



## CSC380: Principles of Data Science

### Predictive Modeling and Classification 2

1

### Announcements

2

- Withdrawn DDL: Oct 29
- HW5 due this Friday, Oct 27 by 11:59 pm
- Final exam: will spend two lectures for the final review
  - We will reuse some midterm problems with a bit of variations.

2

## Outline

- Decision tree variations
- KNN
- Model selections and evaluations

3

## How to construct a tree

4

- Assign all training instances to the root of the tree. Set current node to root node.
- For each feature:
  - a. Partition all data instances at the node by the value of the feature.
  - b. Compute the accuracy from the partitioning.
- Identify feature that results in the highest accuracy. Set this feature to be the splitting criterion at the current node.

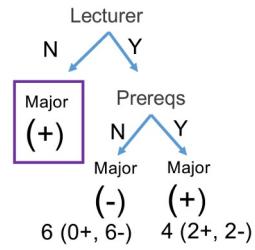
Rating	Prereqs	Lecturer	HasLabs	
	Easy?	At?	Sys?	Thy?
+2	y	y	n	y
+2	y	y	n	y
+2	n	y	n	n
+2	n	n	n	y
+2	n	y	y	n
+1	y	y	n	n
+1	y	y	n	y
+1	n	y	n	y
0	n	n	n	n
0	y	n	n	y
0	n	y	n	y
0	y	y	y	y
-1	y	y	y	n
-1	n	n	y	y
-1	n	n	y	n
-1	y	n	y	n
-2	n	n	y	y
-2	n	y	y	n
-2	y	n	y	n
-2	y	n	y	n

4

2

## How to construct a tree

- Partition all instances according to attribute value of the best feature.
- Denote each partition as a child node of the current node.
- For each child node:
  - If the child node is “pure” (has instances from only one class) tag it as a leaf and return.
  - If not set the child node as the current node and recurse to step 2.



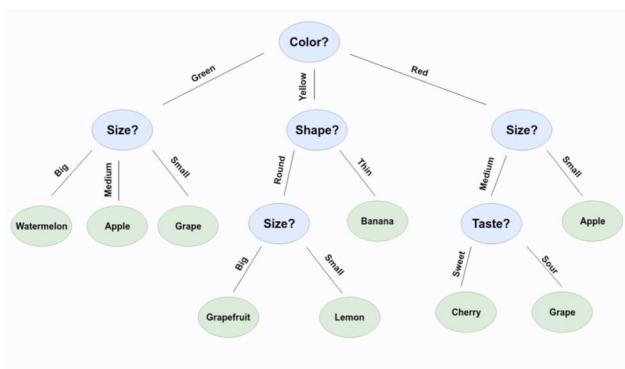
5

6

## Types of features

- Binary
- Categorical: values in  $\{1, \dots, C\}$  e.g., occupation, blood type
  - Option 1: Instead of 2 children, have  $C$  children.
  - Option 2: Derive  $C$  features of the form “feature= $c$ ?” for every  $c \in C$ .  
↑ binary features!

Option 1:



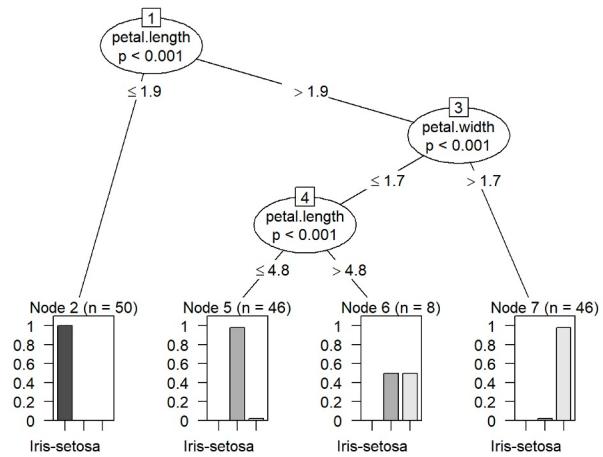
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3

