Computer Science

CSC380: Principles of Data Science

Nonlinear Models 1

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1

1

Outline

3

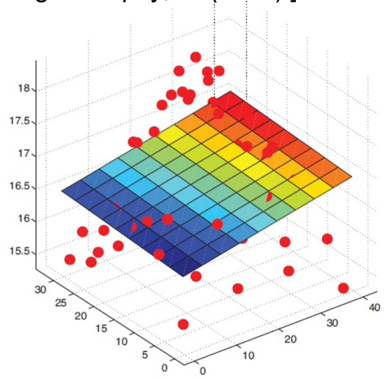
- Basis Functions
- Support Vector Machine
- Neural Networks

3

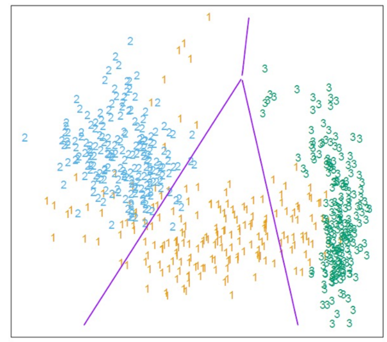
Linear Models

4

[Image: Murphy, K. (2012)]



[Image: Hastie et al. (2001)]



Linear Regression Fit a *linear function* to the data,

$$y = w^T x$$

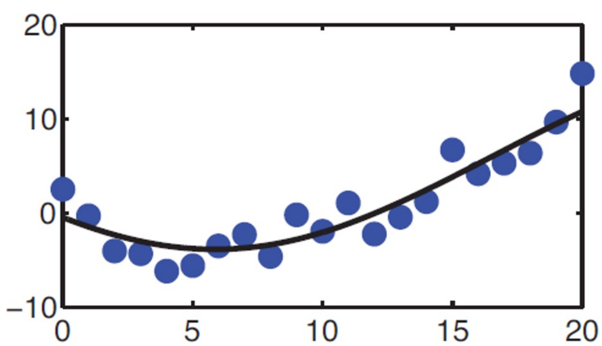
Logistic Regression Learn a decision boundary that is *linear in the data*,

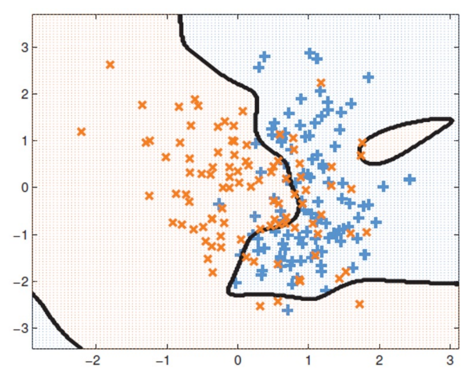
$$y = \mathbf{I}\{w^\top x \geq 0\}$$

4

Nonlinear Data

5





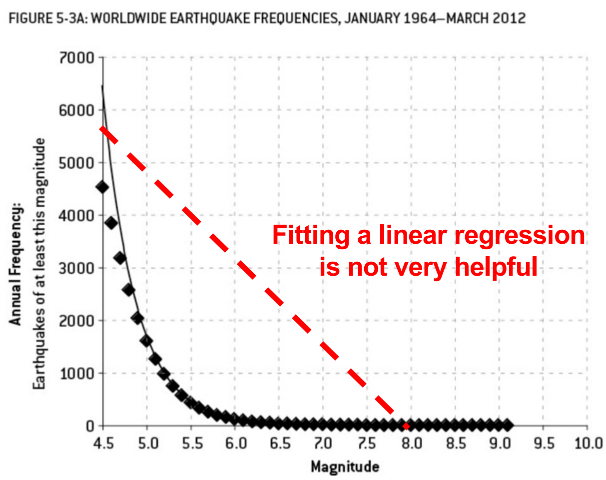
What if our data are *not* well-described by a linear function?

What if classes cannot be well-distinguished by a linear function?

5

Example: Earthquake Prediction 6

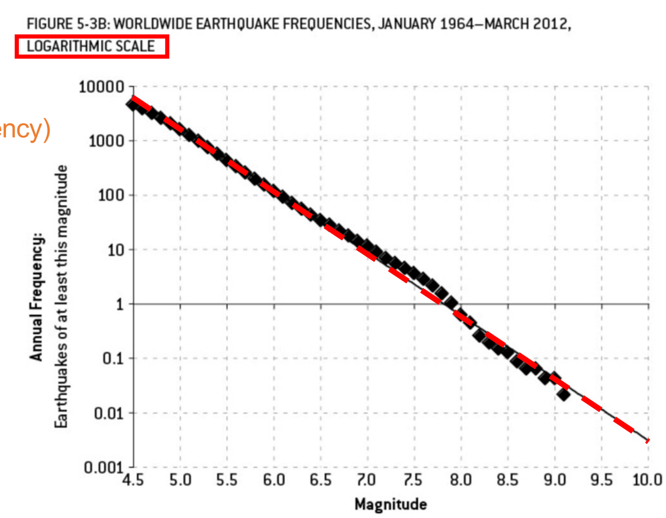
Suppose that we want to predict the number of earthquakes that occur of a certain magnitude. Our data are given by,



6

Example: Earthquake Prediction 7

Suppose that we want to predict the number of earthquakes that occur of a certain magnitude. Our data are given by,



But plotting outputs on a logarithmic scale reveals a strong linear relationship…

it's like $y = e^{-ax+b}$

7

Beyond linearity: Transformation in x

8

- Recall: for 1d problem, we embedded the feature: $x' = (x, 1) \in \mathbb{R}^2$ so we can encode the intercept term.

$$\phi_0(x) = 1 \quad \phi_1(x) = x \quad y = \boldsymbol{w}^\top \Phi_{\text{lin}}(x) = \phi_0(x)w_0 + \phi_1(x)w_1 = w_0 + w_1 x$$

8

Beyond linearity: Transformation in x

9

- Recall: for 1d problem, we embedded the feature: $x' = (x, 1) \in \mathbb{R}^2$ so we can encode the intercept term.

$$\phi_0(x) = 1 \quad \phi_1(x) = x \quad y = \boldsymbol{w}^\top \Phi_{\text{lin}}(x) = \phi_0(x)w_0 + \phi_1(x)w_1 = w_0 + w_1 x$$

- Actually, the embedding trick is stronger.
  - $(x^2, x, 1)$: 2nd order polynomial with respect to x
  - $(x^d, x^{d-1}, \dots, 1)$: d-th order polynomial (= degree d)

