Lecture 4: Sequence Methods

Part 1: Part of Speech Tagging and Hidden Markov Models

Part of Speech (POS) Tagging

INPUT:
Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT:
Profits/N soared/V at/P Boeing/N Co./N ,/, easily/ADV topping/V forecasts/N on/P Wall/N Street/N ,/, as/P their/POSS CEO/N Alan/N Mulally/N announced/V first/ADJ quarter/N results/N ./.

N	=	Noun
V	=	Verb
P	=	Preposition
Adv	=	Adverb
Adj	=	Adjective

...

Part of Speech (POS) Tagging

Training set:

1 Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.

2 Mr./NNP Vinken/NNP is/VBZ chairman/NN of/IN Elsevier/NNP N.V./NNP ,/, the/DT Dutch/NNP publishing/VBG group/NN ./.

3 Rudolph/NNP Agnew/NNP ,/, 55/CD years/NNS old/JJ and/CC chairman/NN of/IN Consolidated/NNP Gold/NNP Fields/NNP PLC/NNP ,/, was/VBD named/VBN a/DT nonexecutive/JJ director/NN of/IN this/DT British/JJ industrial/JJ conglomerate/NN ./.

...

38,219 It/PRP is/VBZ also/RB pulling/VBG 20/CD people/NNS out/IN of/IN Puerto/NNP Rico/NNP ,/, who/WP were/VBD helping/VBG Huricane/NNP Hugo/NNP victims/NNS ,/, and/CC sending/VBG them/PRP to/TO San/NNP Francisco/NNP instead/RB ./.

- From the training set, induce a function/algorithm that maps new sentences to their tag sequences.

Tagging – the Supervised Setup

Training set:
1 Pierre/NNP Vinken/NNP ,/, 61/CD years/NNS old/JJ ,/, will/MD join/VB the/DT board/NN as/IN a/DT nonexecutive/JJ director/NN Nov./NNP 29/CD ./.
2 Mr./NNP Vinken/NNP is/VBZ chairman/NN of/IN Elsevier/NNP N.V./NNP ,/, the/DT Dutch/NNP publishing/VBG group/NN ./.
3 Rudolph/NNP Agnew/NNP ,/, 55/CD years/NNS old/JJ and/CC chairman/NN of/IN Consolidated/NNP Gold/NNP Fields/NNP PLC/NNP ,/, was/VBD named/VBN a/DT nonexecutive/JJ director/NN of/IN this/DT British/JJ industrial/JJ conglomerate/NN ./.
...
38,219 It/PRP is/VBZ also/RB pulling/VBG 20/CD people/NNS out/IN of/IN Puerto/NNP Rico/NNP ,/, who/WP were/VBD helping/VBG Huricane/NNP Hugo/NNP victims/NNS ,/, and/CC sending/VBG them/PRP to/TO San/NNP Francisco/NNP instead/RB ./.

Evaluation:

$$accuracy = \frac{\#\, test\ set\ words\ with\ correct\ tag}{\#test\ set\ words}$$

Word tokens as opposed to *word types*. That is of the *word type* dog appears 5 times in the test set, it will be counted 5 times by the accuracy measure

Reminder: Named Entity Recognition (NER)

INPUT: Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT: Profits soared at [Company Boeing Co.], easily topping forecasts on [Location Wall Street], as their CEO [Person Alan Mulally] announced first quarter results.

NER as a Sequence Labeling Task

INPUT:

Profits soared at Boeing Co., easily topping forecasts on Wall Street, as their CEO Alan Mulally announced first quarter results.

OUTPUT:

Profits/NA soared/NA at/NA Boeing/SC Co./CC ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

NA	= No entity
SC	= Start Company
CC	= Continue Company
SL	= Start Location
CL	= Continue Location

...

Task Definition: Part of Speech Tags

- Parts of Speech are word categories that have similar morphological or syntactic behavior
- For instance, we can define an adjective as a word which appears as a modifier in a Noun Phrase, as a predicate in a copula clause or in a comparative construction
    - The _________ dog (*black/red/hungry/tall*)
    - The dog is __________ (*black/red/hungry/tall*)
    - X / X+er / X + est (*black/red/hungry/tall*)
  - Circular definition! syntactic environments are defined in terms of grammatical categories

(Morphological Paradigms)

- Another defining principles for grammatical categories is the morphology
- The morphology of a language expresses some semantic distinctions explicitly
- Common distinctions expressed through morphology:
  - Tense (in English: morphology + auxiliary verbs, in Hebrew: morphology)
    - Present/Past, Perfect/Imperfect, Progressive/Non-progressive etc.
  - Modality (in Hebrew + English: secondary verbs)
    - What might, should, must be etc.
  - Evidentiality (in Hebrew + English, but common in many other languages)
    - Something I saw for myself, a rumor, reported speech etc.
  - And many other distinctions

Part of Speech Tags

- Wordforms often have more than one possible POS: *back*
  - The *back* door = *Adj*
  - On my *back* = *Noun*
  - Win the voters *back* = Adverb
  - Promised to *back* the bill = Verb

- The POS tagging problem is to determine the (single) POS tag for a particular word token (instance)

Sources of information

- What are the main sources of information for POS tagging?
  1. Knowledge of word probabilities
     - *man* is rarely used as a verb...
  2. Knowledge of neighboring words

Bill	saw	that	man	yesterday
name	**verb (past)**	**det.**	**noun**	**adverb**
verb	*verb*	*det.*	*verb*	*adverb*
verb	*noun*	*conj.*	*verb*	*adverb*

Word-level Classification

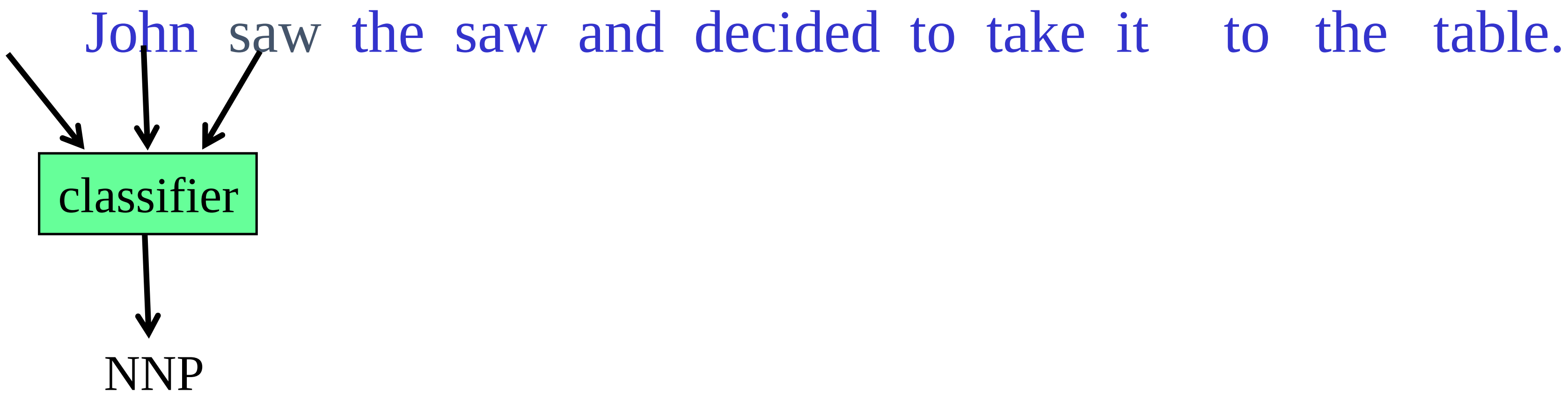
- We can do pretty well by classifying each word on its own
- Features:
  - Lowercase / uppercase
  - Prefixes / suffixes ('-ed' → verb, '-ly' → adverb, 'un-' → adjective)
  - Non-letter characters (periods → acronyms, only numbers → quantifier)

Bill	saw	that	man	yesterday
name	**verb (past)**	**det.**	**noun**	**adverb**
verb	*verb*	*det.*	*verb*	*adverb*
verb	*noun*	*conj.*	*verb*	*adverb*

Not necessarily a bad option. Think of: "find Mary before coming back"

