

CSC 433/533

Computer Graphics

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Thanks: Joshua Levine

Lecture 15

Triangle Meshes

Oct. 13, 2020

Today's Agenda

- Goals for today:
 - Midterm chat
 - Finish distributed ray shooting
 - Discuss how complex shapes are stored and rendered

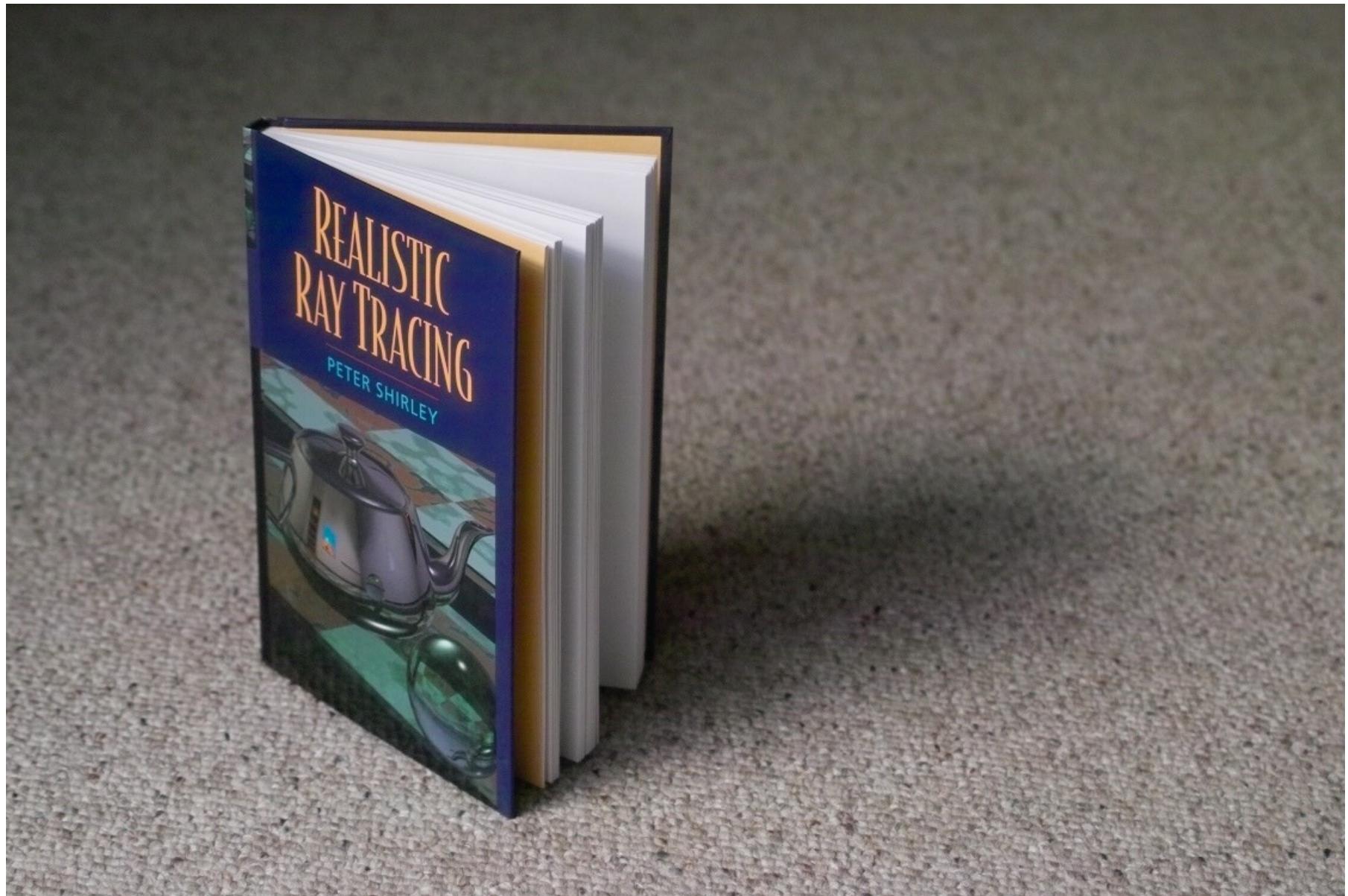
Midterm

- Introduction
 - Basics (Ch.1)
 - Color and Images (Ch.3)
 - Perception (Ch.19)
- Image Processing
 - Signal Processing Concepts (Ch.9)
 - Tone Reproduction (Ch.21.1-21.3)
- Ray Tracing Concepts
 - Vectors (Ch.2.1-2.4)
 - Ray Tracing (Ch.4)
 - Shading (Ch.10)
 - Advanced Ray Tracing (Ch.13)
- Shapes
 - Meshes (Ch.12.1)
 - Accel. Structures (Ch.12.3)

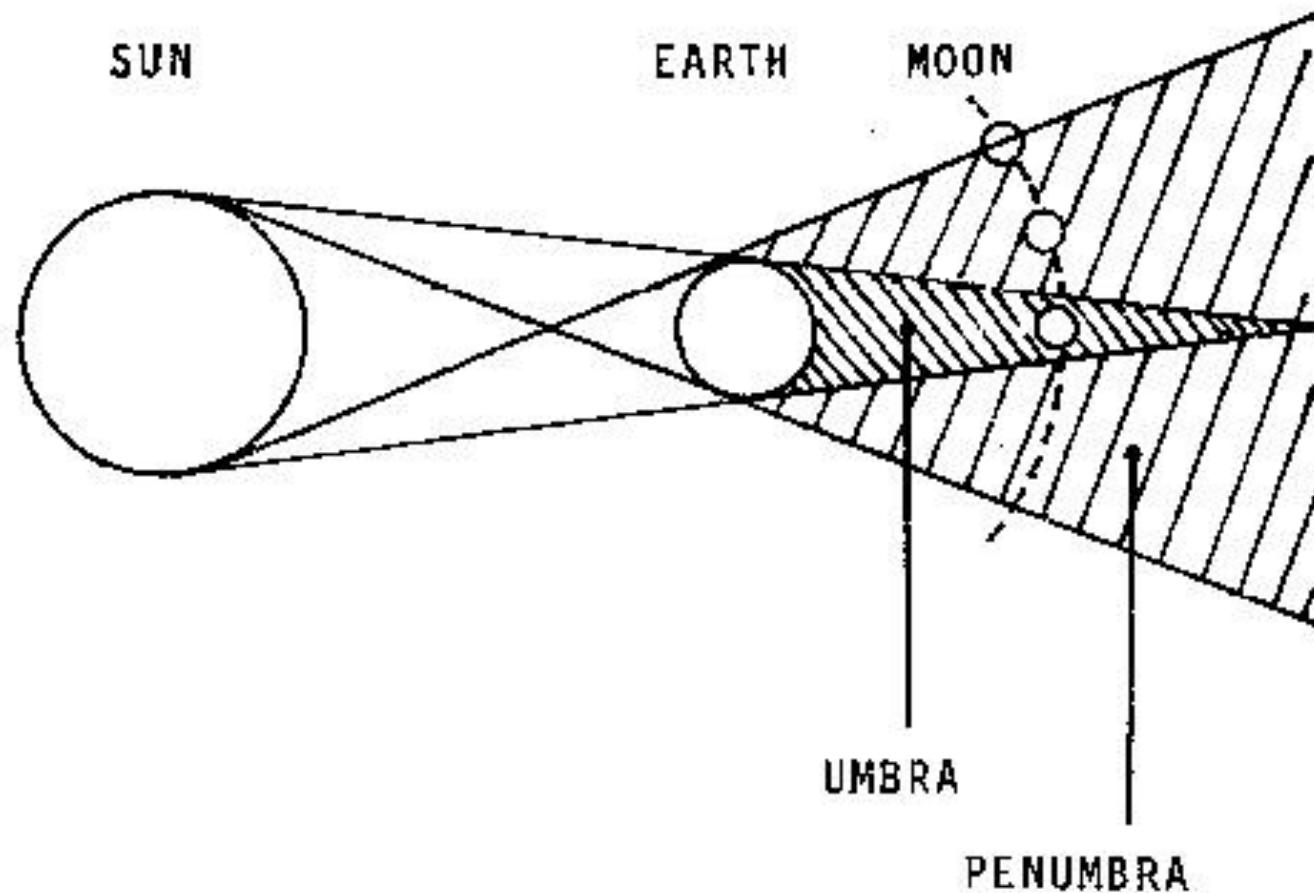
Midterm Format

- Closed book, closed notes: Only need a pen or pencil.
- Usually need to answer 4 out of 5 questions

Soft Shadows



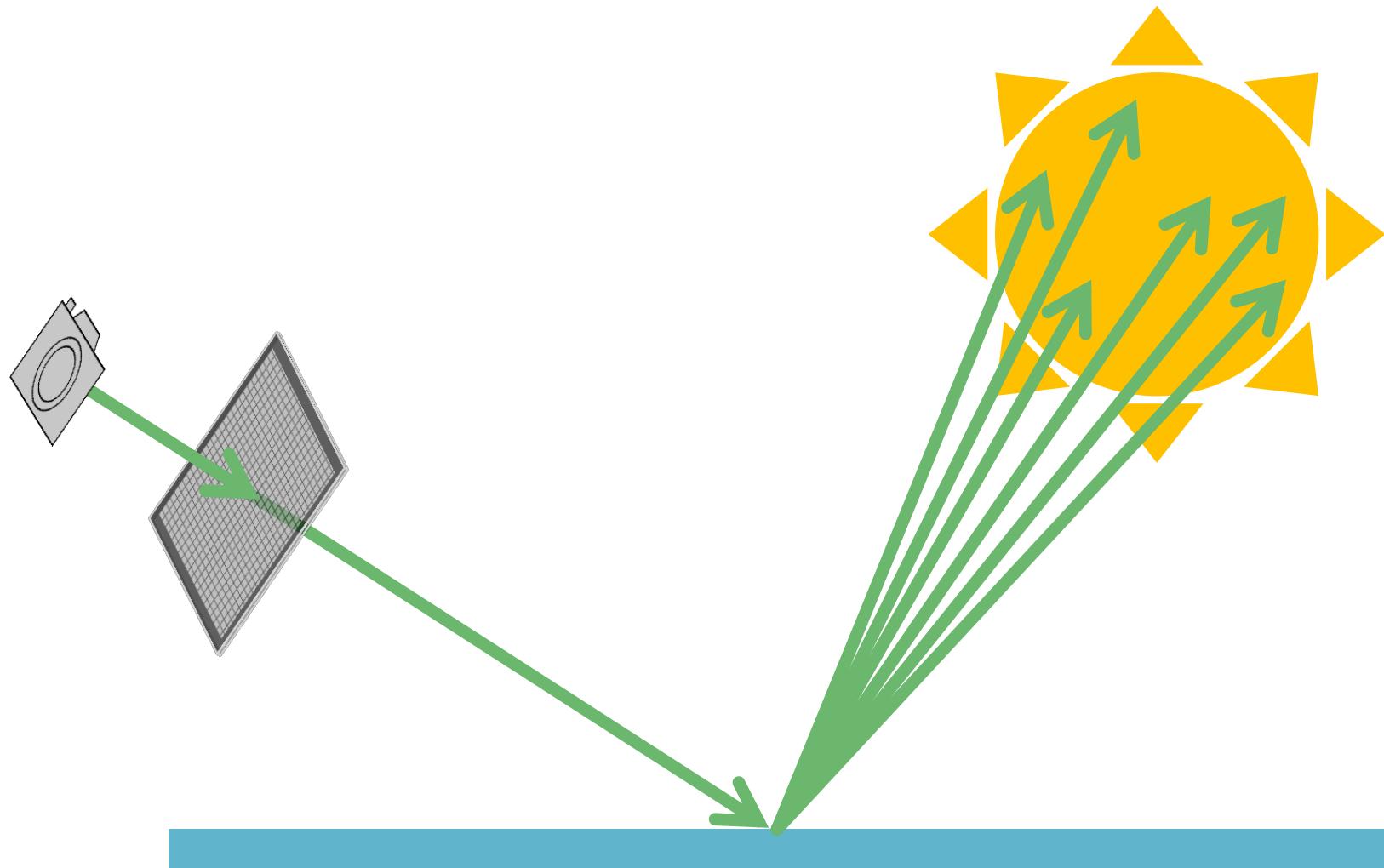
What Causes Soft Shadows



<http://user.online.be/felixverbelen/lunecl.jpg>

Lights aren't all point sources

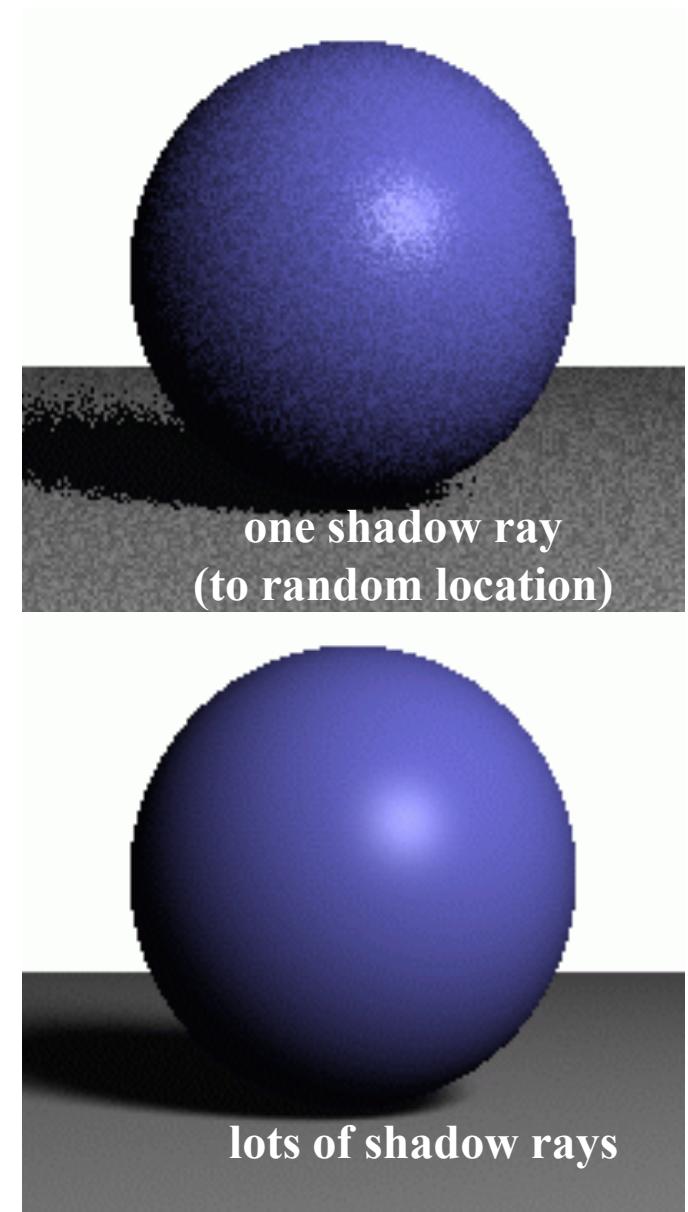
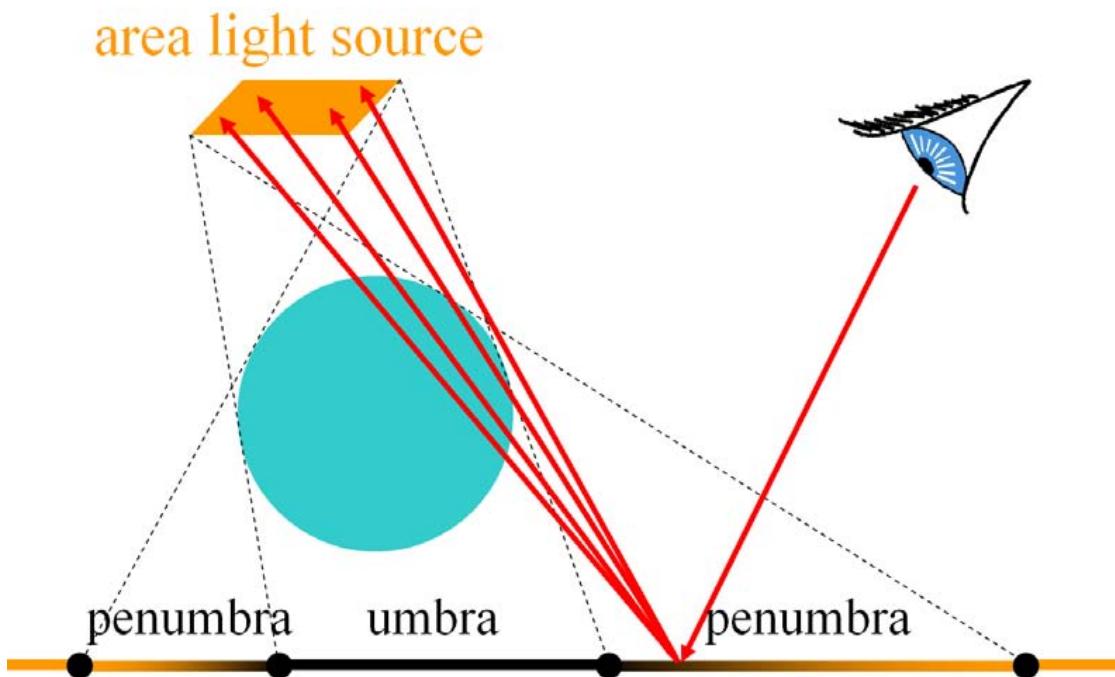
Distribution Soft Shadows



