

CSC 433/533

Computer Graphics

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Lecture 11

Ray Tracing 3

Today's Agenda

- Reminders:
 - A03, questions?
- Goals for today:
 - Continue discussing Lighting and Shading

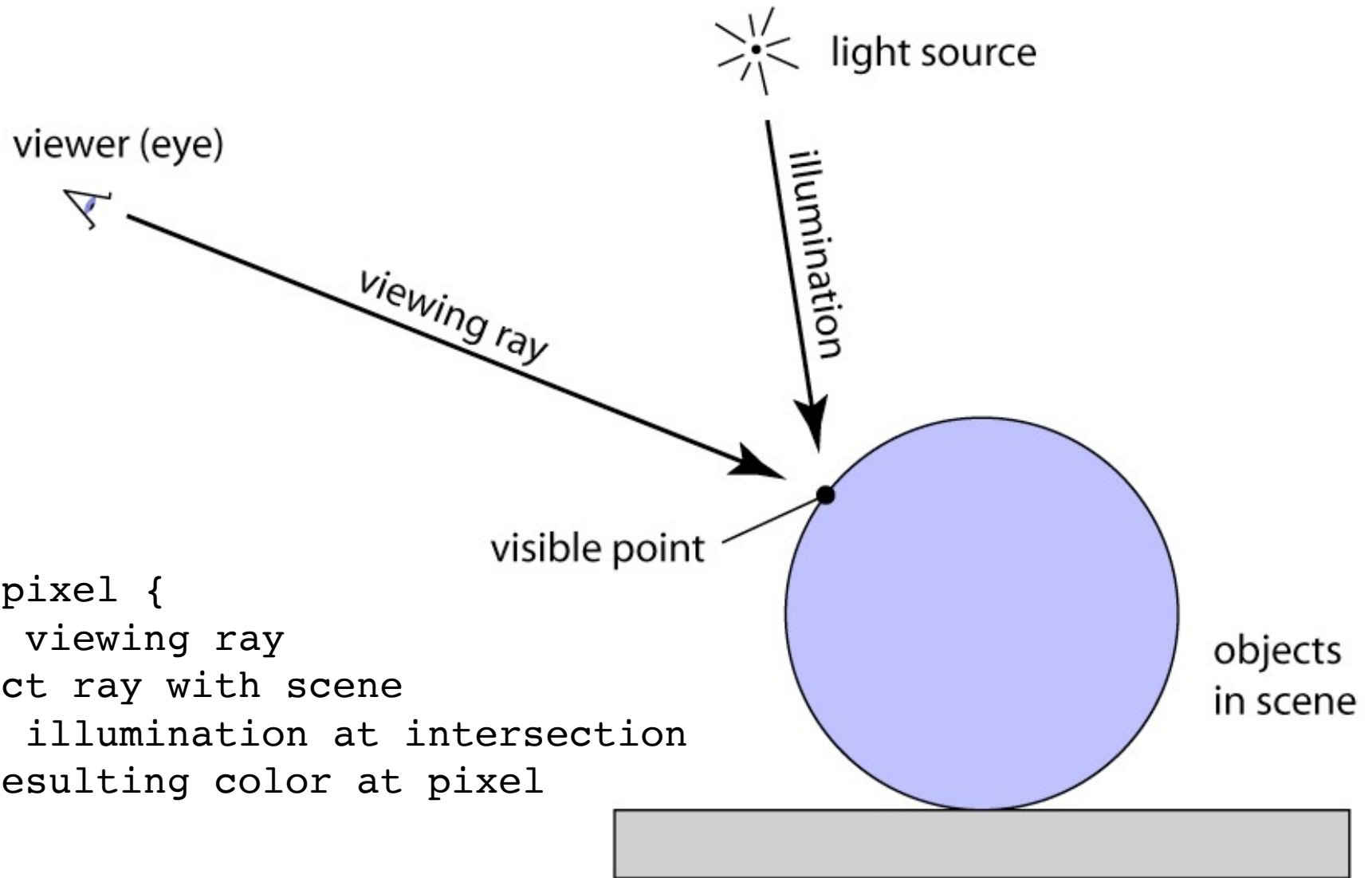
More Examples Of What Can Be Done With Ray Tracing

- <https://developer.nvidia.com/optix>
- <https://embree.github.io/gallery.html>



Last Time

Ray Tracing Algorithm



Ray-Sphere Intersection

- Two conditions must be satisfied:
 - Must be on a ray: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
 - Must be on a sphere: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{c}) \cdot (\mathbf{p} - \mathbf{c}) - R^2 = 0$
- Can substitute the equations and solve for t in $f(\mathbf{p}(t))$:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{c}) \cdot (\mathbf{o} + t\mathbf{d} - \mathbf{c}) - R^2 = 0$$

- Solving for t is a quadratic equation

Ray-Plane Intersection

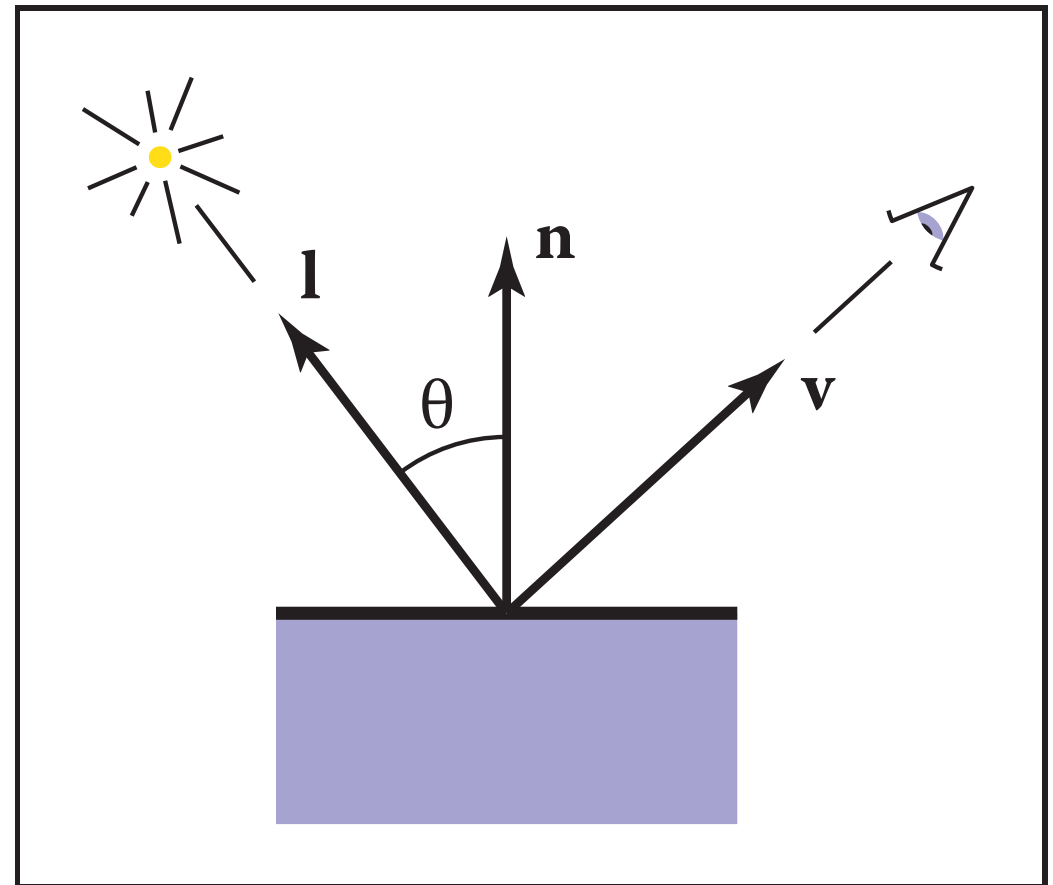
- Two conditions must be satisfied:
 - Must be on a ray: $\mathbf{p}(t) = \mathbf{o} + t\mathbf{d}$
 - Must be on the plane: $f(\mathbf{p}) = (\mathbf{p} - \mathbf{a}) \cdot \mathbf{n} = 0$
- Can substitute the equations and solve for t in $f(\mathbf{p}(t))$:

$$(\mathbf{o} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$

- This means that $t = ((\mathbf{a} - \mathbf{o}) \cdot \mathbf{n}) / (\mathbf{d} \cdot \mathbf{n})$

Shading

- Goal: Compute light reflected toward camera
- Inputs:
 - eye direction
 - light direction (for each of many lights)
 - surface normal
 - surface parameters (color, shininess, ...)



