CSC 433/533
Computer Graphics

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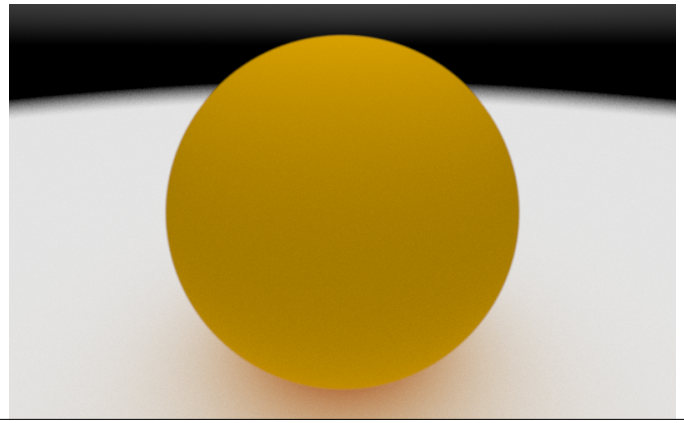
Credit: Joshua Levine

Lecture 13
Global Illumination

Oct. 9, 2019

Indirect Lighting

- The world isn't split in "light sources" vs "non-light sources"



Indirect Lighting

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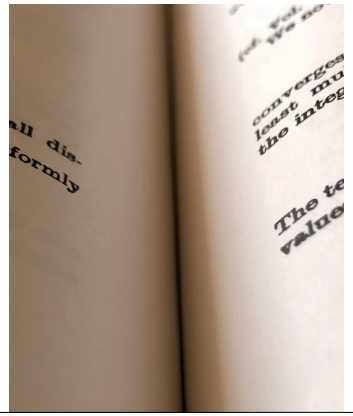
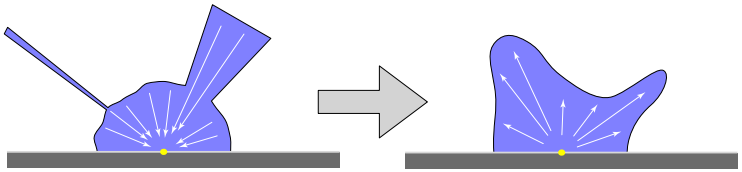


Light reflection: full picture

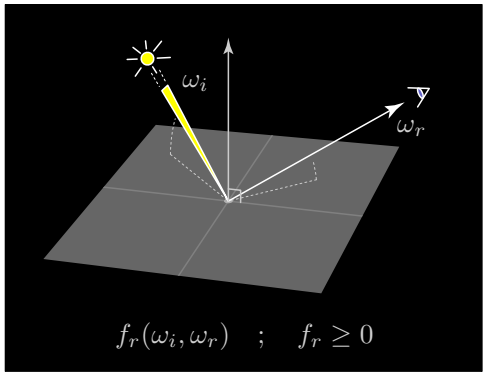
- **when writing a shader, think like a bug standing on the surface**
  - bug sees an incident distribution of light arriving at the surface
  - physics question: what is the outgoing distribution of light?



incident distribution
(function of direction)

reflected distribution
(function of direction)

BRDF



Bidirectional Reflectance Distribution Function

Surface illumination integral (as sum)

- **BRDF tells you how light from a single direction is reflected**
- **Light coming from a small source behaves similarly**
- **What about light coming from everywhere?**
  - approximate incoming light with many small sources on a sphere (the little bug can't tell the difference…)
  - reflected light is sum of reflected light due to each source (each source has its size Ω_k, brightness L_k, and direction ω_k)

$$L_r(\omega_r) = \sum_k \Omega_k L_k f_r(\omega_k, \omega_r) |\omega_k \cdot \mathbf{n}|$$

reflected light in direction ω_r | "intensity" of light source k | BRDF | cosine factor

Surface illumination integral

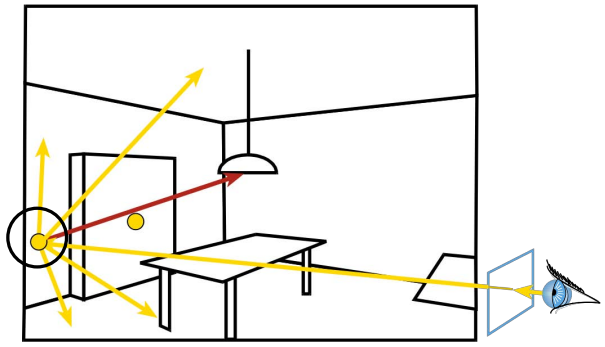
- **Take the limit as the little area sources get smaller**
  - collection of separate brightnesses L_k becomes a function $L_i(\omega_i)$
  - size of sources turns into an integration measure $d\sigma$

$$L_r(\omega_r) = \int_{S^2_+} L_i(\omega_i) f_r(\omega_i, \omega_r) |\omega_i \cdot \mathbf{n}| d\sigma(\omega_i)$$