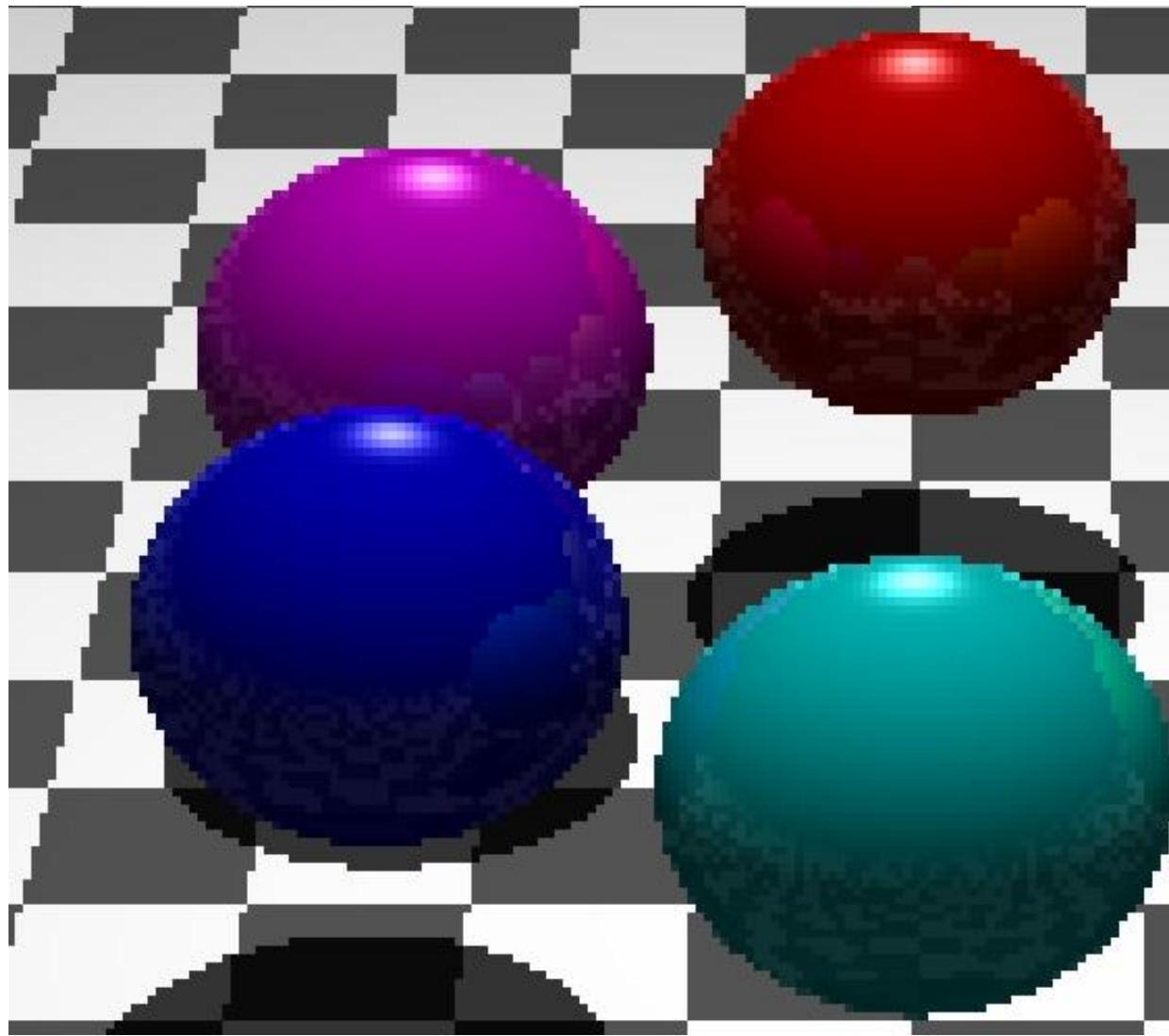
Randomly sample light rays

Computing Soft Shadows

- Model light sources as spanning an area
- Sample random positions on area light source and average rays



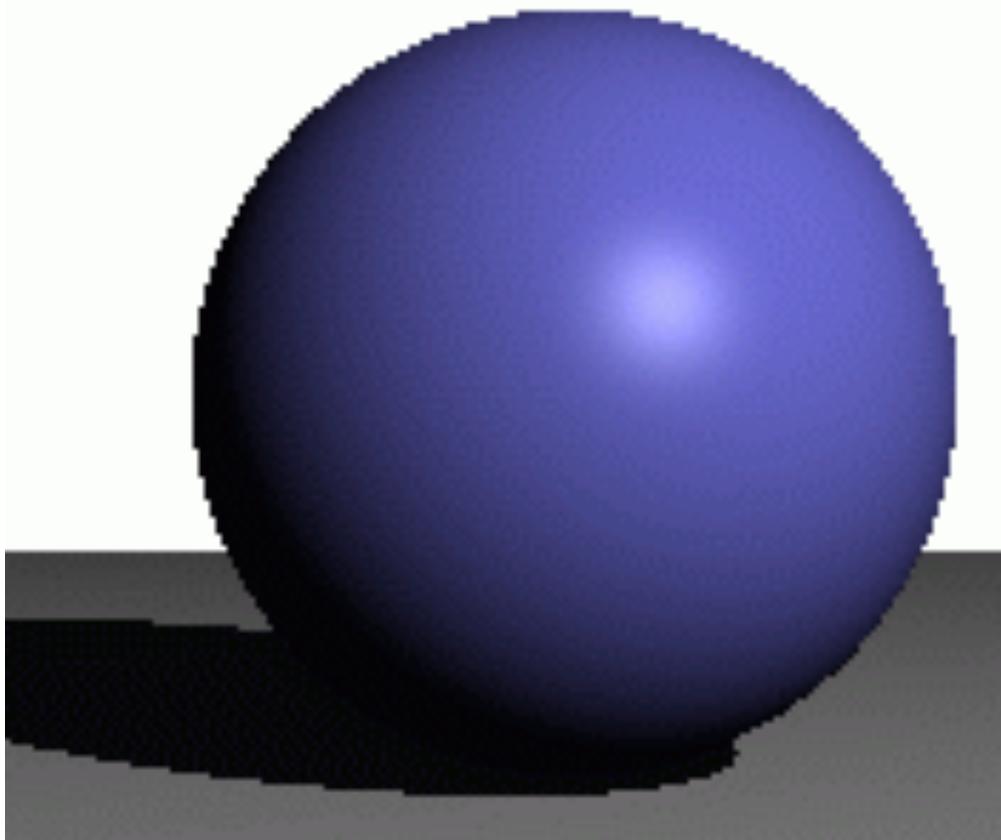
Problem: Aliasing



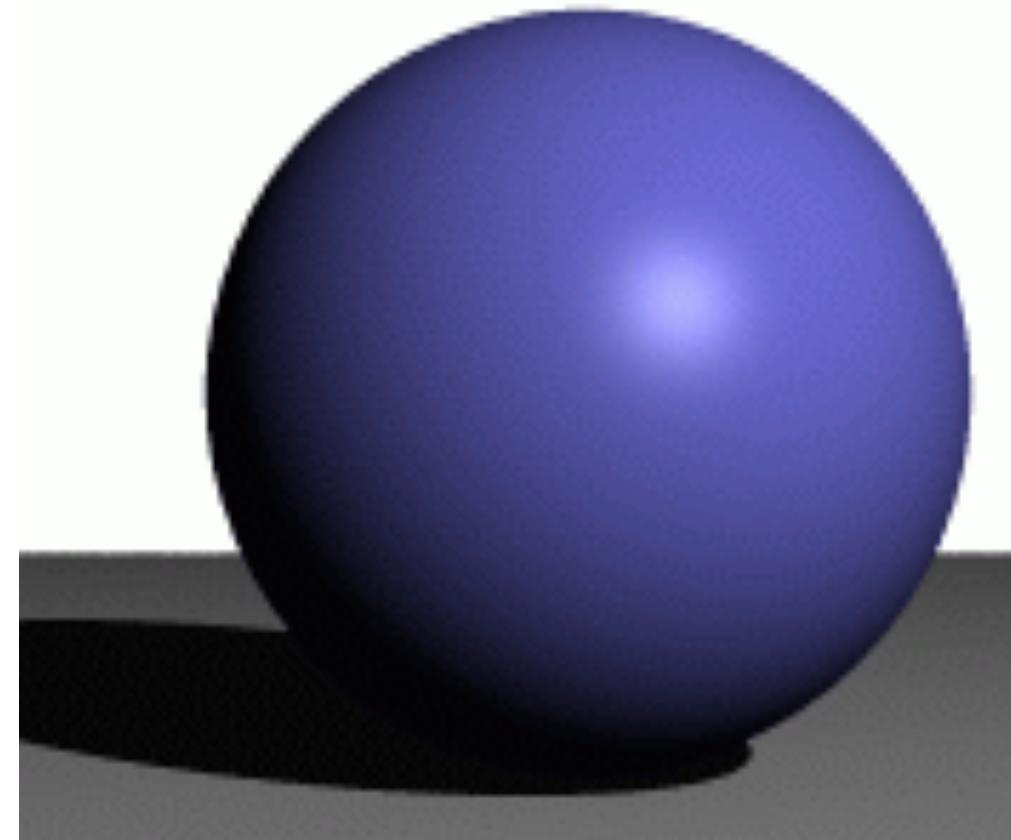
Antialiasing w/ Supersampling

- Cast multiple rays per pixel, average result

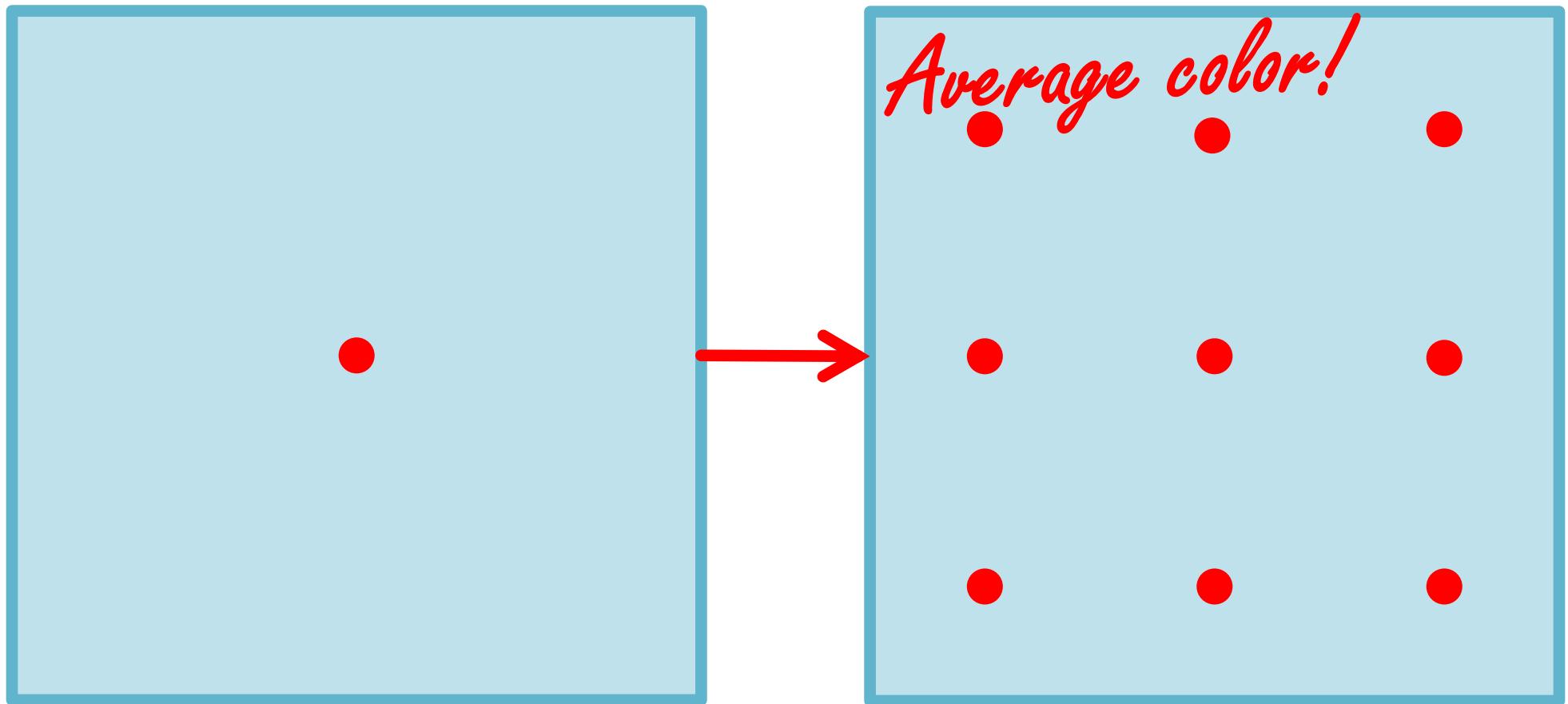
jaggies



w/ antialiasing

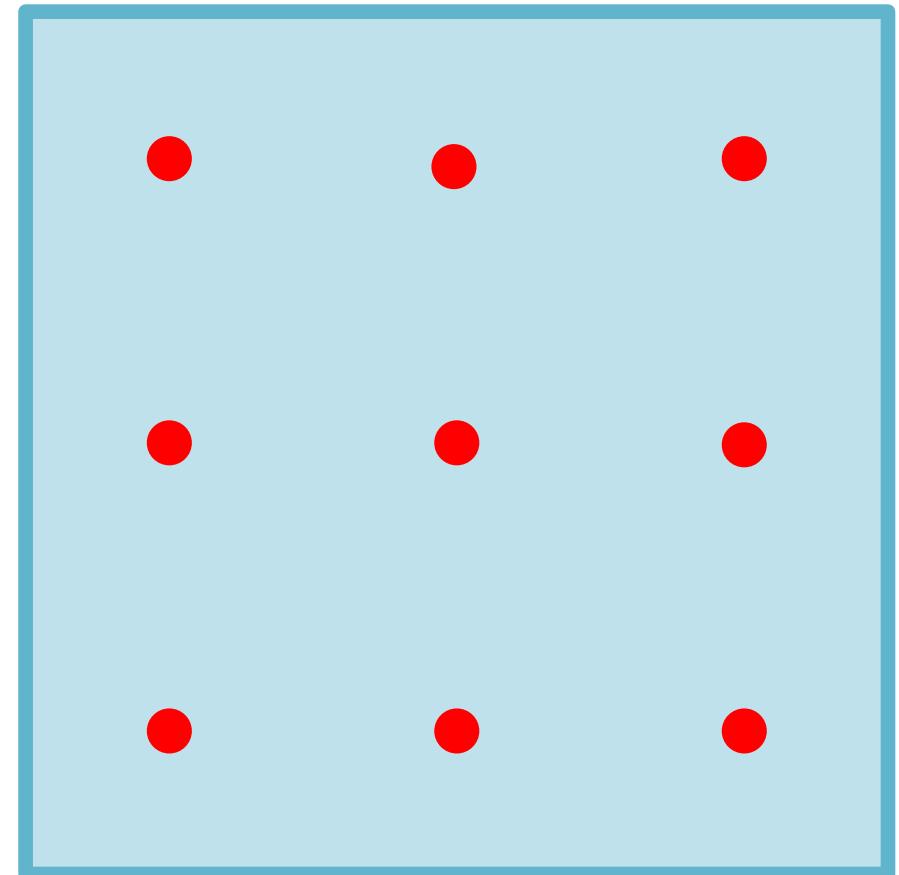


Distribution Antialiasing



Multiple rays per pixel

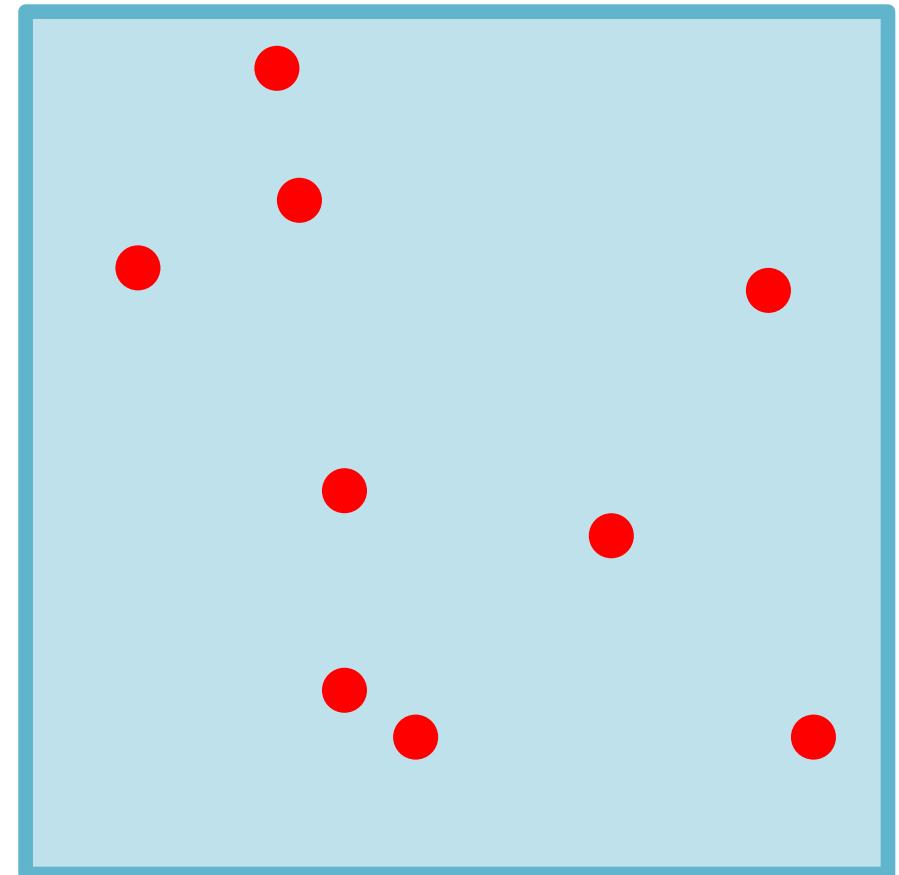
Distribution Antialiasing w/ Regular Sampling



http://upload.wikimedia.org/wikipedia/commons/f/fb/Moire_pattern_of_bricks_small.jpg