Images Without Shading

- With only eye-ray generation and scene intersection

```
for each pixel p in Image {  
  let hit_surf = undefined;  
  ...  
  
  scene-surfaces.forEach( function(surf) {  
    if (surf.intersect(eye, dir, ...)) {  
      hit_surf = surf;  
      ...  
    }  
  });  
  
  c = hit_surf.ambient;  
  Image.update(p, c);  
}
```



**Each surface
storing a single
ambient color**

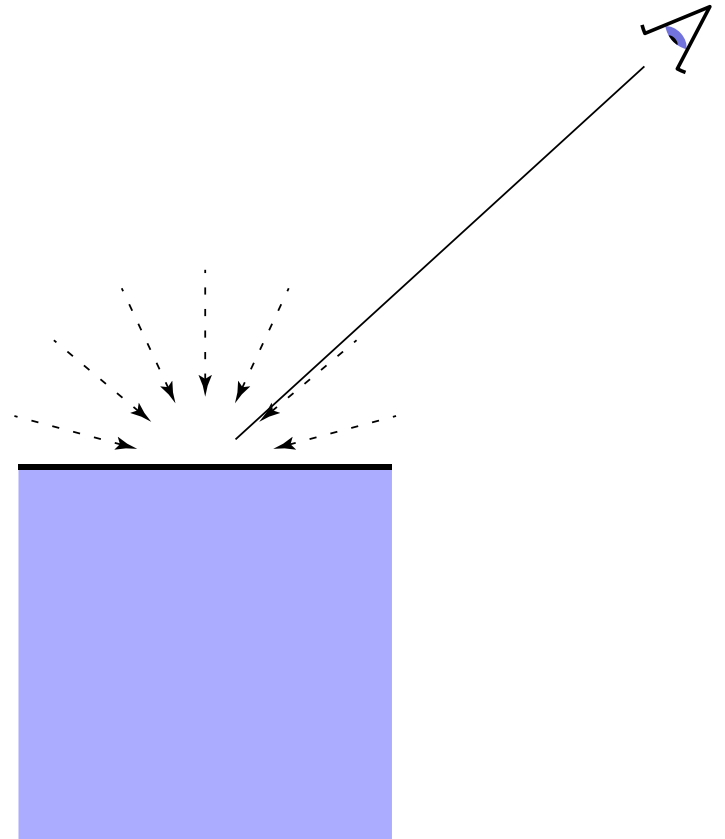
Ambient Shading

- Shading that does not depend on anything
- Idea: add constant color to account for disregarded illumination and fill in black shadows

$$L_a = k_a I_a$$

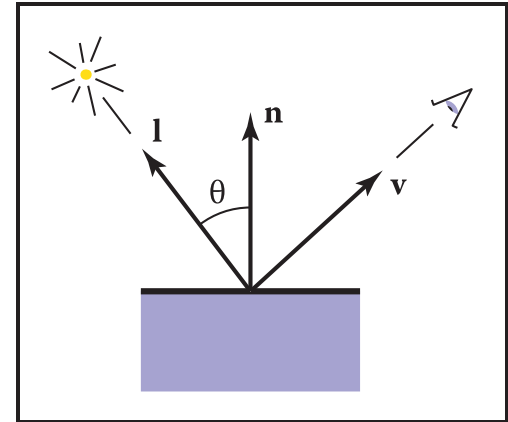
ambient
coefficient
(of surface)

ambient
light intensity



Lambertian (Diffuse) Shading

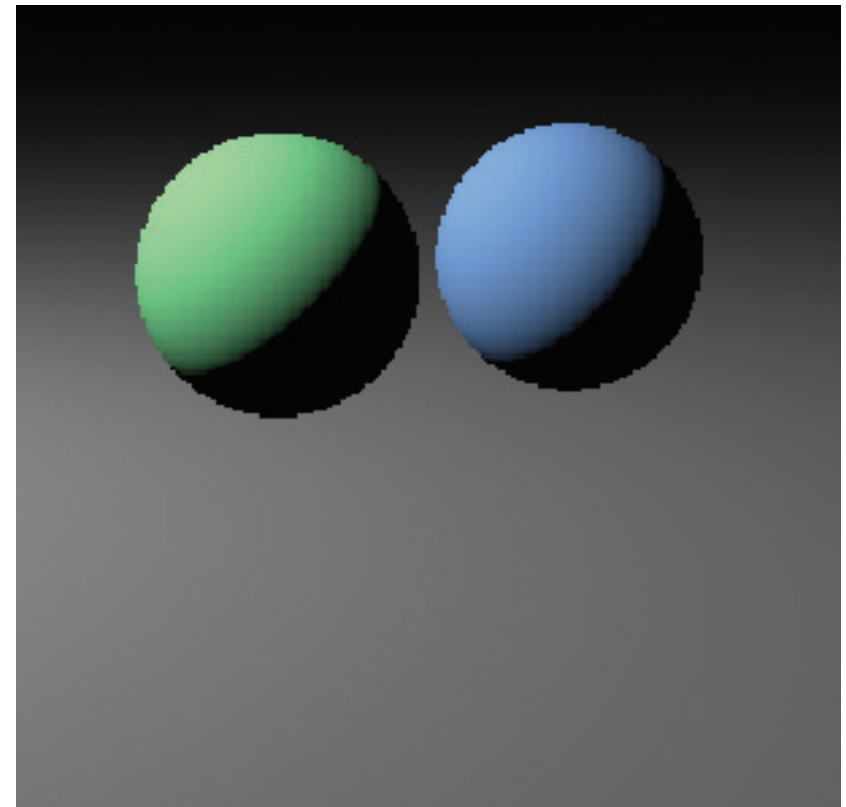
- Simple model: amount of energy from a light source depends on the direction at which the light ray hits the surface
- Results in shading that is *view independent*



$$L_d = k_d I \max(0, \mathbf{n} \cdot \mathbf{l})$$

Diagram illustrating the components of the Lambertian shading equation:

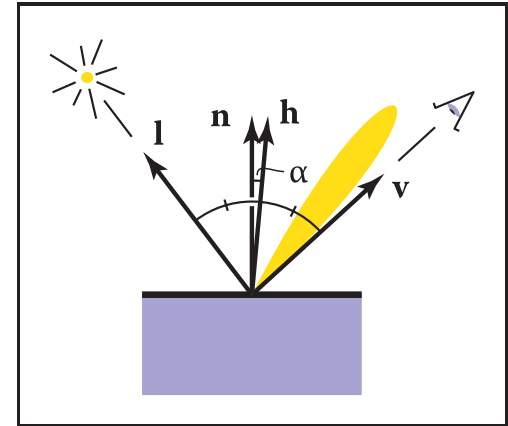
- k_d : diffuse coefficient
- I : intensity/color of light
- $\max(0, \mathbf{n} \cdot \mathbf{l})$: $\cos \theta$