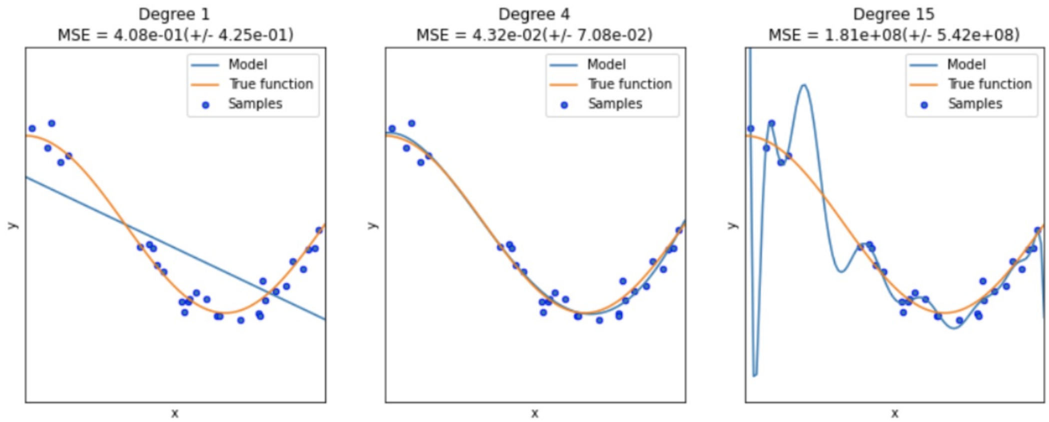
$$\phi_0(x) = 1 \quad \phi_1(x) = x \quad \phi_2(x) = x^2$$

$$y = \boldsymbol{w}^\top \Phi_{\text{lin}}(x) = \phi_0(x)w_0 + \phi_1(x)w_1 + \phi_2(x)w_2 = w_0 + w_1 x + w_2 x^2$$

9

Feature embedding trick

10



$(x, 1)$ $(x^4, x^3, x^2, x, 1)$ $(x^{15}, \ldots, x^3, x^2, x, 1)$

higher-order polynomial = higher complexity = prone to overfitting!

from https://datascience.foundation/sciencewhitepaper/underfitting-and-overfitting-in-machine-learning

10

Basis Functions

11

- A **basis function** can be any function of the input features X
- Define a set of B basis functions $\phi_1(x), \ldots, \phi_B(x)$
- Fit a linear regression model in terms of basis functions,

$$y = \sum_{b=1}^{B} w_b \phi_b(x) = w^T \phi(x)$$

notation:
$\phi(x) := [\phi_1(x), \ldots, \phi_B(x)]$

- The model is *linear in the transformed basis/induced features* $\phi(x)$.
- The model is *nonlinear in the data* X

11

Linear Regression

12

Recall the ordinary least squares solution is given by,

$$\mathbf{X} = \begin{pmatrix} 1 & x_{11} & \dots & x_{1D} \\ 1 & x_{21} & \dots & x_{2D} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{m1} & \dots & x_{mD} \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} \qquad w^{\text{OLS}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$$

Design Matrix
(each training input on a column)

Vector of
Training labels

Can similarly solve in terms of basis functions,

$$\mathbf{\Phi} = \begin{pmatrix} 1 & \phi_1(x_1) & \dots & \phi_B(x_1) \\ 1 & \phi_1(x_2) & \dots & \phi_B(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \phi_1(x_m) & \dots & \phi_B(x_m) \end{pmatrix} \qquad w^{\text{OLS}} = (\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T\mathbf{y}$$

12

`sklearn.preprocessing`.PolynomialFeatures

13

degree : *int or tuple (min_degree, max_degree), default=2*

If a single int is given, it specifies the maximal degree of the polynomial features. If a tuple `(min_degree, max_degree)` is passed, then `min_degree` is the minimum and `max_degree` is the maximum polynomial degree of the generated features. Note that `min_degree=0` and `min_degree=1` are equivalent as outputting the degree zero term is determined by `include_bias`.

interaction_only : *bool, default=False*

If `True`, only interaction features are produced: features that are products of at most `degree` *distinct* input features, i.e. terms with power of 2 or higher of the same input feature are excluded:

- included: `x[0]`, `x[1]`, `x[0] * x[1]`, etc.
- excluded: `x[0] ** 2`, `x[0] ** 2 * x[1]`, etc.

include_bias : *bool, default=True*

If `True` (default), then include a bias column, the feature in which all polynomial powers are zero (i.e. a column of ones - acts as an intercept term in a linear model).

order : *{'C', 'F'}, default='C'*

Order of output array in the dense case. `'F'` order is faster to compute, but may slow down subsequent estimators.

13

Example: Polynomial Basis Functions

14

Create three two-dimensional data points [0,1], [2,3], [4,5]:

```
>>> X = np.arange(6).reshape(3, 2)
>>> X
array([[0, 1],
       [2, 3],
       [4, 5]])
```

Compute quadratic features $(1, x_1, x_2, x_1^2, x_1 x_2, x_2^2)$,

```
>>> poly = PolynomialFeatures(degree=2)
>>> poly.fit_transform(X)
array([[ 1.,  0.,  1.,  0.,  0.,  1.],
       [ 1.,  2.,  3.,  4.,  6.,  9.],
       [ 1.,  4.,  5., 16., 20., 25.]])
```

These are now our new data and ready to fit a model…

14

Example: Polynomial Basis Functions

15

Create a 3-rd order polynomial (cubic) function,

```
f = lambda x: (x-1)*(x-2)*(x-3)
import numpy.random as ra
ra.seed(20)
train_x = np.arange(5)
train_y = f(train_x) + 1*ra.randn(len(train_x))
train_y
```

✓ 0.3s

```
array([-5.11610689,  0.19586502,  0.35753652, -2.34326191,  4.91516741])
```

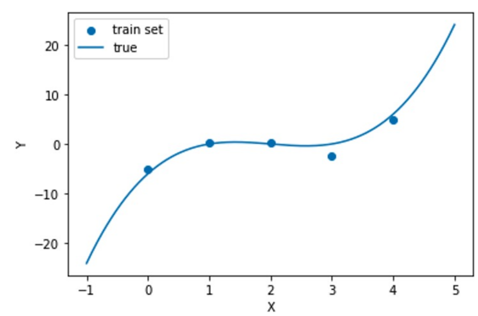
Plot train set and the actual function

```
test_x = np.linspace(-1,5,400)

from matplotlib import pyplot as plt
plt.scatter(train_x,train_y)

plt.plot(test_x, f(test_x))
plt.legend(['train set', 'true'])
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```

✓ 0.4s



15

Example: Polynomial Basis Functions 16

Create cubic features $(1, x, x^2, x^3)$

