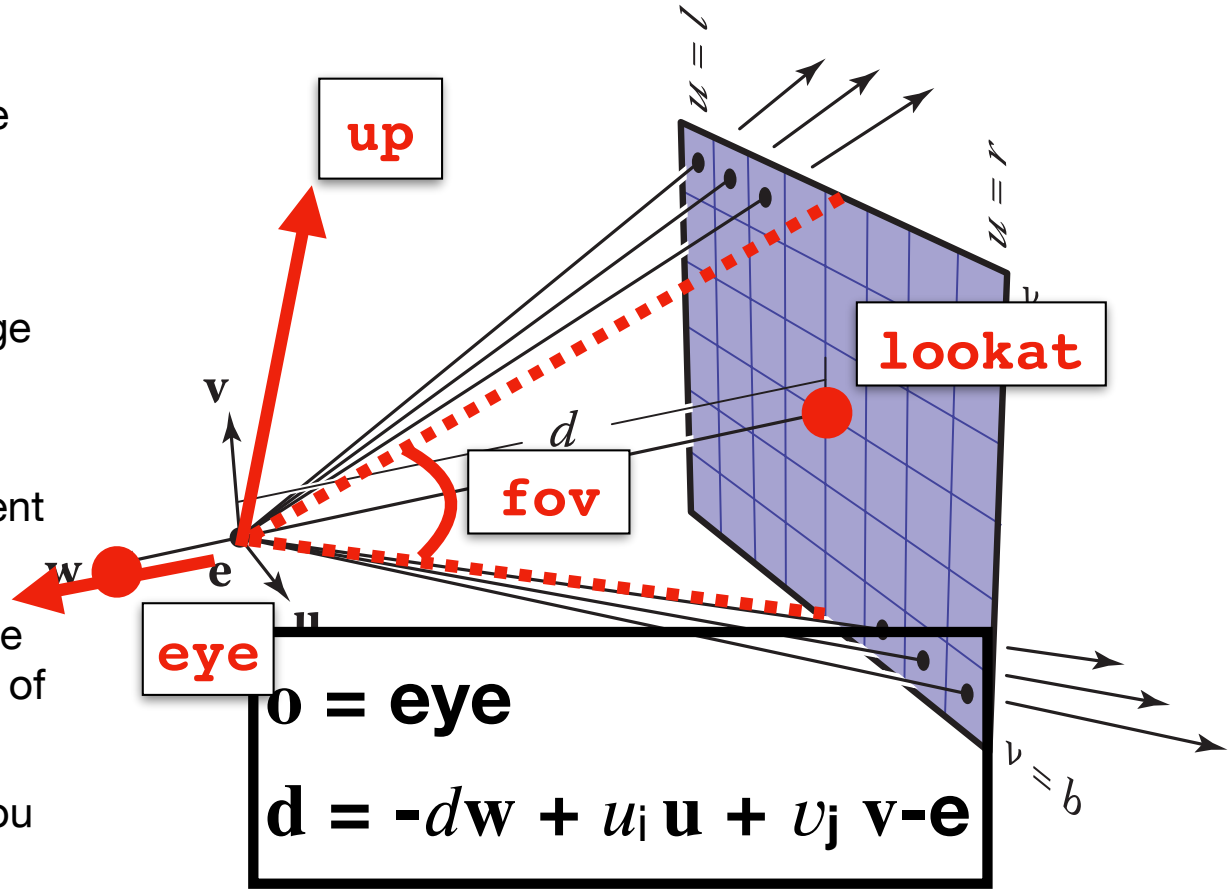
$$\mathbf{o} = \mathbf{e}$$

$$\mathbf{d} = -D\mathbf{w} + u\mathbf{u} + v\mathbf{v}$$

In the Assignment

- An `eye` position
- A position to `lookat`, which is centered in the image
  - **w** can be defined use `eye` and `lookat` as well as d
- An `up` vector, not necessarily **v**! (but the vectors v up w in the same plane)
- A `fov_angle` for the **vertical** FOV
- The FOV defines the **height** of the image plane in world space
- Each pixel is a square, but number of pixels rows vs columns might be different
- You can then use this to compute the **width** of the image plane in world space using the aspect ratio (`rows/columns`) of the image
- Using the number of `rows/columns` you can then sample u, v