Blinn-Phong (Specular) Shading

- Symmetric arrangement captured by examining the half vector \mathbf{h} between \mathbf{v} and \mathbf{l}

$$\mathbf{h} = (\mathbf{v} + \mathbf{l}) / \|\mathbf{v} + \mathbf{l}\|$$

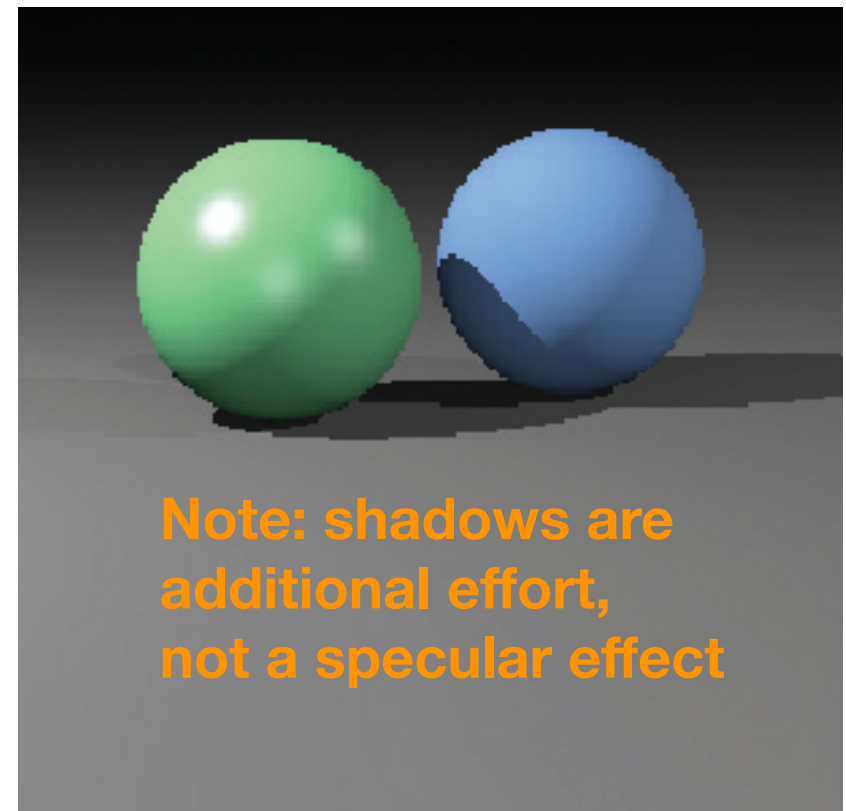


- When $\mathbf{n} \cdot \mathbf{h}$ is maximal, most reflection

$$L_s = k_s I \max(0, \mathbf{n} \cdot \mathbf{h})^p$$

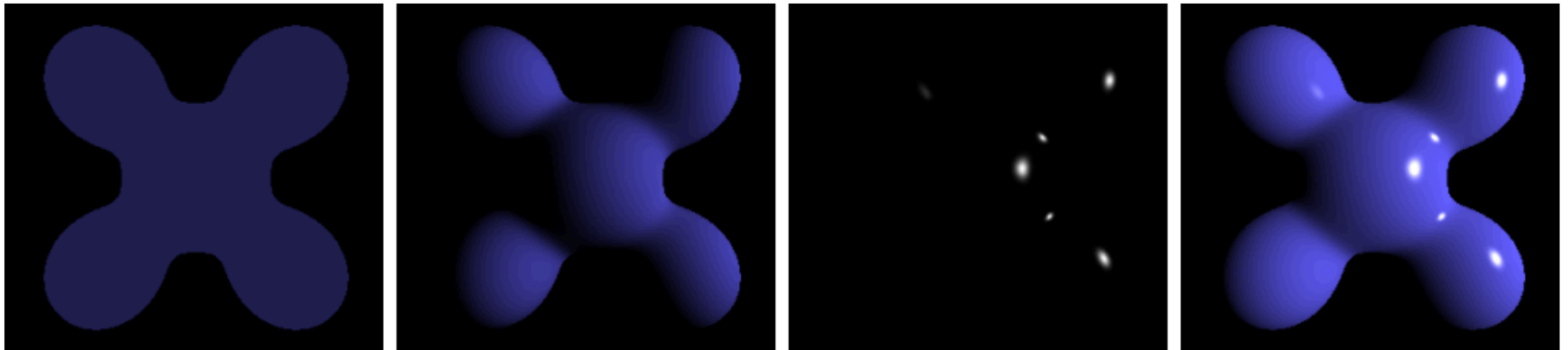
specular
coefficient

Phong
exponent



Note: shadows are
additional effort,
not a specular effect

Blinn-Phong Decomposed



Ambient + Diffuse + Specular = Phong Reflection

Simple Ray Tracer

```
function ray_cast(eye, dir, near, far) {
  let hit_surf = undefined; let hit_rec = undefined;
  let t_min = 0; let hit_t = Infinity;
  let color = background; //default background color

  scene-surfaces.forEach( function(surf) {
    let intersect_rec = surf.hit(eye, dir, t_min, hit_t);
    if (intersect_rec.hit) {
      hit_surf = surf;
      hit_t = intersect_rec.t;
      hit_rec = intersect_rec;
    }
  });

  if (hit_surf !== undefined) {
    color = hit_surf.kA * Ia;
    scene-lights.forEach( function(light) {
      //compute  $l_i$ ,  $h_i$ 

      color = color + hit_surf.kD *  $I_i$  * max(0,  $n \cdot l_i$ ) + hit_surf.kS *  $I_i$  * max(0,  $n \cdot h_i$ )p;
    });
  }

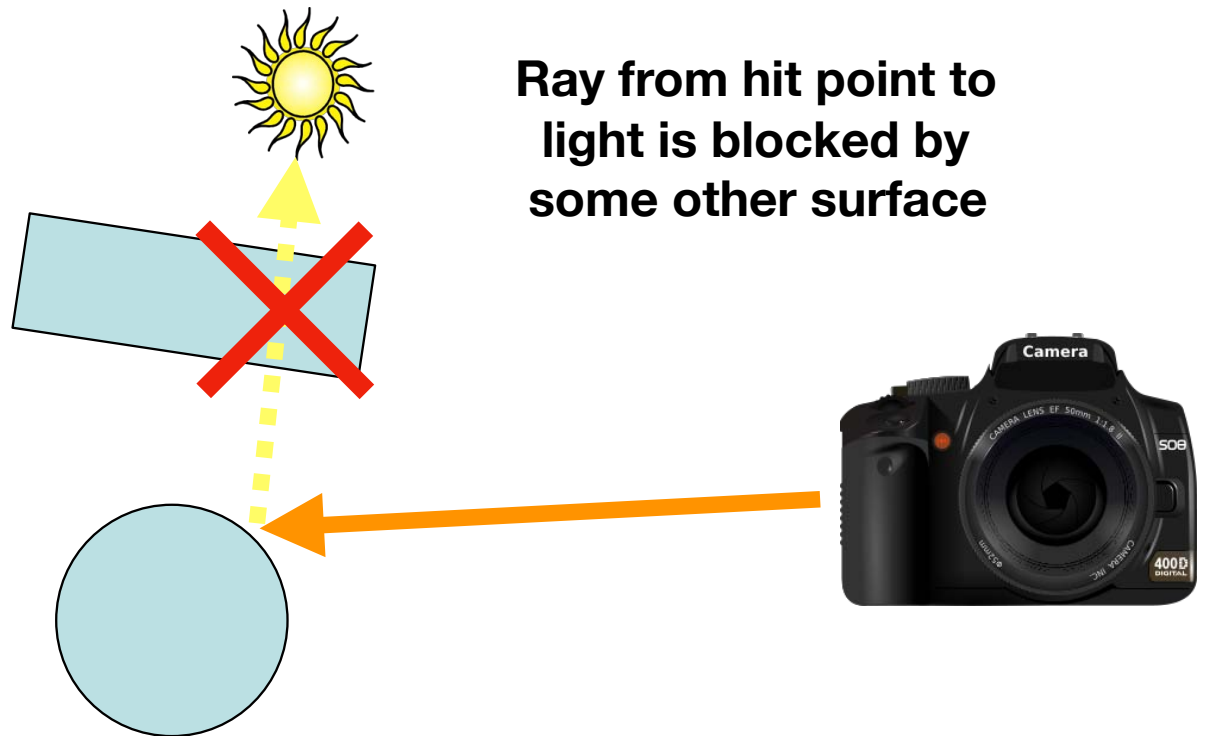
  return color;
}
```

```
for each pixel p in Image {
  let [eye, dir] = camera.compute_ray(p);
  let c = ray_cast(eye, dir, 0, Infinity);
  image.update(p, c);
}
```