```
poly = PolynomialFeatures(degree=3)
train_xx = poly.fit_transform(train_x[:,np.newaxis])
train_xx
```

✓ 0.4s turns train_x (length 5 array) into a matrix (5 by 1 matrix)

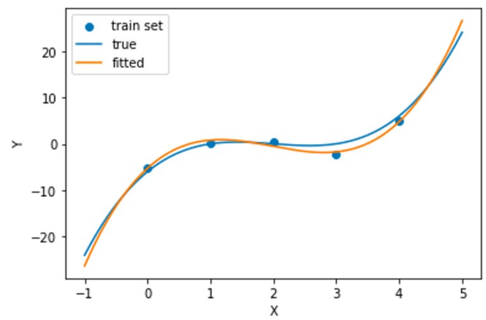
```
array([[ 1.,  0.,  0.,  0.],
       [ 1.,  1.,  1.,  1.],
       [ 1.,  2.,  4.,  8.],
       [ 1.,  3.,  9., 27.],
       [ 1.,  4., 16., 64.]])
```

Perform linear regression; plot it

```
from matplotlib import pyplot as plt
from sklearn.linear_model import LinearRegression
model = LinearRegression().fit(train_xx, train_y)
test_x = np.linspace(-1,5,400)
test_xx = poly.fit_transform(test_x[:,np.newaxis])
pred_y = model.predict(test_xx)

plt.scatter(train_x,train_y)
plt.plot(test_x, f(test_x))
plt.plot(test_x, pred_y)
plt.legend(['train set', 'true', 'fitted'])
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```

✓ 0.2s



16

Other extensions 17

- Fit x_i to $\log(y_i)$ (note that the sum of error is not so easily explained)
- Assume data in 3D. Allow fitting a function of the form
  $$z_i = w_0 + w_1 x_i + w_2 x_i^2 + w_3 y_i$$
  (so quadratic in x but linear in y)

17

Data Preprocessing

18

- Generally the first step in data science involves *preprocessing* or transforming data in some way
  - Filling in missing values (imputation)
  - Centering / normalizing / standardizing
  - Etc.
- We then fit our models to this preprocessed data
- One way to view preprocessing is simply as computing some basis function $\phi(x)$, nothing more

18

Basis Functions

19

PROs

- More flexible modeling that is nonlinear in the original data
- Increases model expressivity

CONs

- Typically requires **more parameters** to be learned
- More sensitive to **overfitting** training data (due to expressivity)
- Requires more **regularization** to avoid overfitting
- Need to find *good* basis functions (feature engineering)

19

Outline

20

- Basis Functions
- Support Vector Machine
- Neural Networks

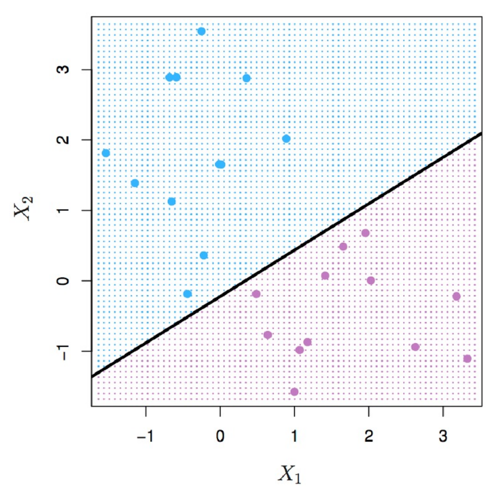
20

Linear Decision Boundary

21

Forget about the 'regression' point of view for now..

At the end of the day, we just want a line that separates the two classes well.



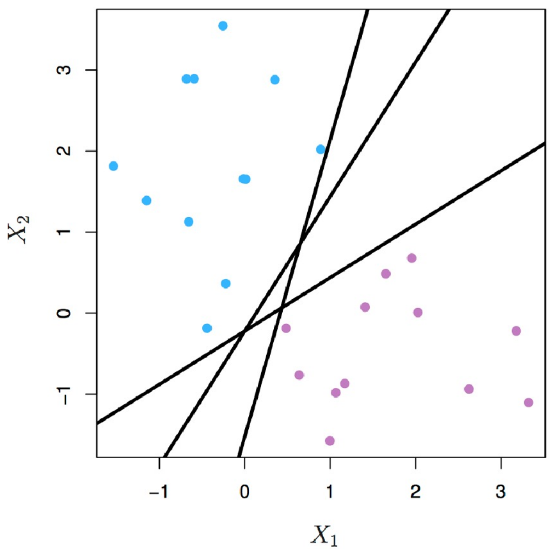
[Source: http://www-bcf.usc.edu/~gareth/ISL/]

21

Linear Decision Boundary

22

Note: Any boundary that separates classes is equivalently good on training data

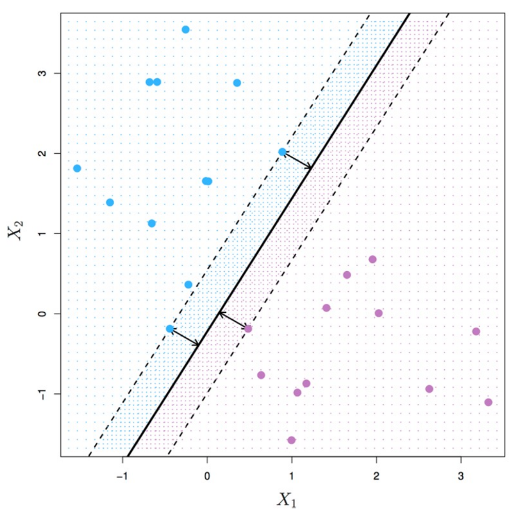


Q: but if you have to choose one, which one will you choose?