We use a terminology that is very common. Not always most intuitive



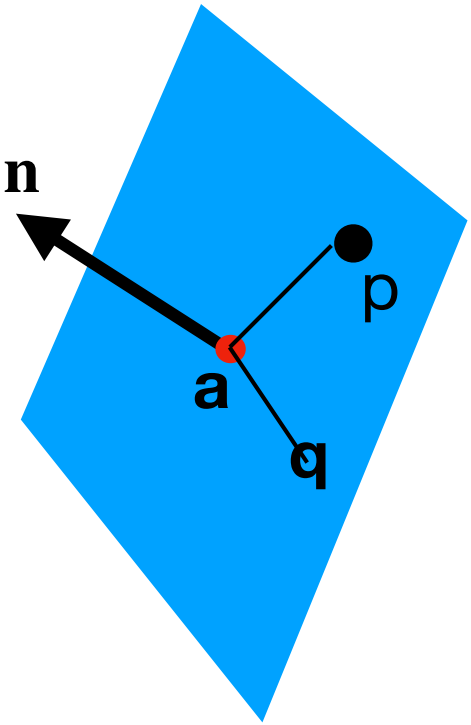
$$u_i = i \cdot 2/N_{cols}, \qquad i = 0, \pm 1, \pm 2, \ldots \pm N_{cols}/2$$

Intersecting Objects

```
for each pixel {
  compute viewing ray
  intersect ray with scene
  compute illumination at intersection
  store resulting color at pixel
}
```

Defining a Plane

- Let h be a plane with normal **n,** and containing a point **a.** Let p be some other point. Then p is on this plane if and only if (iff) $\mathbf{p} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$
- Proof. Consider the segment p-a. p is on the plane iff p-a is orthogonal to n. Using the property of dot product $(\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = |\mathbf{p} - \mathbf{a}||\mathbf{n}| \cos \alpha$
- Here α is the angle between them. Now cos(90)=0. So if p on this plane then $\mathbf{p} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ implying
- If **p n** > **a n** then **p** lives on the "front" side of the plane (in the direction pointed to by the normal
- **p n-an** < 0 means that **p** lives on the "back" side.
- Sometimes used as **f(p)=0** iff ``p on the plane". So the function f(p) is **f(p)=(p-a)n**
- If we have 3 points a,p,q all on the plane, then we can compute a normal $\mathbf{n} = (\mathbf{p} - \mathbf{a}) \times (\mathbf{q} - \mathbf{a})$. (cross product).
- Warning: The term "normal" does not mean that it was normalized.



Ray-Plane Intersection

- A ray $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
- Two conditions must be satisfied:
  - Must be on a ray: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
  - Must be on the plane: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$
- Can substitute the equations and solve for t in $f(\mathbf{p}(t))$:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

- This means that $t_{\text{hit}} = ((\mathbf{a} - \mathbf{o}) \cdot \mathbf{n}) / (\mathbf{d} \cdot \mathbf{n})$. The intersection point is $\mathbf{o} + t_{hit}\mathbf{d}$

Defining a Sphere

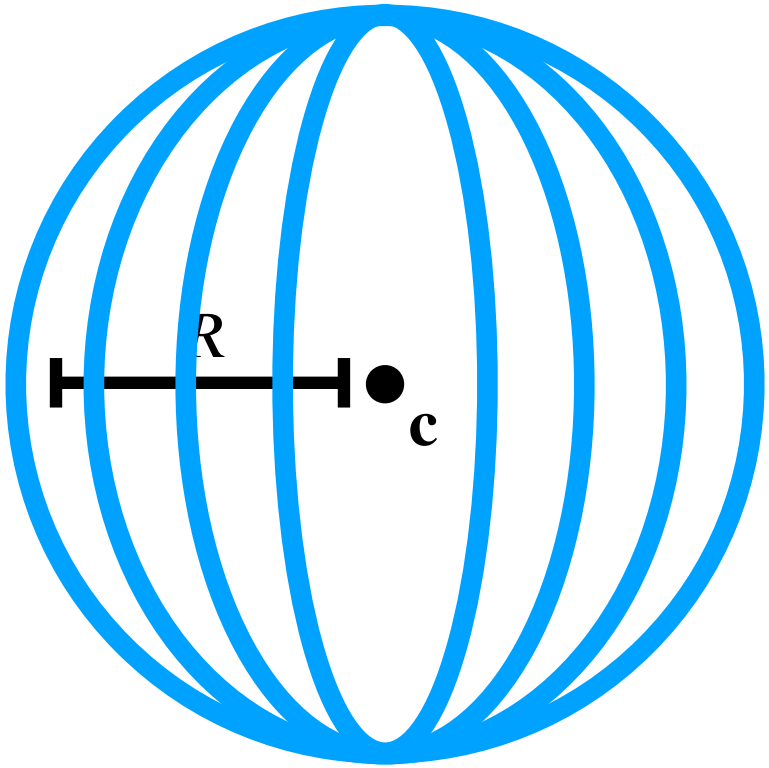
- For every vector v, the length of v is |