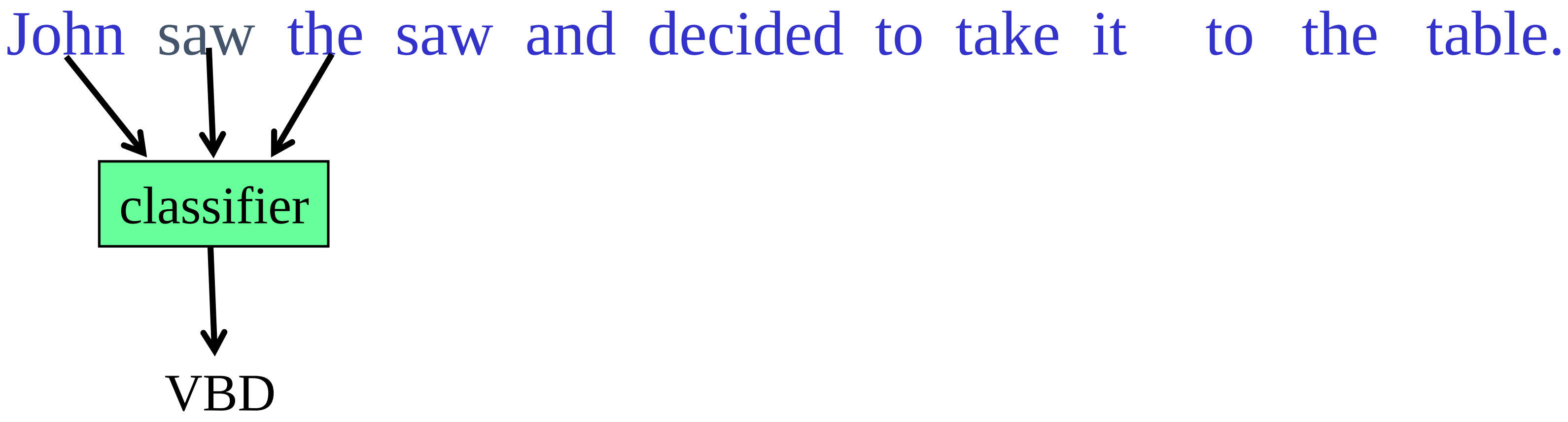
Sequence Labeling as Classification

- **Option 1:** classify each token independently but use as input features, information about the surrounding tokens (sliding window)