Multiple rays per pixel

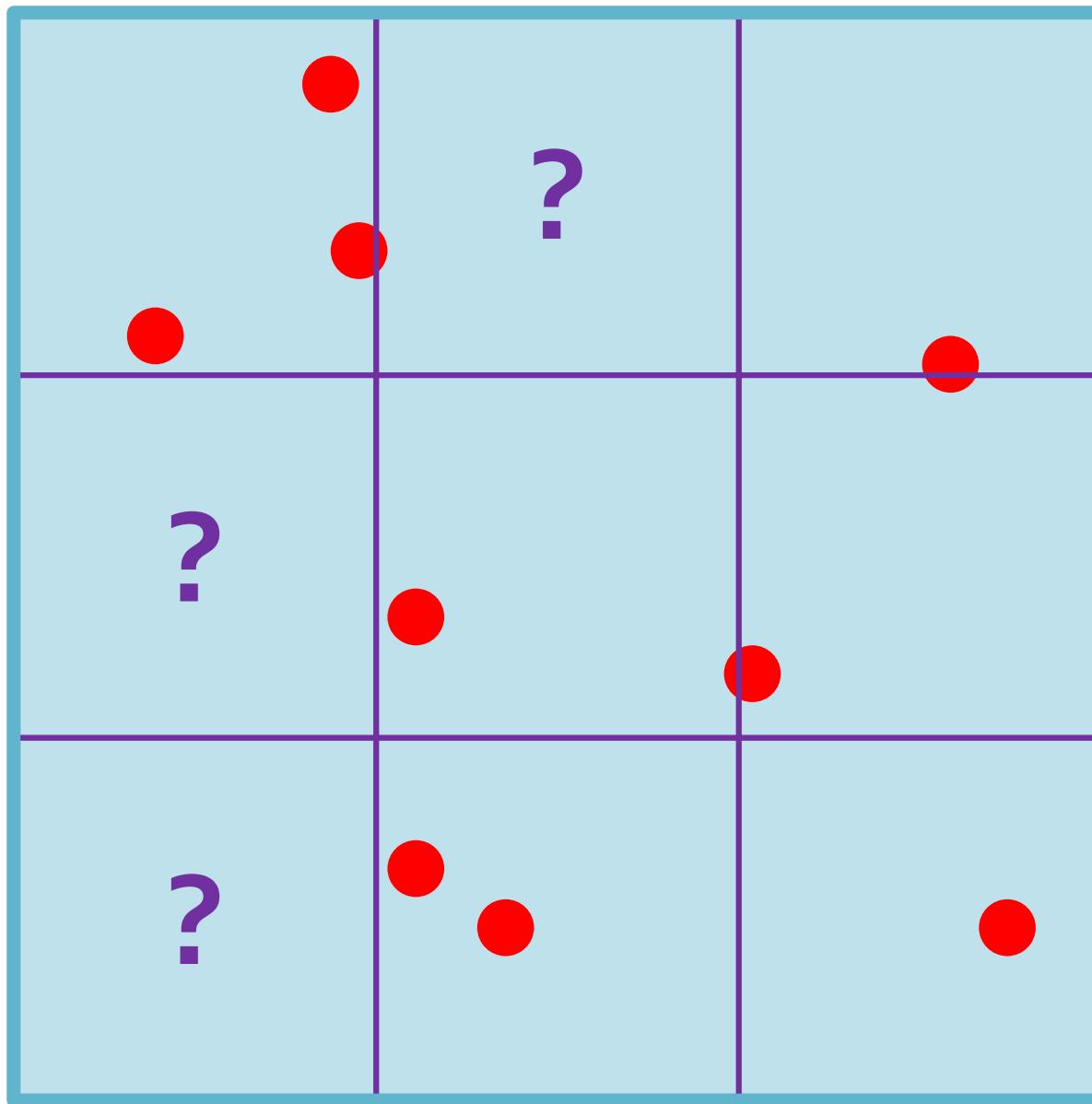
Distribution Antialiasing w/ Random Sampling



http://en.wikipedia.org/wiki/File:Moire_pattern_of_bricks.jpg

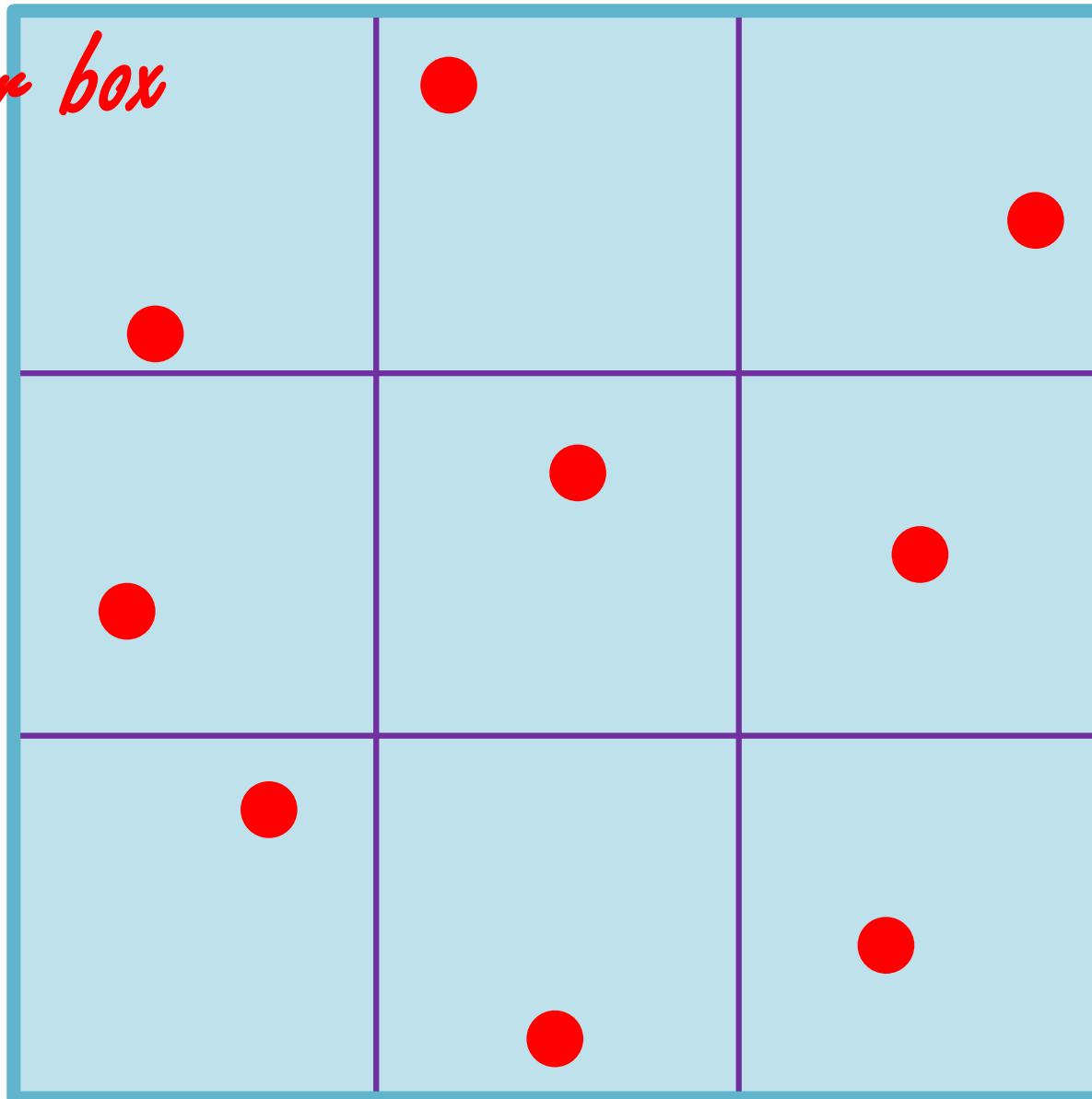
Remove Moiré patterns

Random Sampling Could Miss Regions Without Enough Sampling



Stratified (Jittered) Sampling

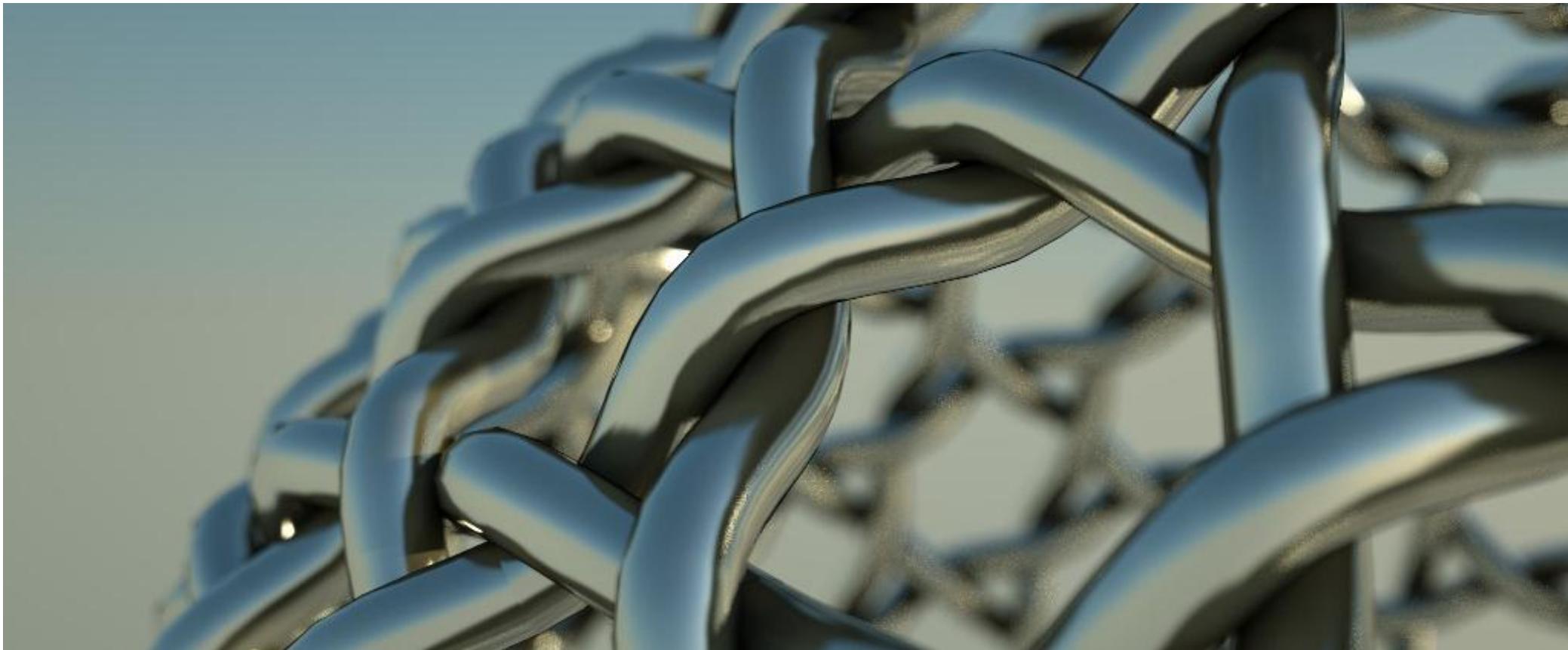
One ray per box



Problem: Focus Real Lenses Have Depth of Field

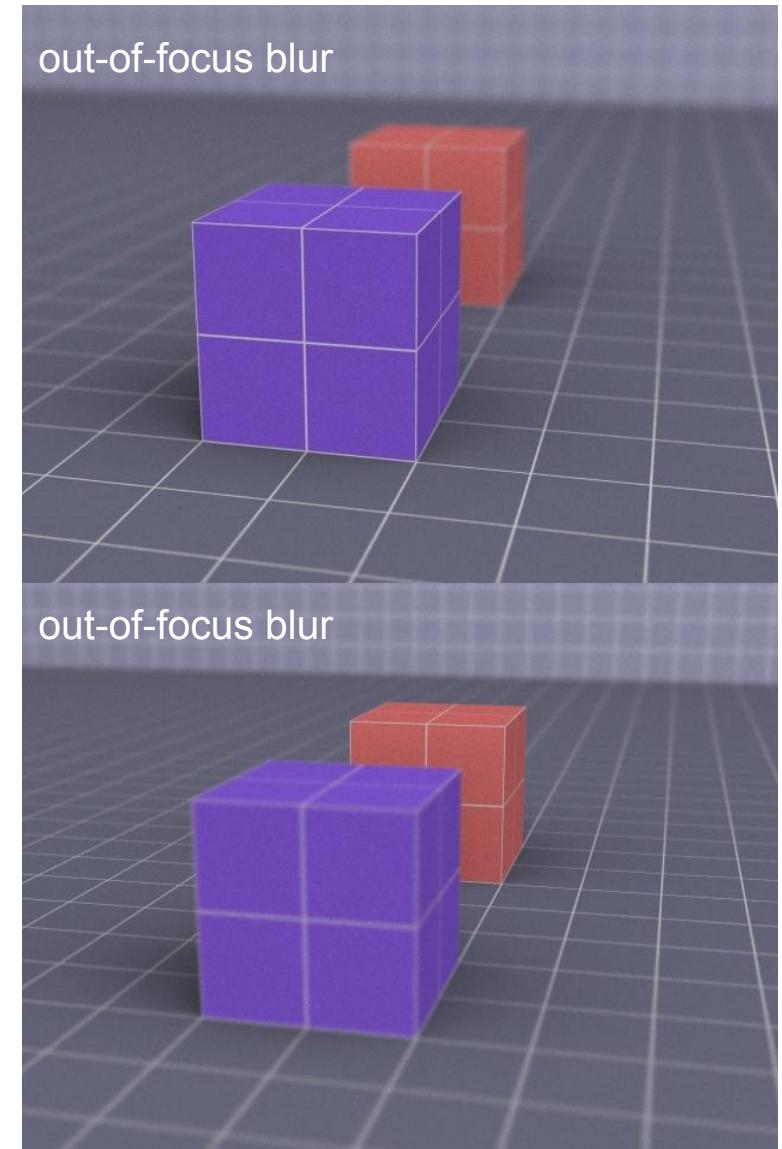
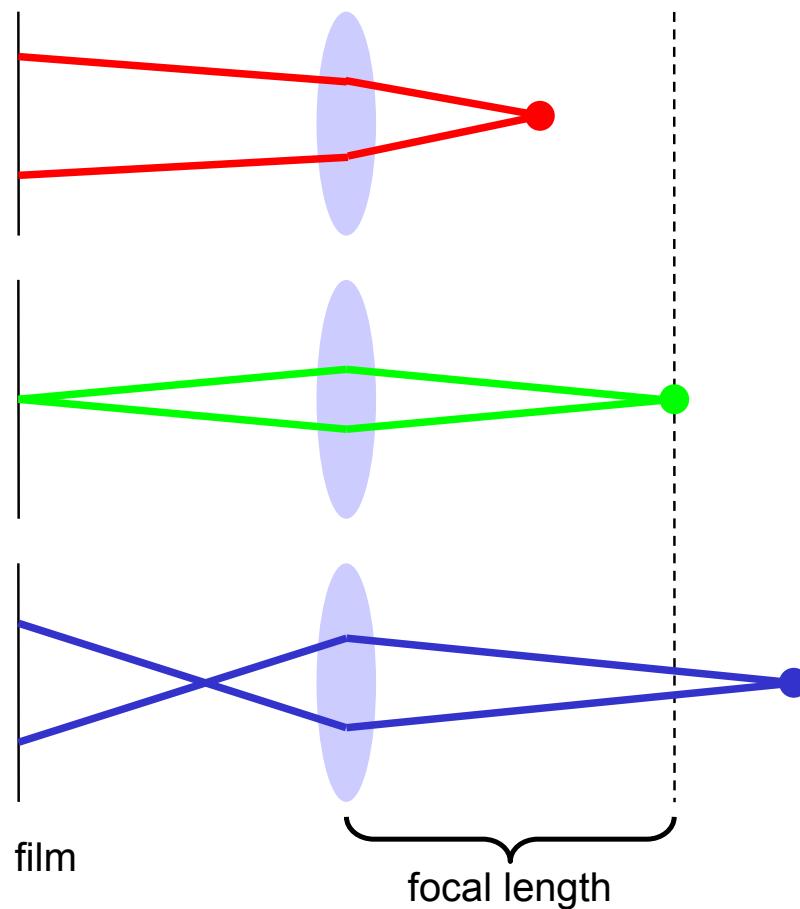


