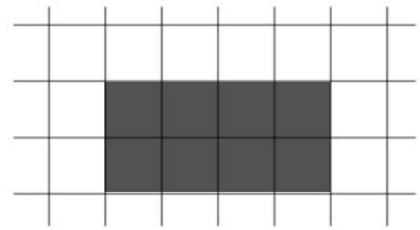
Lecture 03
Transformations in 2D
Short version

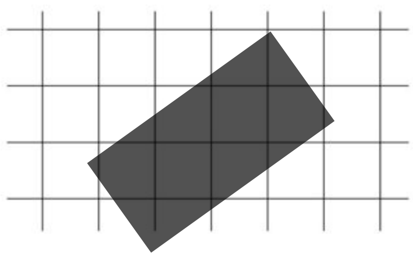
We will discuss transformation in 3D, and with full details, later in the course

About hw1
Aliasing and Anti-Aliasing



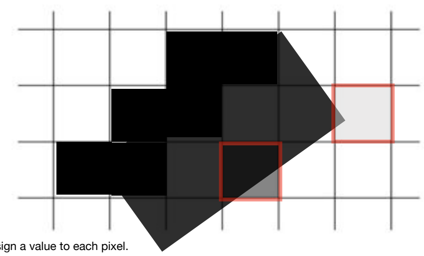
- This about an image where each pixels is fully black or fully white

What if we rotate the rectangle



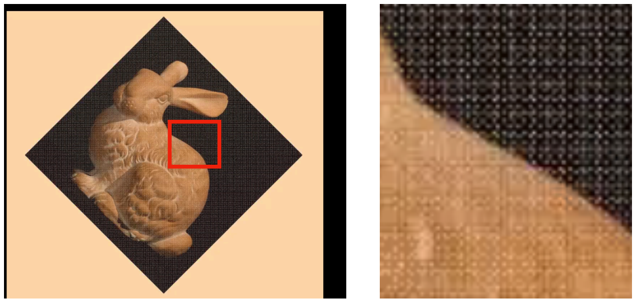
- Some pixels are **partially** covered by the rectangle. Show they be rendered as black, white, or some shade of grey ?

What if we rotate the rectangle



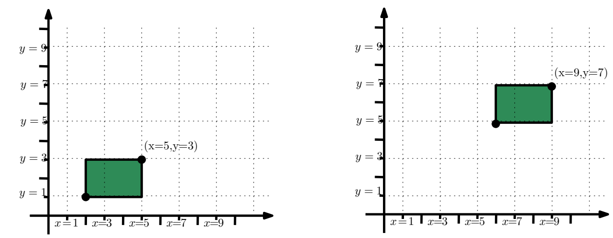
- lWe still need to assign a value to each pixel.
- If we draw each **partially covered** pixel as black, we will obtain a very pixelated shape. This is an example of **aliasing.**
- A possible solution is to render some pixels as gray. For example, based on the portion of its area which is covered. This technique is call antialiasing. Essentially, the color of a pixel might be determined using input from several neighboring pixels.
- We will study much much more about it. Do not worry about it in hw1.
- In hw1, each rendered pixel has the (rgb) value of one (single) input pixel. No averaging or mixing.

Something to be careful about with hw1



Translations (shift) by(α, β)

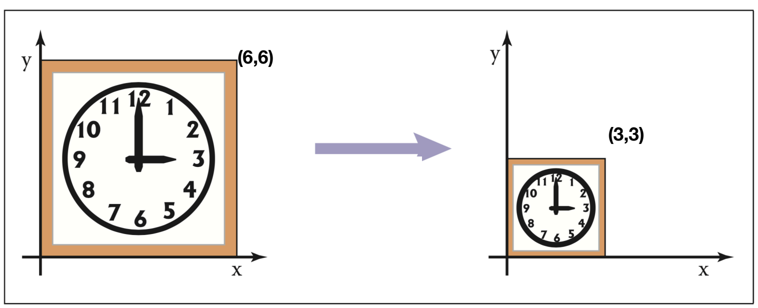
Translation (shift) by $(4, 4)$
$(x, y) \to (x + 4, y + 4)$