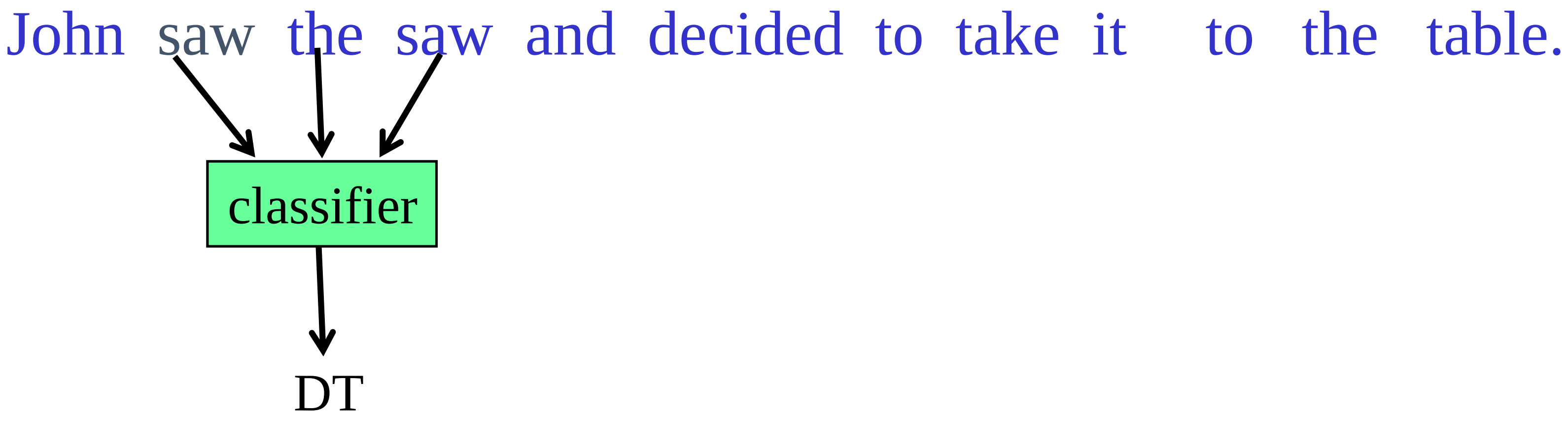


Ray Mooney's slides

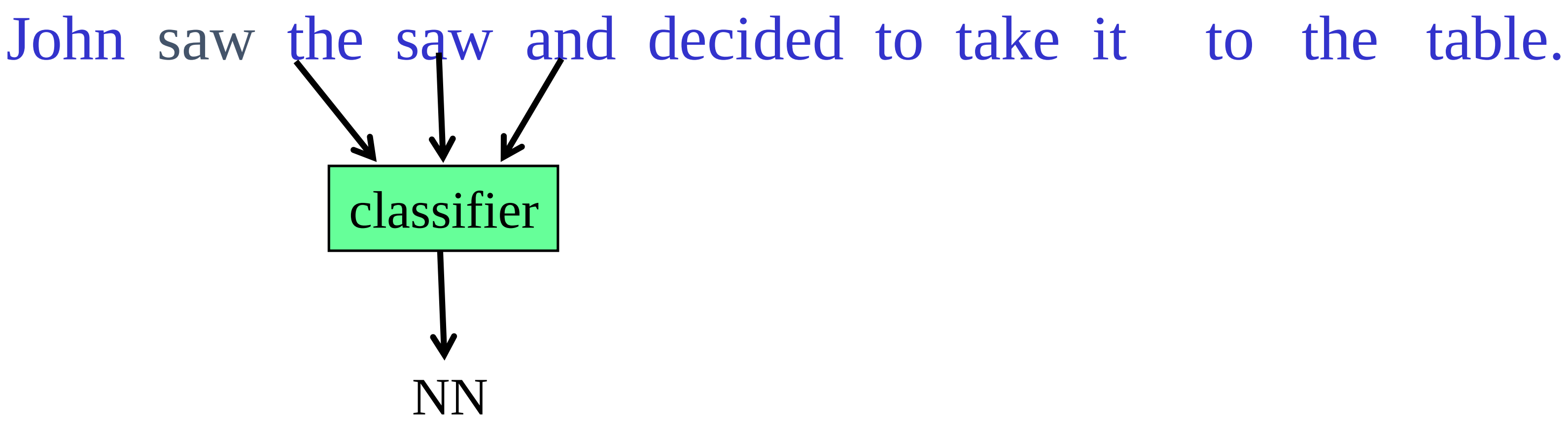
Sequence Labeling as Classification



Sequence Labeling as Classification

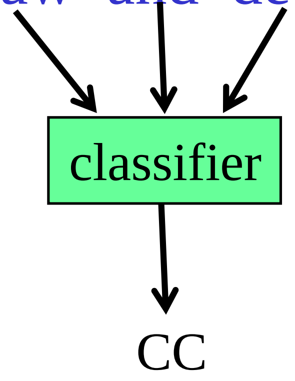


Sequence Labeling as Classification

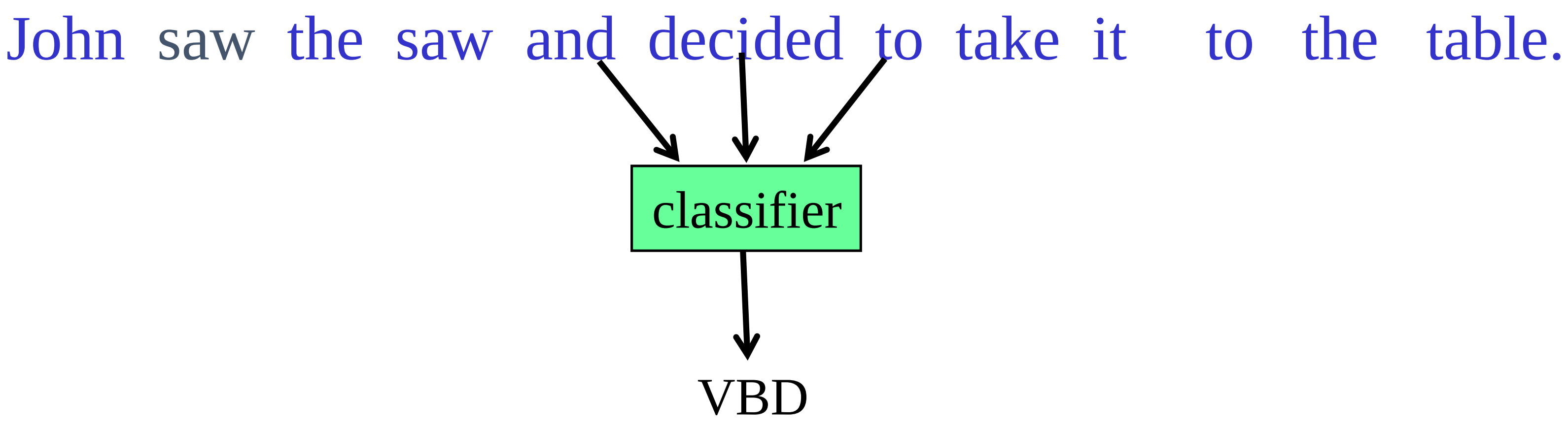


Sequence Labeling as Classification

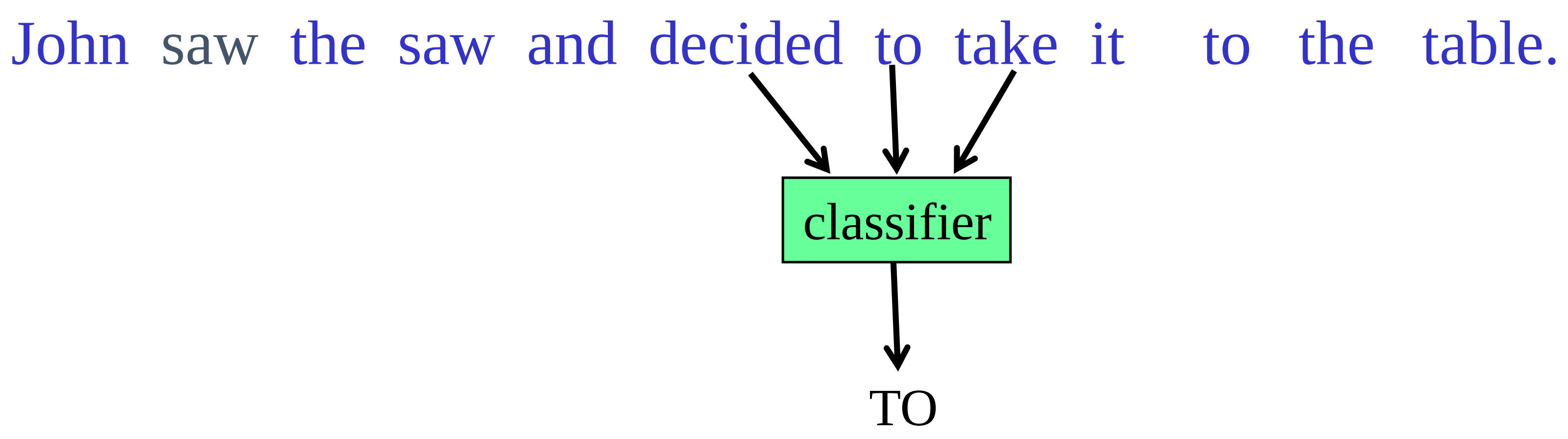
John saw the saw and decided to take it to the table.



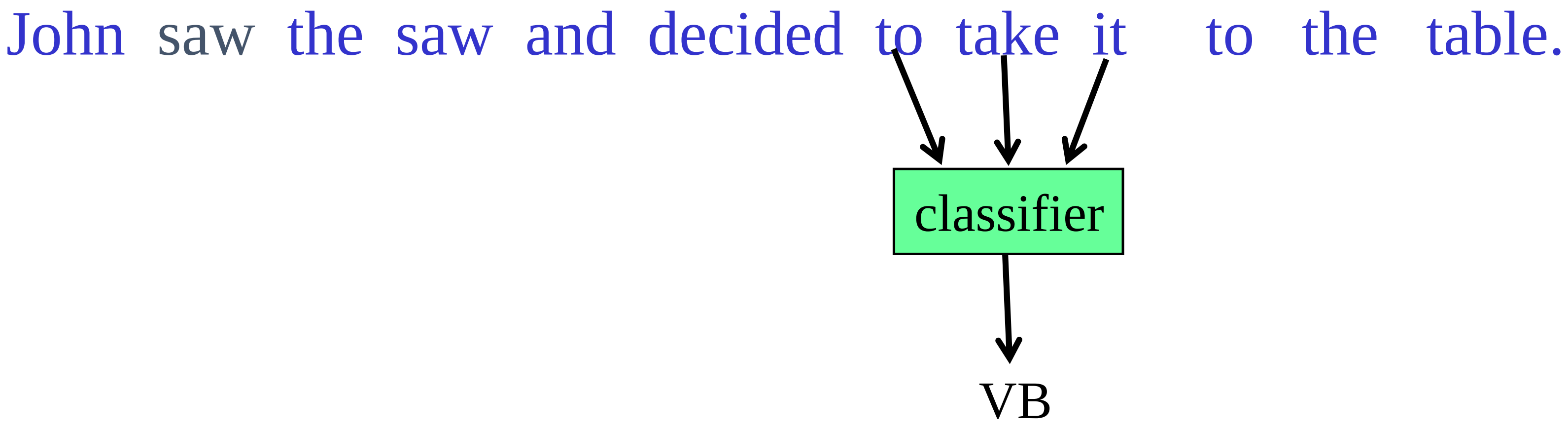
Sequence Labeling as Classification



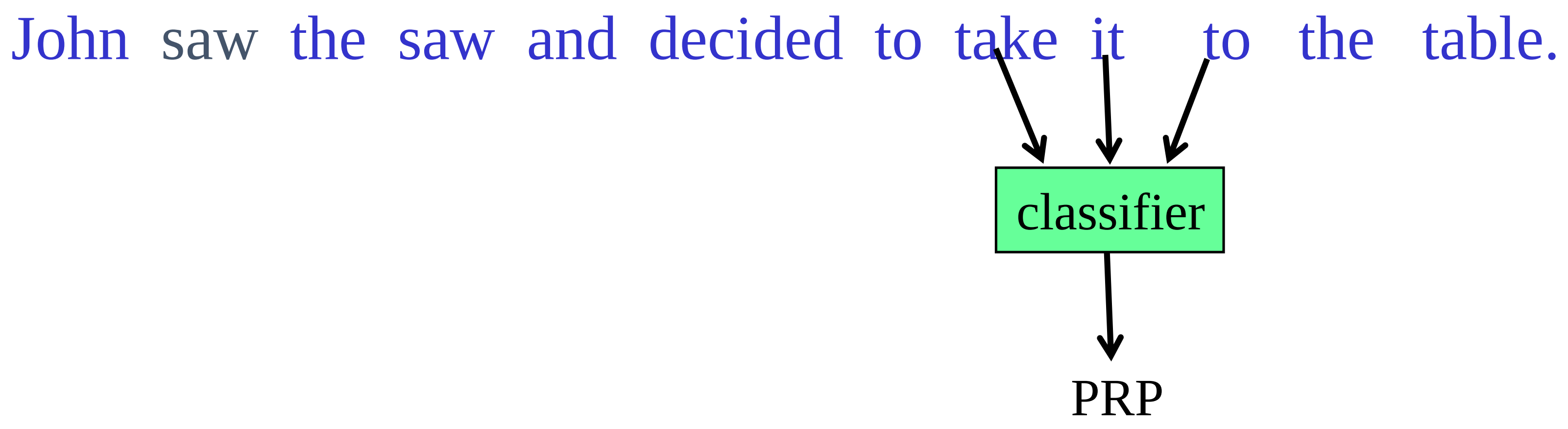
Sequence Labeling as Classification



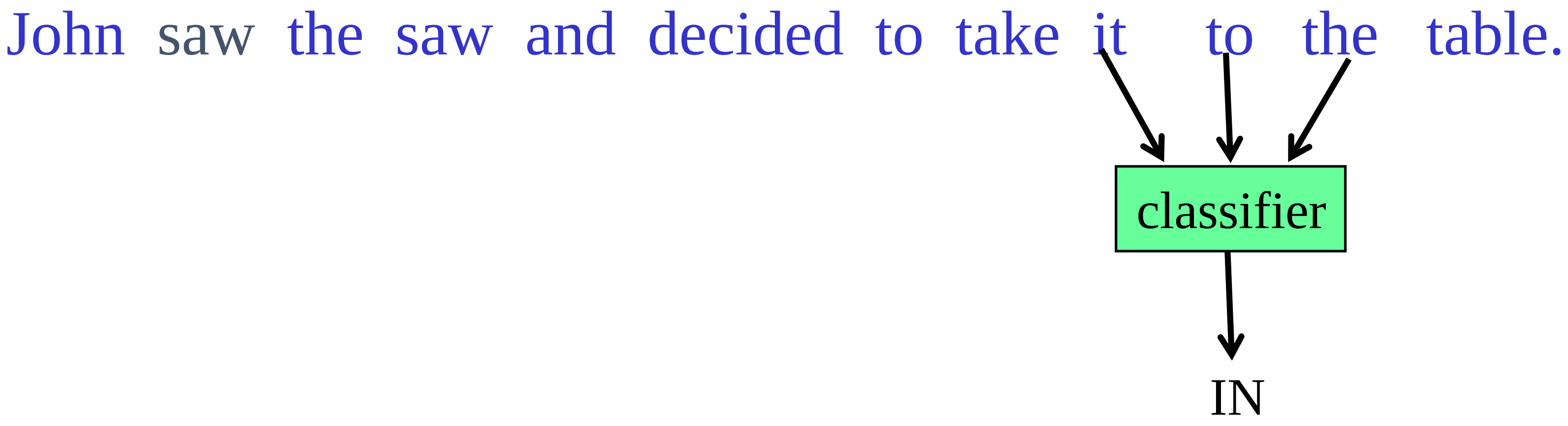
Sequence Labeling as Classification



Sequence Labeling as Classification



Sequence Labeling as Classification



Sequence Labeling as Classification

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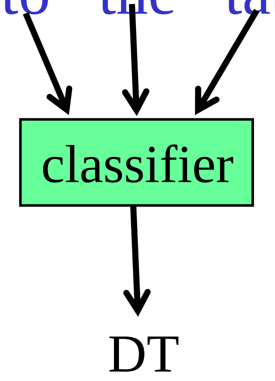