Problem: Focus Real Lenses Have Depth of Field



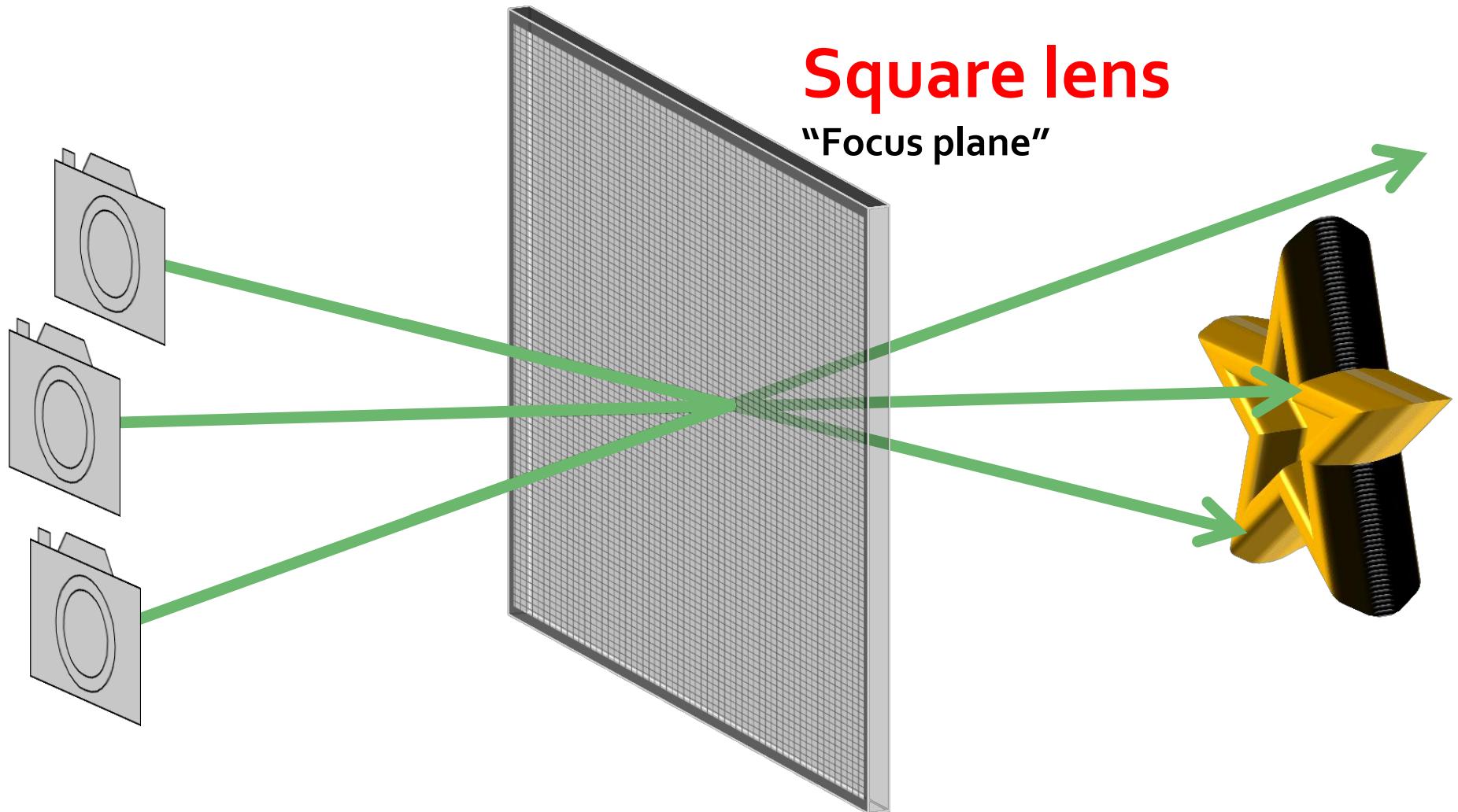
Depth of Field

- Multiple rays per pixel, sample lens aperture



Justin Legakis

Distribution Depth of Field

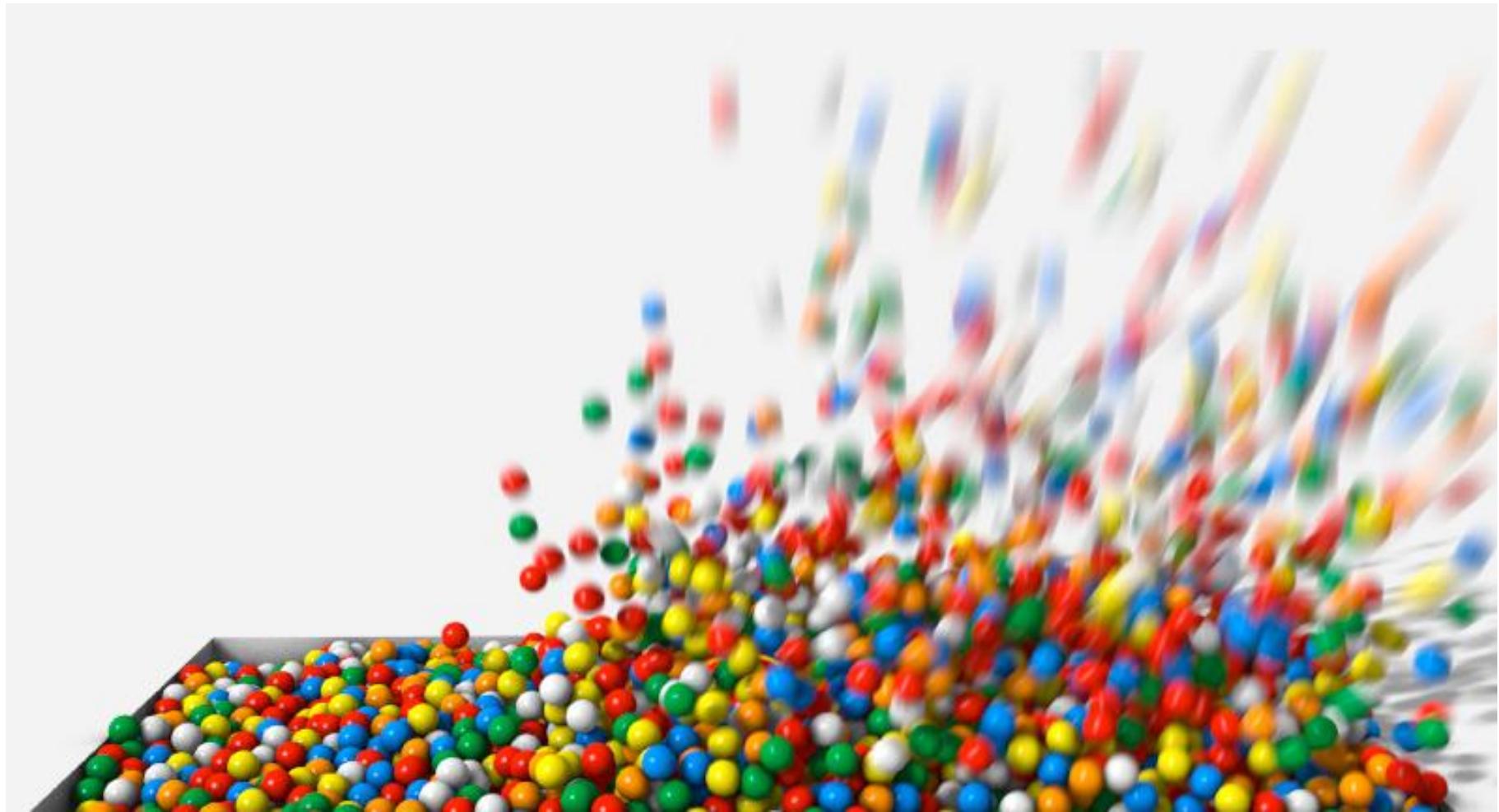


Randomly sample eye positions

Problem: Exposure Time Real Sensors Take Time to Acquire



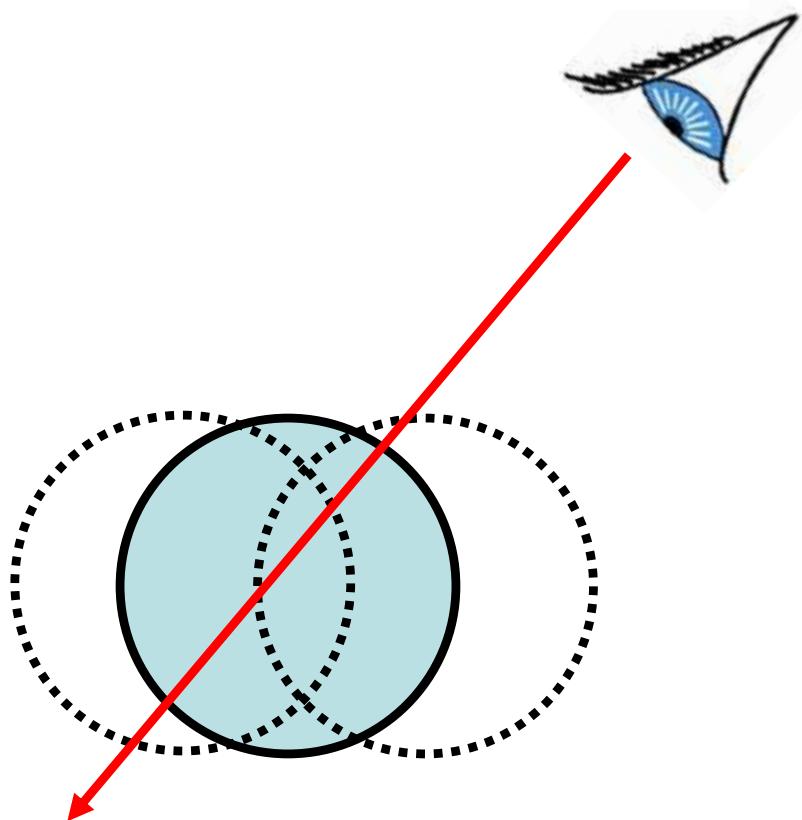
Problem: Exposure Time Real Sensors Take Time to Acquire



<http://www.matkovic.com/anto/3dl-test-balls-01.jpg>

Motion Blur

- Sample objects temporally over a time interval

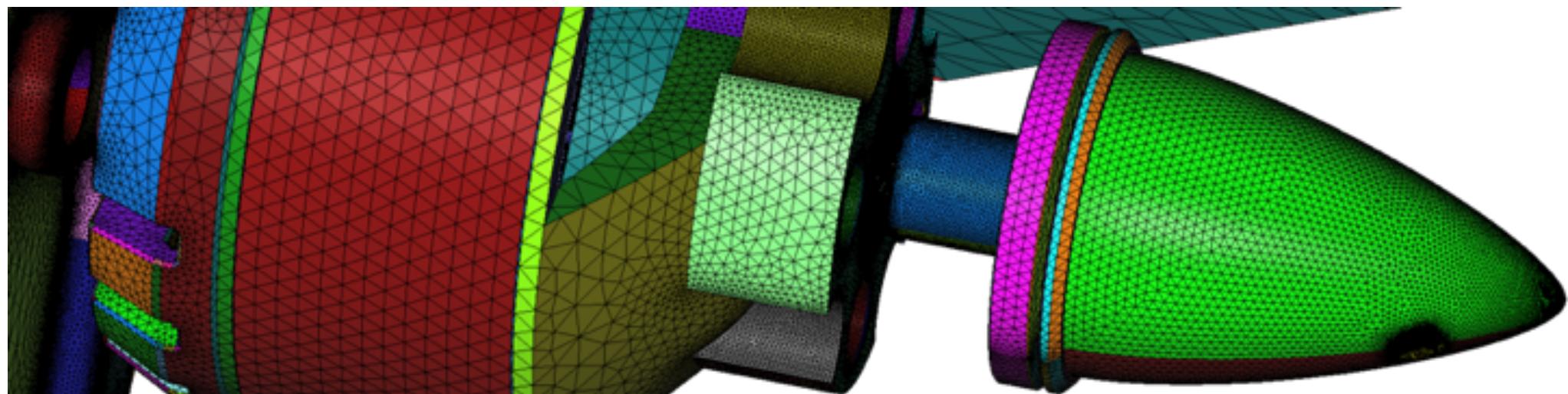
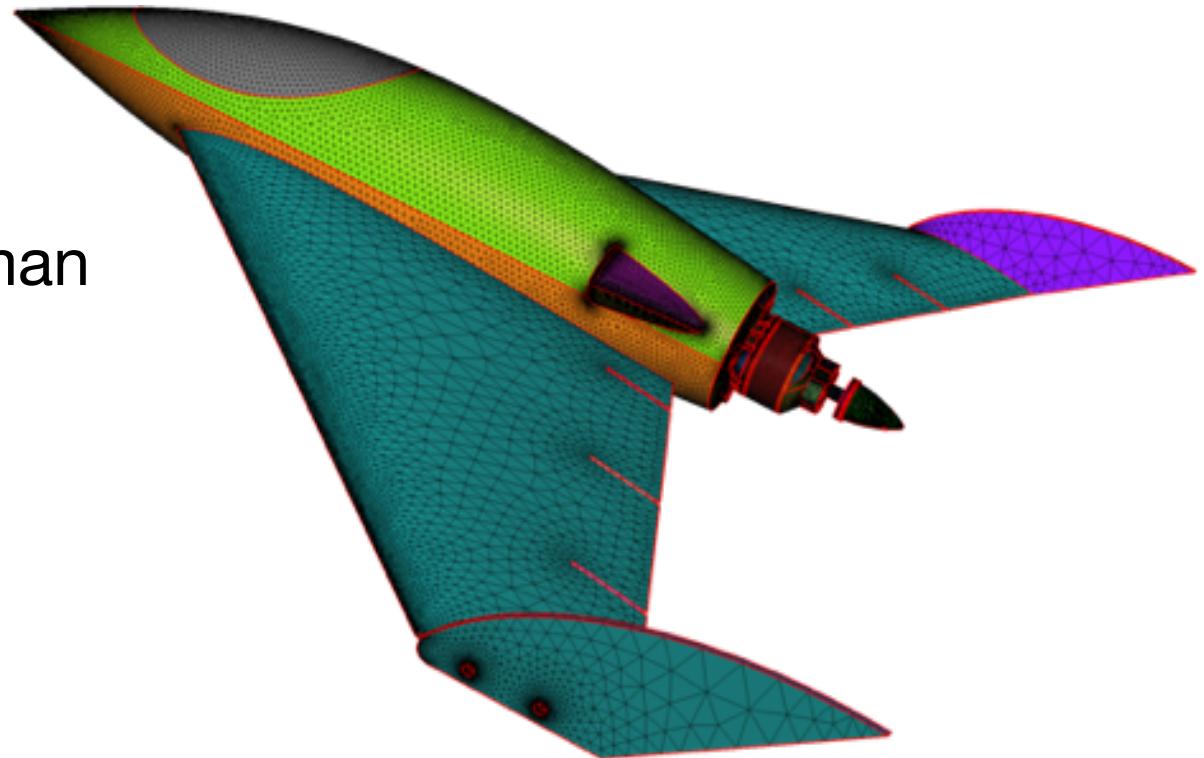


Rob Cook

Next: Triangle Meshes and other data structures

- FOCG, Ch. 12
- Check out recommended reading for some additional references

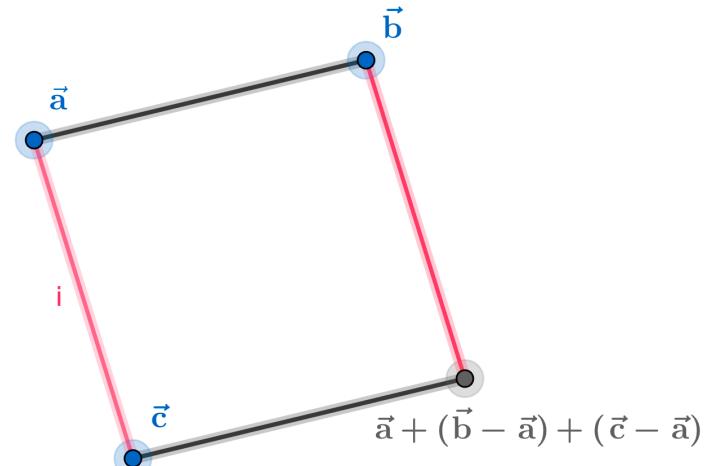
- Patrick Laug & Houman Borouchaki 2013
- 1,844,460 triangles



Modeling Complex Shapes

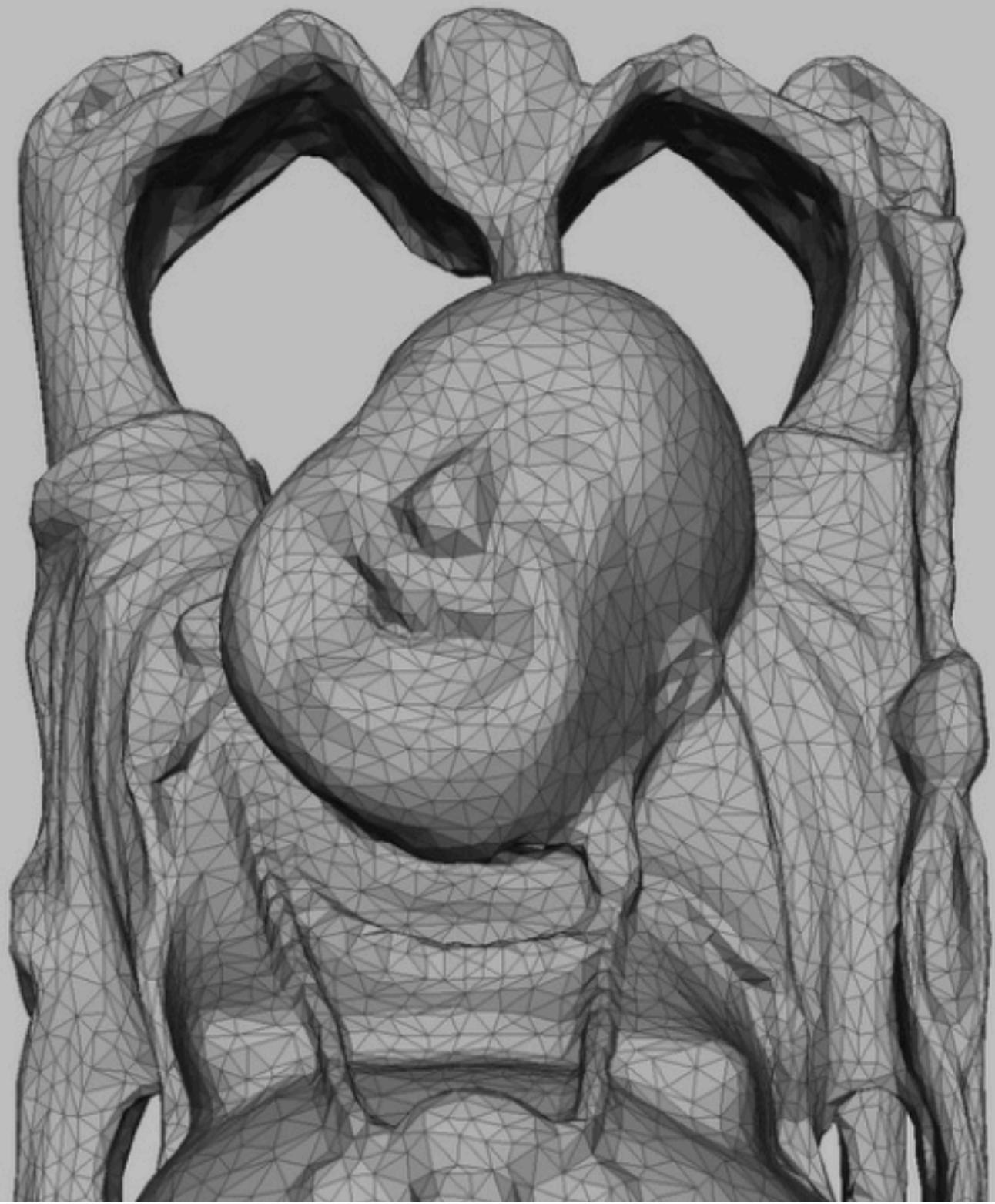
Recall: Shape Models That We Have So Far

- Implicit Shapes ($f(\mathbf{p}) = 0$ for all \mathbf{p} on shape):
 - Sphere: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$
 - Plane: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$
- Parametric Shapes ($\mathbf{p}(t)$ is a point on shape for all t):
 - Rays: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
 - Triangles:
$$p = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}, \quad 0 \leq \alpha, \beta, \gamma \text{ and } \alpha + \beta + \gamma = 1$$
 - Triangle (second form)
$$p = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}), \quad 0 \leq \beta, \gamma \text{ and } \beta + \gamma \leq 1$$
 - Parallelogon $p = \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a}) \quad 0 \leq \beta, \gamma \leq 1$



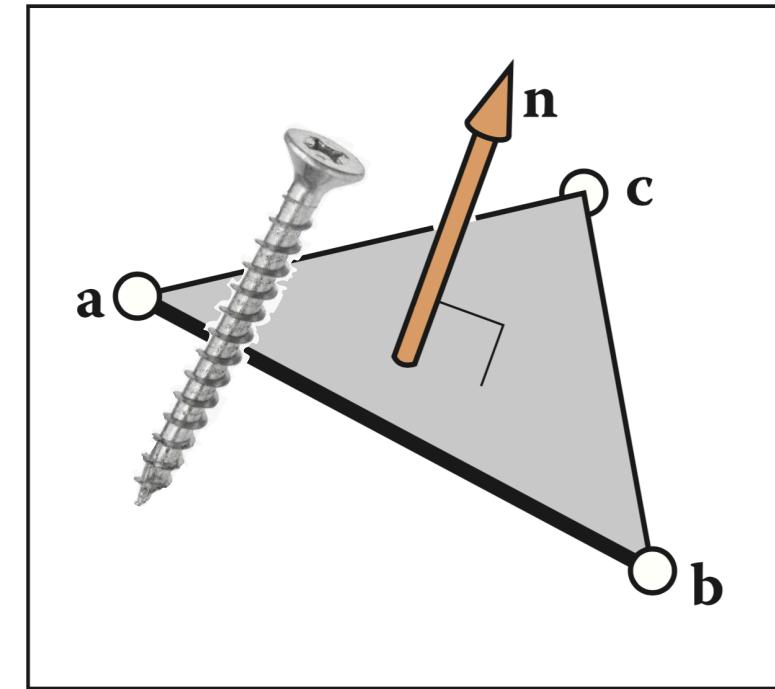
Triangle Meshes

- Are used in a huge number of applications
- Can be used to represent complex shapes by breaking them into simple (perhaps the simplest) two-dimensional elements

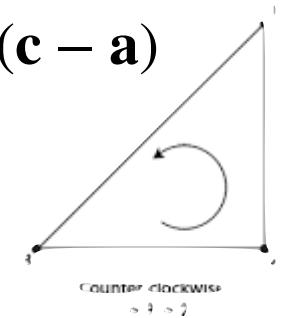


Definition of Triangles

- 3 **vertices** (points **a**, **b**, **c** in 3D space)
- The normal of the triangle is a vector, **n**, that points to its front side
- Convention: vertices listed in counter-clockwise order from the “front” of the triangle

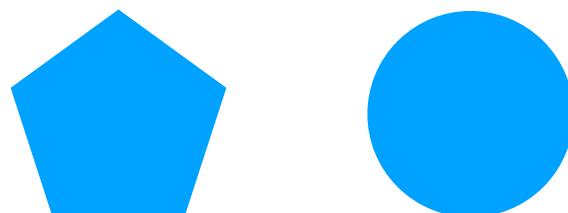


$$\mathbf{n} = (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$$



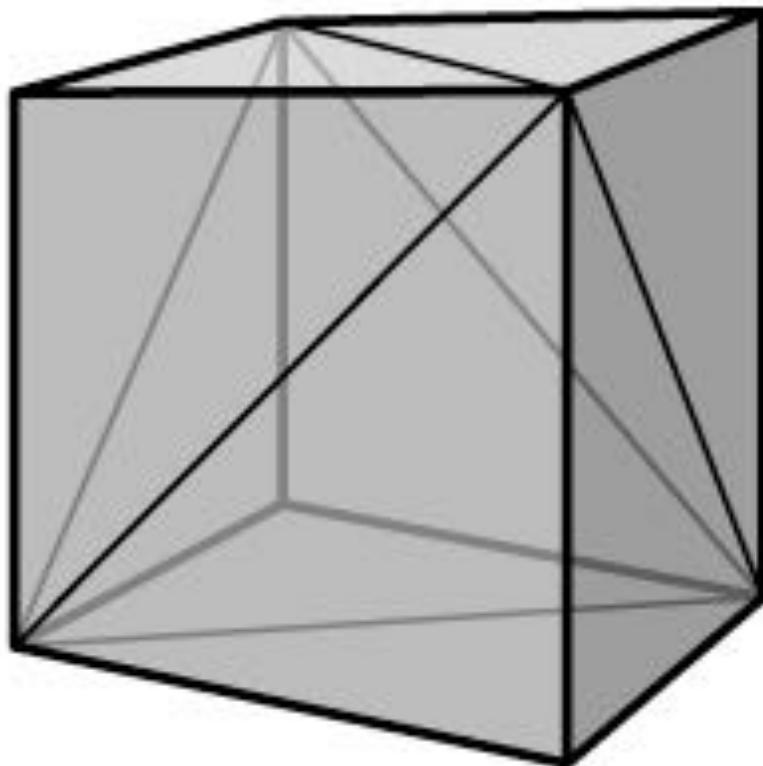
Definition of Triangle Meshes

- In short, a collection of triangles in 3D space that are connected to form a surface
 - Terminology: vertices, edges, triangles
 - Surface is piecewise planar, except where two triangle meet which forms a crease and their shared edge
 - Meshes are often a **piecewise** approximation of a smooth surface. We will study how graphics can hide the artifacts, creates the illusion of a **smooth** surface without increasing their complexity.