- Adding a constant α to the x-coordinate of every point
- Adding a constant β to the y-coordinate of every point
- $(x, y) \to (x + \alpha, y + \beta)$

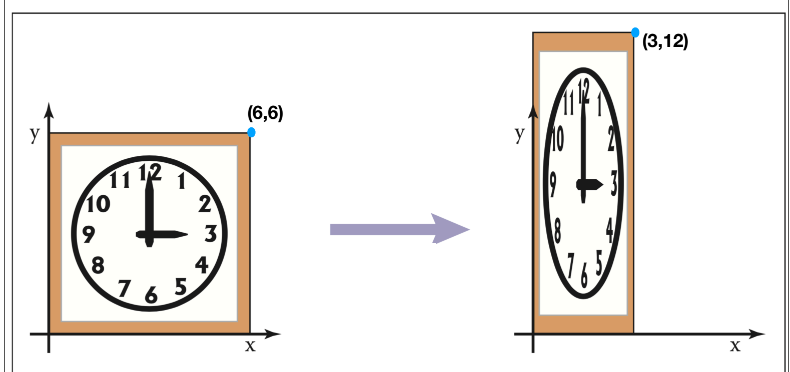
Scaling

- We can use two constants (s_x,s_y) for the x-axis and the y-axis. Then we shift each point *(x,y)* into the point $(s_x \cdot x, \;\; s_y \cdot y)$
- $(x, y) \to (s_x \cdot x, \;\; s_y \cdot y)$
- Example $(x, y) \to (x/2, \; y/2)$

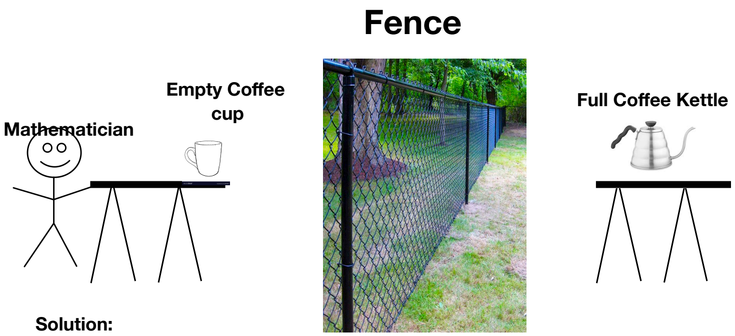


Scaling

- Example: $(x, y) \to (0.5x, \;\; 2y)$

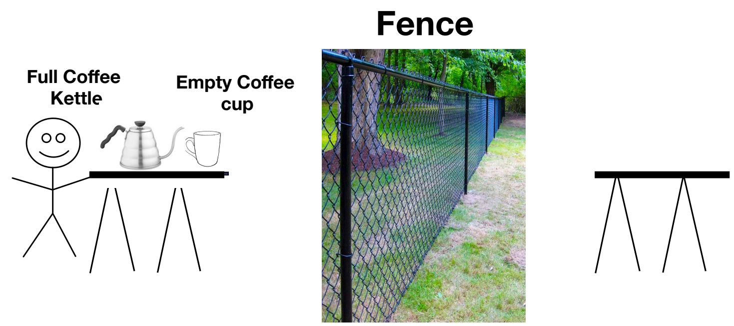


The mathematician and coffee cup non-funny joke
Part 1



Solution:
1. **Walk around the fence,**
2. **fetch coffee kettle,**
3. **walk back pure coffee,**
4. **drink**