Sequence Labeling as Classification

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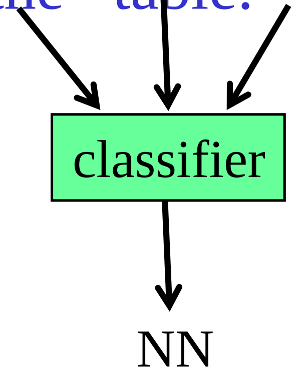
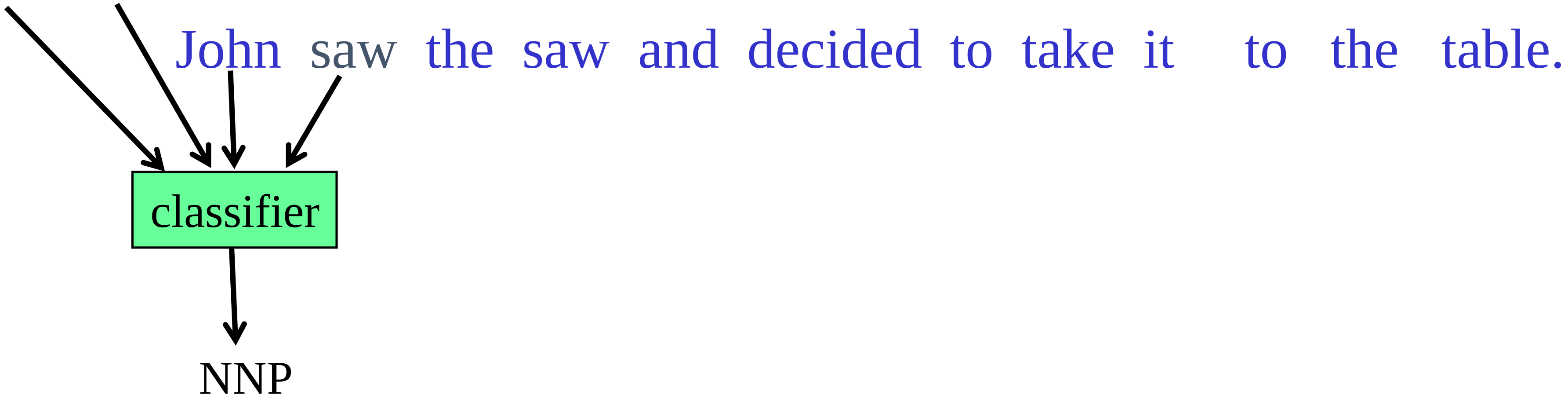


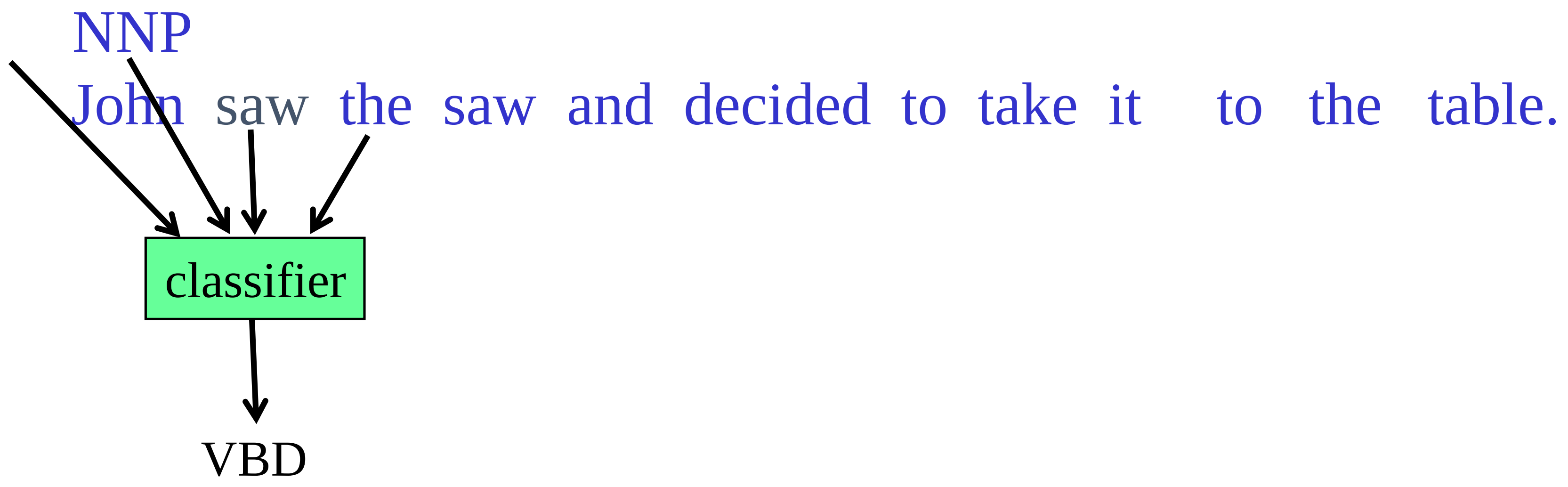
Sequence Labeling as Classification using Outputs as Inputs

- Better input features are usually the categories of the surrounding tokens, but these are not available yet
- Can use category of either the preceding or succeeding tokens by going forward or backward and using previous output