Recursive Ray Tracing

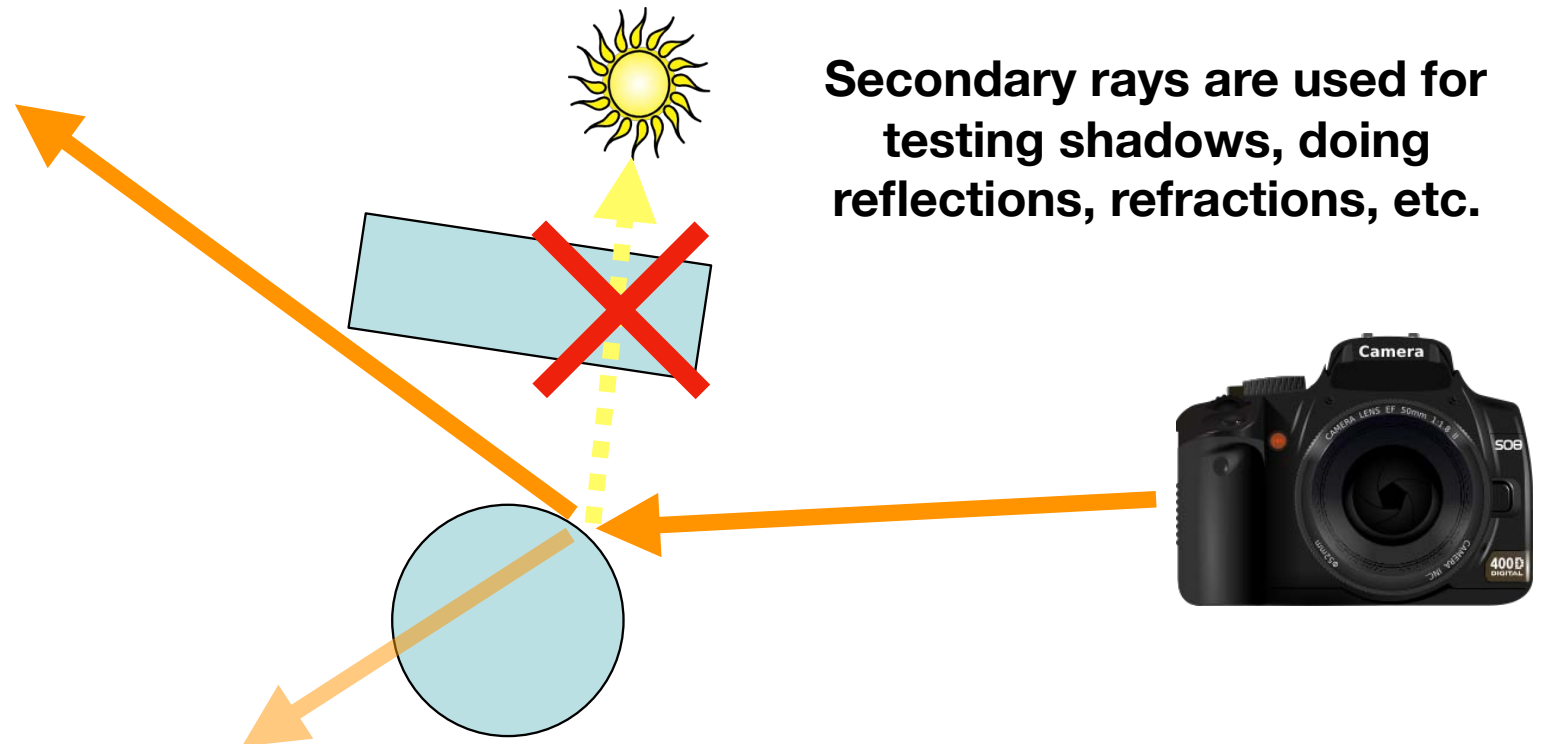
Shadows

- Surface should only be illuminated if nothing blocks the light from hitting the surface
- This can be easily checked by intersecting a new ray with the scene!



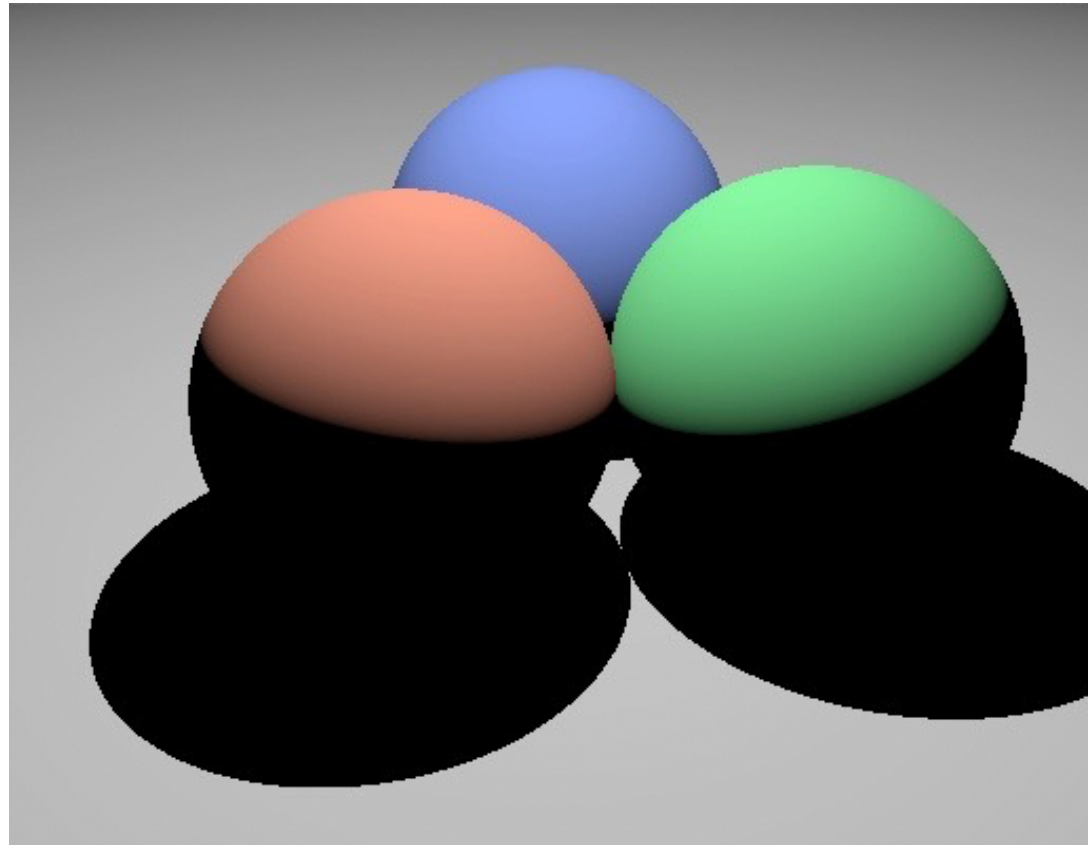
Ray Casting vs Ray Tracing

- Ray casting: tracing rays from eyes only
- Ray tracing: tracing secondary rays



Shadows

- Idea: after finding the closest hit, cast a ray to each light source to determine if it is visible
- Be careful not to intersect with the object itself. Two solutions:
 - Only check for hits against all other surfaces
 - Start shadow rays a tiny distance away from the hit point by adjusting t_{\min}