A Simple Mesh

- How many vertices? How many triangles?



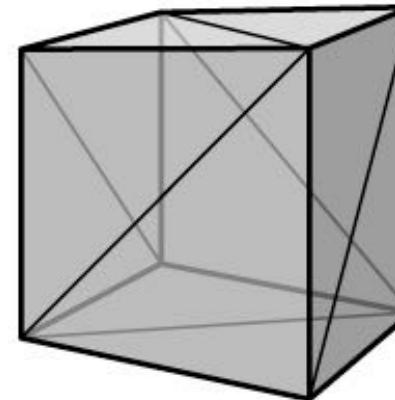
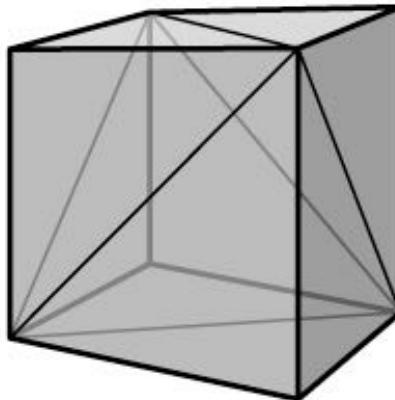
Mesh Topology

Two Considerations for Meshes

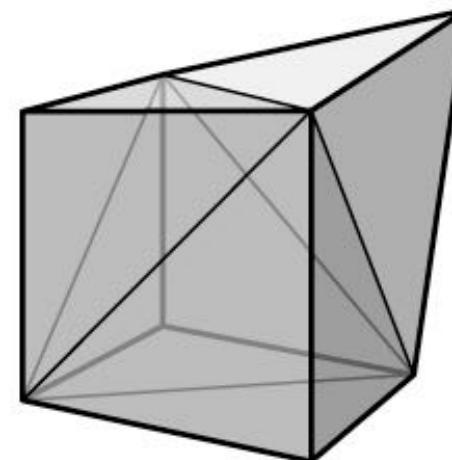
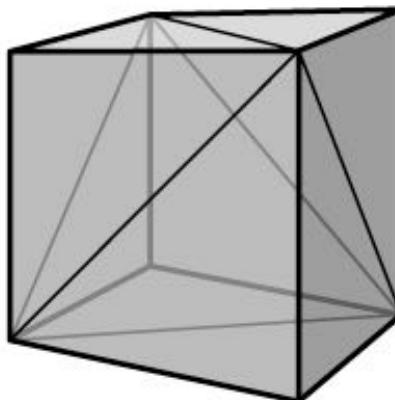
- We typically care about the mesh being a good approximation to a surface:
 - This leads to questions of **mesh geometry**, e.g.: How many triangles? where to place their vertices?
- We also care about how these triangles are connected
 - This leads to questions of **mesh topology**, e.g.: Are there holes in the mesh? How do triangles intersect?
 - Mesh topology can affect assumptions on algorithms that process meshes

Topology vs. Geometry

- Same geometry, different topology



- Same topology, different geometry

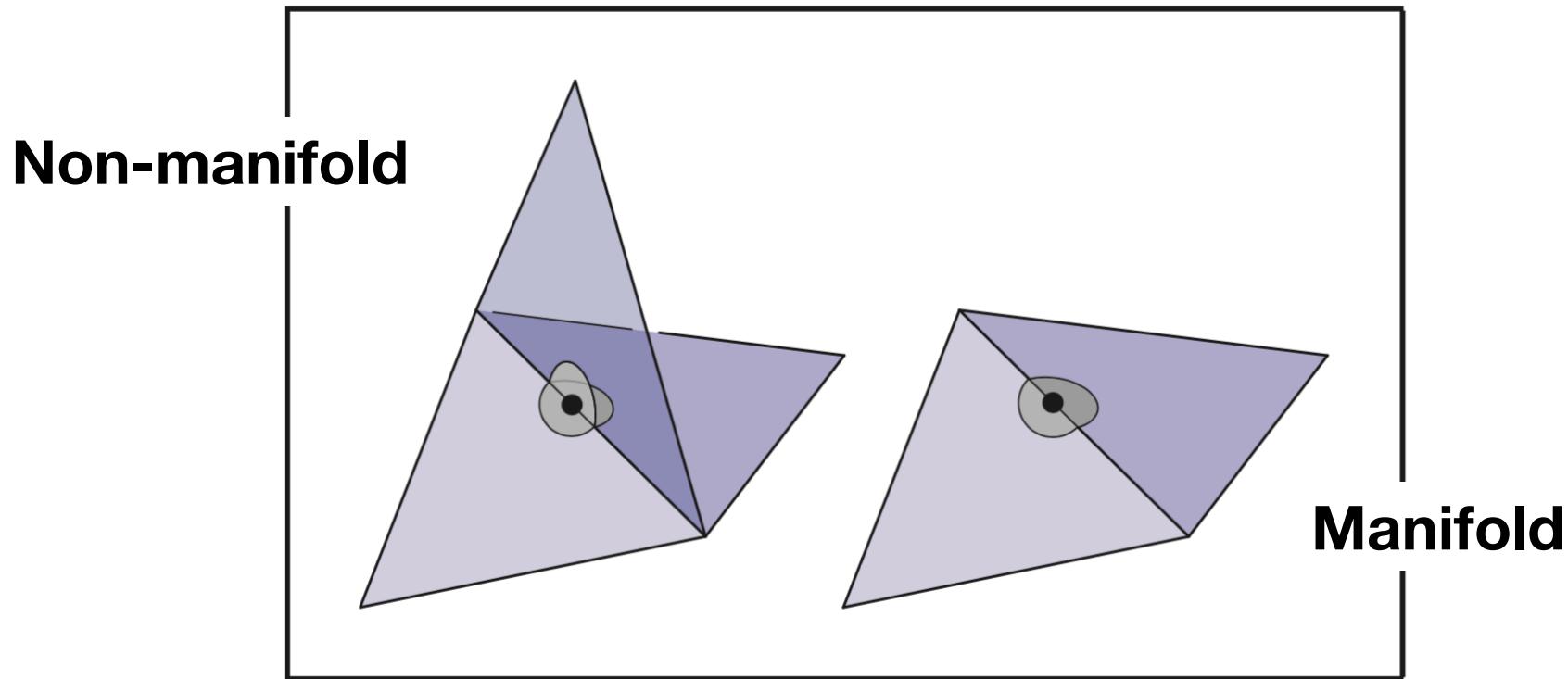


Topological Validity

- Meshes that approximate surfaces should be manifolds
- Definition: A (2-dimensional) **manifold** is a space where every point locally appears to be 2-dimensional space
 - 3 cases: points that are on edges, points that are vertices, and points that are interior to triangles.

When is a Mesh a Manifold?

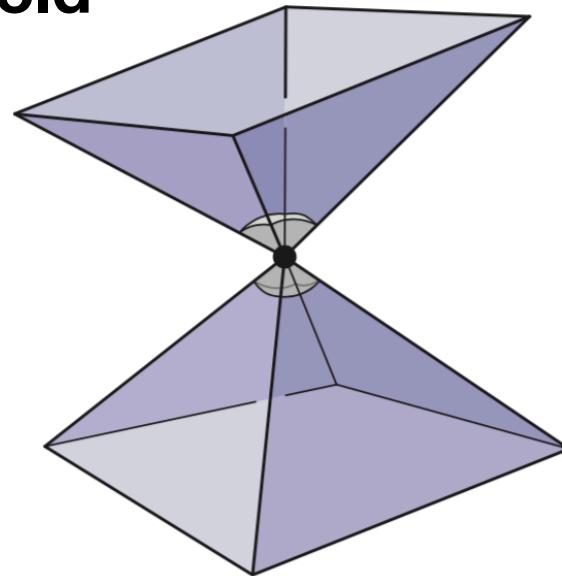
- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Every edge is shared by exactly two triangles



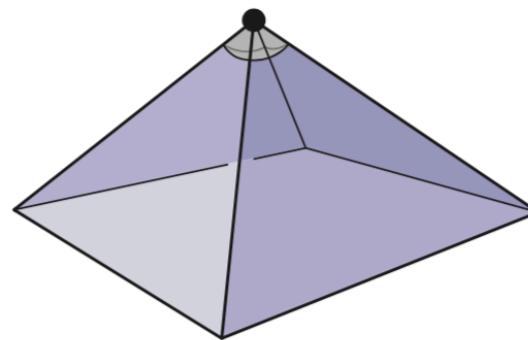
When is a Mesh a Manifold?

- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Every vertex has a single, complete loop of triangles around it

Non-manifold



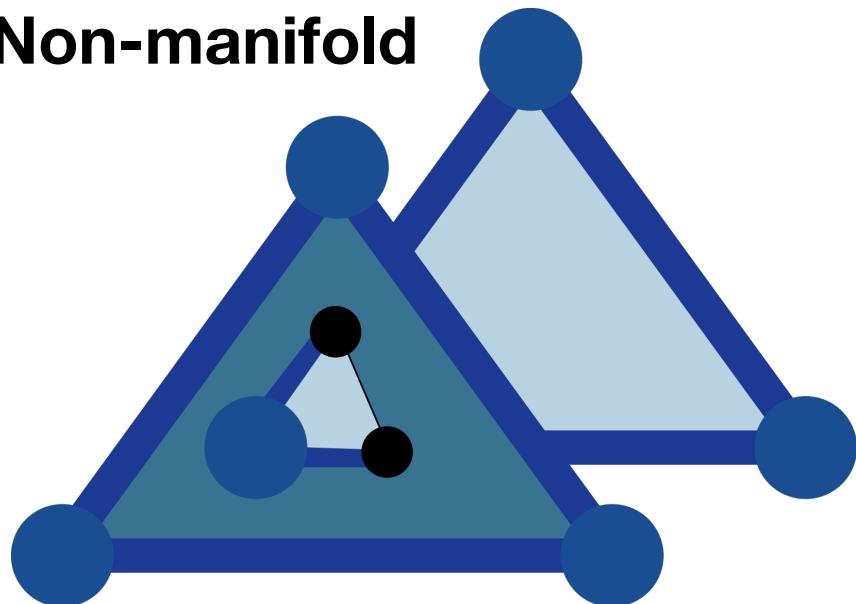
Manifold



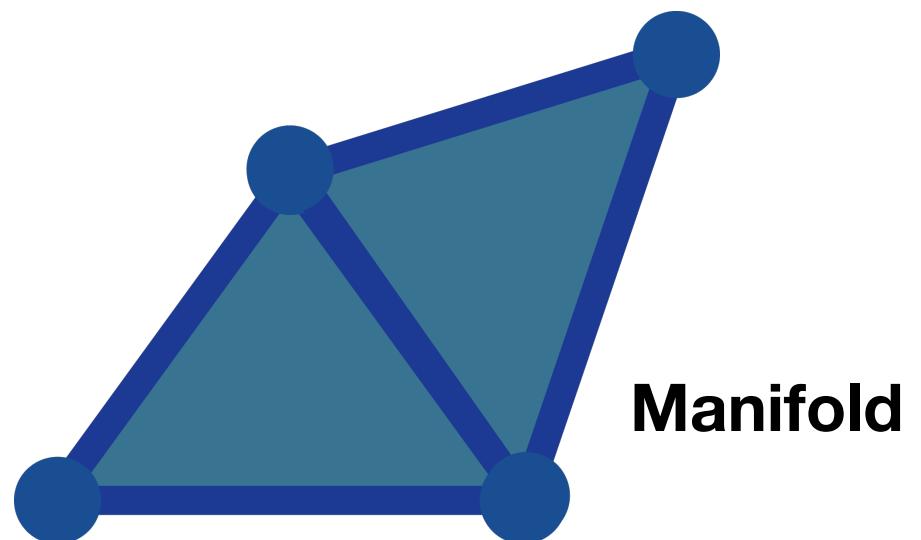
When is a Mesh a Manifold?

- Definition: A (2-dimensional) manifold is a space where every point locally appears to be 2-dimensional space
- Implication: Triangles only intersect at vertices and edges

Non-manifold

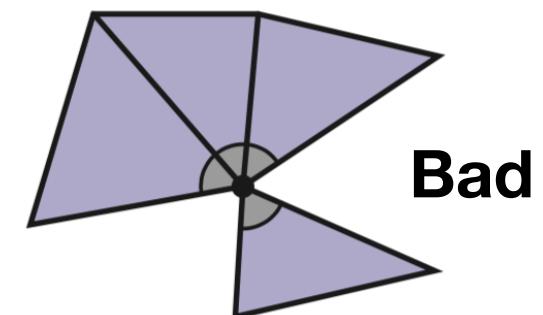
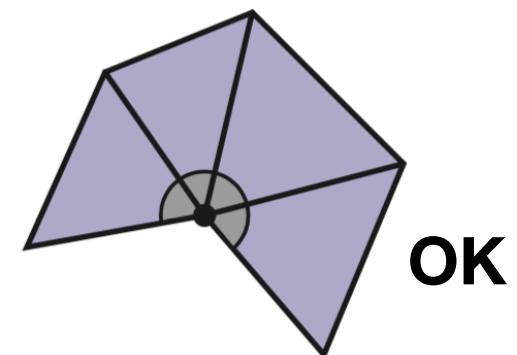
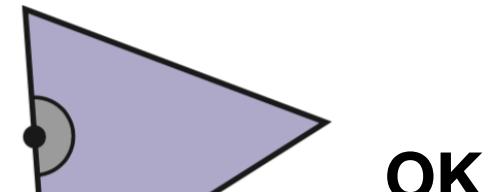


Manifold



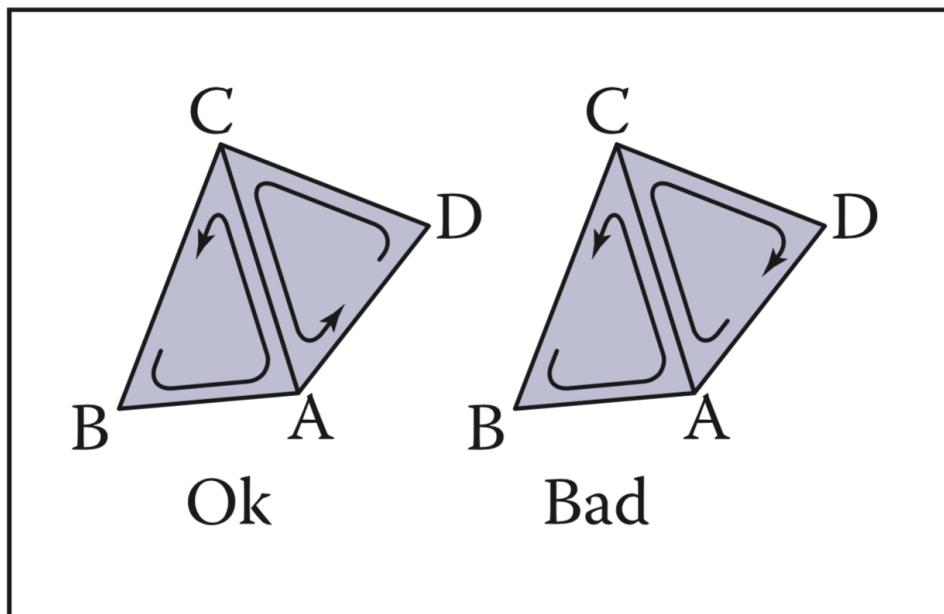
Manifolds with Boundary

- Sometimes, we relax the manifold condition to allow meshes with boundaries.
- Every point on a **manifold with boundary** either locally appears to be 2-dimensional space or 2-dimensional half-space
 - Every edge is used by either one or two triangles
 - Every vertex connects to a single edge-connected set of triangles

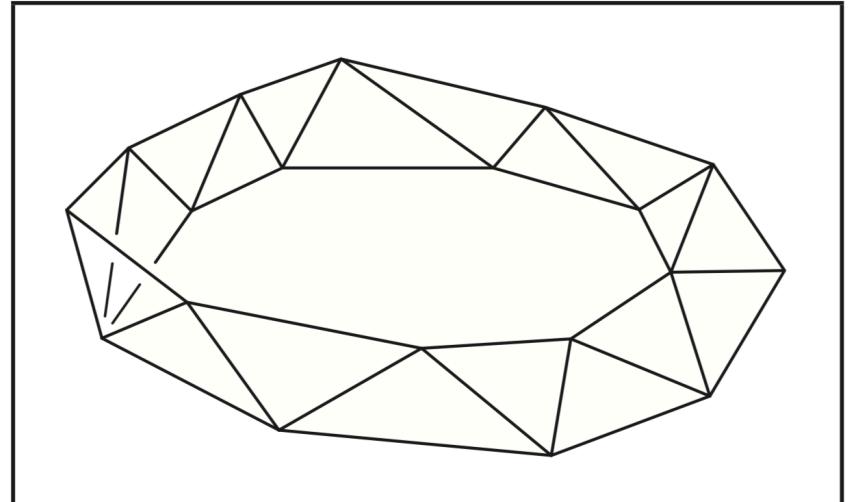


Consistent Orientation

- In many applications, all triangles facing the same way is important
 - Can be used to distinguish inside from outside.
- If consistent: neighboring triangles will appear to disagree on the order of vertices on their shared edge



Möbius strip: Non-orientable



Simple Representations of Triangle Meshes

Important Concerns w/ Representing Triangle Meshes

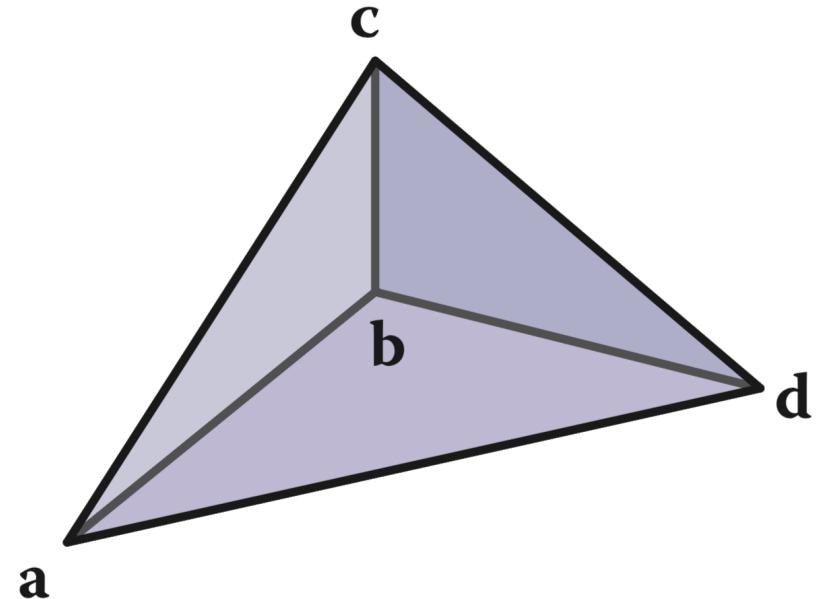
- Efficiency of storage size
 - Many representations store redundant information
- Efficiency of access
 - How quickly can we get the information we need for rendering?
 - How quickly can we get neighborhood information, for mesh modification?