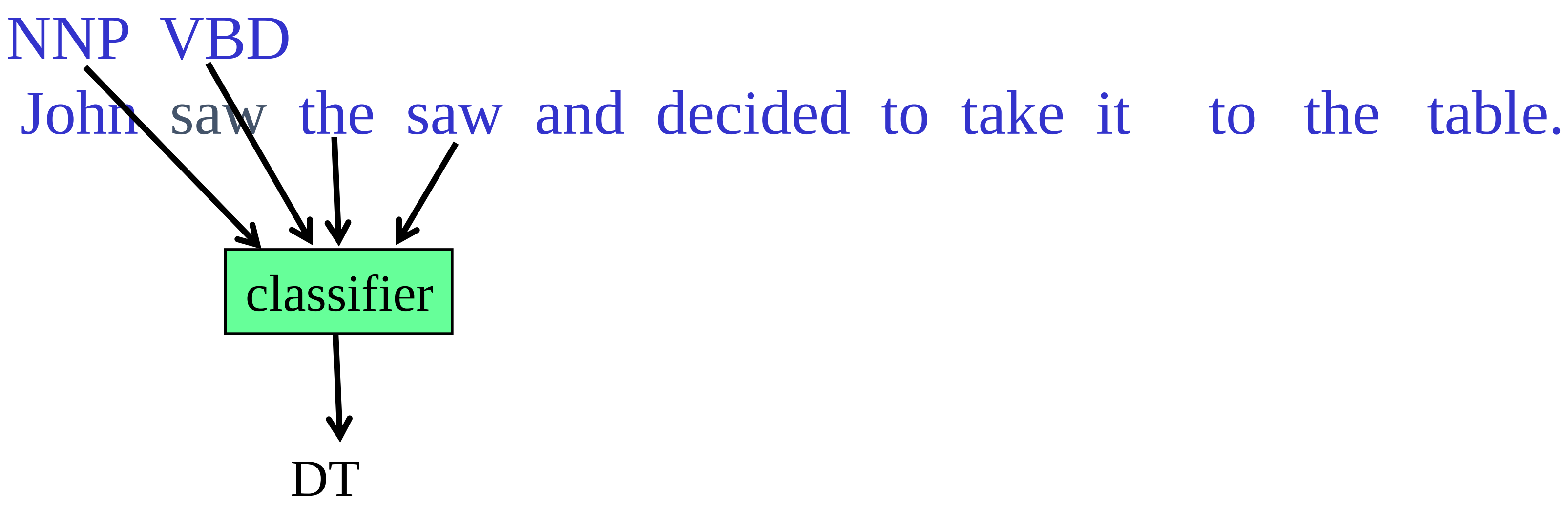
Forward Classification



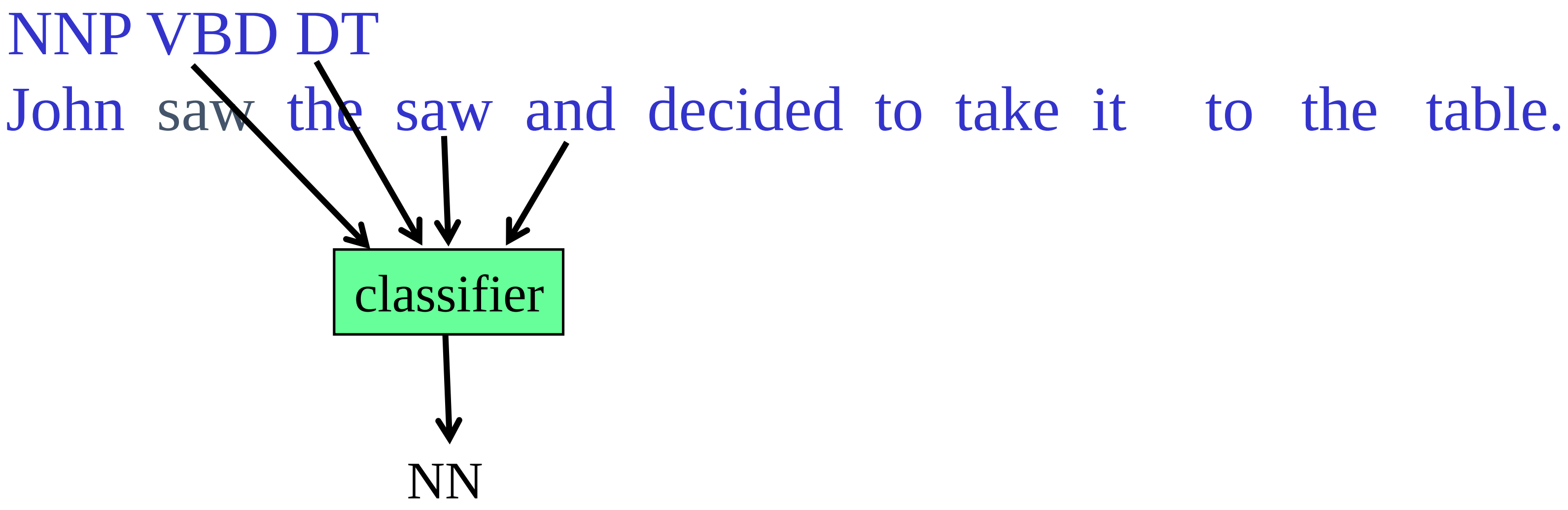
Forward Classification



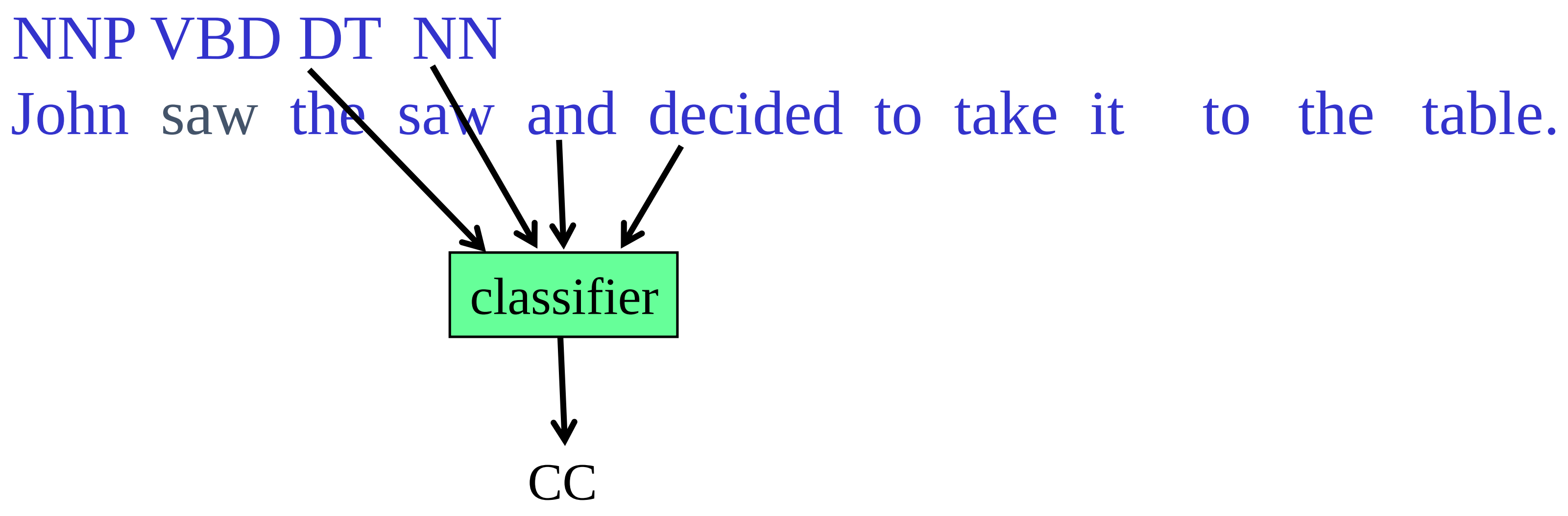
Forward Classification



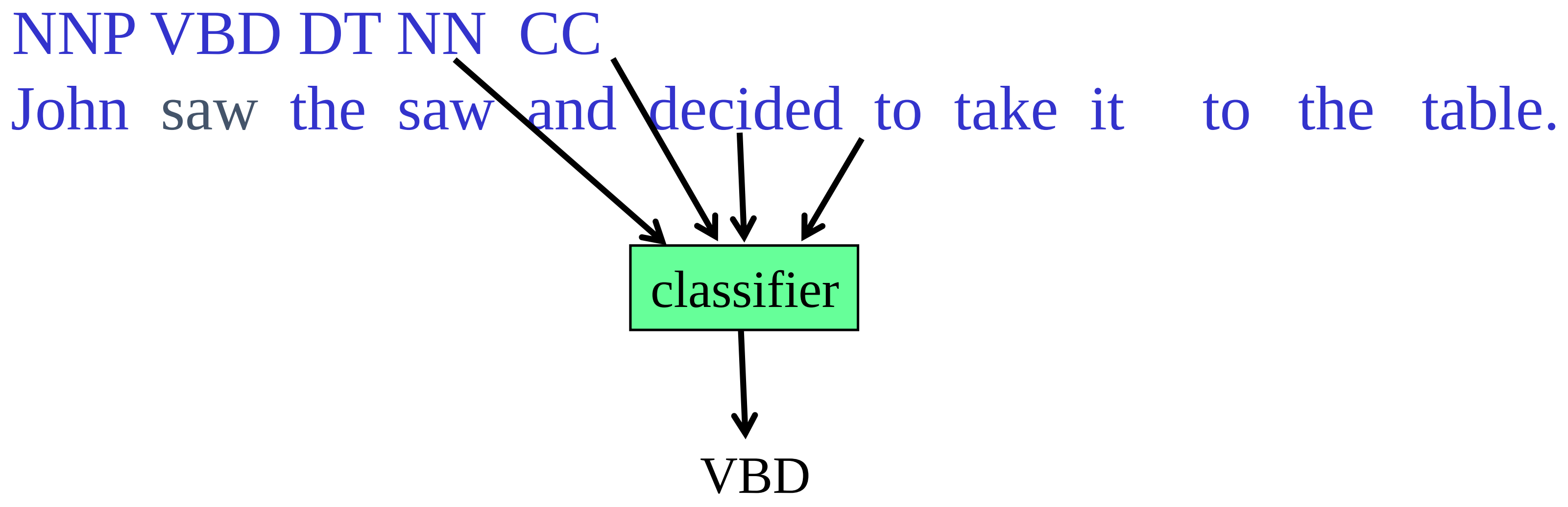
Forward Classification



Forward Classification



Forward Classification