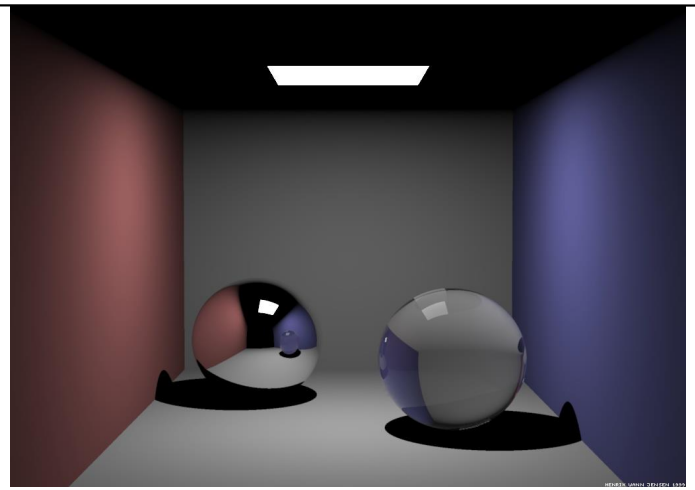
"The light reflected to direction ω_r is the integral, over the positive unit hemisphere, of the incoming light times the BRDF times the incoming cosine factor, with respect to surface area."

Global Illumination

- So far, we've seen only direct lighting (red here)
- We also want indirect lighting
  - Full integral of all directions (multiplied by BRDF)
  - In practice, send tons of random rays



Direct Illumination



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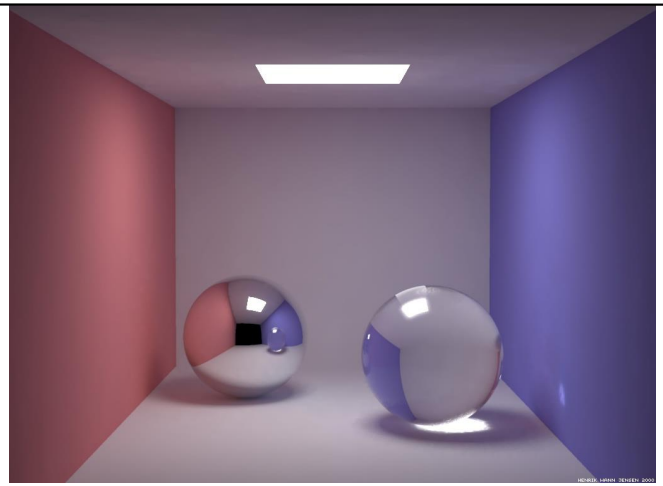
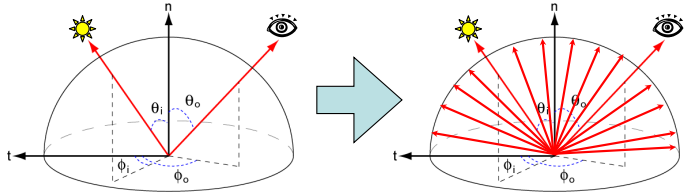
Global Illumination (with Indirect)



Courtesy of Henrik Wann Jensen. Used with permission.

Global Illumination

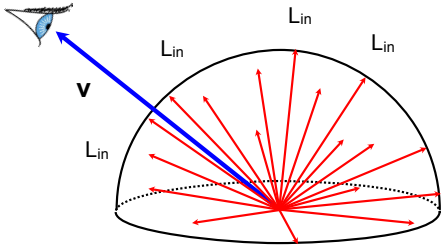
- So far, we only used the BRDF for point lights
  - We just summed over all the point light sources
- BRDF also describes how indirect illumination reflects off surfaces
  - Turns summation into integral over hemisphere
  - As if *every* direction had a light source



Reflectance Equation, Visually

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \, \mathrm{d}l$$

outgoing light to direction v — incident light from direction omega — the BRDF — cosine term

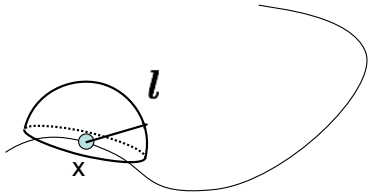


Sum (integrate) over every direction on the hemisphere, modulate incident illumination by BRDF

The Reflectance Equation

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \, \mathrm{d}l$$

- Where does L_{in} come from?