## Types of labels

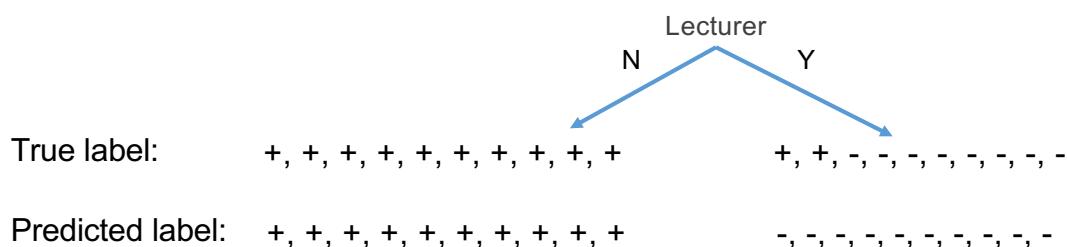
7

- Binary
- Multiclass: What changes do we need to make?
  - Almost none!
  - Just extend the accuracy to multiclass.



7

## Review: criterion is accuracy



True label:      +, +, +, +, +, +, +, +, +, +

Predicted label: +, +, +, +, +, +, +, +, +, +

Accuracy:  $\frac{10}{20} \cdot \frac{10}{10} + \frac{10}{20} \cdot \frac{8}{10} = \frac{18}{20} = 0.9$

- Sum of (fraction of subgroup \* fraction of correct answer in subgroup)
- What if we change it to Sum of (fraction of a subgroup \* some function on that subgroup)?

8

## Different selection criterions

### Notions of uncertainty: general case

Suppose in  $S \subseteq \mathcal{X} \times \mathcal{Y}$ , a  $p_k$  fraction are labeled as  $k$  (for each  $k \in \mathcal{Y}$ ).

#### ① Classification error:

$$u(S) := 1 - \max_{k \in \mathcal{Y}} p_k$$

#### ② Gini index:

$$u(S) := 1 - \sum_{k \in \mathcal{Y}} p_k^2$$

#### ③ Entropy:

$$u(S) := \sum_{k \in \mathcal{Y}} p_k \log \frac{1}{p_k}$$

Each is *maximized* when  $p_k = 1/|\mathcal{Y}|$  for all  $k \in \mathcal{Y}$   
(i.e., equal numbers of each label in  $S$ )

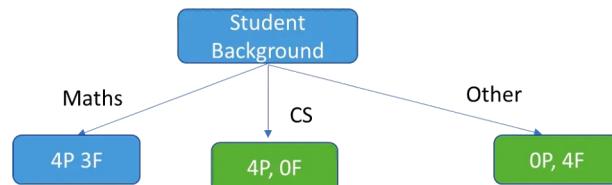
Each is *minimized* when  $p_k = 1$  for a single label  $k \in \mathcal{Y}$   
(so  $S$  is **pure** in label)

9

## Different selection criterions - Gini index

#### ② Gini index:

$$u(S) := 1 - \sum_{k \in \mathcal{Y}} p_k^2$$



$$Gini_{maths} = 1 - \left(\frac{4}{7}\right)^2 - \left(\frac{3}{7}\right)^2 = .4897$$

$$Gini_{CS} = 1 - \left(\frac{4}{4}\right)^2 - \left(\frac{0}{4}\right)^2 = 0$$

$$Gini_{others} = 1 - \left(\frac{0}{4}\right)^2 - \left(\frac{4}{4}\right)^2 = 0$$

$$Gini_{bkgrd} = \frac{7}{15} * .4897 + \frac{4}{15} * 0 + \frac{4}{15} * 0 = .2286$$

Pick the one with lowest  
Gini index as root

<https://towardsdatascience.com/decision-trees-explained-entropy-information-gain-gini-index ccp-pruning-4d78070db36c>

10

11

## k-Nearest Neighbors (k-NN)

11

## Background: Train Error vs Test Error

12

Error :=  $1 - \text{accuracy}$ .

Suppose we have trained a function  $\hat{f}$  on  $D = \{(x^{(i)}, y^{(i)})\}_{i=1}^m$  using a supervised learning algorithm.