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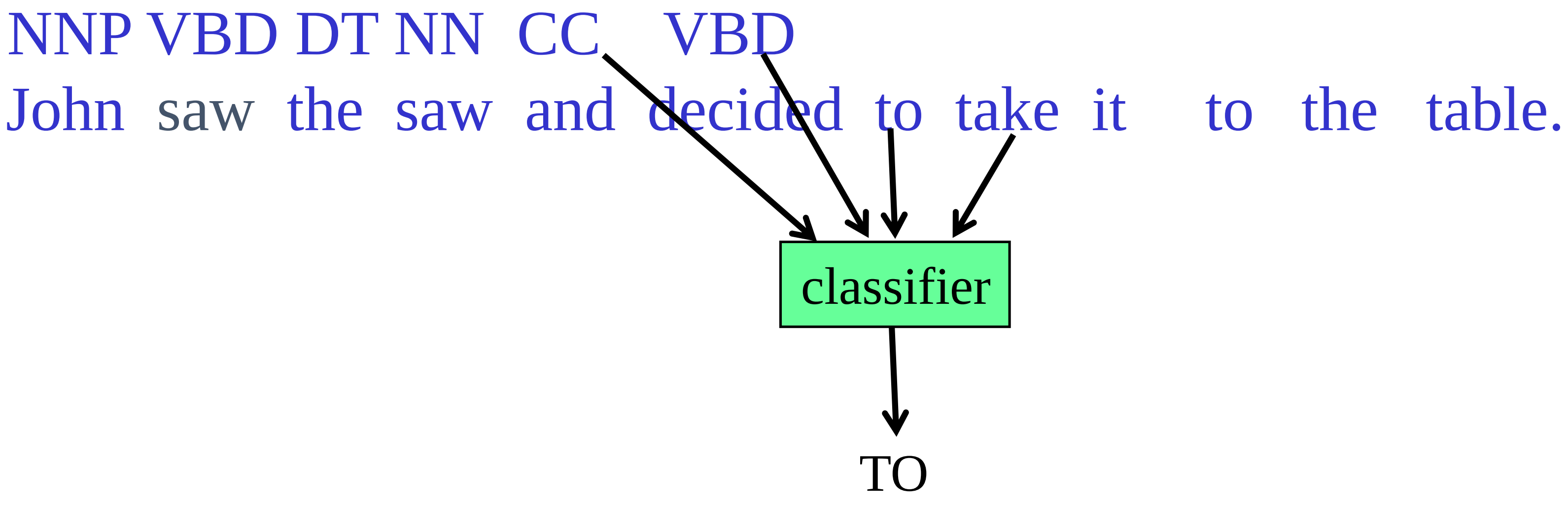
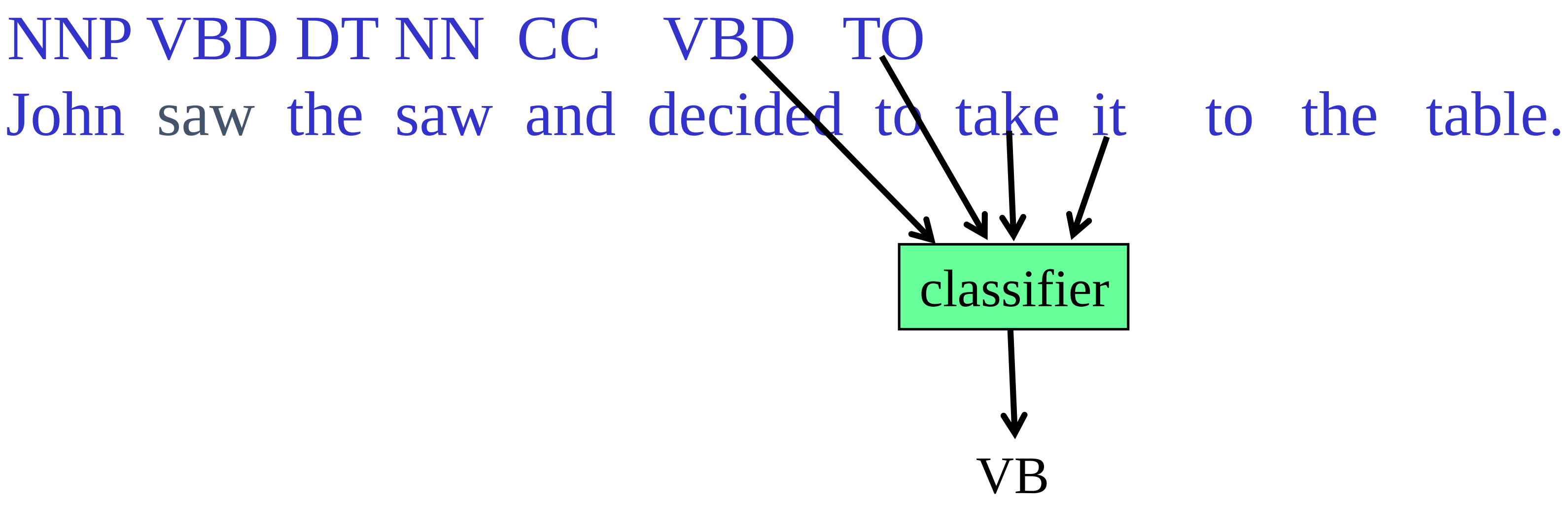


Forward Classification



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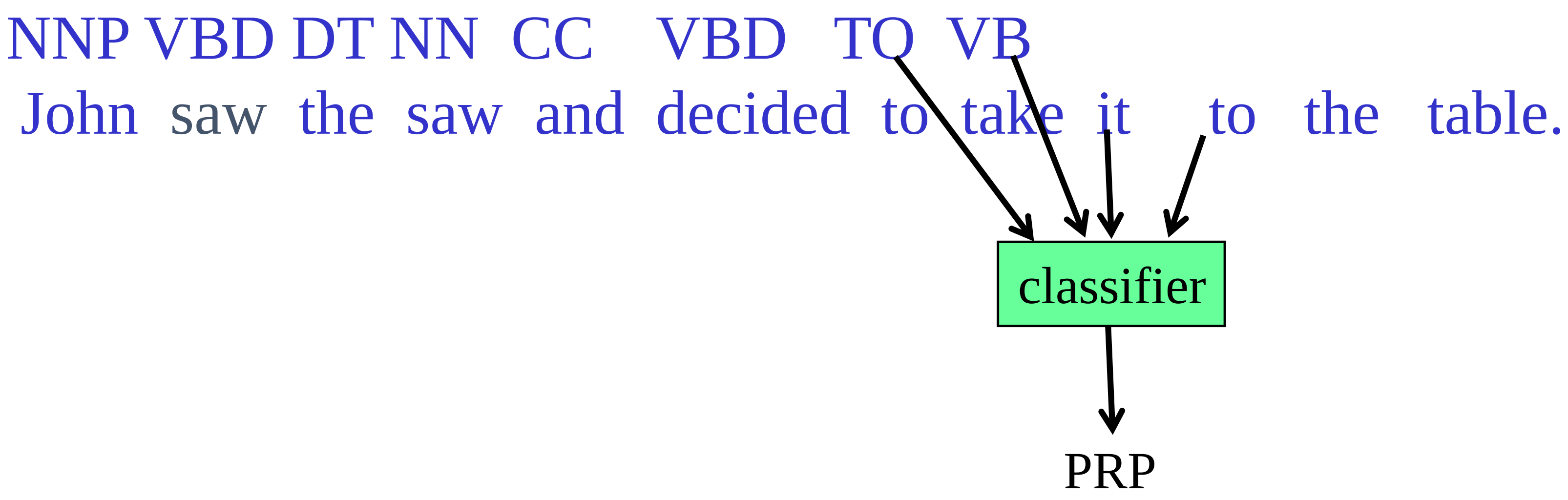

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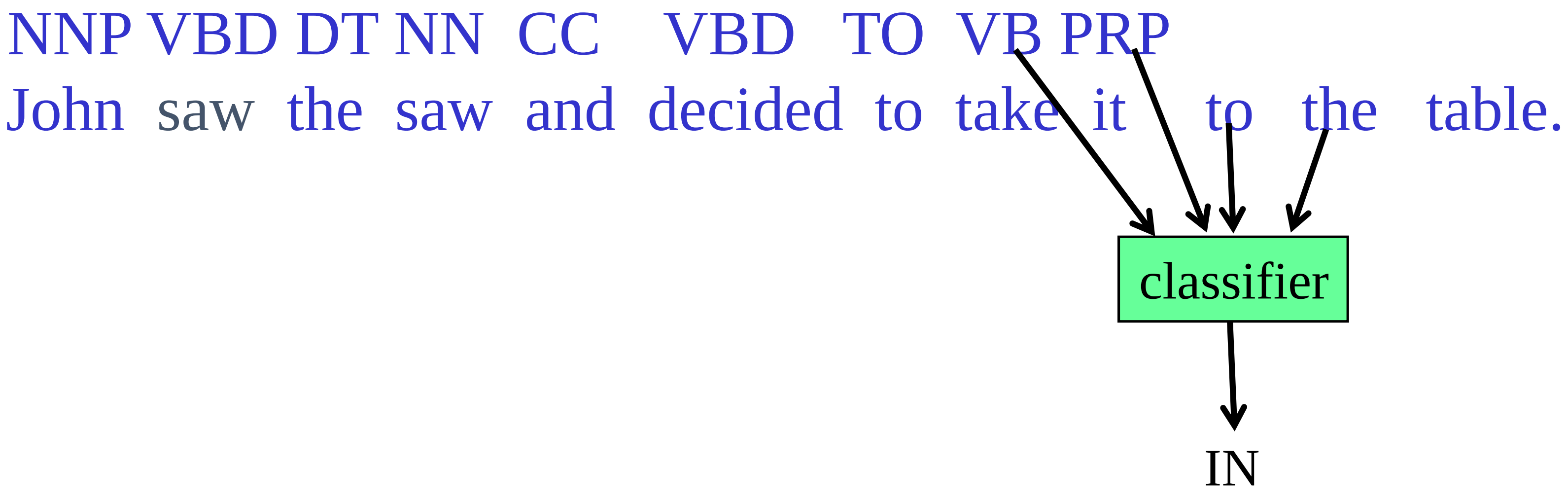
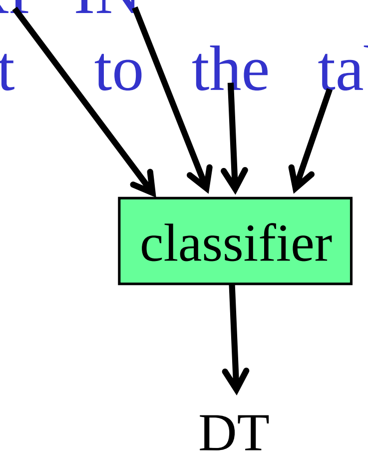