The Reflectance Equation

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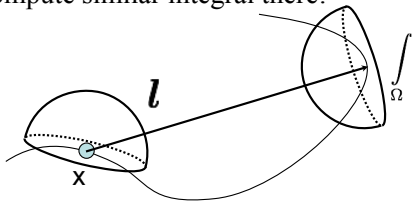
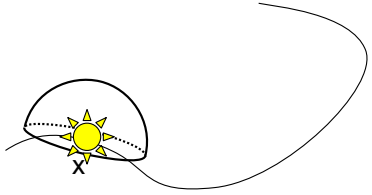
- Where does L_{in} come from?
  - It is the light reflected towards x from the surface point in direction ***l*** ==> must compute similar integral there!
    - Recursive!



The Rendering Equation

$$L_{\text{out}}(x, \boldsymbol{v}) = \int_{\Omega} L_{\text{in}}(\boldsymbol{l})\, f_r(x, \boldsymbol{l}, \boldsymbol{v})\, \cos\theta \, \mathrm{d}l + E_{\text{out}}(x, \boldsymbol{v})$$

- Where does L_{in} come from?
  - It is the light reflected towards x from the surface point in direction ***l*** ==> must compute similar integral there!
    - Recursive!
  - AND if x happens to be a light source, we add its contribution directly



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- The rendering equation describes the appearance of the scene, including direct and indirect illumination
  - An "integral equation", the unknown solution function L is both on the LHS and on the RHS inside the integral
    - Must either discretize or use Monte Carlo integration
  - Originally described by Kajiya and Immel et al. in 1986

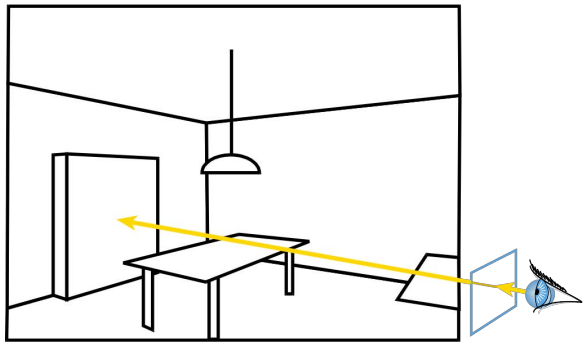
The Rendering Equation

- Analytic solution is usually impossible
- Lots of ways to solve it approximately
- Monte Carlo techniques use random samples for evaluating the integrals
  - We'll look at some simple method in a bit...
- Finite element methods discretize the solution using basis functions (again!)
  - Radiosity, wavelets, precomputed radiance transfer, etc.

How to Compute Global Illumination

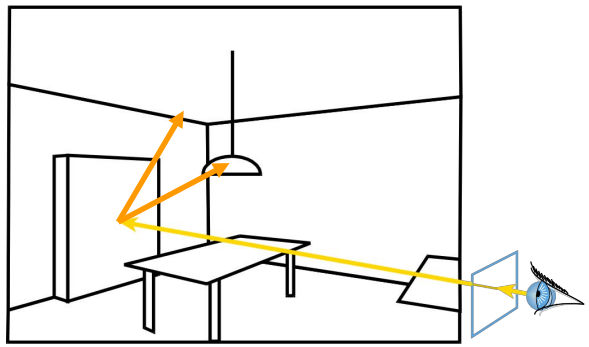
Ray Casting

- Cast a ray from the eye through each pixel



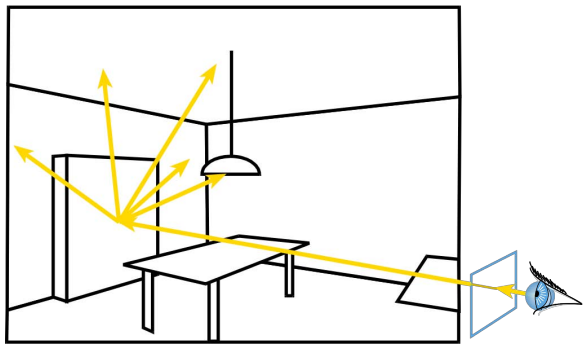
Ray Tracing

- Cast a ray from the eye through each pixel
- Trace secondary rays (shadow, reflection, refraction)