Ray Tracer w/ Shadows

```
function ray_cast(eye, dir, near, far) {  
    ...  
    //initialize color c;  compute hit_surf, hit_position;  
    ...  
  
    if (hit_surf !== undefined) {  
        color = hit_surf.kA * Ia;  
        scene.lights.forEach( function(light) {  
            //compute  $l_i$ ,  $h_i$   
            //check if light is visible from hit point  
            if (light is visible) {  
                color += effect of light;  
            }  
        });  
    }  
  
    return color;  
}
```

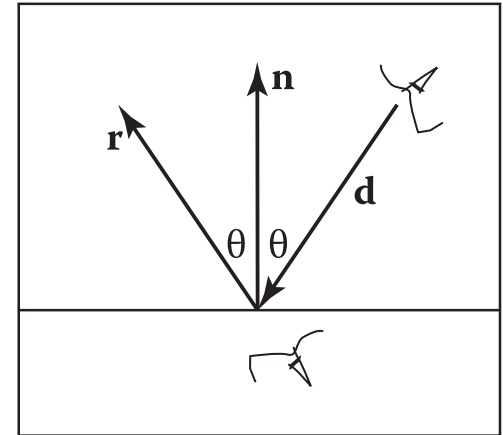
Ray Tracer w/ Shadows

```
function ray_cast(eye, dir, near, far) {  
    ...  
    //initialize color;  compute hit_surf, hit_pos;  
    ...  
  
    if (hit_surf !== undefined) {  
        color = hit_surf.kA * Ia;  
        scene.lights.forEach( function(light) {  
            //compute  $l_i$ ,  $h_i$   
            //check if light can is visible from hit point  
            let shadow_hit = false;  
            scene-surfaces.forEach( function(surf) {  
                let intersect_rec = surf.hit(hit_pos,  $l_i$ , epsilon, ||hit_pos - light.pos||);  
                if (intersect_rec.hit) {  
                    shadow_hit = true;  
                }  
            });  
  
            if (shadow_hit == false) {  
                color += hit_surf.kD *  $I_i$  * max(0,  $n \cdot l_i$ ) + hit_surf.kS *  $I_i$  * max(0,  $n \cdot h_i$ )p;  
            }  
        });  
    }  
  
    return color;  
}
```

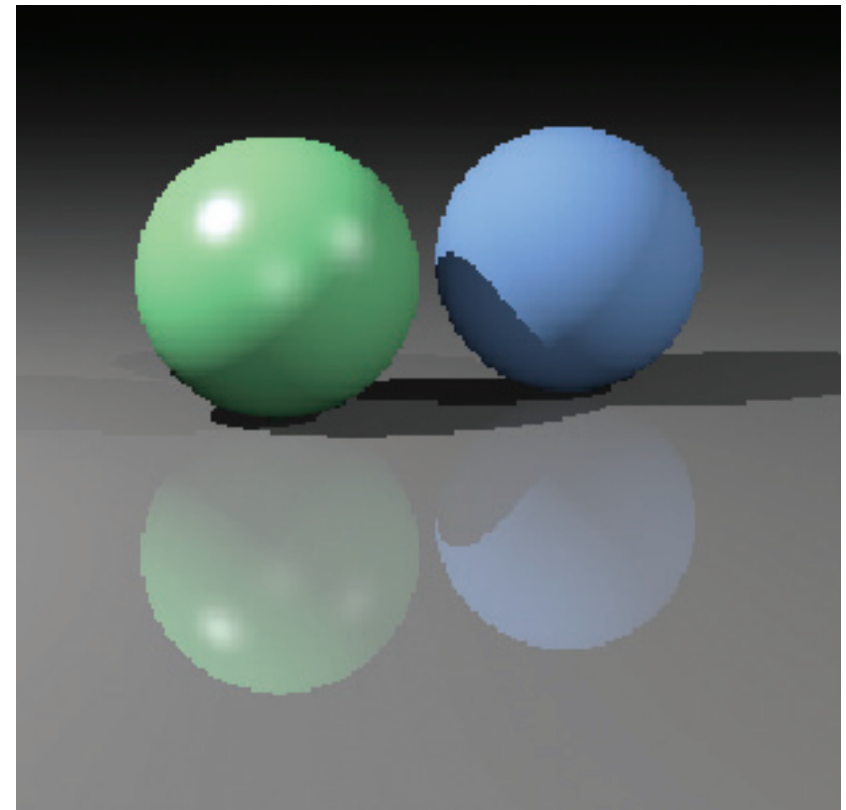
Offset shadow rays a small amount from surface

Cast ray no further than to the light position

Reflection



- Ideal **specular** reflection, or mirror reflection, can be modeled by casting another ray into the scene from the hit point
- Direction $\mathbf{r} = \mathbf{d} - 2(\mathbf{d} \cdot \mathbf{n})\mathbf{n}$
- One can then recursively accumulate some amount of color from whatever object this hits
- $\text{color} += k_m * \text{ray_cast}()$



Recursive Ray Tracer

```
function ray_cast(eye, dir, near, far) {  
    ...  
    //initialize color; compute hit_surf, hit_position;  
    ...  
  
    if (hit_surf is valid) {  
        color = hit_surf.kA * Ia;  
        scene.lights.forEach( function(light) {  
            //compute  $\mathbf{l}_i$ ,  $\mathbf{h}_i$   
            //check for shadow rays to decide if the light illuminates  
            if (ray from hit_position in direction of  $\mathbf{l}_i$  does not hit scene) {  
                color += hit_surf.kD *  $I_i$  * max(0,  $\mathbf{n} \cdot \mathbf{l}_i$ ) + hit_surf.kS *  $I_i$  * max(0,  $\mathbf{n} \cdot \mathbf{h}_i$ )p;  
            }  
        });  
  
        //compute reflect direction  $\mathbf{r}_i$   
        //call ray_cast() recursively for mirror reflections  
        color += hit_surf.kM * ray_cast(hit_position,  $\mathbf{r}_i$ , epsilon, +inf);  
    }  
  
    return color;  
}
```

Like w/ shadows, we
need to offset a bit

Recursive!!