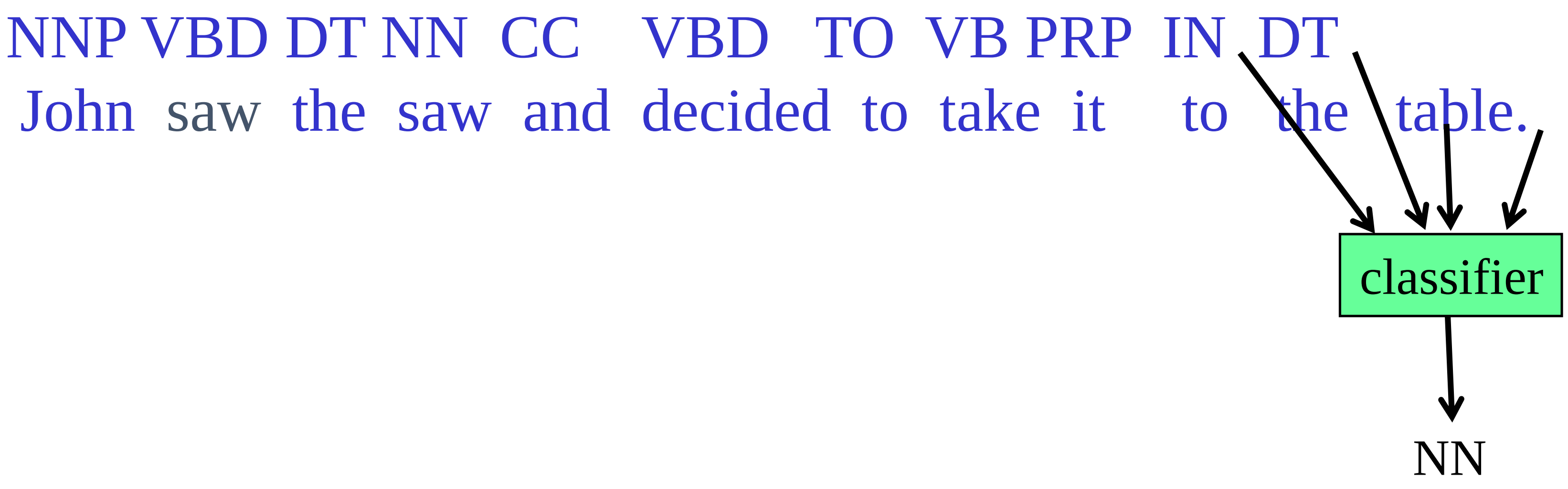
Forward Classification

NNP VBD DT NN CC VBD TO VB PRP IN

John saw the saw and decided to take it to the table.



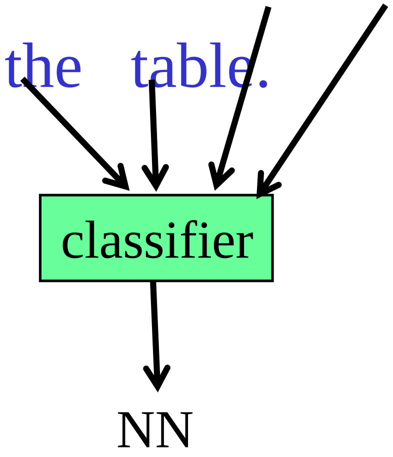
Forward Classification



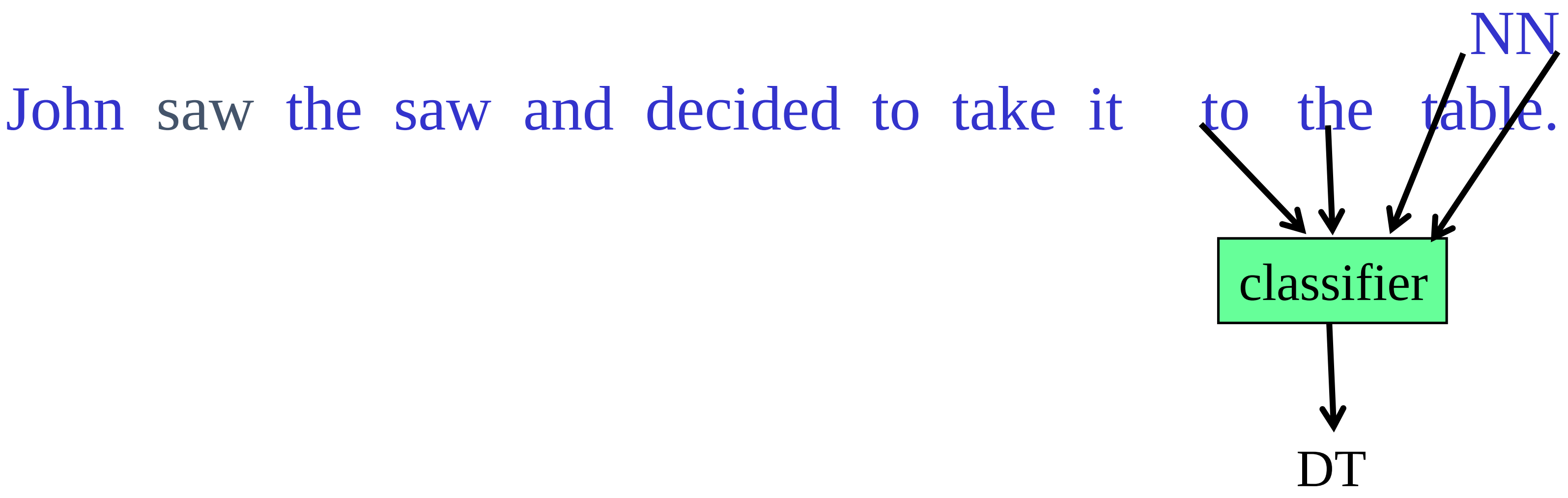
Backward Classification

- Disambiguating “to” in this case would be easier backward

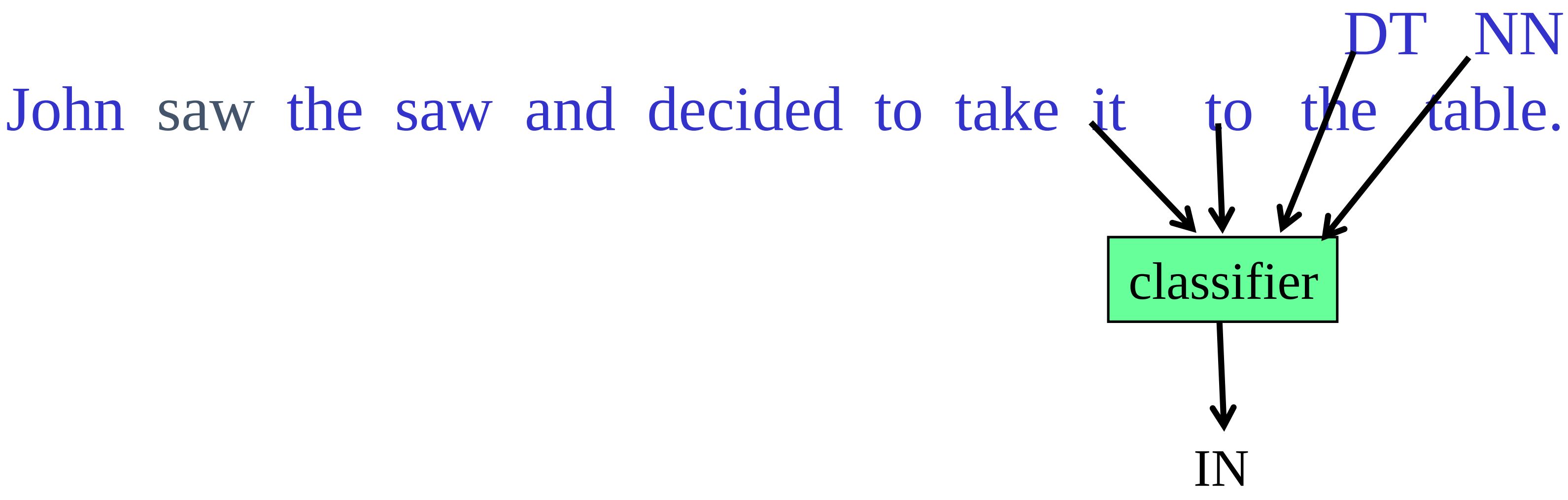
John saw the saw and decided to take it to the table.