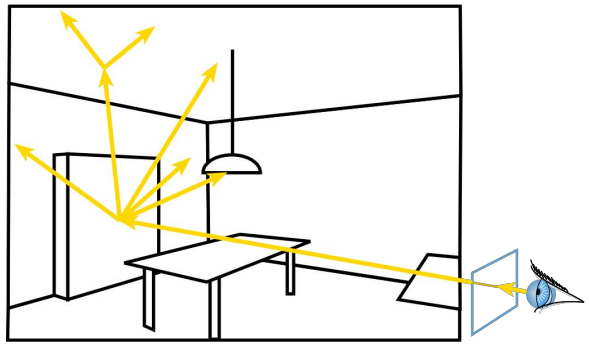
“Monte-Carlo Ray Tracing”

- Cast a ray from the eye through each pixel
- Cast random rays from the hit point to evaluate hemispherical integral using random sampling



“Monte-Carlo Ray Tracing”

- Cast a ray from the eye through each pixel
- Cast random rays from the visible point
- Recurse



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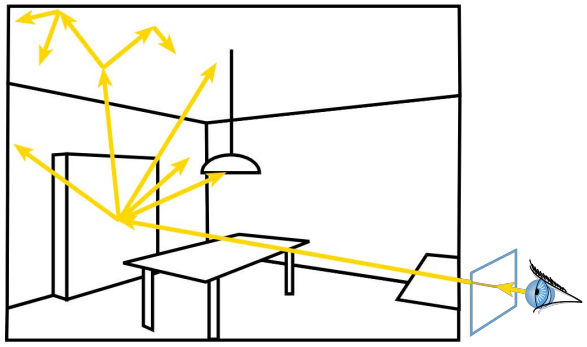
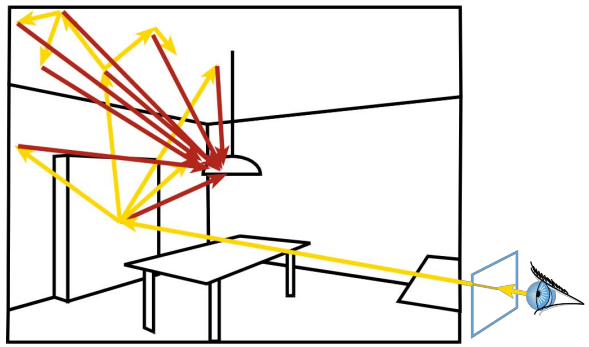
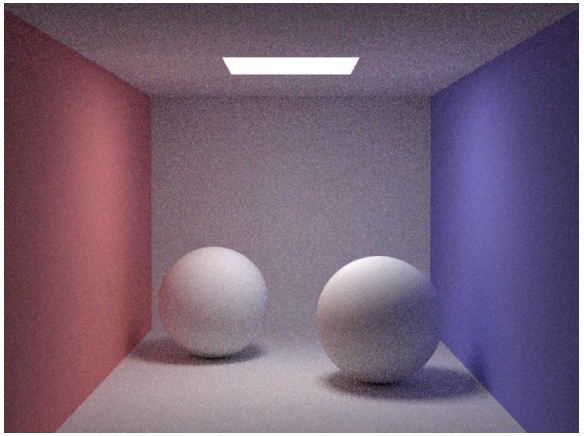


“Monte-Carlo Ray Tracing”

- Systematically sample light sources at each hit
  - Don’t just wait until the rays hit it by chance



Results



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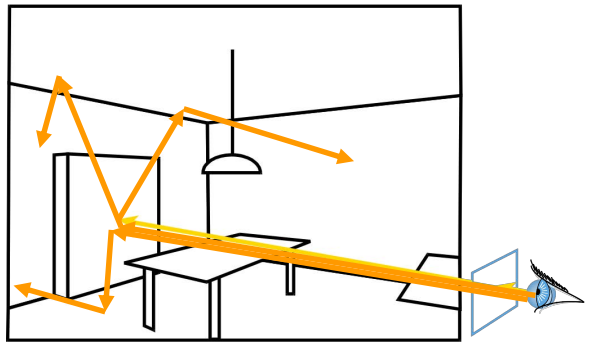
Monte Carlo Path Tracing