Using Separate Triangles

- Use a simple structure to store each triangle:

```
Triangle {  
    vertexPositions[3]; //Vec3  
};
```

- Store a triangle mesh using an array of Triangle
- Problems?



#	Vertex 0	Vertex 1	Vertex 2
0	(a_x, a_y, a_z)	(b_x, b_y, b_z)	(c_x, c_y, c_z)
1	(b_x, b_y, b_z)	(d_x, d_y, d_z)	(c_x, c_y, c_z)
2	(a_x, a_y, a_z)	(d_x, d_y, d_z)	(b_x, b_y, b_z)

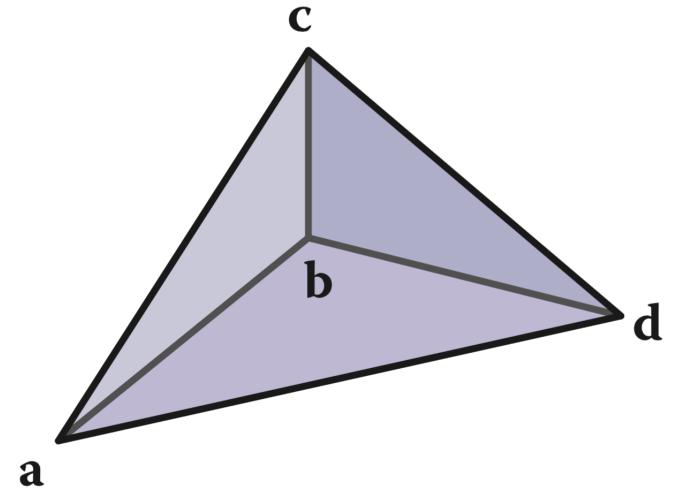
Using Indexed Meshes

- Triangles share a common list of vertices, storing only references/pointers:

```
Triangle {  
    vertices[3]; //object reference or int  
};
```

```
Vertex {  
    position; //Vec3  
};
```

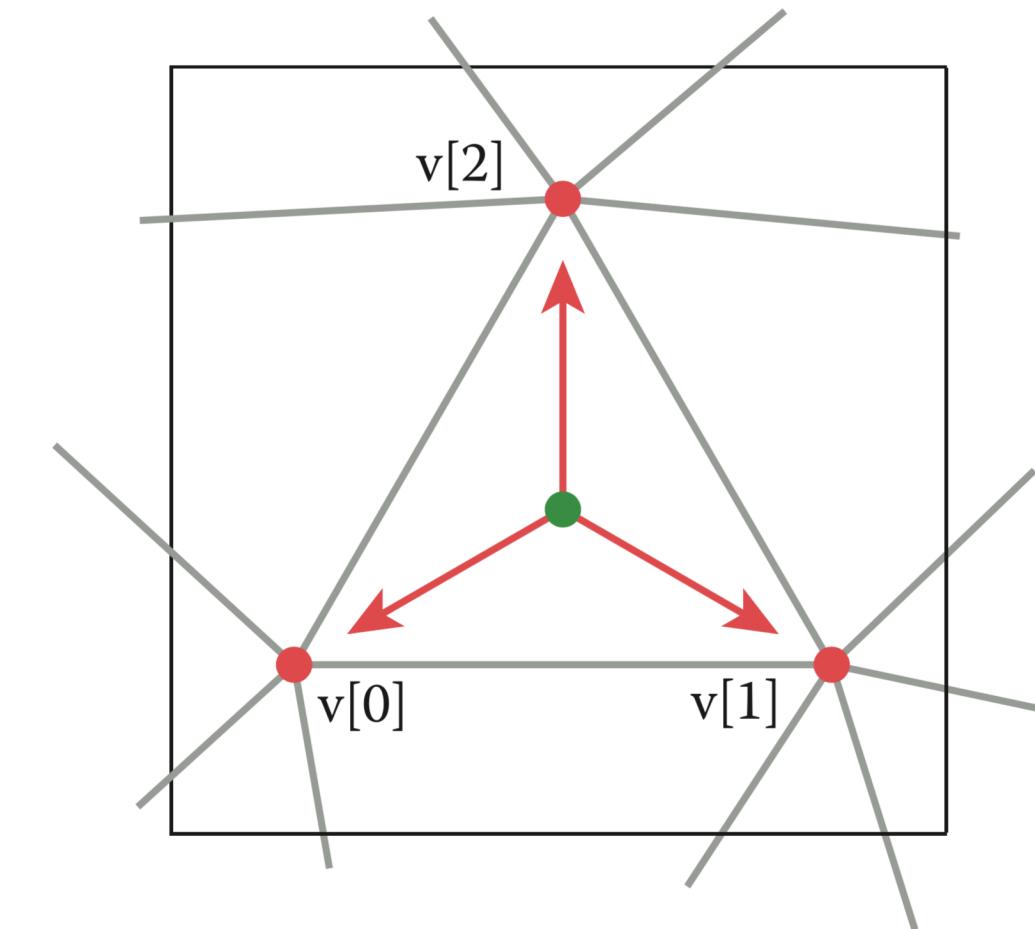
- Store a triangle mesh using two arrays, one of `Vertex` and the other of `Triangle`



Triangles		Vertices	
#	Vertices	#	Position
0	(0, 1, 2)	0	(a_x, a_y, a_z)
1	(1, 3, 2)	1	(b_x, b_y, b_z)
2	(0, 3, 1)	2	(c_x, c_y, c_z)
		3	(d_x, d_y, d_z)

Using Indexed Meshes

- Each triangle thus tracks references to the vertices associated with it

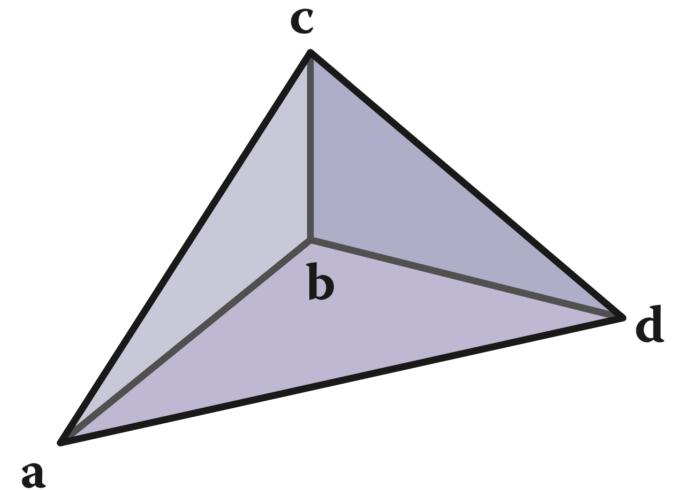


Using Indexed Meshes

- Alternatively one can store using array indices directly:

```
IndexedMesh {  
    vertices[num_verts];      //Vec3  
    triIndices[num_tris];    //int  
};
```

- Plus, it is easy (or at least easier) to see which two triangles share an edge.



Triangles		Vertices	
#	Vertices	#	Position
0	(0, 1, 2)	0	(a_x, a_y, a_z)
1	(1, 3, 2)	1	(b_x, b_y, b_z)
2	(0, 3, 1)	2	(c_x, c_y, c_z)
		3	(d_x, d_y, d_z)

Storage Requirements

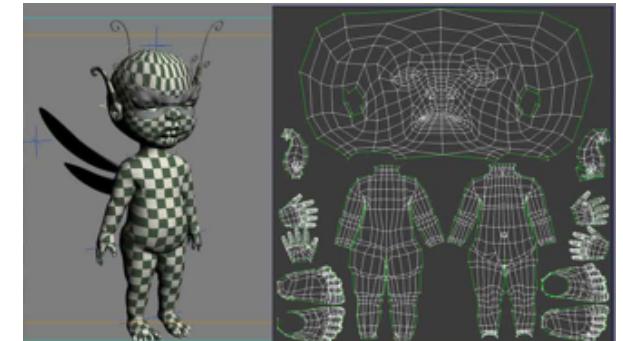
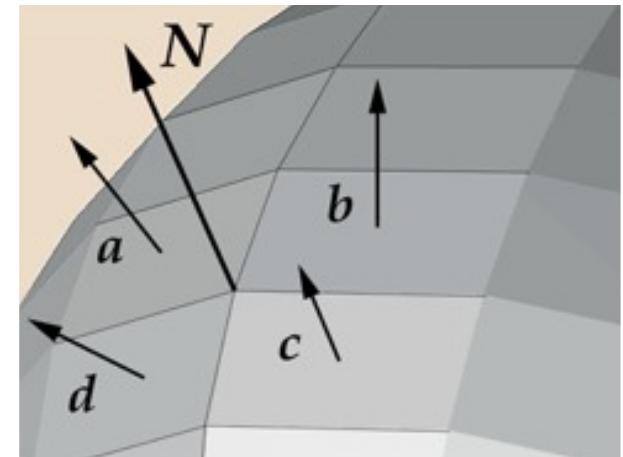
- Let n_v be the number of vertices and n_t be the number of triangles.
 - A single vertex requires 12 bytes (3 floats, 4 bytes/float)
 - Separate triangles:
 - 3 vertices per triangle => $12*3*n_t = 36*n_t$ bytes
 - Indexed Meshes
 - 3 integers per triangle, 3 floats per vertex => $12*n_t + 12*n_v$ bytes

Storage Requirements

- For large meshes, $n_t \approx 2n_v$
- Separate triangles:
 - $36*n_t = 72*n_v$ bytes
- Indexed Meshes
 - $12*n_t + 12*n_v = 36*n_v$ bytes

Data on Meshes

- Typically, we store a variety of data on meshes as well
- Can store this on vertices, triangles, or even edges
- Examples:
 - Colors stored on vertices
 - Normals stored on faces
 - Texture coordinates stored on vertices
- Information stored on vertices is typically interpolated with **barycentric coordinates**



Remember Diffuse Shading

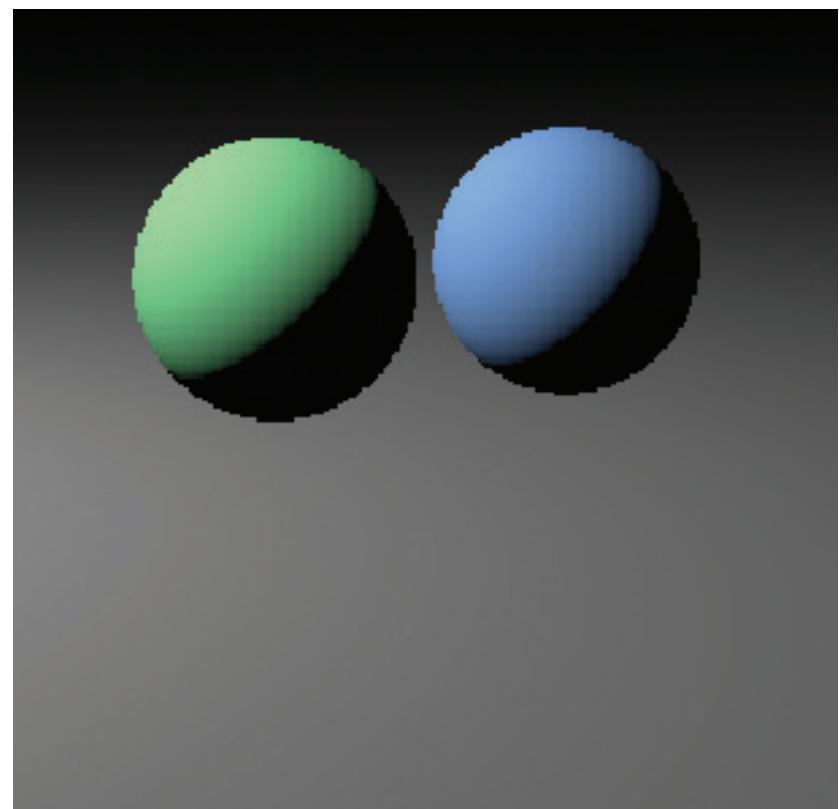
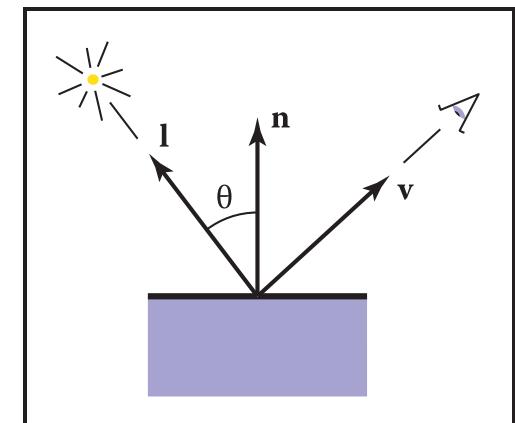
- Simple model: amount of energy from a light source depends on the direction at which the light ray hits the surface
- Results in shading that is *view independent*

$$L_d = k_d I \max(0, \mathbf{n} \cdot \mathbf{l})$$

diffuse coefficient

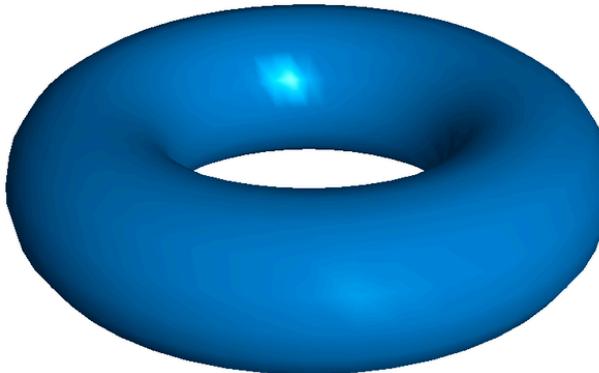
intensity/color of light

$\cos \theta$



Diffuse shading on triangle meshes

- The shading of each triangle is determined by its normal (same normal for all points in the triangle). Edges of triangles are very noticeable. This is called ***flat shading***



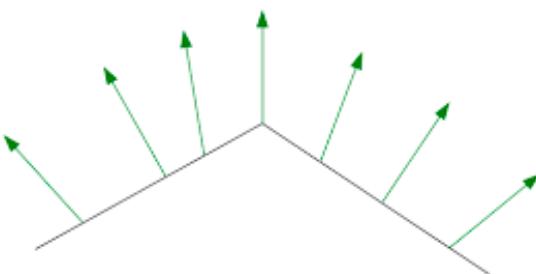
Diffuse shading formula

$$L_d = k_d \cdot I(\vec{n} \cdot \vec{l})$$

\vec{l} -direction to light

Gouraud shading solves it by allowing normals interpolation.

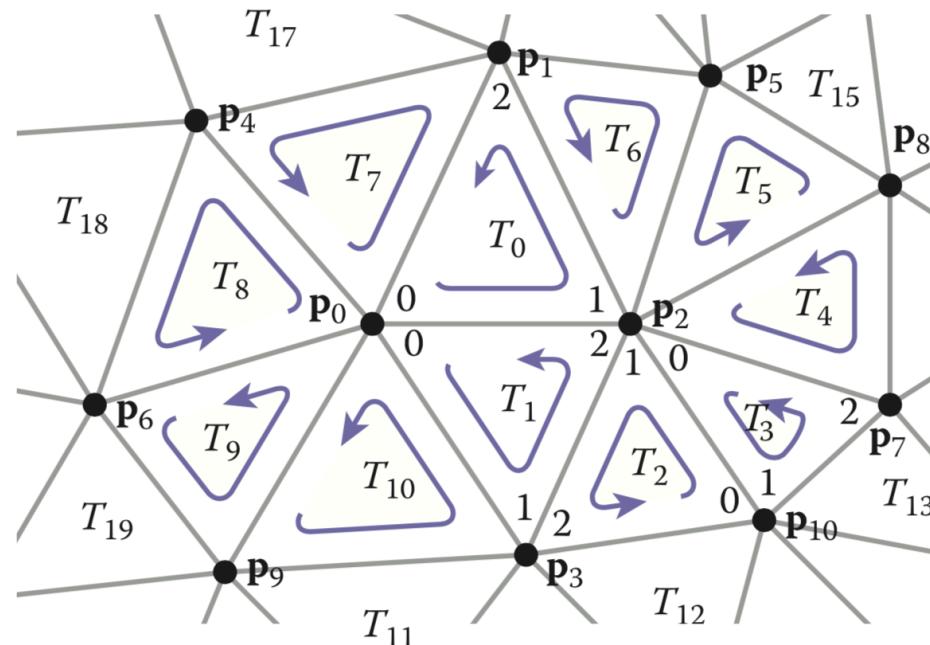
- First the normal at each vertices is computed. It is the “average” of the normal of the adjacent face.
- Next, each pixel p in a triangle uses barycentric coordinates to interpolate its own normal, based on the vertices normals.