- Train error: Evaluate on D.

$$\widehat{\text{err}}_D(\mathbf{f}) := \frac{1}{|D|} \sum_{(x,y) \in D} \mathbf{I}\{\mathbf{f}(x) \neq y\}$$

- Test error: Evaluate on  $D' = \{(x^{(i)}, y^{(i)})\}_{i=1}^{m'}$  not used for training.

Q: Choose one:

- (1) train error  $\geq$  test error   (2) train error  $\approx$  test error   (3) train error  $\leq$  test error

12

## Background: Train Error

Suppose our train set:

Index	Ground Truth	Predicted
0	0	1
1	1	1
2	0	0
3	0	1
4	1	0
5	0	0
6	1	1

$$\frac{1}{6} \cdot (1 + 0 + 0 + 1 + 1 + 0) = \frac{1}{2}$$

13

14

## Background: Workflow of Training a Classifier

Standard practice:

- Given a data set  $D$ , split it into train set  $D_{train}$  and  $D_{test}$ 
  - large data: 90-10 ratio
  - medium data: 80-20 ratio
  - small data: 70-30 ratio

(these are guidelines only)
- Train on  $D_{train}$  and evaluate error rate on  $D_{test}$ . You trust that  $D_{test}$  will be the performance when you deploy the trained classifier.

14

## $k$ -nearest neighbor: main concept

15

- Train set:  $S = \{(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})\}$
- **Idea:** given a new, unseen data point  $x$ , its label should resemble the labels of **nearby points**
- What function?
  - Input:  $x \in \mathbb{R}^d$
  - From  $S$ , find the  $k$  nearest points to  $x$  from  $S$ ; call it  $N(x)$
  - Output: the majority vote of  $\{y_i: i \in N(x)\}$ 
    - For regression, take the average label.

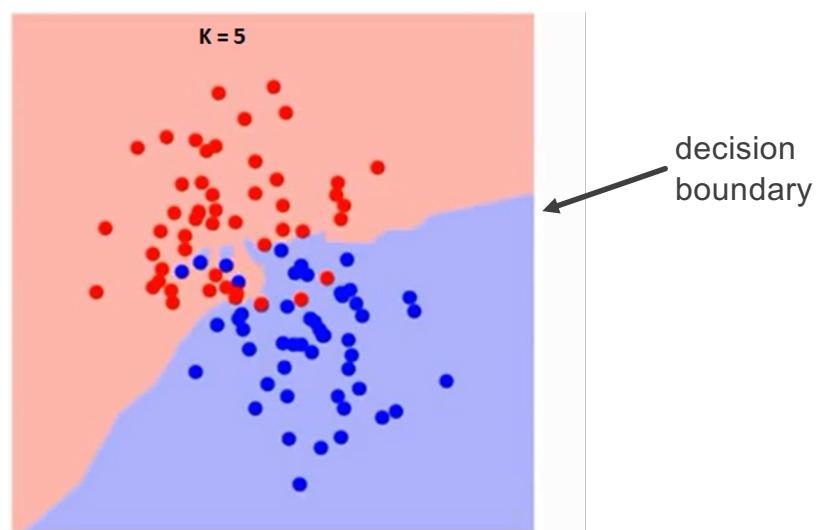
E.g., Euclidean distance



15

## $k$ -NN example

16



16

## Basics

17

How to extract features as real values?

- Binary features: Take 0/1
- Categorical {1,...,C} (e.g., movie genres)
  - Binary vector of length C. Set c-th coordinate 1 and 0 otherwise. one-hot encoding

id	color
1	red
2	blue
3	green
4	blue



id	color_red	color_blue	color_green
1	1	0	0
2	0	1	0
3	0	0	1
4	0	1	0

17

## Basics

18

How to extract features as real values?