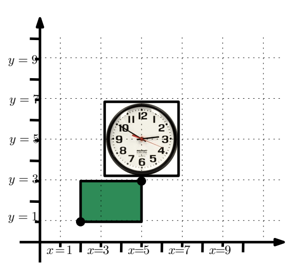
The mathematician and coffee cup non-funny joke
Part 2

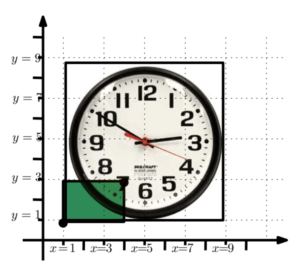


Solution:
1. **Bring the coffee Kettle to the other table, and walk to the left table**
2. **Apply the solution from the previous slide**

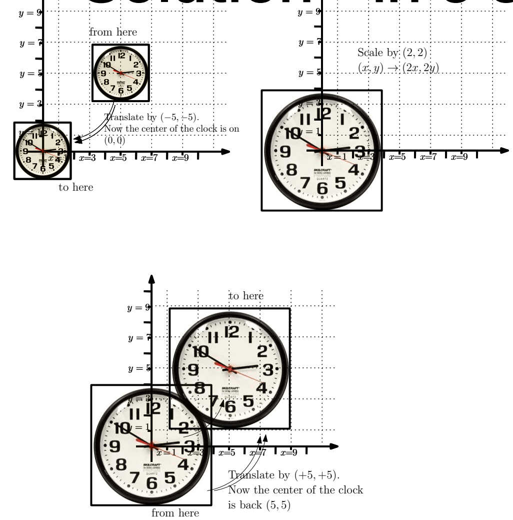
Resize the clock, without changing its center

Problem: scale the clock, but without changing its center and without effecting the green rectangle



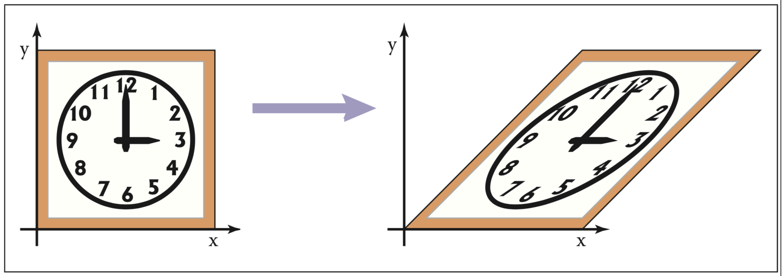


Solution - in 3 steps



Shearing

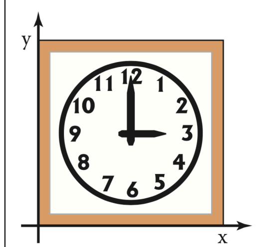
- If we move each point (x,y) into the point
  $(x, y) \rightarrow (x + y,\ y)$

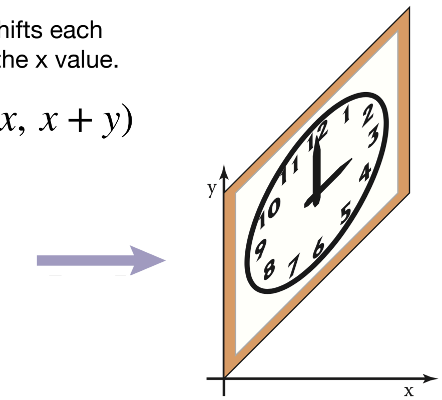


Shearing

- Vertical shearing shifts each column based on the x value.