Mesh File Formats: *.obj

- Widely used format for indexed meshes
- Supports additional data stored on vertices and polygons

verts[0]	x_0, y_0, z_0
verts[1]	x_1, y_1, z_1
verts[2]	x_2, y_2, z_2
verts[3]	x_3, y_3, z_3
\vdots	
tInd[0]	0, 2, 1
tInd[1]	0, 3, 2
tInd[2]	10, 2, 3
tInd[3]	2, 10, 7
\vdots	



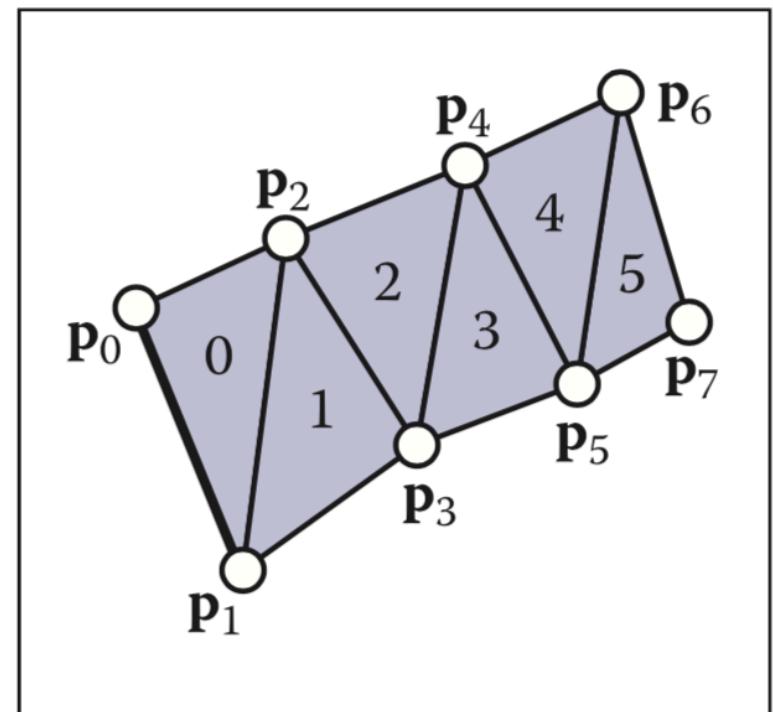
```
#sample .obj file
```

```
v 0.000 0.000 0.000
v 0.500 0.809 0.309
v 1.000 0.000 -0.309
v 0.583 -0.720 0.225
v -0.630 0.750 0.025
...
f 1 3 2
f 1 4 3
f 11 3 4
f 3 11 7
...
```

More Efficient Representations

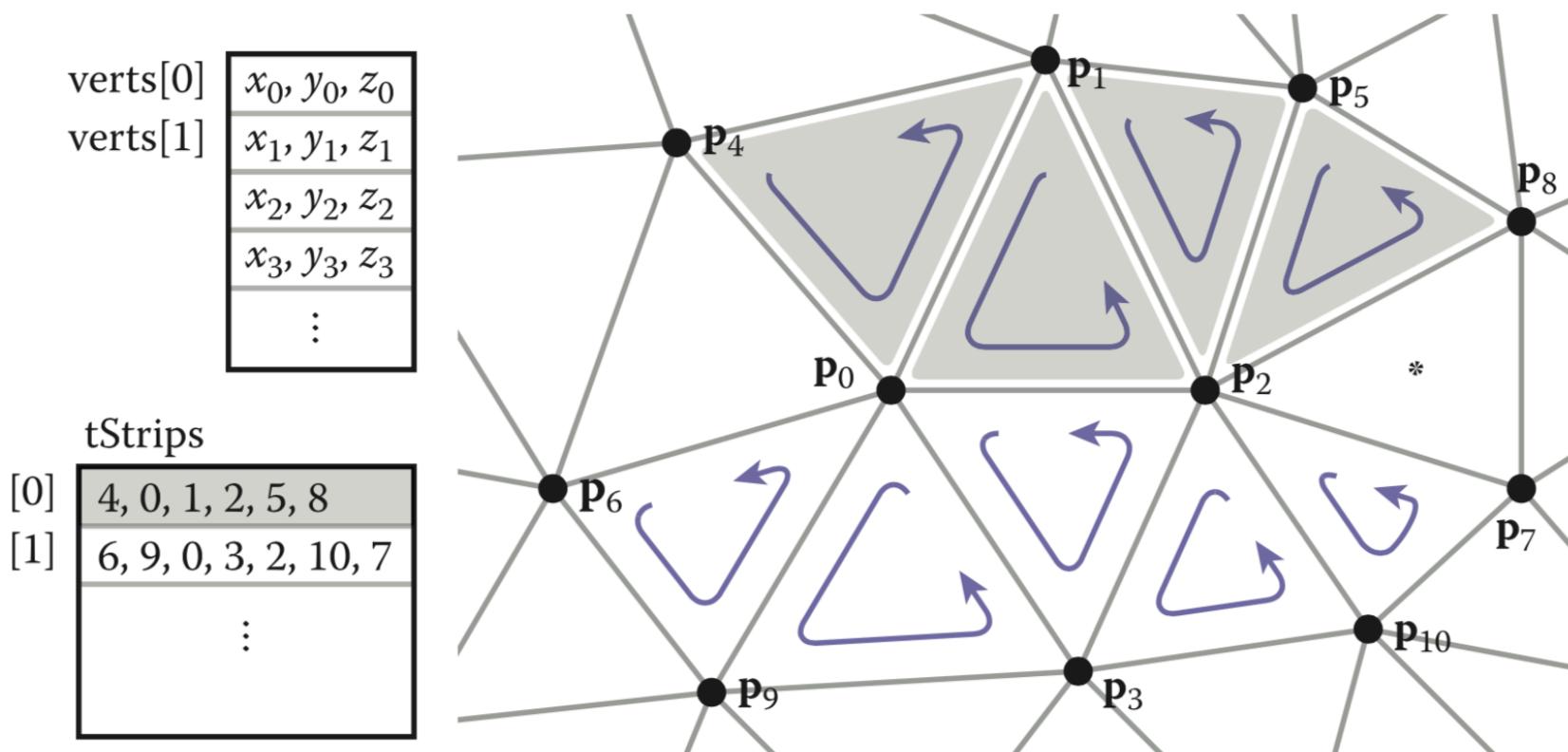
Triangle Strips

- Idea: Rely on the mesh property and group triangles that share common vertices
- Create a new triangle by reusing the last two vertices in the strip
- $[0,1,2,3,4,5,6,7]$ specifies the sequence on the right with triangles $(0,1,2)$, $(1,2,3)$, $(2,3,4)$...
- Have to invert every other for consistent orientation



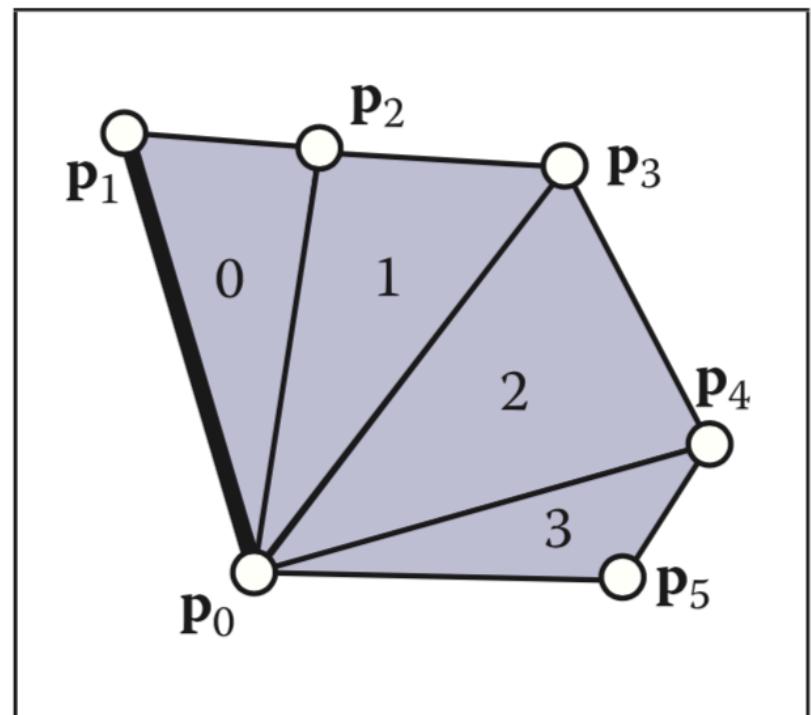
Triangle Strips

- Complex meshes store list of strips
- How long of a strip to use?



Triangle Fans

- Same idea as triangle strips, but keep the earliest vertex in the list instead of the last two
- $[0,1,2,3,4,5]$ specifies the sequence on the right with triangles $(0,1,2)$, $(0,2,3)$, $(0,3,4)$, ...



Mesh Data Structures and Queries

Queries on Meshes

- For face, find all:
 - Vertices
 - Edges
 - Adjacent faces
- For vertex, find all:
 - Incident edges
 - Incident triangles
 - Neighboring vertices
- For edge, find:
 - Two adjacent faces
 - Two adjacent vertices

```
Triangle {  
    v[3]; //Vertex  
    e[3]; //Edge  
    adj[3]; //Triangle
```

}

```
Vertex {  
    t[]; //Triangle  
    e[]; //Edge  
    adj[]; //Vertex
```

}

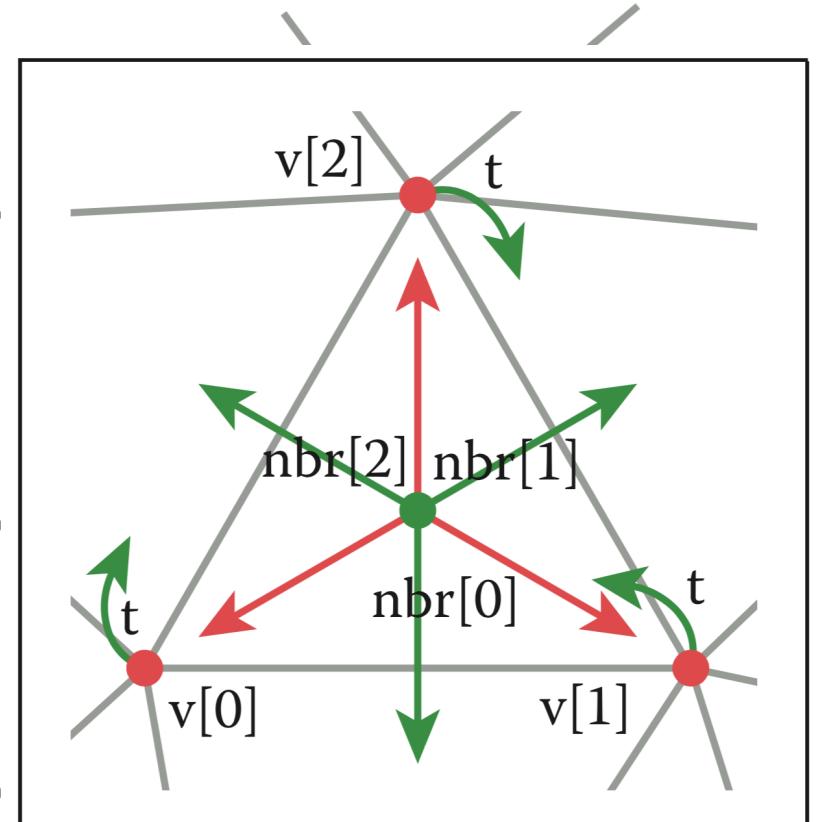
```
Edge {  
    v[2]; //Vertex  
    t[2]; //Triangle
```

}

Can we do better?

Triangle-Neighbor Structure

- Let's try first extending the indexed mesh structure for sharing vertices
- Add pointers, $\text{nbr}[]$, to 3 neighboring triangles
- Add a single pointer, t , for each vertex to one of its adjacent triangles
- Can now enumerate triangles adjacent to vertices



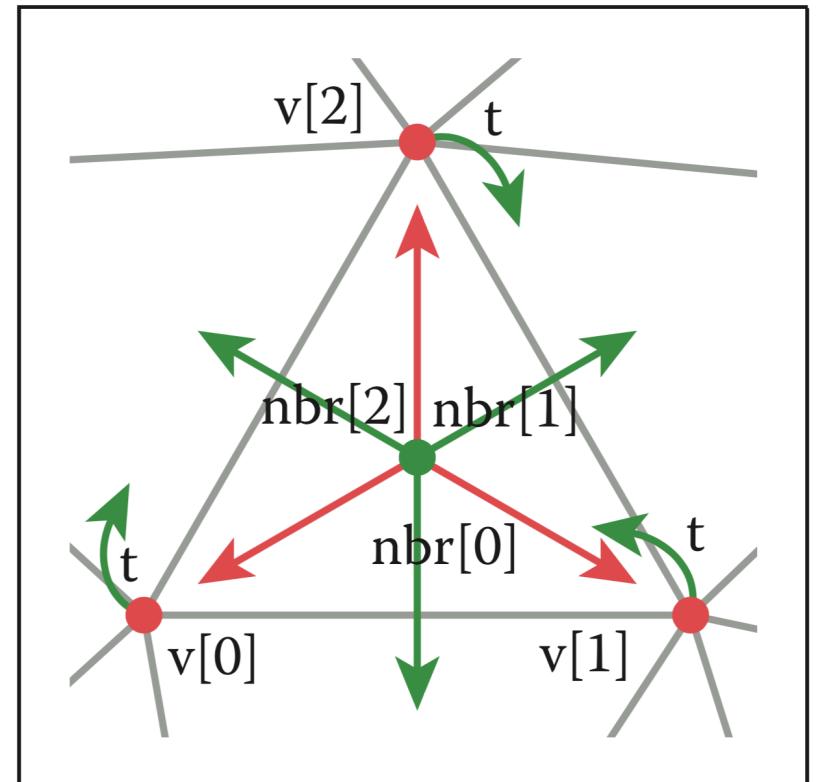
Triangle-Neighbor Structure

```
Triangle {  
    v[ 3];      //Vertex  
    nbr[ 3];    //Triangle  
}
```

```
Vertex {  
    ...  
    t;          //Triangle  
}
```

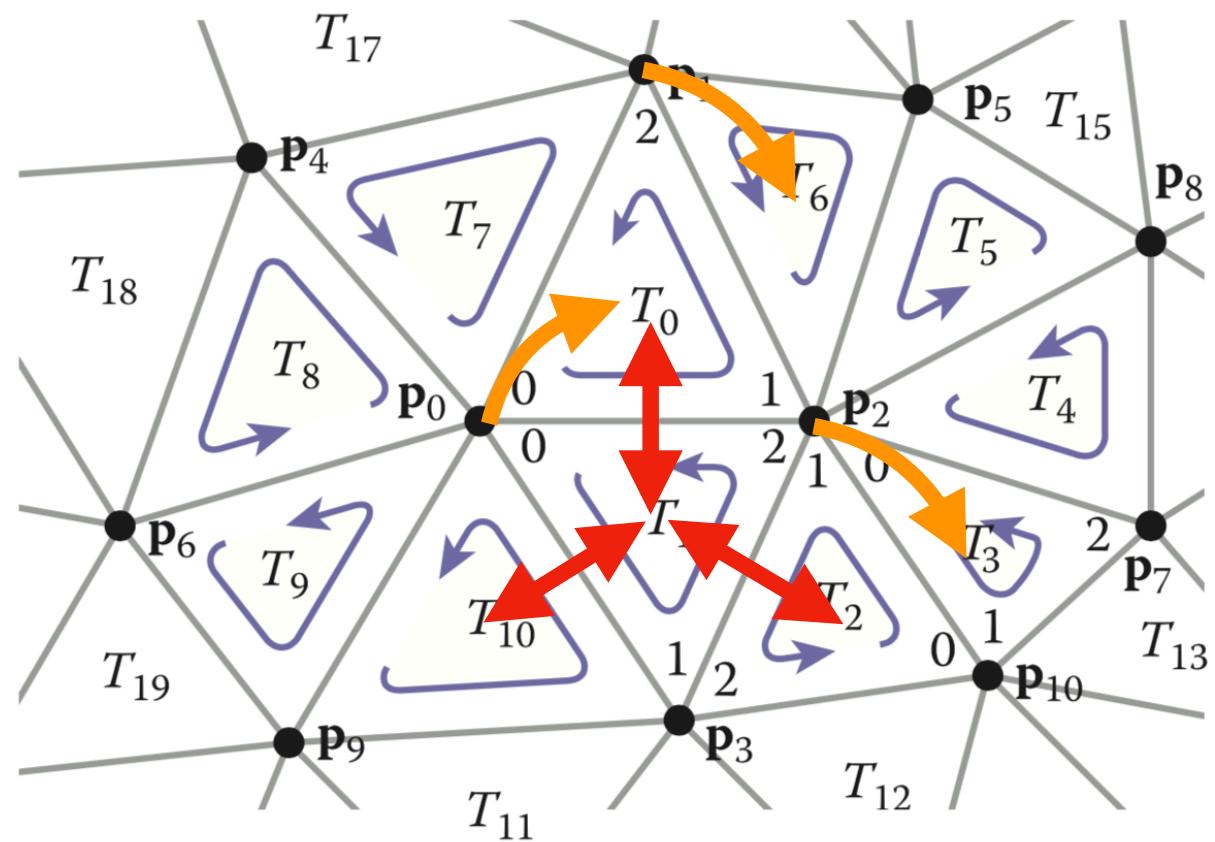
...or...

```
IndexedMesh {  
    ...  
    tInd[num_tris];   //int[ 3]  
    tNbr[num_tris];   //int[ 3]  
    vTri[num_verts];  //int  
};
```



Triangle-Neighbor Structure

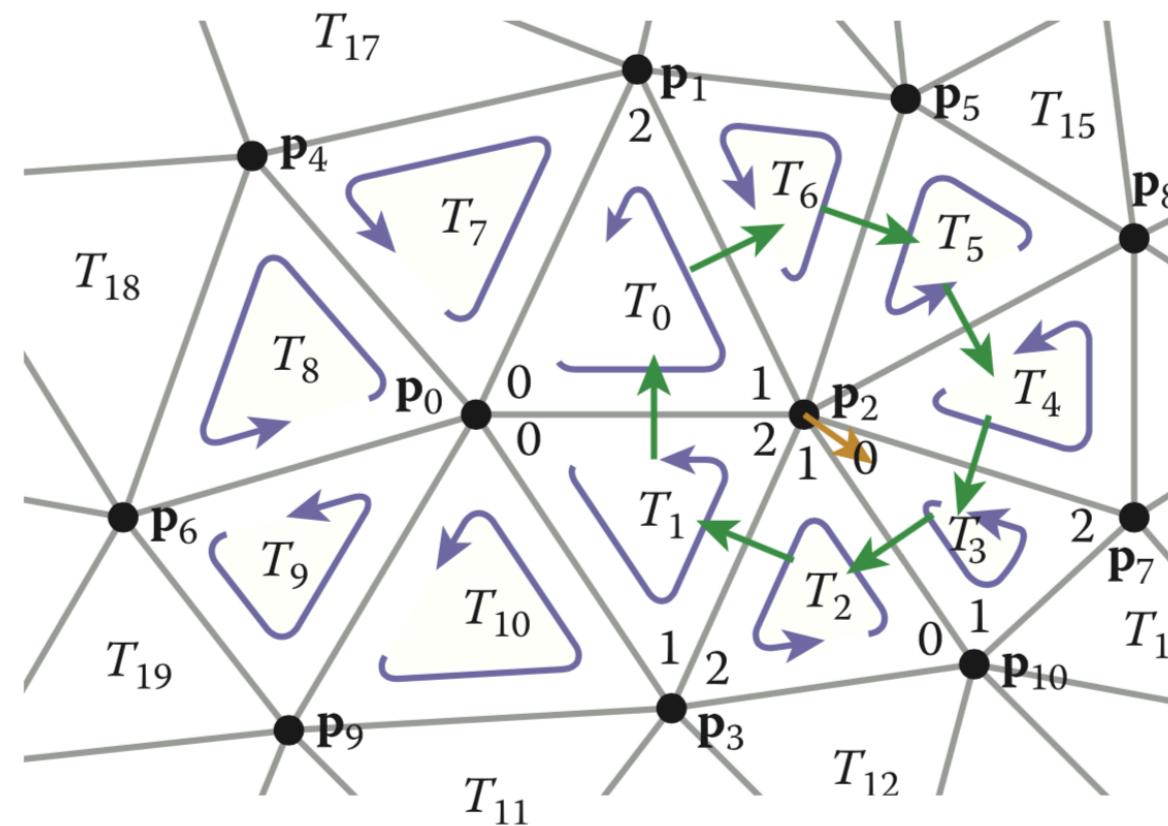
	vTri	tNbr
[0]	0	[0] 1, 6, 7
[1]	6	[1] 10, 2, 0
[2]	3	[2] 3, 1, 12
[3]	1	[3] 2, 13, 4 ⋮
		tInd
[0]		[0] 0, 2, 1
[1]		[1] 0, 3, 2
[2]		[2] 10, 2, 3
[3]		[3] 2, 10, 7 ⋮