- Binary features: Take 0/1
- Categorical {1,...,C} (e.g., movie genres)
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### Distance:

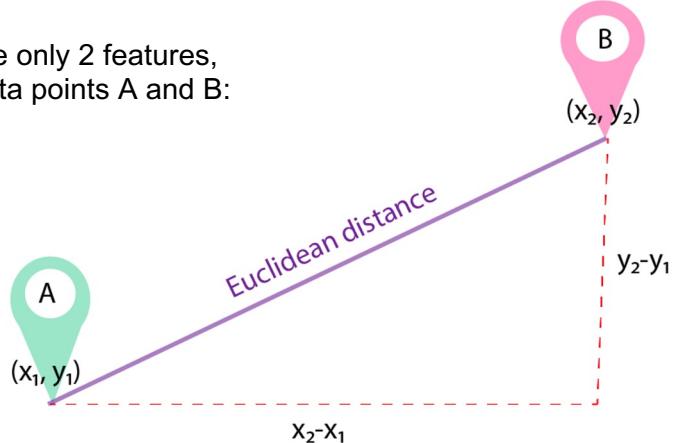
- (popular) Euclidean distance:  $d(x, x') = \sqrt{\sum_{i=1}^d (x_i - x'_i)^2}$
- Manhattan distance :  $d(x, x') = \sum_{i=1}^d |x_i - x'_i|$

A total of d dimensions/features: i from 1 to d

18

## Euclidean distance

Suppose there are only 2 features, and we have 2 data points A and B:



Q: If we shift a feature, would the distance change? no

Q: What about scaling a feature? yes

19

## Issue: features in different scales

Suppose our train set:

Index	Weight (lbs)	Shoe size
0	130	9
1	180	10.5
2	100	6
3	210	12
4	155	7.5
5	170	9.5
6	90	7

Features in different scales can be problematic:  
“weight” is the dominance in the distance.

20

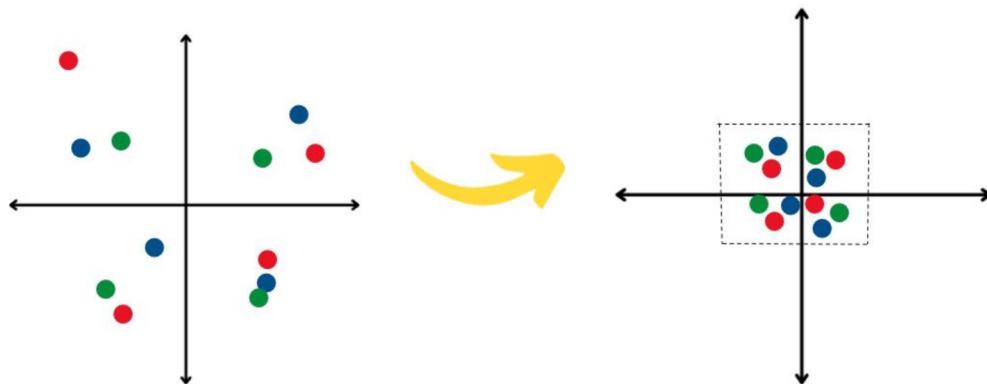
## Make sure features are scaled fairly

21

- Features having different scale can be problematic. (e.g., weights in lbs vs shoe size)
- [Definition] **Standardization**
  - For each feature  $f$ , compute  $\mu_f = \frac{1}{m} \sum_{i=1}^m x_f^{(i)}$ ,  $\sigma_f = \sqrt{\frac{1}{m} \sum_{i=1}^m (x_f^{(i)} - \mu_f)^2}$
  - Then, transform the data by  $\forall f \in \{1, \dots, d\}, \forall i \in \{1, \dots, m\}, x_f^{(i)} \leftarrow \frac{x_f^{(i)} - \mu_f}{\sigma_f}$   
after transformation, each feature has mean 0 and variance 1
- Be sure to keep the “standardize” function and apply it to the test points.
  - Save  $\{(\mu_f, \sigma_f)\}_{f=1}^d$
  - For test point  $x^*$ , apply  $x_f^* \leftarrow \frac{x_f^* - \mu_f}{\sigma_f}, \forall f$

21

## Standardization



After transformation, each feature has mean 0 and variance 1.