- Trace only one secondary ray per recursion
  - Otherwise number of rays explodes!
- But send many primary rays per pixel (antialiasing)



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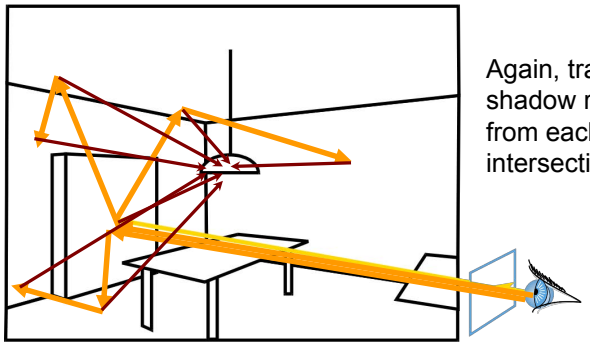
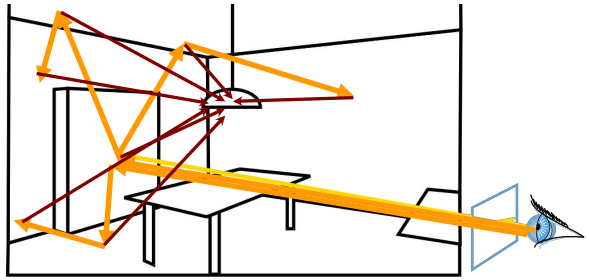


Again, trace shadow rays from each intersection

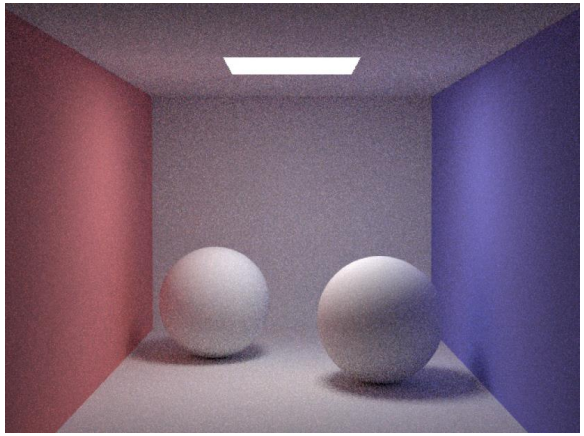
Monte Carlo Path Tracing

- We shoot one path from the eye at a time
  - Connect every surface point on the way to the light by a shadow ray
  - We are randomly sampling the space of all possible light paths between the source and the camera



Path Tracing Results

- 10 paths/pixel



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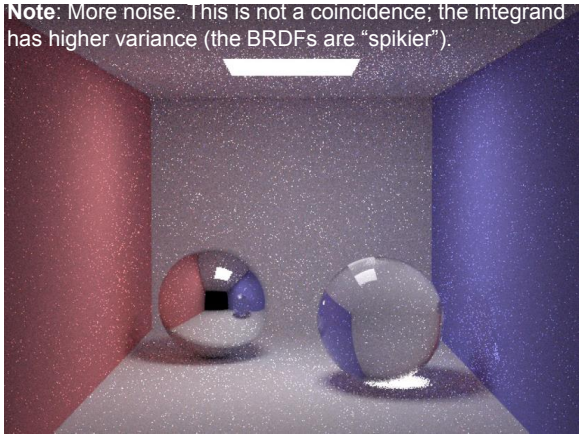
Path Tracing Results: Glossy Scene

- 10 paths/pixel



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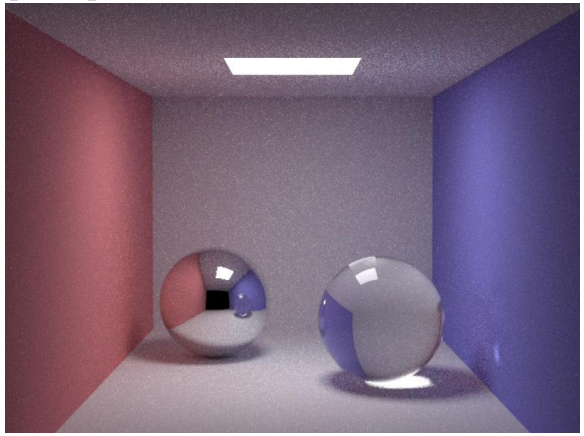
Path Tracing Results: Glossy Scene