22

Classifier Margin

23



*The **margin** measures minimum distance between each class and the decision boundary*

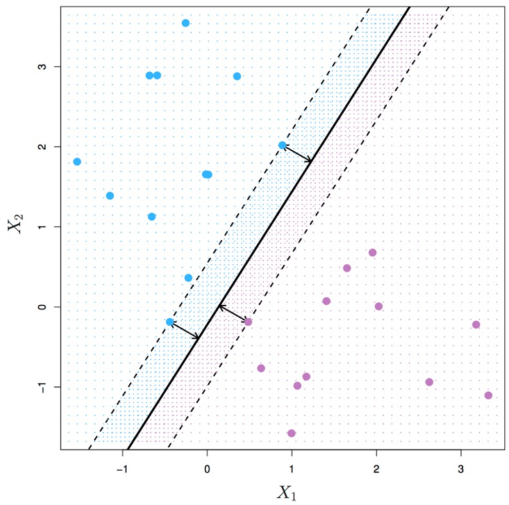
Observation Decision boundaries with larger margins are more likely to generalize to unseen data

Idea Learn the classifier with the largest margin that still separates the data…

…we call this a ***max-margin classifier***

23

Max-Margin Classifier (Linear Separable Case)



For now, let's focus on the case where the data is **linearly separable**

(Otherwise, there is no margin to talk about!)

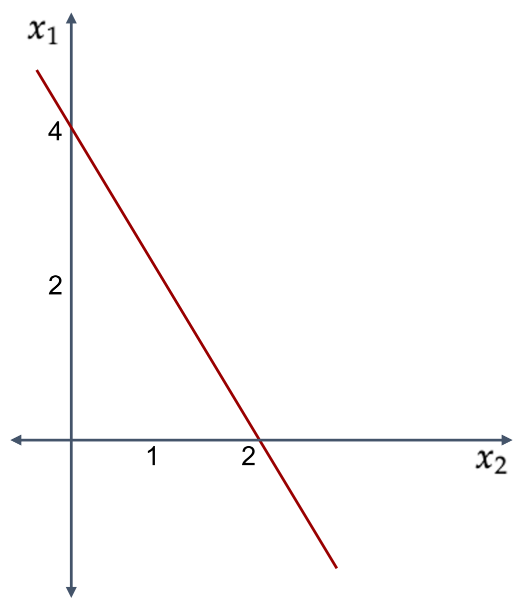
Hyperplane

A linear discriminant function in D dimensions is given by a hyperplane, defined as follows:

$$\begin{aligned} h(\mathbf{x}) &= \mathbf{w}^T\mathbf{x} + b \\ &= w_1x_1 + w_2x_2 + \cdots + w_dx_d + b \end{aligned}$$

For points that lie on the hyperlane, we have:

$$h(\mathbf{x}) = \mathbf{w}^T\mathbf{x} + b = 0$$



Separating Hyperplane

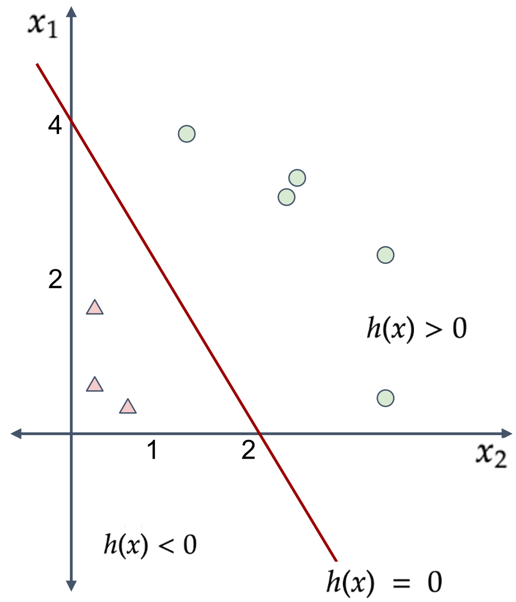
26

A hyperplane h(x) splits the original d-dimensional space into two half-spaces. If the input dataset is linearly separable:

$$y = \begin{cases} +1 & \text{if } h(\mathbf{x}) > 0 \\ -1 & \text{if } h(\mathbf{x}) < 0 \end{cases}$$

Example:

$$h(x) = x_1 + 2x_2 - 4$$



26

Separating Hyperplane: weight vector

27

Let a_1 and a_2 be two arbitrary points that lie on the hyperplane, we have:

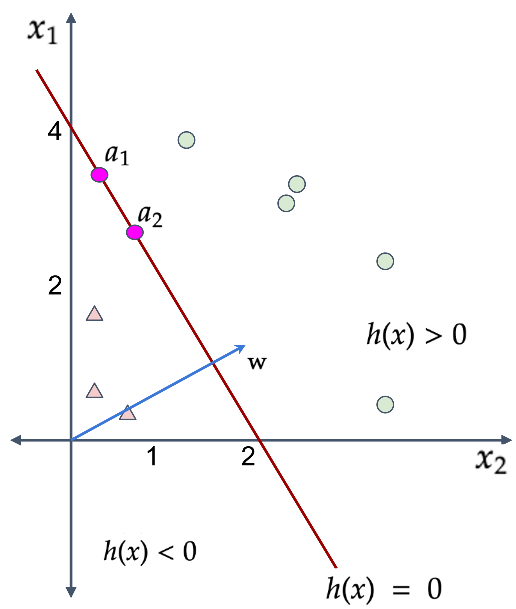
$$h(\mathbf{a}_1) = \mathbf{w}^T\mathbf{a}_1 + b = 0$$

$$h(\mathbf{a}_2) = \mathbf{w}^T\mathbf{a}_2 + b = 0$$

Subtracting one from the other:

$$\mathbf{w}^T(\mathbf{a}_1 - \mathbf{a}_2) = 0$$

The weight vector **w** is orthogonal to the hyperplane.



27

Distance of a Point to the Hyperplane

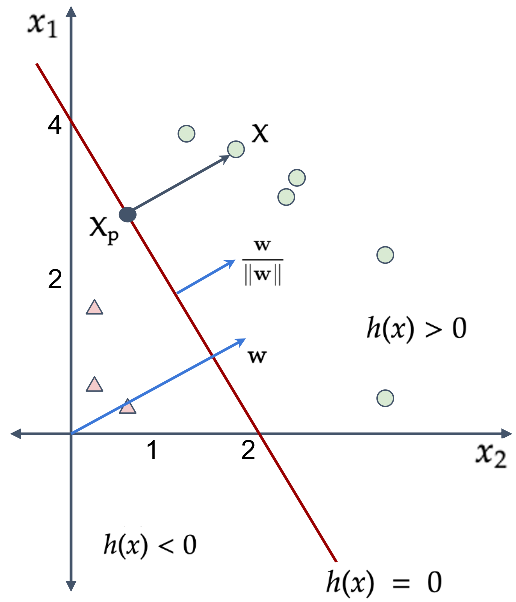
28

Consider a point X not on the hyperplane. Let X_p be the projection of X on the hyperplane.