22

## k-NN Summary

23

- Given: labeled data D
- Training
  - Compute and save  $\{(\mu_f, \sigma_f)\}_{f=1}^d$
  - Compute and save standardization of D
- Test
  - Given  $x^*$ , apply standardization  $x_f^* \leftarrow \frac{x_f^* - \mu_f}{\sigma_f}, \forall f$
  - Compute k nearest neighbors  $N(x^*)$
  - Predict by majority vote label in  $N(x^*)$  (average label for regression tasks)

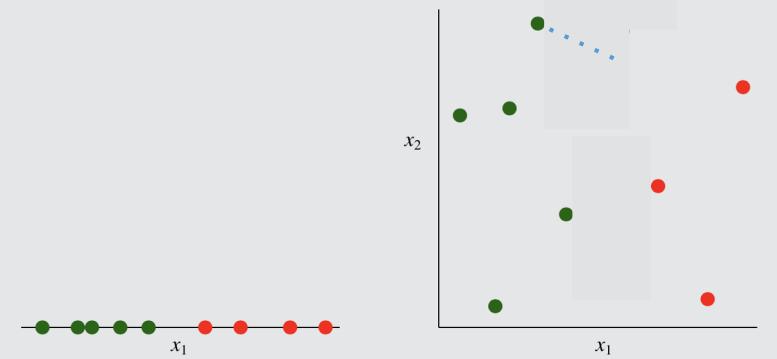
23

## Issues 1: irrelevant features

24

here's a case in which there is one relevant feature  $x_1$  and a 1-NN rule classifies each instance correctly

consider the effect of an irrelevant feature  $x_2$  on distances and nearest neighbors



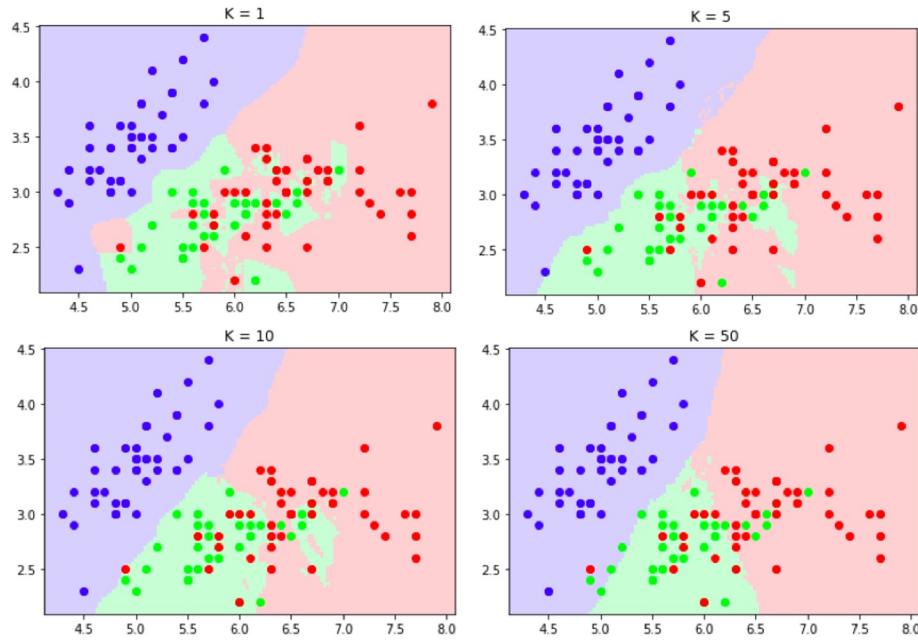
Q: how did we deal with irrelevant features in decision trees?

not all features are used because (a) we stop adding features when having 0 local accuracy, (b) prune the tree

24

## Issue 2: choosing k

25



25

## Comparison

26

	Decision Tree	k-NN
• Interpretability	good	bad
• Sensitivity to irrelevant features	low	high
• train time	$O(dm^2 + dm \log m)$	$O(dm)$
• test time	depth of the tree worst: $O(\min\{d, m\})$ best: $\log(m)$	$O(m(d + \log(m)))$ <span style="color:red">bad</span>
• test time space complexity	worst: $O(m)$ in general: much smaller	$O(dm)$