$$|\mathbf{v}| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{x^2 + y^2 + z^2}$$

- We can define a sphere of radius R, centered at position $\mathbf{c}$, using the implicit form |p-c|=R, implying

$$f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$$

- Any point $\mathbf{p}$ that satisfies the above lives on the sphere



Ray-Sphere Intersection

- Two conditions must be satisfied:
  - Must be on a ray: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
  - Must be on a sphere: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$
- Can substitute the equations and solve for t in $f(\mathbf{p}(t))$:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$$

- Solving for t is a quadratic equation

Ray-Sphere Intersection

- Solve $(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$ for t:
- Rearrange terms:

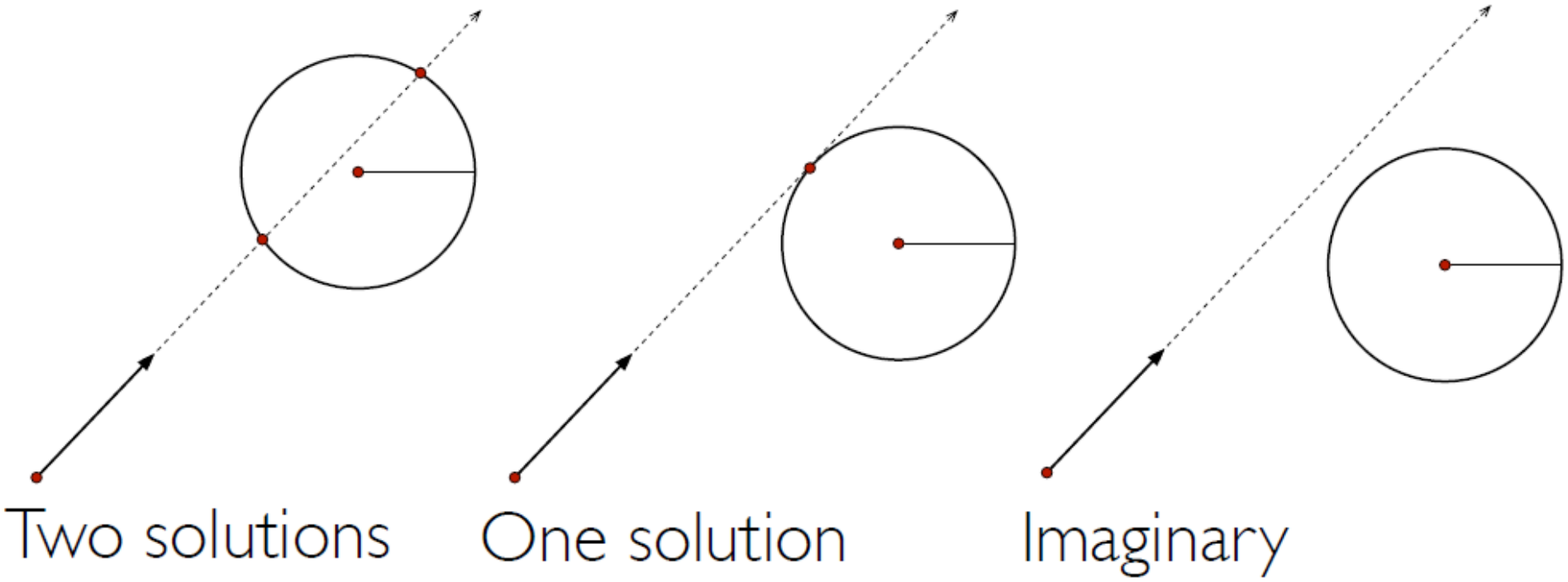
$$(\mathbf{d} \cdot \mathbf{d})t^2 + (2\mathbf{d} \cdot (\mathbf{o} - \mathbf{c}))t + (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2 = 0$$