- 100 paths/pixel



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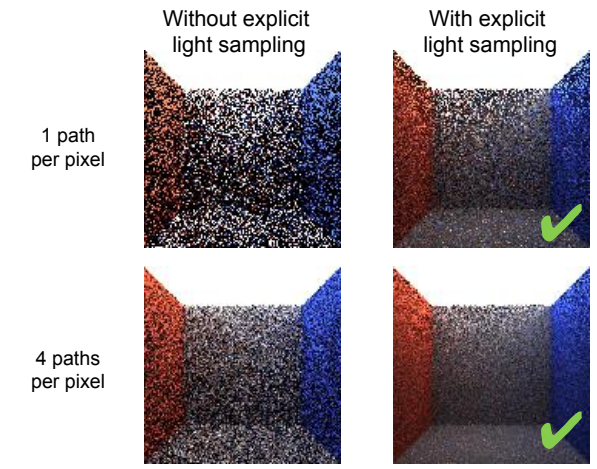
Importance of Sampling the Light



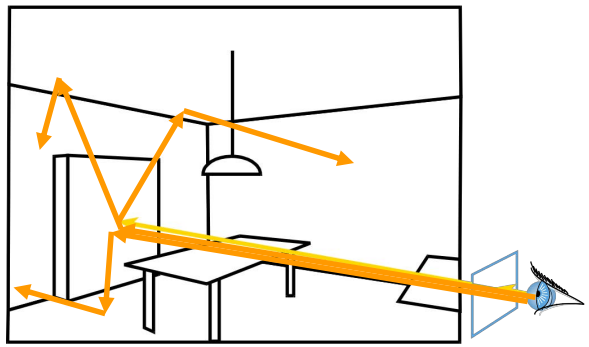
Questions?

- Vintage path tracing by Kajiya

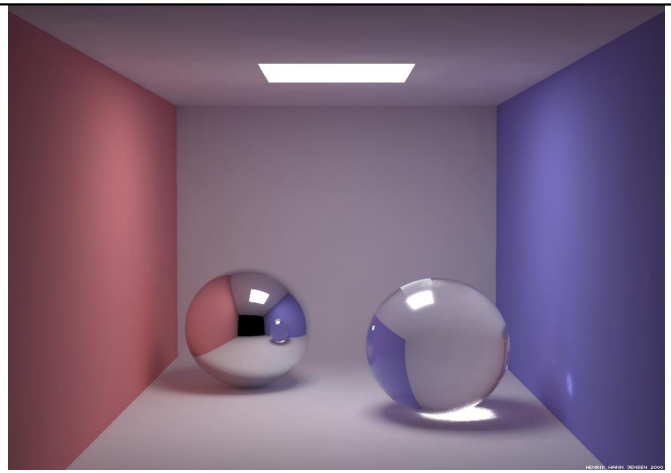
Irradiance Caching

Path Tracing is costly

- Needs tons of rays per pixel!



Global Illumination (with Indirect)



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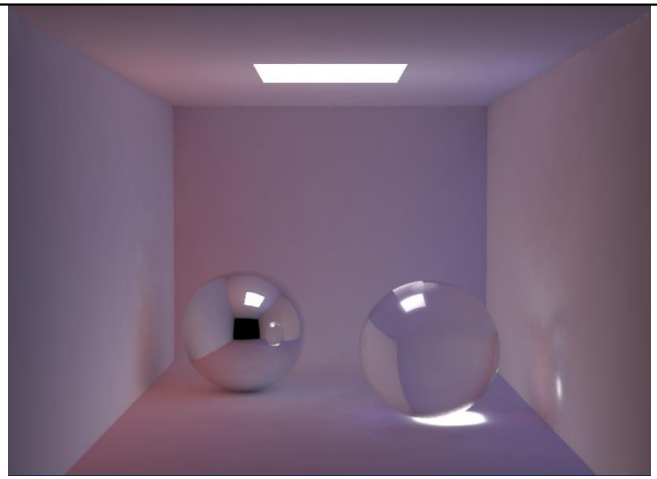
Indirect Lighting is Mostly Smooth