26

27

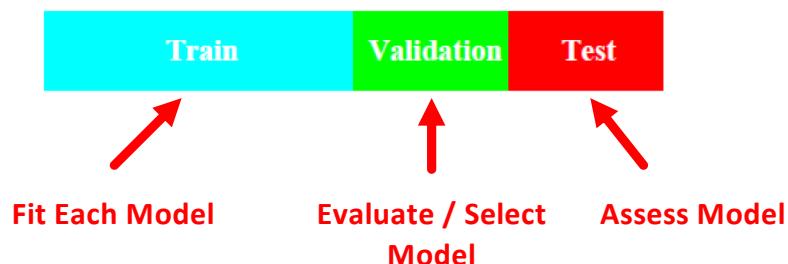
## Model Selection and Evaluation

27

28

## Model Selection / Assessment

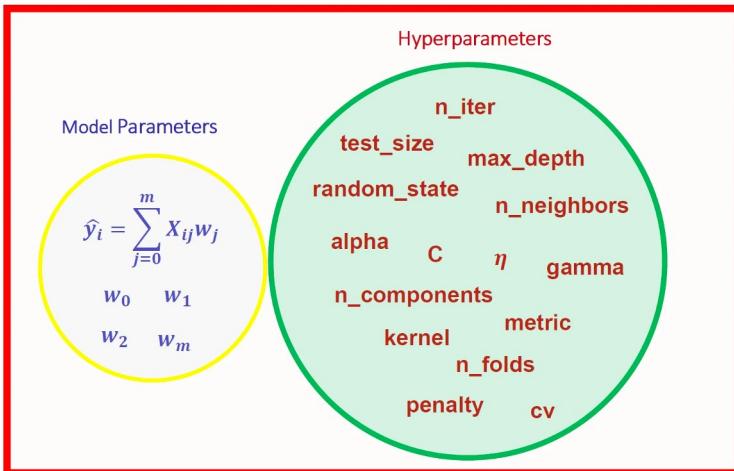
Partition your data into Train-Validation-Test sets



- Ideally, Test set is kept in a “vault” and only peek at it once model is selected
- Small dataset: 50% Training, 25% Validation, 25% Test (very loose rule set by statisticians)
- For large data (say a few thousands), 80-10-10 is usually fine.

28

## Hyperparameters vs Parameters



**hyperparameter:** parameters of the model that are not trained automatically by ML algorithms (e.g., k in KNN).

**parameters:** those that are trained automatically (e.g., tree structures in decision tree)

29

## Tuning hyperparameters

30

### Validation set method:

- For each hyperparameter  $h \in H$ 
  - Train  $\hat{f}$  on train set with  $h$
  - Compute the error rate of  $\hat{f}$  on validation set
- Choose the best performing hyperparameter  $h^*$
- Use  $h^*$  to retrain the final model  $\hat{f}^*$  with both train and validation set.
- Finally, evaluate  $\hat{f}^*$  on test set to estimate its future performance.

### Pro tip

- Do not use arithmetic grids; use geometric grids.
- |       |                             |
|-------|-----------------------------|
| Don't | $k = 1, 3, 5, 7, 9, \dots$  |
| Do    | $k = 1, 2, 4, 8, 16, \dots$ |

30

# Tuning hyperparameters

31

## K-fold cross validation

- Randomly partition train set  $\mathcal{S}$  into  $K$  disjoint sets; call them  $\text{fold}_1, \dots, \text{fold}_K$
- For each hyperparameter  $h \in \{1, \dots, H\}$ 
  - For each  $k \in \{1, \dots, K\}$ 
    - train  $\hat{f}_k^h$  with  $\mathcal{S} \setminus \text{fold}_k$
    - measure error rate  $e_{h,k}$  of  $\hat{f}_k^h$  on  $\text{fold}_k$
  - Compute the average error of the above:  $\widehat{\text{err}}^h = \frac{1}{K} \sum_{k=1}^K e_{h,k}$
- Choose  $\hat{h} = \arg \min_h \widehat{\text{err}}^h$
- Train  $\hat{f}^*$  using  $\mathcal{S}$  (all the training points) with hyperparameter  $h$
- Finally, evaluate  $\hat{f}^*$  on test set to estimate its future performance.