Let r be the steps need to walk from X_p to X.

$$\mathbf{x} = \mathbf{x}_p + r\frac{\mathbf{w}}{\|\mathbf{w}\|}$$

Q: how many steps/direct distance do we need to walk?



28

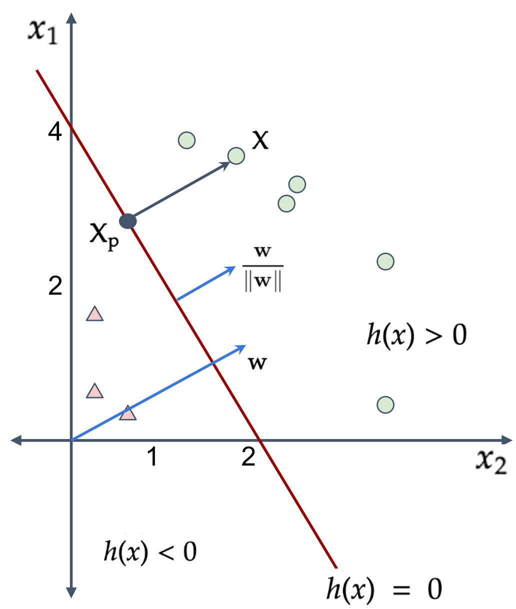
Distance of a Point to the Hyperplane

29

Consider a point X not on the hyperplane. Let X_p be the projection of X on the hyperplane.

Let r be the steps need to walk from X_p to X.

$$\begin{aligned} h(\mathbf{x}) &= h(\mathbf{x}_p + r\frac{\mathbf{w}}{\|\mathbf{w}\|}) \\ &= \mathbf{w}^T\left(\mathbf{x}_p + r\frac{\mathbf{w}}{\|\mathbf{w}\|}\right) + b \\ &= \underbrace{\mathbf{w}^T\mathbf{x}_p + b}_{h(\mathbf{x}_p)} + r\frac{\mathbf{w}^T\mathbf{w}}{\|\mathbf{w}\|} \\ &= \underbrace{h(\mathbf{x}_p)}_{0} + r\|\mathbf{w}\| \\ &= r\|\mathbf{w}\| \end{aligned} \qquad r = \frac{h(\mathbf{x})}{\|\mathbf{w}\|}$$



29

Distance of a Point to the Hyperplane

30

Q: What is the direct distance from origin (x=0) to the hyperplane?

$$r = \frac{h(\mathbf{x})}{\|\mathbf{w}\|} \qquad r = \frac{h(\mathbf{0})}{\|\mathbf{w}\|} = \frac{\mathbf{w}^T\mathbf{0} + b}{\|\mathbf{w}\|} = \frac{b}{\|\mathbf{w}\|}$$

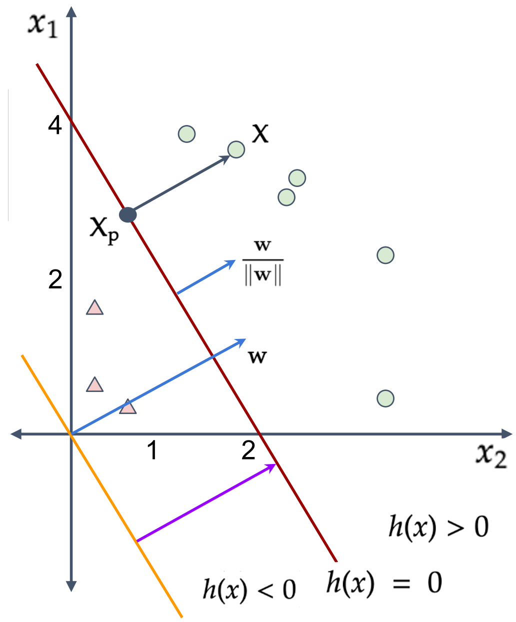
Example:

$$h(x) = x_1 + 2x_2 - 4$$

$$w^T x + b = (1 \;\; 2)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - 4$$

$$\frac{b}{\|w\|} = -\frac{4}{\sqrt{5}}$$

Q: how to deal with negative distance?



30

Distance of a Point to the Hyperplane

31

Q: How to deal with negative distance?

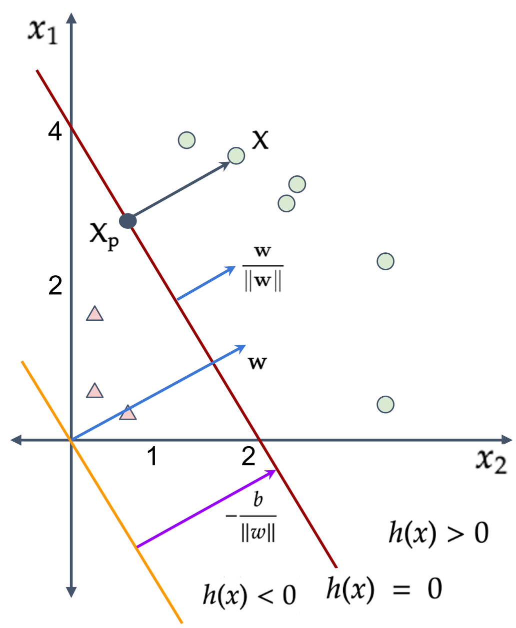
$$r = \frac{h(\mathbf{x})}{\|\mathbf{w}\|}$$

$$y = \begin{cases} +1 & \text{if } h(\mathbf{x}) > 0 \\ -1 & \text{if } h(\mathbf{x}) < 0 \end{cases}$$

$$\delta = y\, r = \frac{y\, h(\mathbf{x})}{\|\mathbf{w}\|}$$

Example (when point is the origin):

$$(-1) \cdot \frac{b}{\|w\|} = \frac{4}{\sqrt{5}}$$



31

Margin and Support Vectors

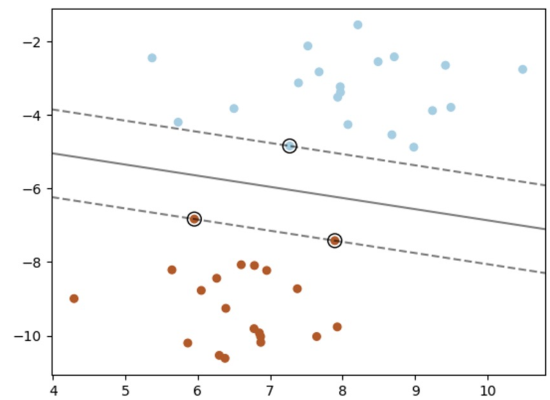
32

Over all the n points, the ***margin*** of the linear classifier is the minimum distance of a point from the separating hyperplane:

$$\delta^* = \min_{\mathbf{x}_i} \left\{ \frac{y_i(\mathbf{w}^T\mathbf{x}_i + b)}{\|\mathbf{w}\|} \right\}$$

All the points that achieve this minimum distance are called ***support vectors***.

$$\delta^* = \frac{y^*(\mathbf{w}^T\mathbf{x}^* + b)}{\|\mathbf{w}\|}$$



32

Max-Margin Classifier (Linear Separable Case)

33

For training data $\{(x^{(i)}, y^{(i)})\}_{i=1}^m$, a classifier $f(x) = w^\top x + b$ with 0 train error will satisfy

$$y^{(i)} f(x^{(i)}) = y^{(i)}(w^\top x^{(i)} + b) > 0$$

↓ negative margin when misclassifying it!