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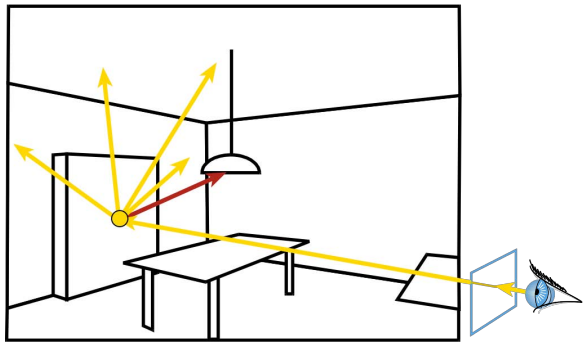
Irradiance Caching

- Indirect illumination is smooth



Irradiance Caching

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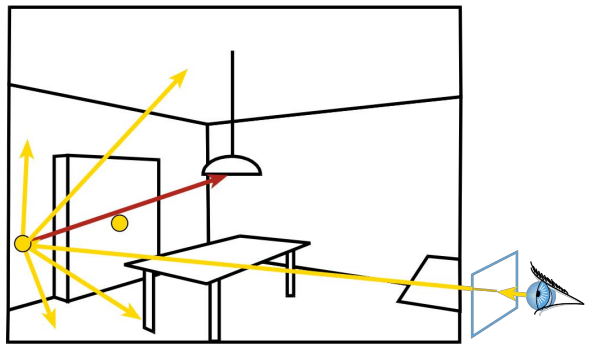
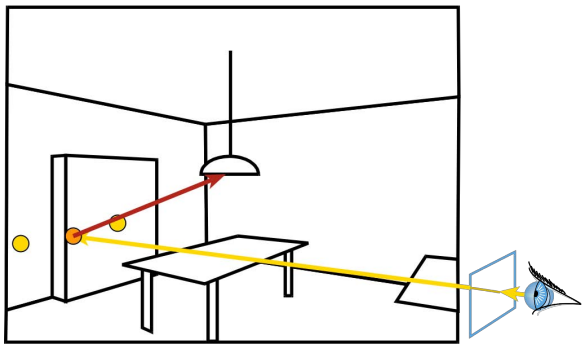


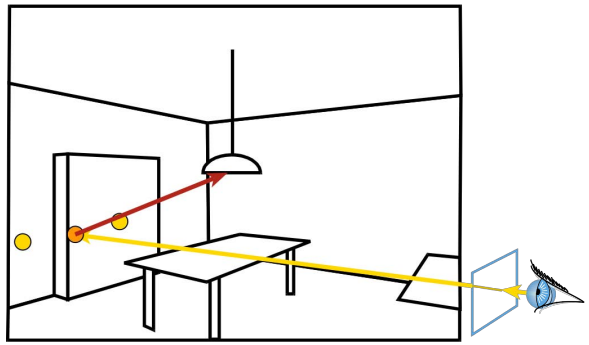
Irradiance Caching

- Indirect illumination is smooth
  ==> Sample sparsely, interpolate nearby values



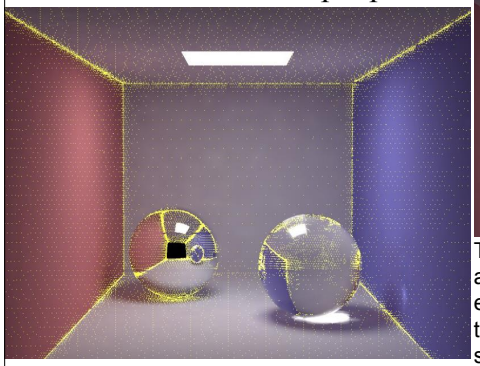
Irradiance Caching

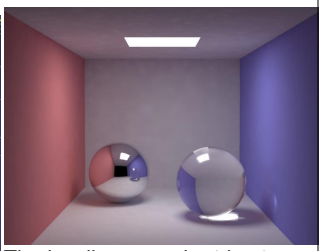
- Store the indirect illumination
- Interpolate existing cached values
- But do full calculation for direct lighting



Irradiance Caching

- Yellow dots:
  indirect diffuse sample points



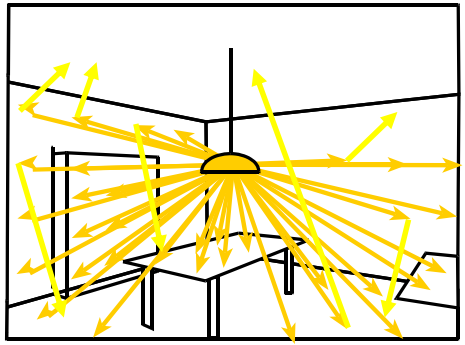


The irradiance cache tries to adapt sampling density to expected frequency content of the indirect illumination (denser sampling near geometry)