Use when (1) the dataset is small (2) ML algorithm's retraining time complexity is low (e.g., kNN)

31

## 5-fold cross validation



32

## Scikit-Learn

33

Python library for machine learning. Install using Anaconda:



```
$ conda install -c conda-forge scikit-learn
```

Or using PyPi:

```
$ pip install -U scikit-learn
```

33

## Preprocessing : Encoding Labels

34

Oftentimes, categorical labels come as strings, which aren't easily modeled (e.g., with Naïve Bayes),

```
>>> le = preprocessing.LabelEncoder()
>>> le.fit(["paris", "paris", "tokyo", "amsterdam"])
LabelEncoder()
>>> list(le.classes_)
['amsterdam', 'paris', 'tokyo']
>>> le.transform(["tokyo", "tokyo", "paris"])
array([2, 2, 1]...)
>>> list(le.inverse_transform([2, 2, 1]))
['tokyo', 'tokyo', 'paris']
```

LabelEncoder transforms these into integer values, e.g. for categorical distributions

`fit()` is doing the heavy work: create the mapping from string to integers

Can *undo* using `inverse_transform` so we don't have to store two copies of the data

34

## Preprocessing : Z-Score

35

Typical ML workflow starts with *pre-processing* or *transforming* data into some useful form, which Scikit-Learn calls *transformers*:

```
>>> from sklearn.preprocessing import StandardScaler
>>> data = [[0, 0], [0, 0], [1, 1], [1, 1]]
>>> scaler = StandardScaler()
>>> print(scaler.fit(data))
StandardScaler()
>>> print(scaler.mean_)
[0.5 0.5]
>>> print(scaler.transform(data))
[[ -1. -1.]
 [-1. -1.]
 [ 1.  1.]
 [ 1.  1.]]
```

Standardization:

$$z = \frac{x-\mu}{\sigma}$$

with mean:

$$\mu = \frac{1}{N} \sum_{i=1}^N (x_i)$$

and standard deviation:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

- Features are standardized independently (columns of X)

35

## Cross-Validation

36

Easily do cross validation for model selection / evaluation...

```
>>> from sklearn import datasets, linear_model
>>> from sklearn.model_selection import cross_val_score
>>> diabetes = datasets.load_diabetes()
>>> X = diabetes.data[:150]
>>> y = diabetes.target[:150]
>>> lasso = linear_model.Lasso() Linear Model trained with Lasso prior as regularizer
>>> print(cross_val_score(lasso, X, y, cv=3))
[0.3315057  0.08022103  0.03531816]
```

36

## Scikit-Learn

37



Models can be fit using the `fit()` function.  
E.g., Random Forest Classifier,

```
>>> from sklearn.ensemble import RandomForestClassifier
>>> clf = RandomForestClassifier(random_state=0)
>>> X = [[1, 2, 3], # 2 samples, 3 features
...      [11, 12, 13]]
>>> y = [0, 1] # classes of each sample
>>> clf.fit(X, y)
RandomForestClassifier(random_state=0)
```

`fit()` Generally accepts 2 inputs

- Sample matrix X—typically 2d array (`n_samples, n_features`)
- Target values Y—real numbers for regression, integer for classification

37

## k-Nearest Neighbors

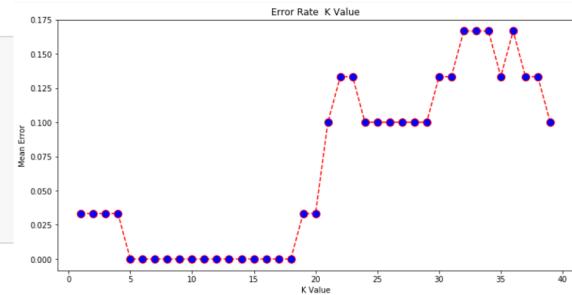
38

Train / evaluate the KNN classifier for each value K,

```
from sklearn.neighbors import KNeighborsClassifier
error = []

# Calculating error for K values between 1 and 40
for i in range(1, 40):
    knn = KNeighborsClassifier(n_neighbors=i)
    knn.fit(X_train, y_train)
    pred_i = knn.predict(X_val)
    error.append(np.mean(pred_i != y_val))

    ↑ vector
    operation!
```



Print error:

in practice: use geometric grid like 1,2,4,8,...