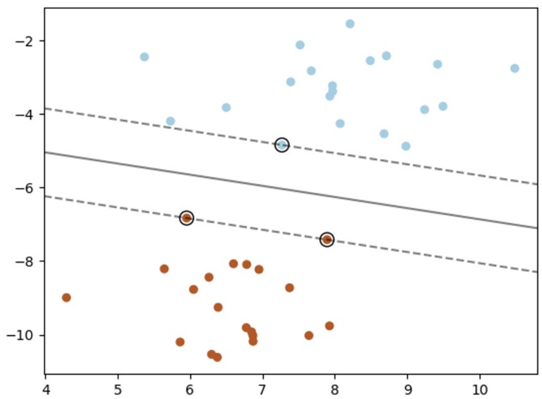
The distance for ($x^{(i)}$, $y^{(i)}$) to separating hyperplane $\dfrac{y^{(i)}(w^\top x^{(i)} + b)}{\|w\|}$

The margin of a classifier $f(x)$ is $\min_i \dfrac{y^{(i)}(w^\top x^{(i)} + b)}{\|w\|}$

Find f that maximize margin $\arg\max_{w,b} \min_i \dfrac{y^{(i)}(w^\top x^{(i)} + b)}{\|w\|}$

33

Max-Margin Classifier (Linear Separable Case) 34



Maximize the minimum margin

$$\overbrace{\arg\max_{w,b} \underbrace{\min_i \frac{y^{(i)}(w^\top x^{(i)} + b)}{\|w\|}}_{\text{Minimum margin over all training data}}}^{\text{Maximize the minimum margin}}$$

Minimum margin over all training data

Find the parameters (w,b) that **maximize** the **smallest margin** over all the training data

34

Canonical Hyperplane 35

Issue: infinite equivalent hyperplanes result in infinite solutions:

- Multiplying on both sides by some scalars yields an equivalent hyperplane

$$s\ h(\mathbf{x}) = s\ \mathbf{w}^T\mathbf{x} + s\ b$$

Example of equivalent hyperplanes:

$$h(x) = x_1 + 2x_2 - 4$$
$$h(x) = 2x_1 + 4x_2 - 8$$

Maximize the minimum margin

$$\overbrace{\arg\max_{w,b} \underbrace{\min_i \frac{y^{(i)}(w^\top x^{(i)} + b)}{\|w\|}}_{\text{Minimum margin over all training data}}}^{\text{Maximize the minimum margin}}$$

Minimum margin over all training data

35

Canonical Hyperplane 36

Way to solve this issue:

- Choose the scalar s such that the absolute distance of a ***support vector*** from the hyperplane is 1.

$$sy^*(\mathbf{w}^T\mathbf{x}^* + b) = 1$$

$$s = \frac{1}{y^*(\mathbf{w}^T\mathbf{x}^* + b)} = \frac{1}{y^* h(\mathbf{x}^*)}$$

$$y_i\ (\mathbf{w}^T\mathbf{x}_i + b) \geq 1, \text{ for all points } \mathbf{x}_i \in \mathbf{D}$$

$$\arg\max_{w,b}\min_{i} \frac{\overset{1}{\boxed{y^{(i)}(w^\top x^{(i)} + b)}}}{\|w\|}$$

Margin: $\delta^* = \dfrac{1}{\|\mathbf{w}\|}$

Max margin: $h^* = \arg\max_{h} \{\delta_h^*\} = \arg\max_{\mathbf{w},b} \left\{\dfrac{1}{\|\mathbf{w}\|}\right\}$

36

Canonical Hyperplane: an example 37

$$h'(\mathbf{x}) = \begin{pmatrix}5\\2\end{pmatrix}^T \mathbf{x} - 20 = 0$$

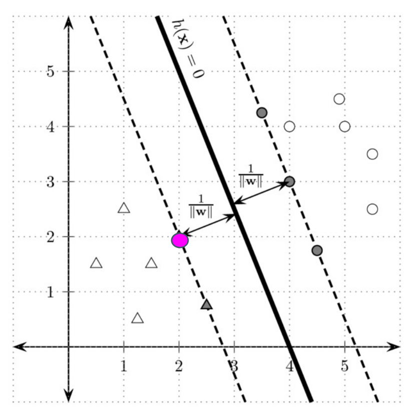
support vector $\mathbf{x}^* = (2,2)$, with class $y^* = -1$

$$s = \frac{1}{y^* h'(\mathbf{x}^*)} = \frac{1}{-1\left(\begin{pmatrix}5\\2\end{pmatrix}^T\begin{pmatrix}2\\2\end{pmatrix} - 20\right)} = \frac{1}{6}$$

$$\mathbf{w} = \frac{1}{6}\begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}5/6\\2/6\end{pmatrix} \qquad b = \frac{-20}{6}$$

$$h(\mathbf{x}) = \begin{pmatrix}5/6\\2/6\end{pmatrix}^T \mathbf{x} - 20/6 = \begin{pmatrix}0.833\\0.333\end{pmatrix}^T \mathbf{x} - 3.33$$

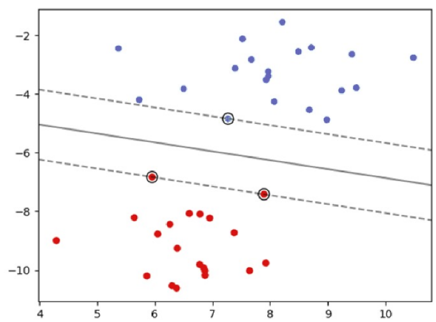
$$\delta^* = \frac{y^*\, h(\mathbf{x}^*)}{\|\mathbf{w}\|} = \frac{-1\left(\begin{pmatrix}5/6\\2/6\end{pmatrix}^T\begin{pmatrix}2\\2\end{pmatrix} - 20/6\right)}{\sqrt{(\frac{5}{6})^2 + (\frac{2}{6})^2}} = \frac{1}{\frac{\sqrt{29}}{6}} = 1.114$$