How much recursion? Typically,
we use a max number of bounces

Mirror Reflection vs. Specular Reflection

- Consider perfectly shiny surface
 - Typically, no highlights! Usually, just reflections of other objects
- Can render this using recursive ray tracing
 - To find out mirror reflection color, ask what color is seen from surface point in reflection direction
- “Glazed” or “Glossy” materials often only have the mirror reflection + diffuse, which suggests

$$L = L_a + L_d + L_m$$

- Where L_m is evaluated by tracing a new ray and replaces L_s

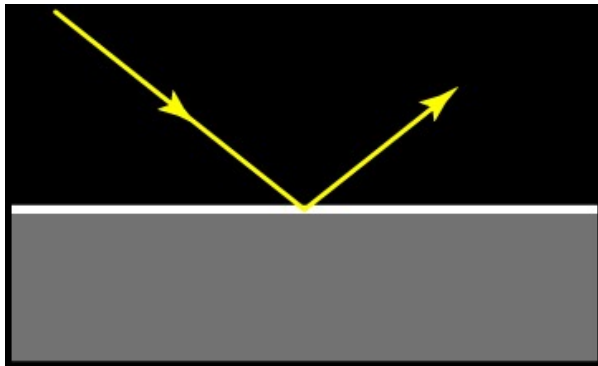
Fancier Shading

- While Phong has long been the heuristic baseline, newer methods are more based on physics:
 - When writing a shader, think like a bug standing on the surface
 - Bug sees an incident distribution of light that is arriving at the surface
 - Physics question: what is the outgoing distribution of light?

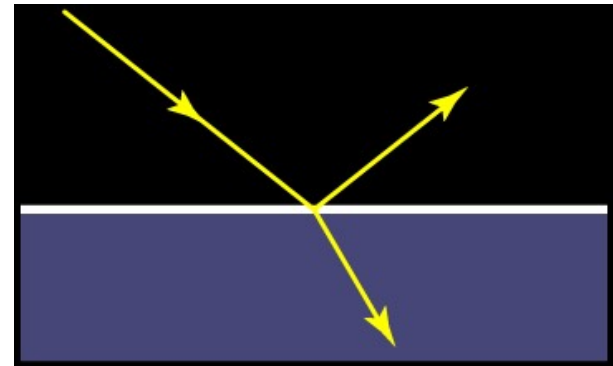
Simple materials



metal



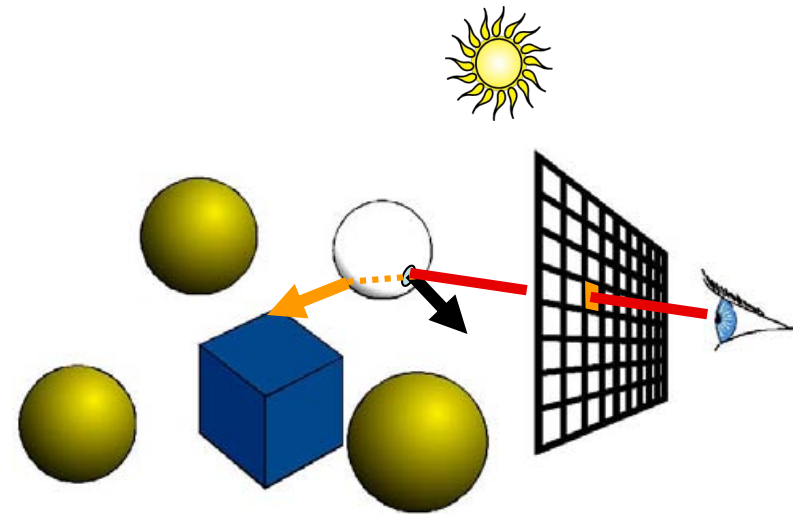
dielectric



Illuminating Dielectrics

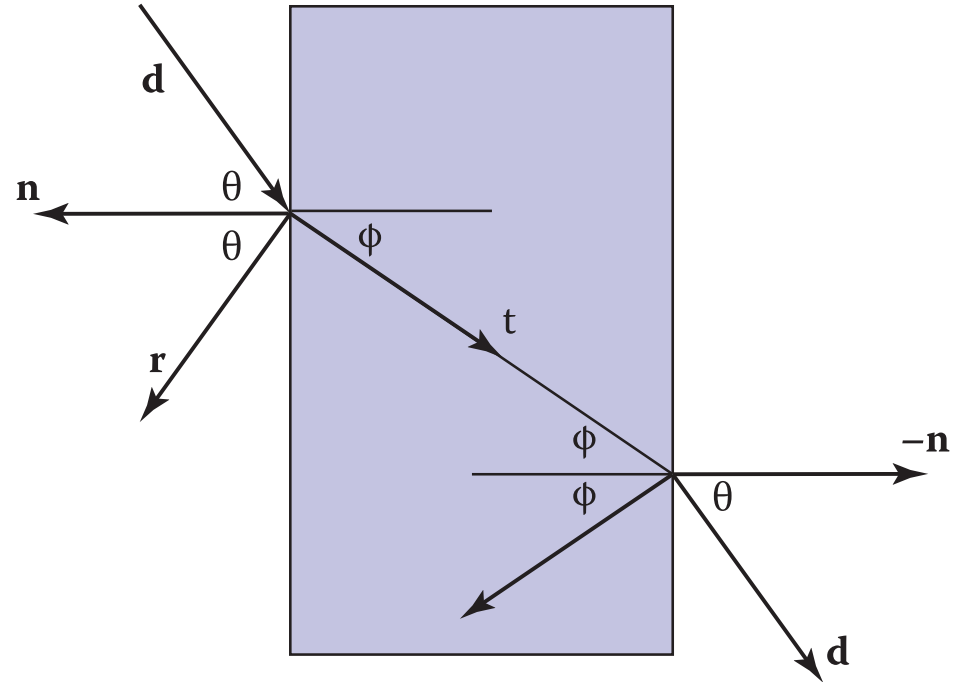
Translucency (Refraction)

- When a ray hits a dielectric surface, some portion of it transmits through the surface, but bends
- Color of the ray can be modulated by a refraction color



Snell's Law

- Governs the angle at which a refracted ray bends
- Computation based on refraction index of original medium, n , versus new index n_t
- $n_t \sin \theta = n \sin \phi$

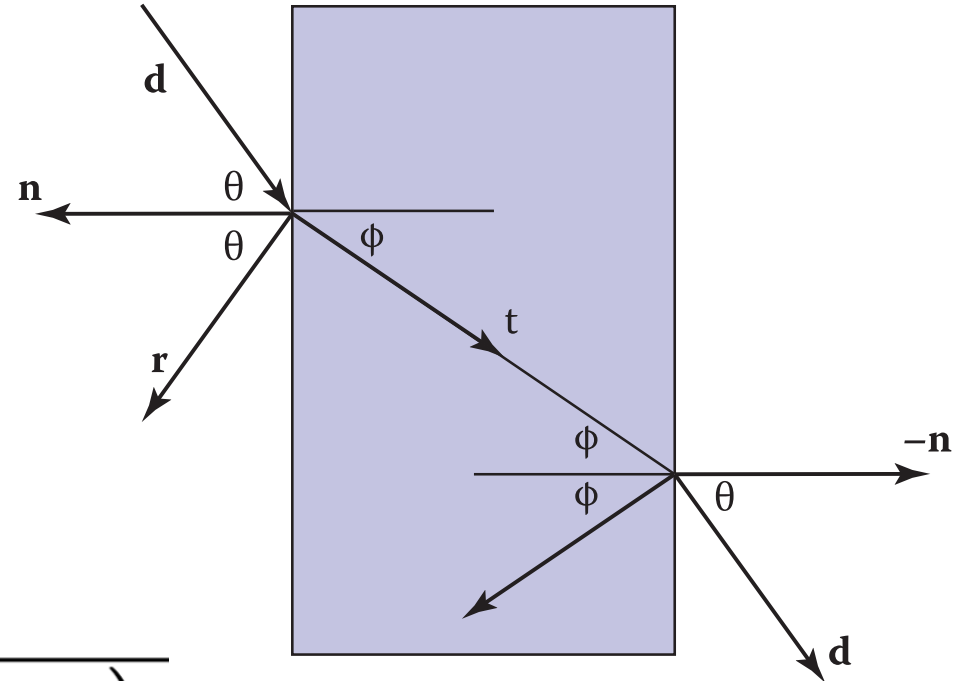


Snell's Law

- Working with cosine's are easier because we can use dot products
- Can derive the vector for the refraction direction \mathbf{t} as