- Solve the quadratic equation $At^2 + Bt + C = 0$ where
  - $A = (\mathbf{d} \cdot \mathbf{d})$
  - $B = 2^*\mathbf{d} \cdot (\mathbf{o} - \mathbf{c})$
  - $C = (\mathbf{o} - \mathbf{c}) \cdot (\mathbf{o} - \mathbf{c}) - R^2$

Discriminant, $D = B^2-4^*A^*C$
Solutions must satisfy:
$t = (-B \pm \sqrt{(D)}) / 2A$

Ray-Sphere Intersection

- Number of intersections dictated by the discriminant
- In the case of two solutions, prefer the one with lower t



Revisiting dot product: projections

Let **u,v** be orthonormal vectors **(orthogonal and unit length). r** is another vector

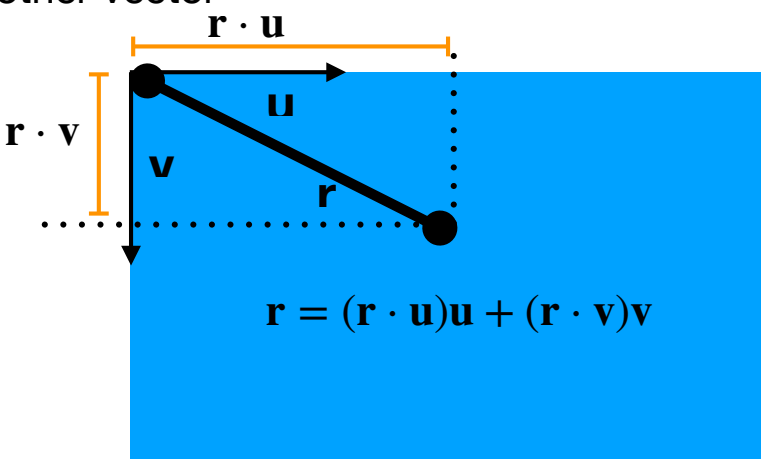
$\mathbf{r} \cdot \mathbf{u} = |\mathbf{r}||\mathbf{u}|\cos\alpha = |\mathbf{r}|\cos\alpha$

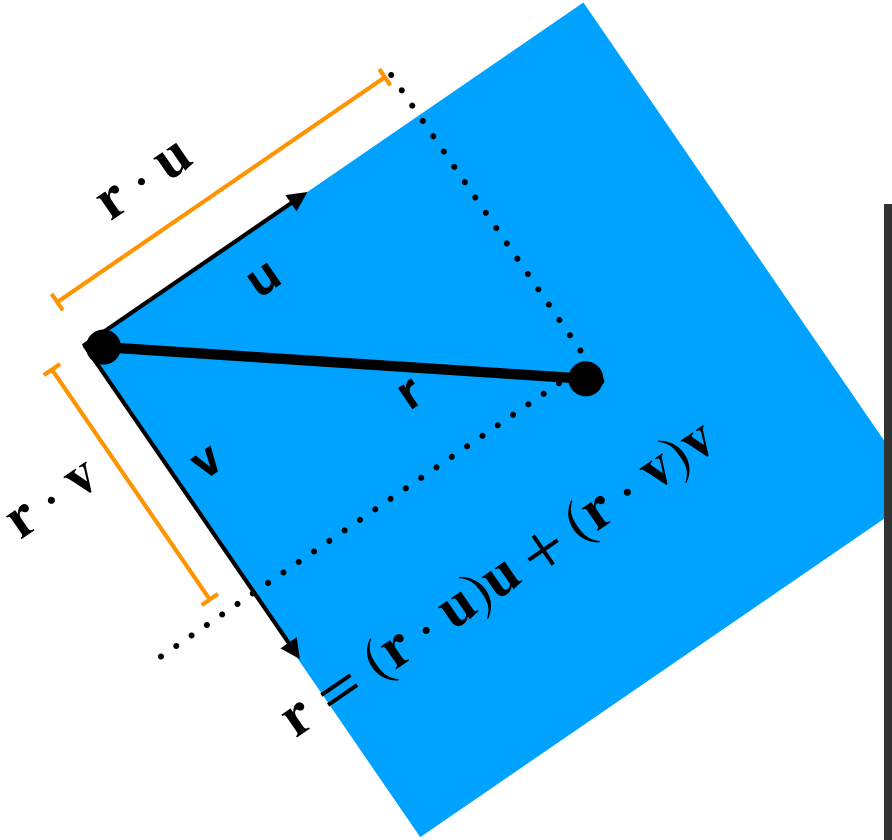
is the projection of **r** on the direction of **u**