Triangle-Neighbor Structure

```
TrianglesOfVertex(v) {  
    t = v.t  
    do {  
        find i where (t.v[i] == v)  
        t = t.nbr[i]  
    } while (t != v.t);  
}
```

- Can optimize by storing pointers to neighboring edges

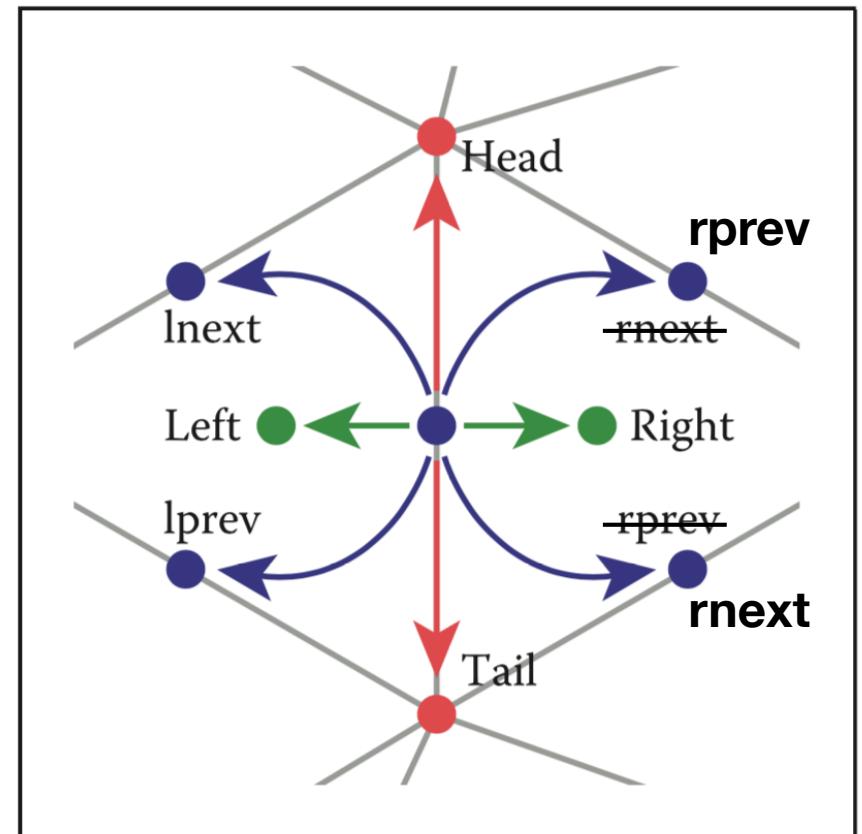


Triangle-Neighbor Structure

- Recall that indexed meshes needed $36*n_v$ bytes and $n_t \approx 2n_v$
- We added an array of triples of indices (per triangle)
 - This increases storage by $3*4*n_t$ or $24*n_v$ bytes
- We also added an array of representative triangle per vertex
 - This increases storage by $4*n_v$ bytes
- Total storage: $36 + 24 + 4 = 64$ bytes per vertex
 - Still not as much as separate triangles

Winged-Edge Structure

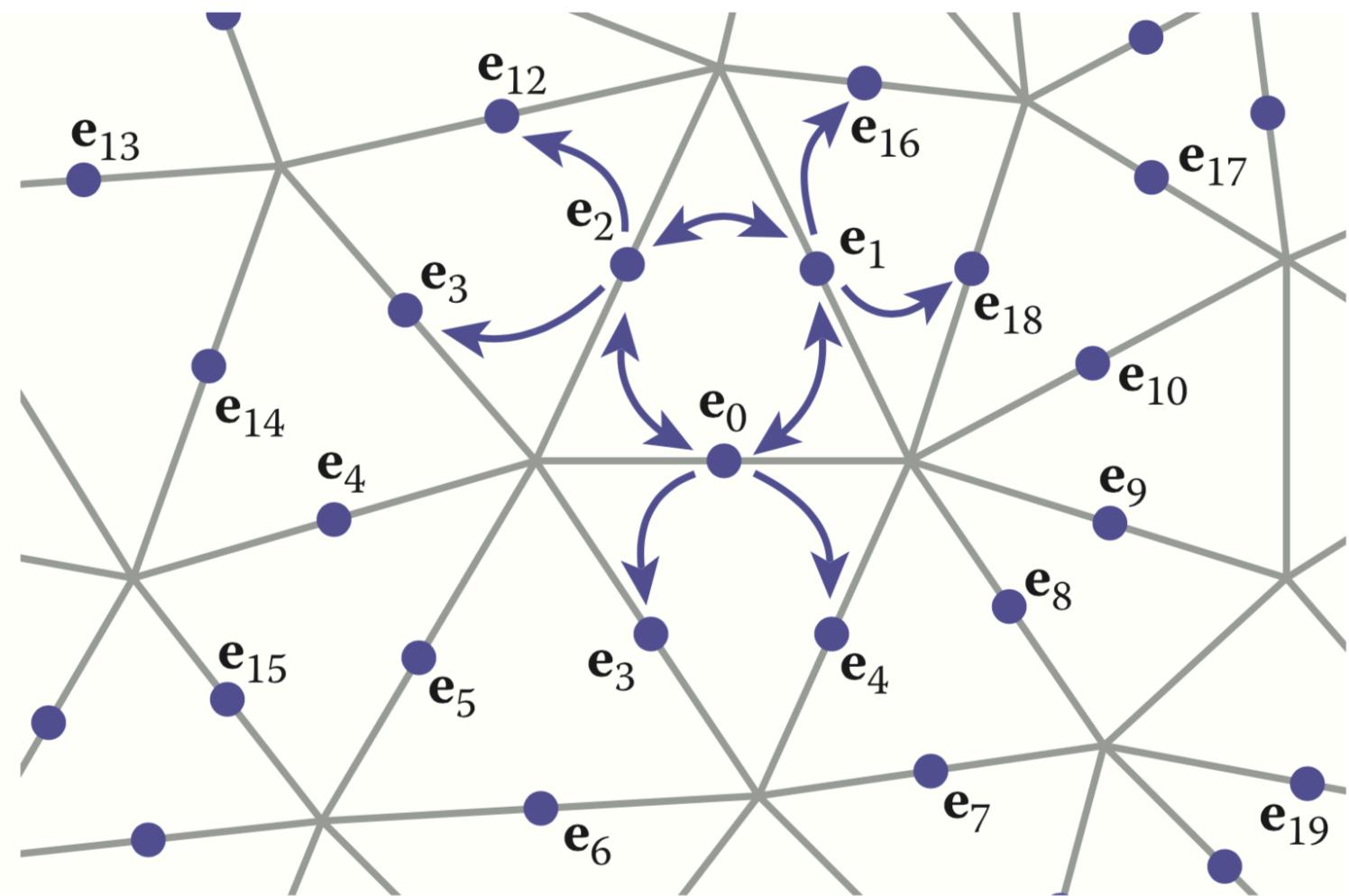
- Widely used mesh structure that focuses on edges instead of triangles
- Edges store pointers to:
 - Head/Tail vertices
 - Left/Right triangles
 - Left/Right “next” edges
 - Left/Right “previous” edges
- Each vertex/triangle stores one pointer to some edge



Winged-Edge Structure

Winged-edge table

	ln	rp	lp	rn
[0]	1	4	2	3
[1]	18	0	16	2
[2]	12	1	3	0
	⋮			



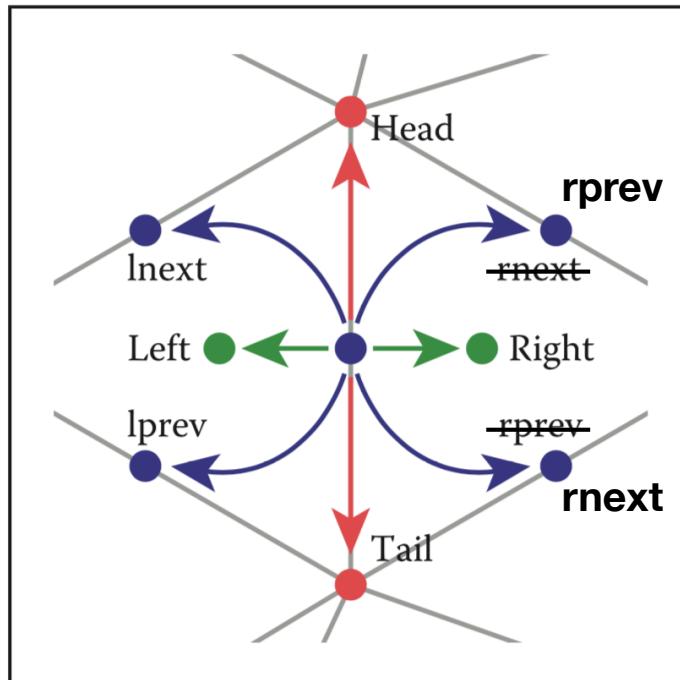
Winged-Edge Structure

```
Edge {  
    lprev, lnext, rprev, rnext; //Edge  
    head, tail; //Vertex  
    left, right; //Face  
}
```

```
Face {  
    ...  
    e; //Edge  
}
```

```
Vertex {  
    ...  
    e; //Edge  
}
```

```
EdgesOfVertex(v) {  
    e = v.e;  
    do {  
        if (e.tail == v) {  
            e = e.lprev;  
        } else {  
            e = e.rprev;  
        }  
    } while (e != v.e);  
}
```

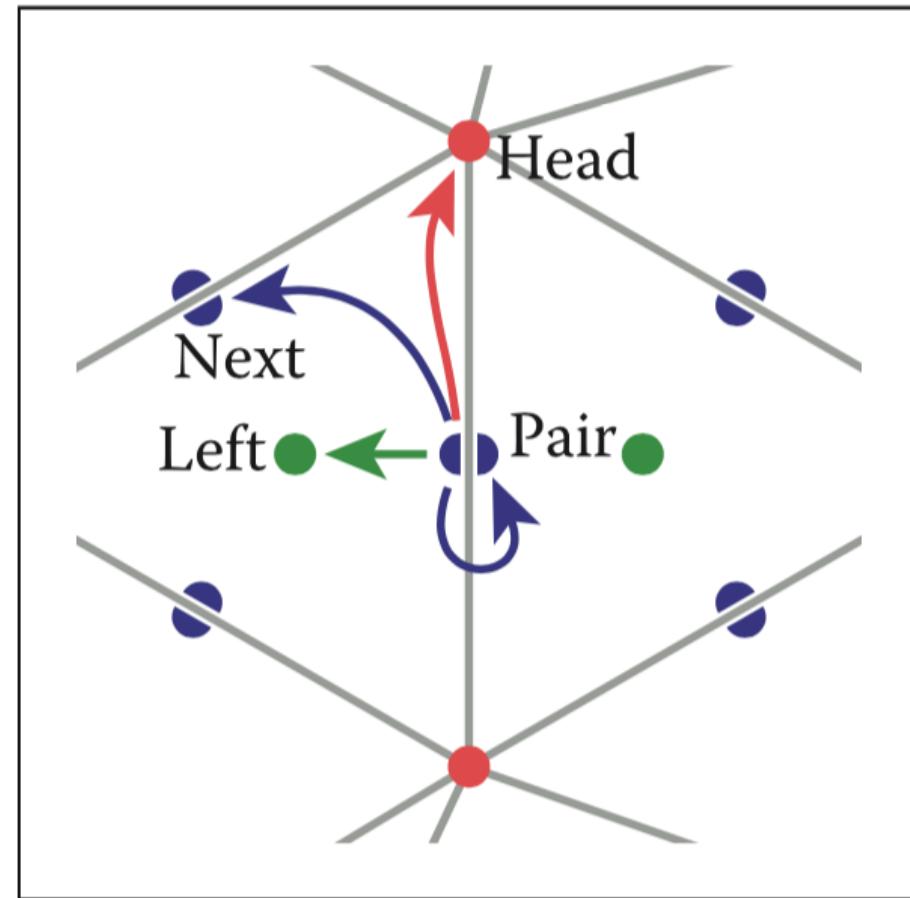


Winged-Edge Storage Requirements

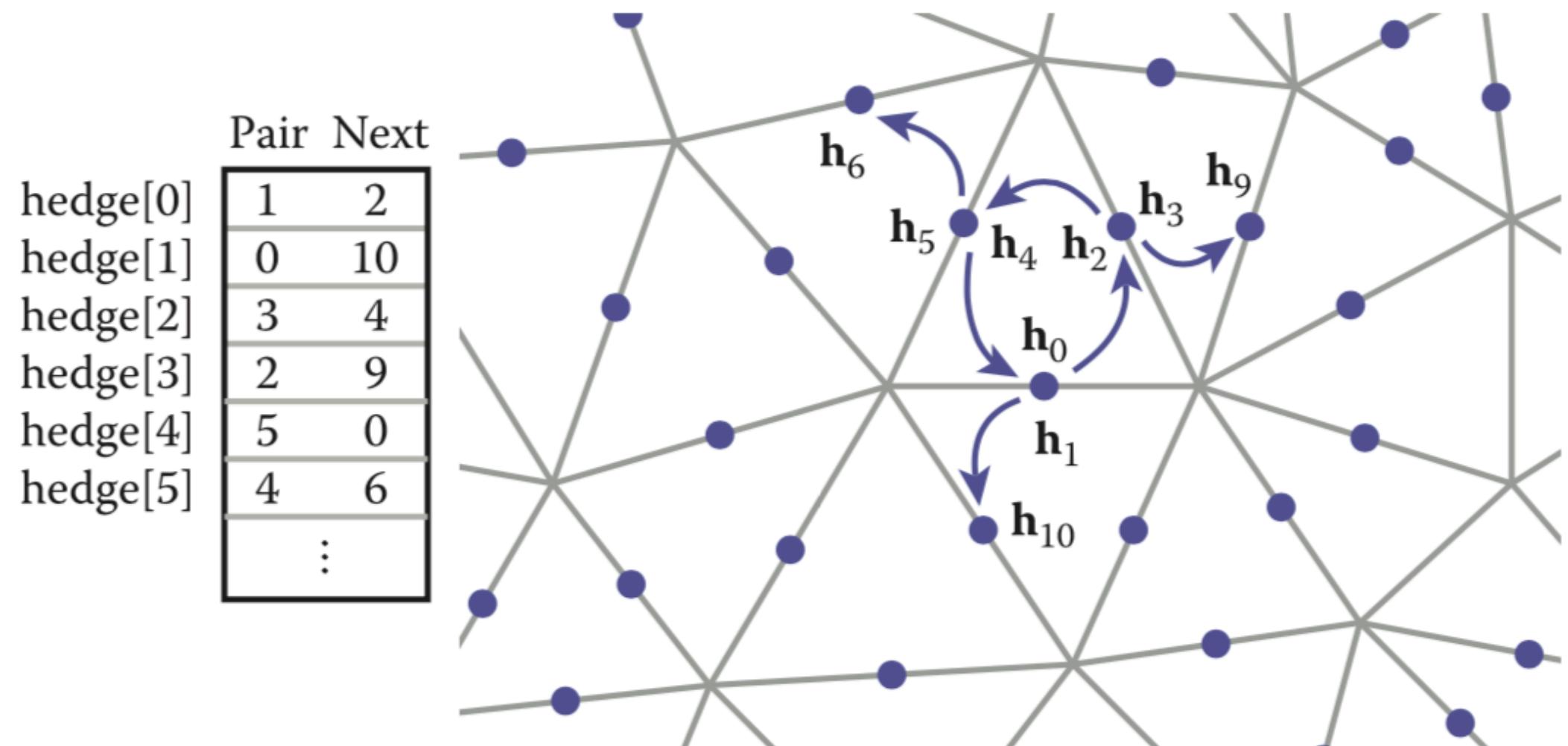
- Vertex data: 3 floats for position, 1 int for edge reference
 - $4*4 = 16n_v$ bytes
- Face data: 1 int for edge reference
 - $4*1 = 4n_t = 8n_v$ bytes.
- Edge data, 8 ints for references
 - $n_e \approx 3n_v$
 - $8*4*3 = 96n_v$ bytes.
- In total, $120n_v$ bytes.

Half-Edge Structure

- Simplifies winged-edge, removes awkwardness of checking which way edges are oriented
- Each **half-edge** store pointers to:
 - Head vertex
 - Left triangle
 - Left “next” edge
 - The opposite “pair” half-edge
- Each vertex/triangle stores one pointer to a half-edge



Half-Edge Structure



Half-Edge Structure

```
HEdge {  
    pair, next; //HEdge  
    v;           //Vertex  
    f;           //Face  
};
```

```
EdgesOfVertex(v) {  
    h = v.h;  
    do {  
        h = h.next.pair;  
    } while (h != v.h);  
}
```

```
EdgesOfVertex(v) {  
    e = v.e;  
    do {  
        if (e.tail == v) {  
            e = e.lprev;  
        } else {  
            e = e.rprev;  
        }  
    } while (e != v.e);  
}
```

Winged-Edge Implementation

Half-Edge Storage Requirements

- Vertex data: 3 floats for position, 1 int for edge reference
 - $4*4 = 16n_v$ bytes
- Face data: 1 int for edge reference
 - $4*1 = 4*n_t = 8n_v$ bytes.
- Edge data, 4 ints for references, but store a pair of half edges for each edge
 - $n_h \approx 6n_v$
 - $8*4*6 = 96n_v$ bytes.
- In total, $120n_v$ bytes.



OpenMesh

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OpenMesh

OpenMesh

A generic and efficient polygon mesh data structure

OpenMesh is a generic and efficient data structure for representing and manipulating polygonal meshes. For more information about OpenMesh and its features take a look at the [Introduction page](#).

On top of OpenMesh we develop OpenFlipper, a flexible geometry modeling and processing framework.

News

- [OpenMesh 6.3 released](#)

Oct. 4, 2016

OpenMesh 6.3 is still fully backward compatible with the 2.x to 5.x branches. We marked some functions which should not be used anymore as deprecated and added hints which should be used instead.

This will be the last release officially supporting C++98 compilers and building the integrated applications with Qt 4.

The update adds a workaround for an gcc optimizer bug causing segfaults when optimizing with '-O3'. If your gcc is affected (gcc 4.x and 5.x) OpenMesh will fallback to

Lec15 Required Reading

- FOCG, Ch. 12.3

Reminder: Assignment 04

Assigned: Wednesday, Oct. 9

Written Due: Monday, Oct. 23, 4:59:59 pm

Program Due: Wednesday, Oct. 28, 4:59:59 pm