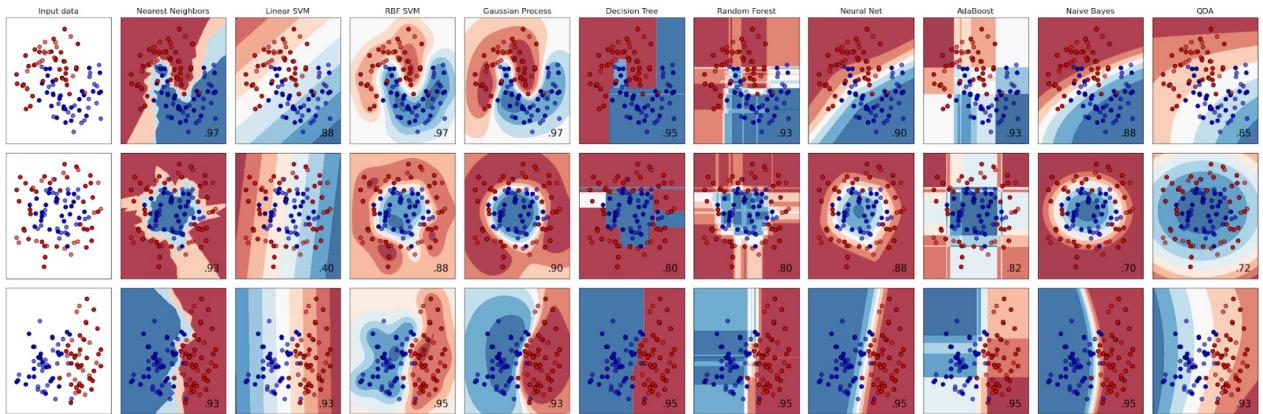
```
plt.figure(figsize=(12, 6))
plt.plot(range(1, 40), error, color='red', linestyle='dashed', marker='o',
         markerfacecolor='blue', markersize=10)
plt.title('Error Rate K Value')
plt.xlabel('K Value')
plt.ylabel('Mean Error')
```

38

# Scikit-Learn

39

Easily try out *all* the classifiers...



See full code.

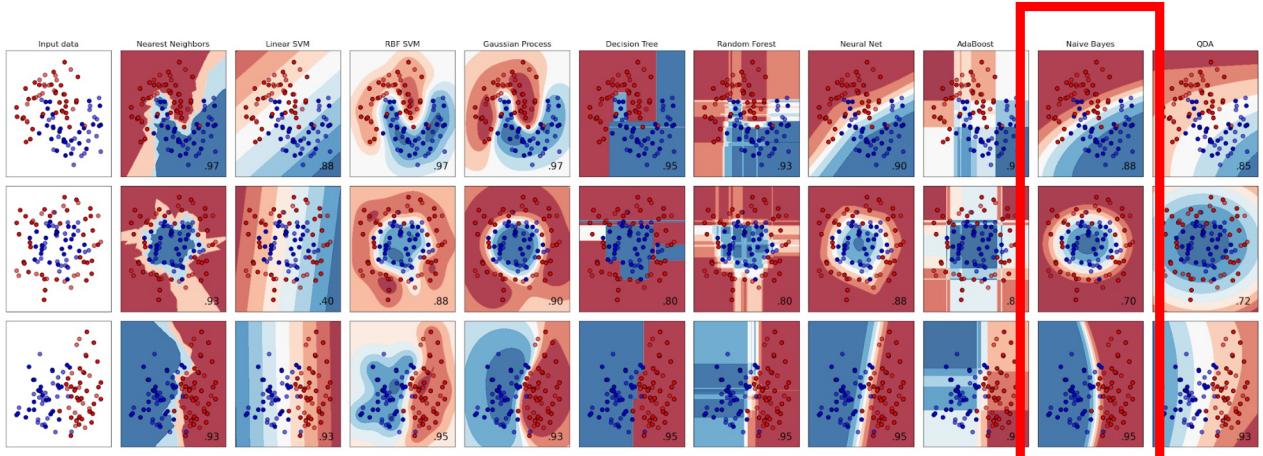
39

# Scikit-Learn

40

Easily try out *all* the classifiers...

Naïve Bayes



See full code.

40