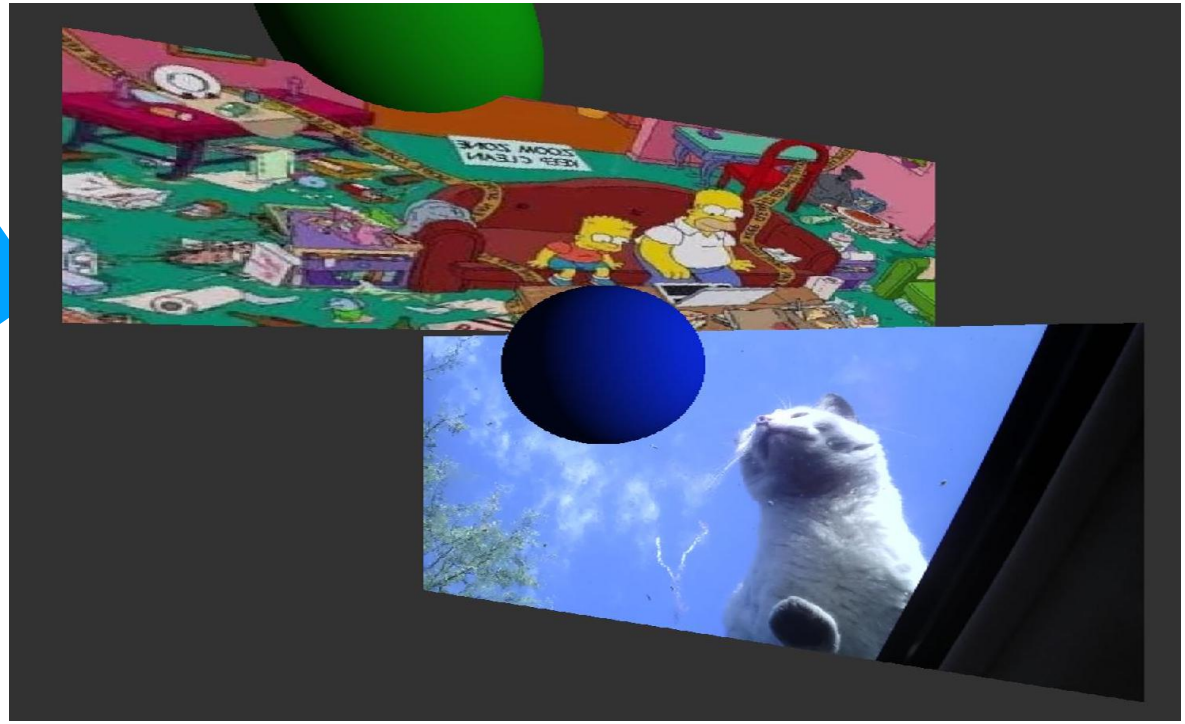
the length of the “shadow” that **r** cast on the line containing **u**

Sounds obvious when the coordinate system is xyz

But also true if the system is rotated







More Examples Of What Can Be Done With Ray Tracing

- https://developer.nvidia.com/optix
- https://embree.github.io/gallery.html

Next Lecture Required Reading

- FOCG, Ch. 4.5-4.9

Assignment 3. Balls and Billboards

Input: JSON file describing locations of billboards and spheres.
Images placed on the billboards.
Output: scene showing what a viewer could see