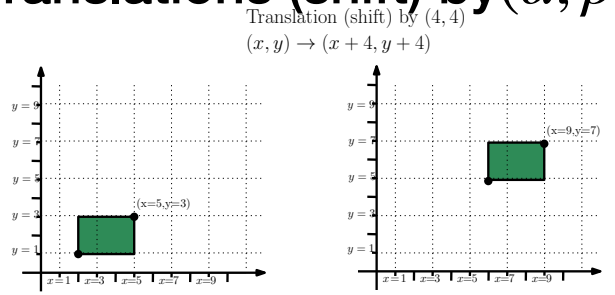
Lecture 03
Transformations in 2D
Short version

We will discuss transformation in 3D, and with full details, later in the course

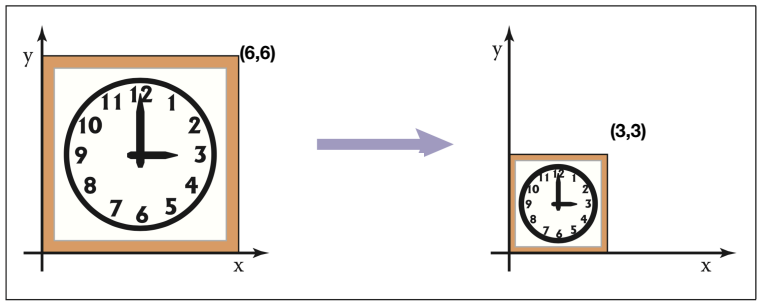
Translations (shift) by (α, β)



- Adding a constant α to the x-coordinate of every point
- Adding a constant β to the x-coordinate of every point
- $(x, y) \rightarrow (x + \alpha, y + \beta)$

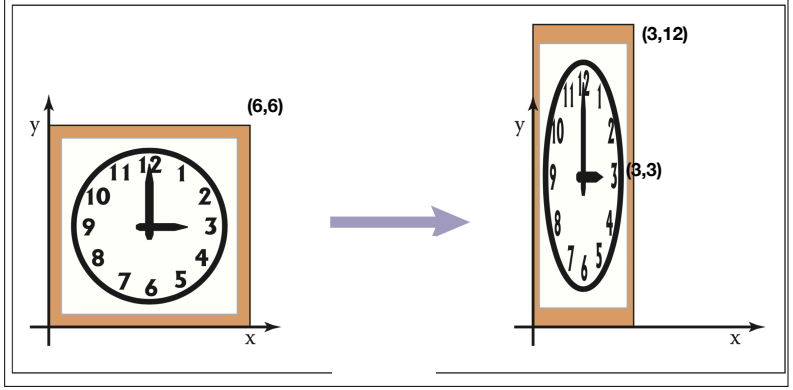
Scaling

- We can use different constants (s_x, s_y) for the x-axis vs. the y-axis. Then we shift each point (x,y) into the point
- $(x, y) \rightarrow (s_x \cdot x, \quad s_y \cdot y)$



Scaling

- Let s be a constant. If we move each point (x,y) into the point $(x, y) \rightarrow (s \cdot x, \quad s \cdot y)$ we scaled the image by s.



The mathematician and coffee cup non-funny joke Part 1

Fence

Solution:
1. **Walk around the fence,**
2. **fetch coffee kettle,**
3. **walk back pure coffee,**
4. **drink**

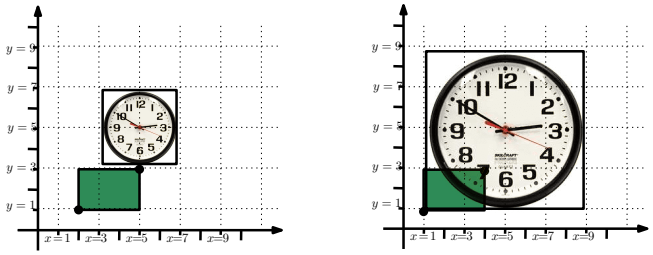
The mathematician and coffee cup non-funny joke Part 2

Fence

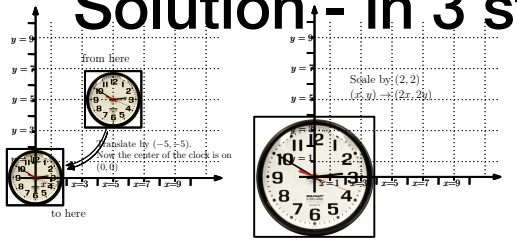
Solution:
1. **Bring the coffee Kettle to the other table**
2. **Apply the solution from the previous slide**

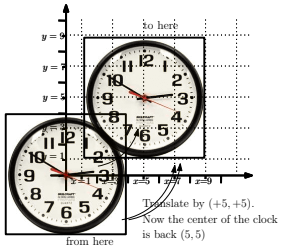
Scale the clock, without changing its center

Problem: scale the clock, but without changing its center and without effecting the green rectangle