$$(x, y) \rightarrow (x,\ x + y)$$

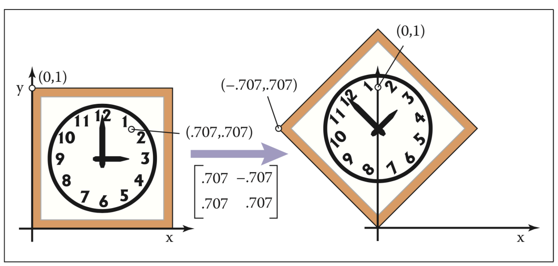




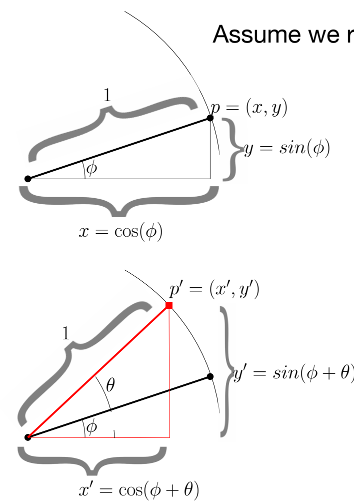
Rotation

- Rotate counterclockwise by an angle ϕ about the origin.

$$(x, y) \rightarrow (\underbrace{x\cos\phi - y\sin\phi}_{\textbf{New } x},\ \underbrace{x\sin\phi + y\cos\phi}_{\textbf{New } y})$$



Assume we rotate *p* by an angle θ CCW

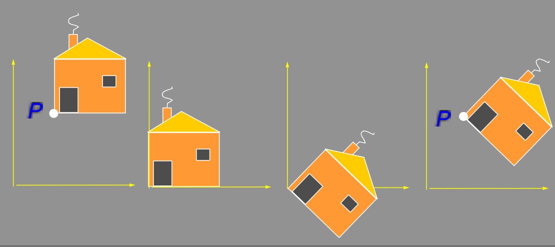


$$\cos(\phi + \theta) =$$

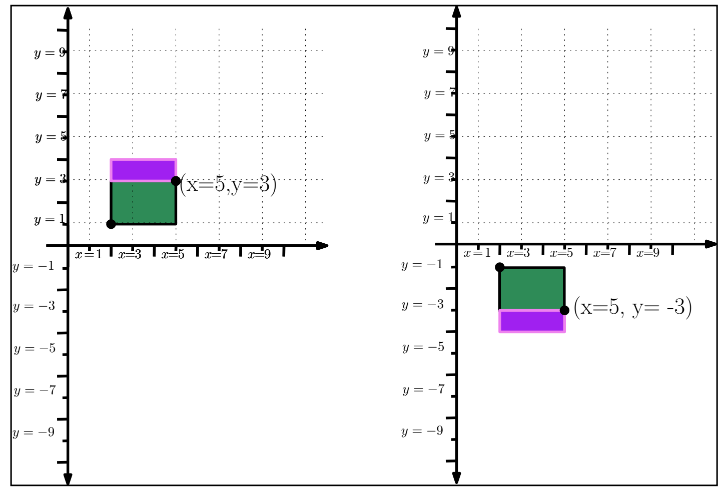
$$\underbrace{\cos(\phi)}_{=x}\cos(\theta) - \underbrace{\sin(\phi)}_{=y}\sin(\theta)$$

Transformation Composition

- What operation rotates by θ around $P = (p_x, p_y)$?
  - Translate P to origin
  - Rotate around origin by θ
  - Translate back

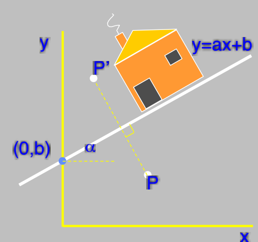


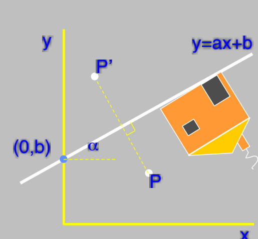
Reflection on the x-axes: $(x, y) \rightarrow (x, -y)$



Arbitrary Reflection - promo
We will get back to it later in the semester

$\alpha = \tan^{-1}(a)$





Shift by (0,-b)

Rotate by $-\alpha$

Reflect through x

Rotate by α

Shift by (0,b)

Very scarrrrry….

Unless we represent transformation by matrices

And then it is trivial.