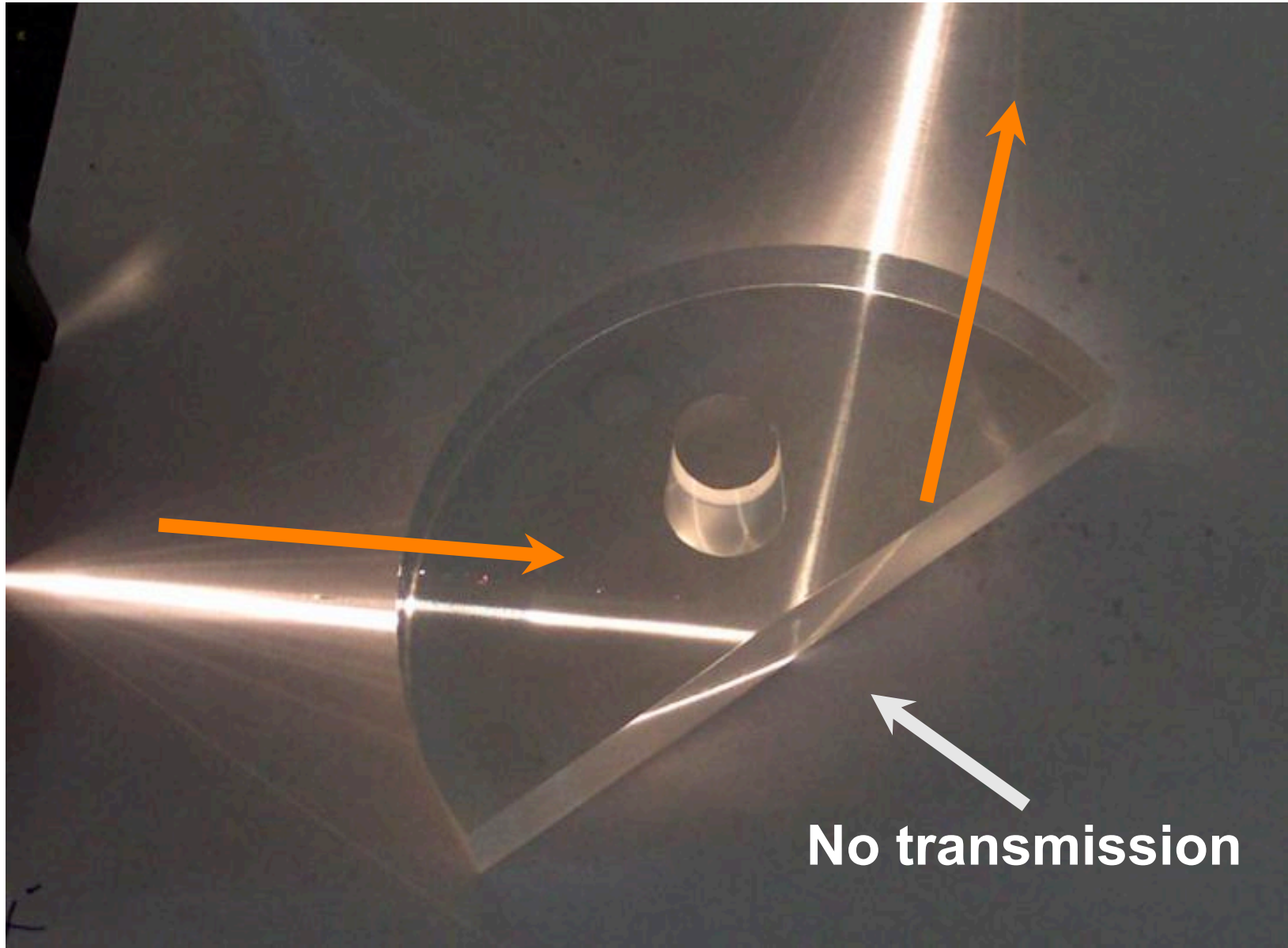
$$\mathbf{t} = \frac{n(\mathbf{d} + \mathbf{n} \cos \theta)}{n_t} - \mathbf{n} \cos \phi$$

$$= \frac{n(\mathbf{d} - \mathbf{n}(\mathbf{d} \cdot \mathbf{n}))}{n_t} - \mathbf{n} \sqrt{1 - \frac{n^2 (1 - (\mathbf{d} \cdot \mathbf{n})^2)}{n_t^2}}$$



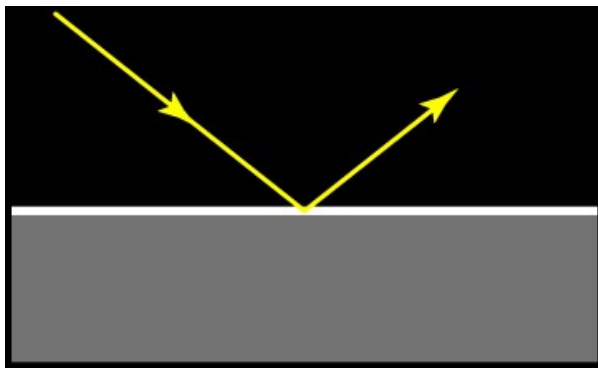
What happens if this is negative?

Total Internal Reflection

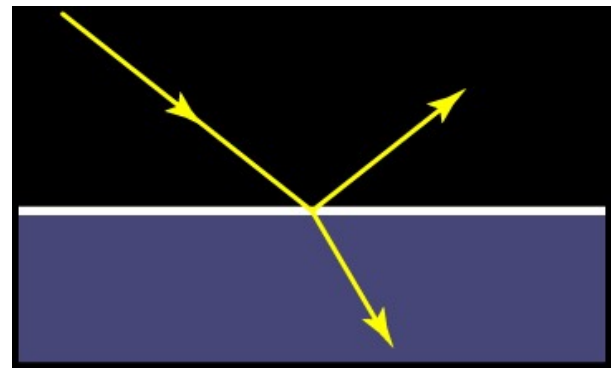


Specular reflection

- Smooth surfaces of pure materials have ideal specular reflection (said this before)
 - Metals (conductors) and dielectrics (insulators) behave differently
- Reflectance (fraction of light reflected) depends on angle