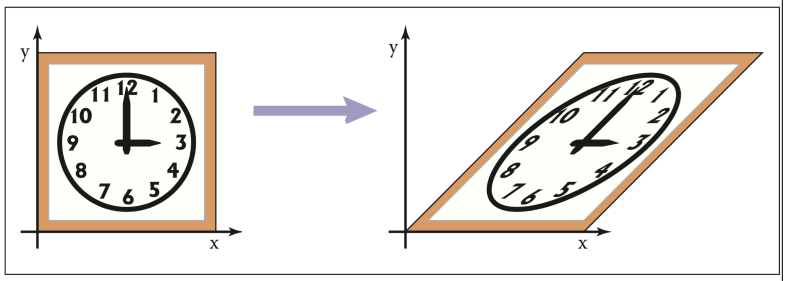
Solution - in 3 steps





Shearing

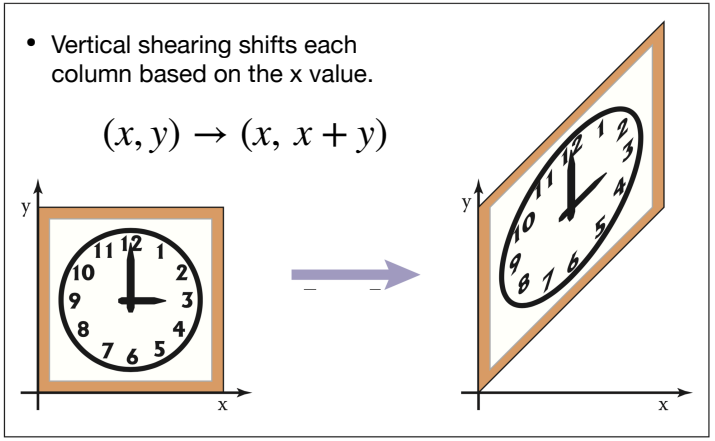
- If we move each point (x,y) into the point $(x, y) \rightarrow (x + y,\ y)$ we scaled the image by s.



Shearing

- Vertical shearing shifts each column based on the x value.

$$(x, y) \rightarrow (x,\ x + y)$$

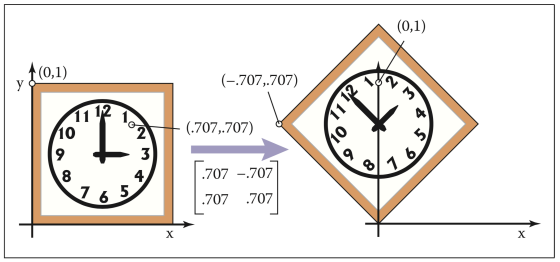


Rotation

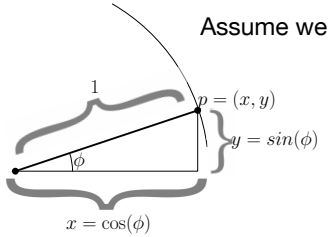
- Rotate counterclockwise by an angle ϕ about the origin.

$$(x, y) \rightarrow (x\cos\phi - y\sin\phi,\ \ x\sin\phi + y\cos\phi)$$

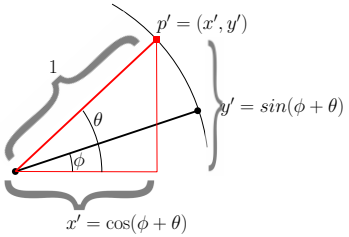
New *x* **New *y***



Assume we rotate *p* by an angle θ CCW

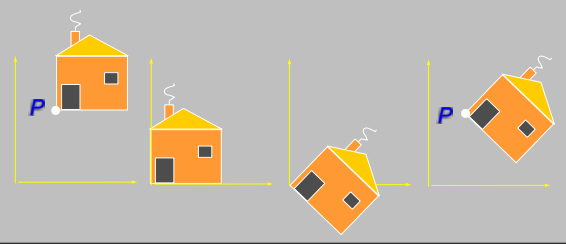


$$\cos(\phi + \theta) =$$
$$\underbrace{\cos(\phi)\cos(\theta)}_{=x} - \underbrace{\sin(\phi)\sin(\theta)}_{=y}$$

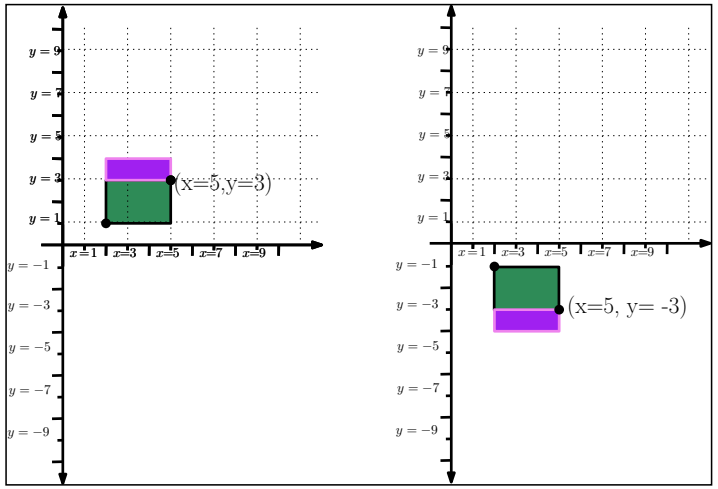


Transformation Composition

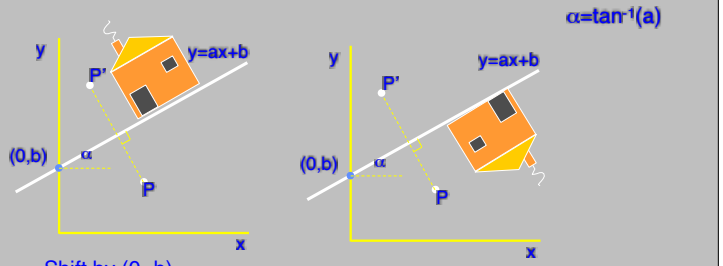
- What operation rotates by θ around $P = (p_x, p_y)$?
  - Translate P to origin
  - Rotate around origin by θ
  - Translate back



Reflection on the x-axes: $(x, y) \rightarrow (x, -y)$



Arbitrary Reflection - promo
We will get back to it later in the semester



Shift by (0,-b)
Rotate by $-\alpha$
Reflect through x
Rotate by α
Shift by (0,b)

Very scarrrrry….
Unless we represent transformation by matrices
And then it is trivial.