Photon Mapping

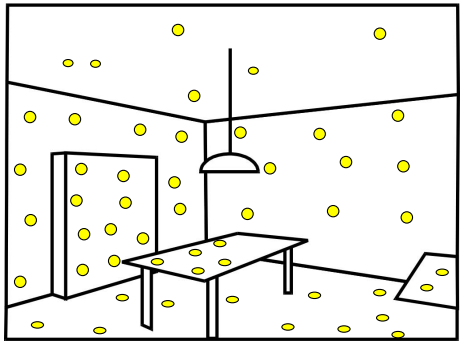
Photon Mapping

- Preprocess: cast rays from light sources, let them bounce around randomly in the scene
- Store “photons”



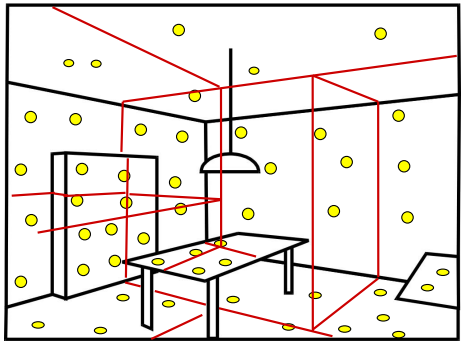
Photon Mapping

- Preprocess: cast rays from light sources
- Store photons (position + light power + incoming direction)



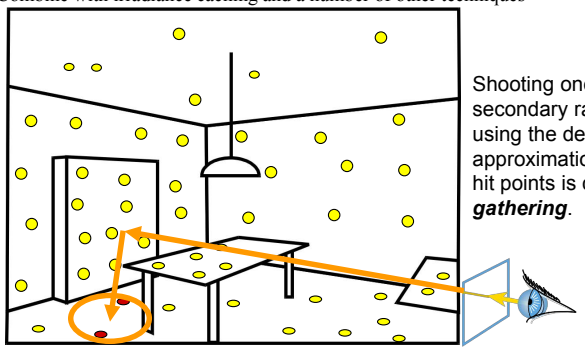
The Photon Map

- Efficiently store photons for fast access
- Use hierarchical spatial structure (kd-tree)



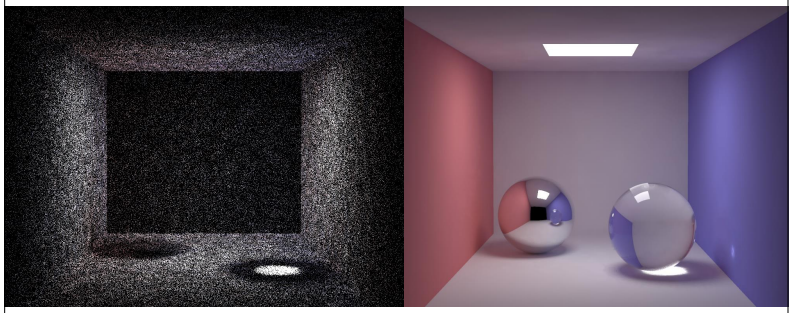
Photon Mapping - Rendering

- Cast primary rays
- For secondary rays
  - reconstruct irradiance using adjacent stored photon
  - Take the k closest photons
- Combine with irradiance caching and a number of other techniques



Shooting one bounce of secondary rays and using the density approximation at those hit points is called ***final gathering***.

Photon Map Results



Courtesy of Henrik Wann Jensen. Used with permission.

More Global Illumination Coolness

- Many materials exhibit *subsurface scattering*
  - Light doesn't just reflect off the surface
  - Light enters, scatters around, and exits at another point
  - Examples: Skin, marble, milk

Images: Jensen et al.

Courtesy of Henrik Wann Jensen. Used with permission.

That Was Just the Beginning

- Tons and tons of other Monte Carlo techniques
  - Bidirectional Path Tracing
    - Shoot random paths not just from camera but also light, connect the path vertices by shadow rays
  - Metropolis Light Transport
- And Finite Element Methods
  - Use basis functions instead of random sampling
  - Radiosity (with hierarchies & wavelets)
  - Precomputed Radiance Transfer
- This would warrant a class of its own!

Questions?

- Images by Veach and Guibas, SIGGRAPH 95

Naïve sampling strategy

Optimal sampling strategy

Lec14 Required Reading

- FOCG, Ch. 12.1