37

Support Vector Machine (Hard Margin) 38

… it leads to



$$\min_{w,b} \frac{1}{2}\|w\|^2$$
subject to
$$y^{(i)}(w^\top x^{(i)} + b) \geq 1 \qquad \text{for } i = 1, \ldots, m$$

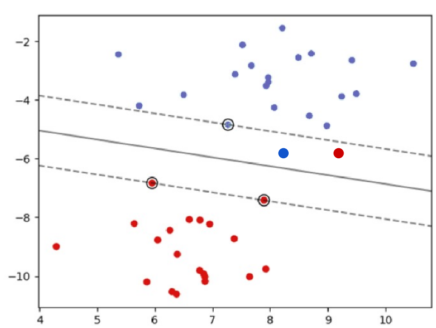
This is a convex (quadratic) optimization problem that can be solved efficiently

- Data are D-dimensional *vectors*
- Margins determined by nearest data points called *support vectors*
- We call this a *support vector machine* (SVM)

38

Support Vector Machine (Soft Margin) 39

If the data is linearly not separable,



error

$$\min_{w,b} \frac{1}{2}\|w\|^2 + C \cdot \sum_{i=1}^{m} \xi_i$$
subject to
$$y^{(i)}(w^\top x^{(i)} + b) \geq 1 - \xi_i$$
$$\xi_i \geq 0 \quad \text{for } i = 1, \ldots, m$$

Tradeoff between margin and the error!

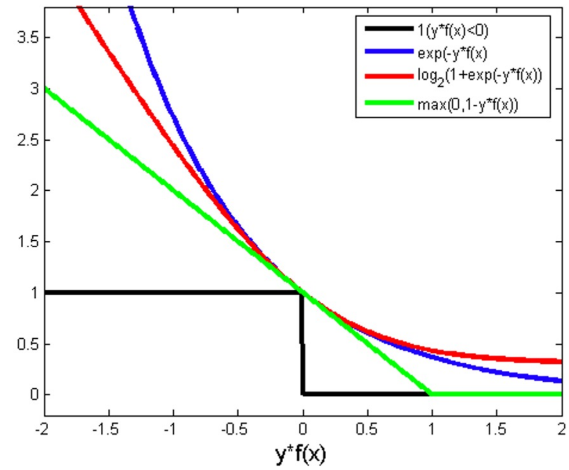
39

Support Vector Machine

$$\text{minimize } \frac{1}{2}\|w\|^2 + C \cdot \sum_{i=1}^{m} \xi_i$$
subject to
$$y^{(i)}(w^\top x^{(i)} + b) \geq 1 - \xi_i$$
$$\xi_i \geq 0 \quad \text{for } i = 1, \ldots, m$$

Equivalent formulation

$$\min_{\mathbf{w,b}} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\left(1 - y^{(i)}(w^\top x^{(i)} + b)\right)_+$$



$$\ell\left(f; x^{(i)}, y^{(i)}\right) = \left(1 - y^{(i)} f\left(x^{(i)}\right)\right)_+$$

$(X)_+ \coloneqq \max(X, 0)$

SVM - Soft Margin: an example

$$\textit{Hinge loss} = \max\left(0,\ 1 - y_i\left(w^T x_i + b\right)\right)$$

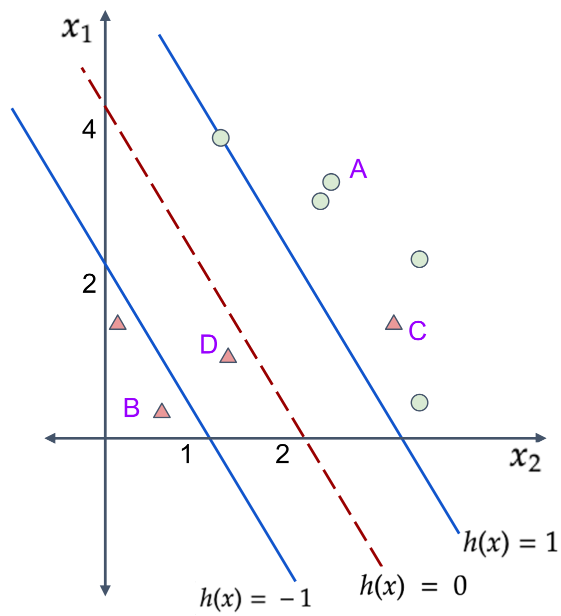
$A:\ \max(0,\ 1 - 1 \cdot (>1)) \rightarrow 0$

$B:\ \max(0, 1 - (-1) \cdot (<-1)) \rightarrow 0$

$C:\ \max(0,\ 1 - (-1) \cdot (>1)) \rightarrow\ >1$

$D:\ \max(0, 1 - (-1) \cdot (\text{between } [-1, 0]))$

$\rightarrow \text{between } [0, 1]$



General Principle

42

$$\arg\min_{w,b} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\left(1 - y^{(i)}(w^\top x^{(i)} + b)\right)_+$$

=> by setting $C = 1/\lambda$, it's equivalent to solve

$$\arg\min_{w,b} \frac{\lambda}{2}\|w\|^2 + \sum_{i=1}^{m}\left(1 - y^{(i)}(w^\top x^{(i)} + b)\right)_+$$

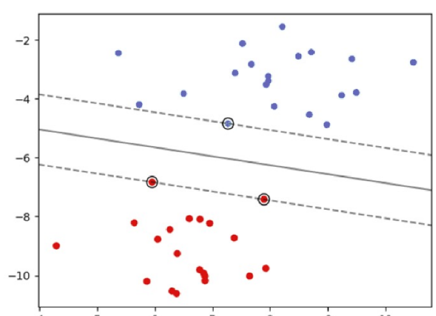
SVM belongs to the general loss-oriented formulation!

$$\text{Model} = \arg\min_{\text{model}} \text{Loss}(\text{Model}, \text{Data}) + \lambda \cdot \text{Regularizer}(\text{Model})$$

42

Support Vectors

43



$$\min_{w,b} \frac{1}{2}\|w\|^2 + C \cdot \sum_{i=1}^{m} \xi_i$$

subject to

$$y^{(i)}(w^\top x^{(i)} + b) \geq 1 - \xi_i$$

$$\xi_i \geq 0 \quad \text{for } i = 1, \ldots, m$$

Those data points achieving equality $y^{(i)}(w^\top x^{(i)} + b) = 1 - \xi_i$ are called **support vectors**.