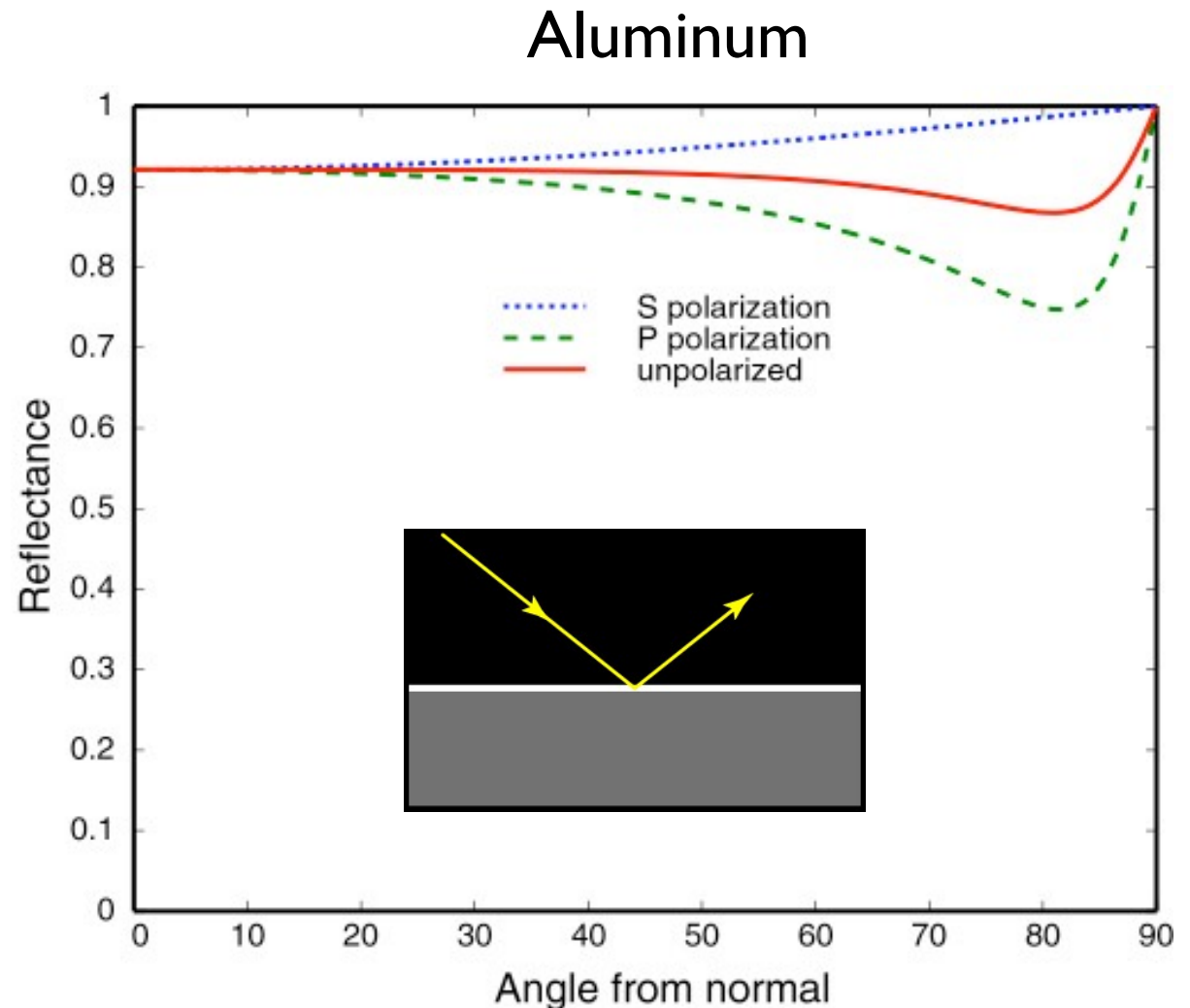
metal



dielectric

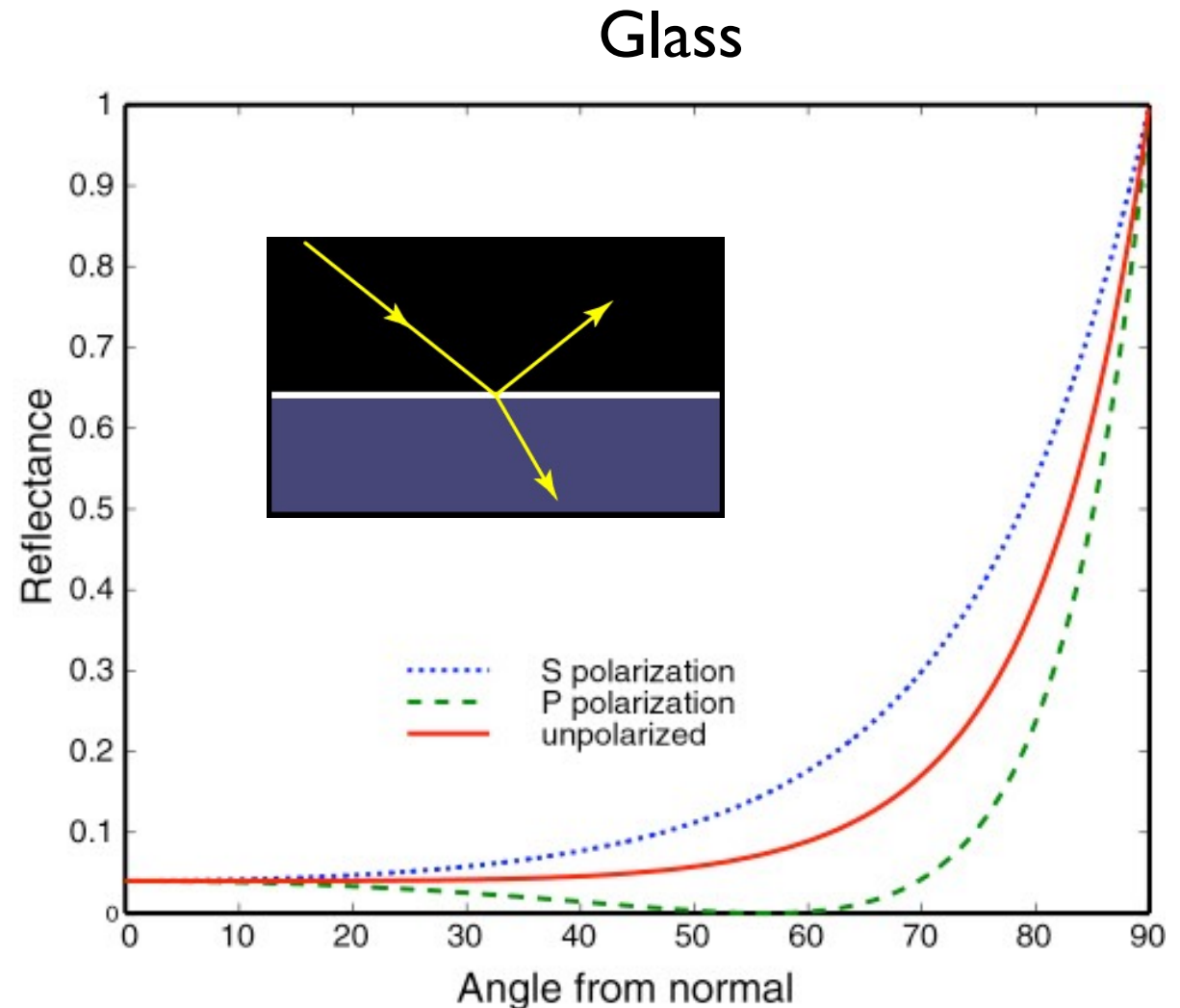
Specular Reflection from Metal

- Reflectance does depend on angle, but not much
- We typically ignore this for rendering



Specular Reflection from Dielectrics like Glass/Water

- Significant dependence on angle
- About 4% at normal incidence
- Nearly 100% at grazing (rest of the light is transmitted)
- Getting this right has a strong affect on appearance



Fresnel Equation Models the Reflectivity of Dielectrics

- Can be used to predict how much light reflects from a smooth interface (usually, one material is air/empty space)
- R is the fraction that is reflected
- (1-R) is the fraction that is transmitted

$$F_p = \frac{\eta_2 \cos \theta_1 - \eta_1 \cos \theta_2}{\eta_2 \cos \theta_1 + \eta_1 \cos \theta_2}$$

$$F_s = \frac{\eta_1 \cos \theta_1 - \eta_2 \cos \theta_2}{\eta_1 \cos \theta_1 + \eta_2 \cos \theta_2}$$

$$R = \frac{1}{2} (F_p^2 + F_s^2)$$

Lec12 Required Reading

- FOCG, Ch. 13