Turns out, if you knew support vectors already, solving the optimization problem above with **just the support vectors as train set** leads to the same solution.

⇒ Leave-one-out cross validation can be done fast!

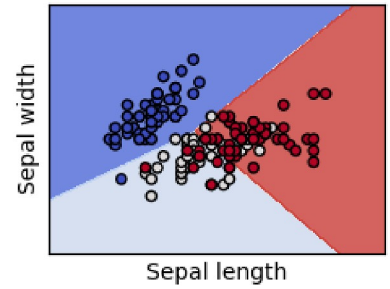
43

SVM in Scikit-Learn

44

SVM with linear decision boundaries,

`sklearn.svm.LinearSVC`



Call options include…

penalty : *{'l1', 'l2'}, default='l2'*

Specifies the norm used in the penalization. The 'l2' penalty is the standard used in SVC. The 'l1' leads to `coef_` vectors that are sparse.

dual : *bool, default=True*

Select the algorithm to either solve the dual or primal optimization problem. Prefer dual=False when n_samples > n_features.

C : *float, default=1.0*

Regularization parameter. The strength of the regularization is inversely proportional to C. Must be strictly positive.

44

`sklearn.svm`.SVC

45

kernel : *{'linear', 'poly', 'rbf', 'sigmoid', 'precomputed'}, default='rbf'*

Specifies the kernel type to be used in the algorithm. It must be one of 'linear', 'poly', 'rbf', 'sigmoid', 'precomputed' or a callable. If none is given, 'rbf' will be used. If a callable is given it is used to pre-compute the kernel matrix from data matrices; that matrix should be an array of shape `(n_samples, n_samples)`.

gamma : *{'scale', 'auto'} or float, default='scale'*

Kernel coefficient for 'rbf', 'poly' and 'sigmoid'.

for RBF,
small γ: complex decision boundary
large γ: more like linear decision boundary

- if `gamma='scale'` (default) is passed then it uses 1 / (n_features * X.var()) as value of gamma,
- if 'auto', uses 1 / n_features.

max_iter : *int, default=-1*

Hard limit on iterations within solver, or -1 for no limit.

verbose : *bool, default=False*

Enable verbose output. Note that this setting takes advantage of a per-process runtime setting in libsvm that, if enabled, may not work properly in a multithreaded context.

class_weight : *dict or 'balanced', default=None*

Set the parameter C of class i to class_weight[i]*C for SVC. If not given, all classes are supposed to have weight one. The "balanced" mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as `n_samples / (n_classes * np.bincount(y))`.

45

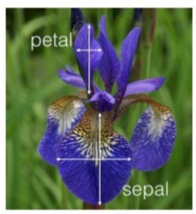
Example: Fisher’s Iris Dataset

Classify among 3 species of Iris flowers…

Iris setosa

Iris versicolor

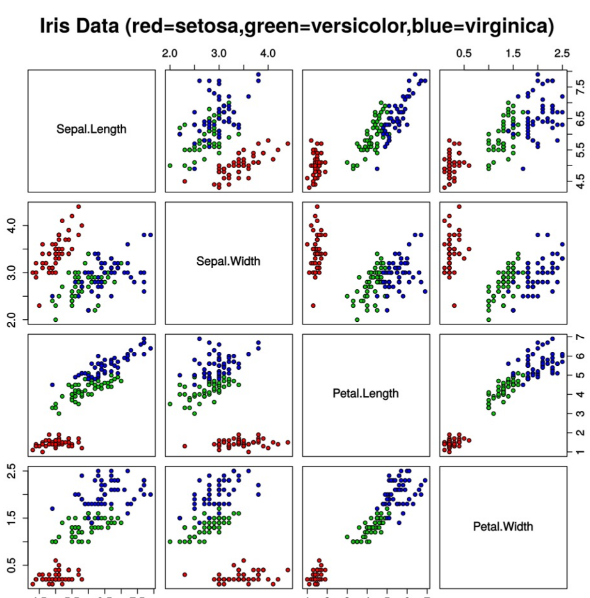
Iris virginica



Four features (in centimeters)

- Petal length / width
- Sepal length / width

Example: Fisher’s Iris Dataset



Fairly easy to separate ***setosa*** *from others using a* *<u>linear classifier</u>*

Need to use nonlinear basis / kernel representation to better separate other classes

Example: Fisher's Iris Dataset

Train 8-degree polynomial kernel SVM classifier,

```
from sklearn.svm import SVC
svclassifier = SVC(kernel='poly', degree=8)
svclassifier.fit(X_train, y_train)
```

Generate predictions on held-out test data,

```
y_pred = svclassifier.predict(X_test)
```

Show confusion matrix and classification accuracy,

```
print(confusion_matrix(y_test, y_pred))
print(classification_report(y_test, y_pred))
```

```
[[11  0  0]
 [ 0 12  1]
 [ 0  0  6]]
```

	precision	recall	f1-score	support
Iris-setosa	1.00	1.00	1.00	11
Iris-versicolor	1.00	0.92	0.96	13
Iris-virginica	0.86	1.00	0.92	6
avg / total	0.97	0.97	0.97	30

[Source: https://stackabuse.com/implementing-svm-and-kernel-svm-with-pythons-scikit-learn/]

48

Trick for Multi-Class

- Recall: logistic regression had a very natural extension to multi-class.
- What about SVM?

  … Researchers have found a few, but it was not any better than a simple trick below.

binary: $p(y=1 \mid x) = \frac{1}{1+e^{-w^\top x}}$

multi-class: $p(y=j \mid x) = \frac{\exp\left({w^{(j)}}^\top x\right)}{\sum_{c=1}^{C} \exp\left({w^{(c)}}^\top x\right)}$

[One-vs-the-rest trick]

- Given: dataset $D = \{(x^{(i)}, y^{(i)})\}_{i=1}^{m}$
- For each class $c \in \{1, \dots, C\}$
  - Define label $z^{(i)} \in \{-1,1\}$ where 1 for class c and -1 for other classes, for all i=1,…,m.
  - Train a classifier f_c with $\{(x^{(i)}, z^{(i)})\}_{i=1}^{m}$
- To classify x^*, compute $\hat{y} = \arg\max_{c \in \{1,\dots,C\}} \text{decision\_value}(f_c(x^*))$

49