Backward Classification



Backward Classification



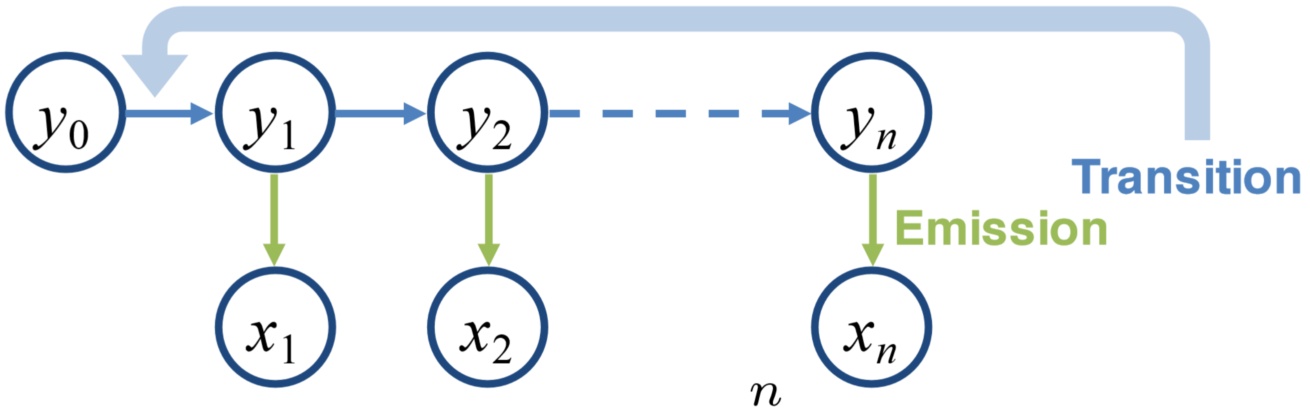
Problems with Sequence Labeling as Classification

- Not easy to integrate information from category of tokens on both sides

- Difficult to propagate uncertainty between decisions and "collectively" determine the most likely joint assignment of categories to all of the tokens in a sequence

Probabilistic Sequence Models

- Probabilistic sequence models allow integrating uncertainty over multiple, interdependent classifications and collectively determine the most likely global assignment
- Classic solution: Hidden Markov Models



$$p(x_1 \ldots x_n, y_1 \ldots y_n) = q(STOP|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i)$$

Markov Models

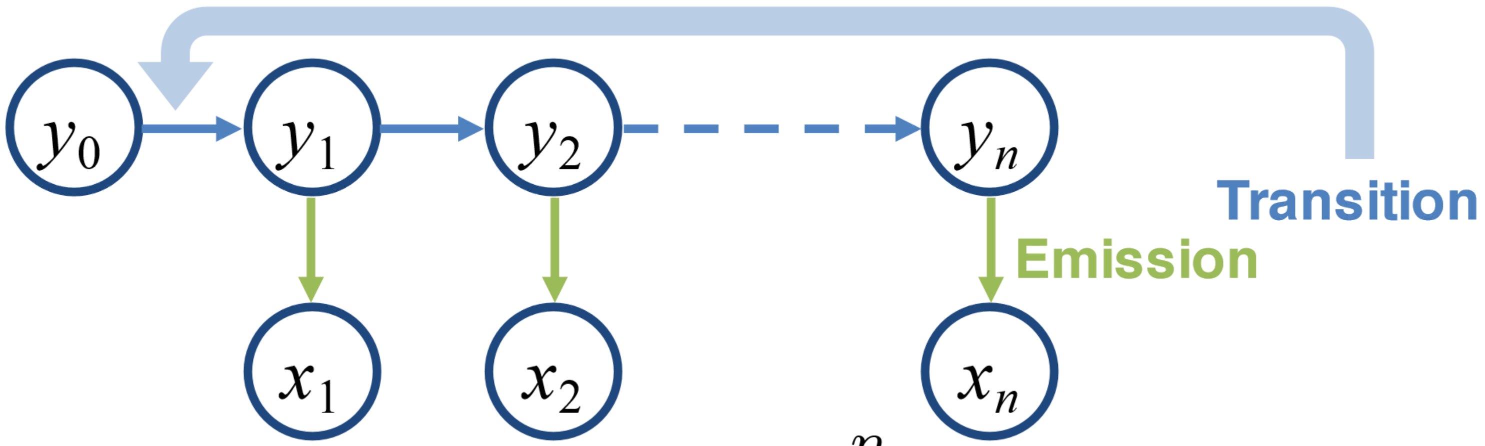
- Recall: Markov models assume the probability of a sequence can be given as a product of the *transition probabilities*

$$\Pr(x_1 x_2 \ldots xn) = \prod_i \Pr(xi | x_{i-1})$$

- For brevity of notation, we sometimes assume that x_0 is a fixed START symbol
- Equivalent to a finite state machine with probabilistic state transitions

Hidden Markov Model

- In a hidden Markov model there are two sequences. One is a Markov chain (the y_i's) and one is *emitted* from the Markov chain (x_i's)



$$p(x_1 \dots x_n, y_1 \dots y_n) = q(STOP|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i)$$

Hidden Markov Model

- Two important modeling assumptions:
  - The word (emitted value) x_i is independent of the rest of the variables given y_i
  - The POS sequence is Markovian, so a POS tag is independent of past tags given the previous tag
- POS tags don't meet either assumption
  - POS tags have dependencies that cannot be bounded in distance
    - The verb *give* usually takes two arguments, where the first can be arbitrarily long
  - Words are dependent on other words in the sentence, even given their POS
    - If you heard the verb *elapsed*, then it is likely the word *time, minutes, seconds or hours* also appeared in the sentence
- **Still, HMMs are a practical option**

Hidden Markov Models (HMMs)

- We have an input sentence $x = x_1, x_2, \ldots, x_n$ (x_i is the i'th word in the sentence)
- We have a tag sequence $y = y_1, y_2, \ldots, y_n$ (y_i is the i'th tag in the sentence)
- We'll use an HMM to define

$$p(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n)$$

  for any sentence $x_1 \ldots x_n$ and tag sequence $y_1 \ldots y_n$ of the same length.
- Then the most likely tag sequence for x is

$$\arg\max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1, y_2, \ldots, y_n)$$

Trigram HMM

For any sentence $x_1 \ldots x_n$ where $x_i \in \mathcal{V}$ for $i = 1 \ldots n$, and any tag sequence $y_1 \ldots y_{n+1}$ where $y_i \in \mathcal{S}$ for $i = 1 \ldots n$, and $y_{n+1} =$ STOP, the joint probability of the sentence and tag sequence is

$$p(x_1 \ldots x_n, y_1 \ldots y_{n+1}) = \prod_{i=1}^{n+1} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{n} e(x_i | y_i)$$

where we have assumed that $x_0 = x_{-1} = *$.

Trigram HMM

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where we have assumed that $x_0 = x_{-1} = *$.

Parameters of the model:

- $q(s|u, v)$ for any $s \in \mathcal{S} \cup \{\text{STOP}\}$, $u, v \in \mathcal{S} \cup \{*\}$
- $e(x|s)$ for any $s \in \mathcal{S}$, $x \in \mathcal{V}$

Trigram HMM

An Example

If we have $n = 3$, $x_1 \ldots x_3$ equal to the sentence *the dog laughs*, and $y_1 \ldots y_4$ equal to the tag sequence D N V STOP, then

$$p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

$$= \ ?$$

Trigram HMM

An Example

If we have $n = 3$, $x_1 \ldots x_3$ equal to the sentence *the dog laughs*, and $y_1 \ldots y_4$ equal to the tag sequence $\mathtt{D\ N\ V\ STOP}$, then

$$
\begin{aligned}
& p(x_1 \ldots x_n, y_1 \ldots y_{n+1}) \\
= \; & q(\mathtt{D}|*, *) \times q(\mathtt{N}|*, \mathtt{D}) \times q(\mathtt{V}|\mathtt{D}, \mathtt{N}) \times q(\mathtt{STOP}|\mathtt{N}, \mathtt{V}) \\
& \times e(\textit{the}|\mathtt{D}) \times e(\textit{dog}|\mathtt{N}) \times e(\textit{laughs}|\mathtt{V})
\end{aligned}
$$

- STOP is a special tag that terminates the sequence
- We take $y_0 = y_{-1} = *$, where * is a special "padding" symbol

Trigram HMM

Smoothed Estimation

$$q(\mathsf{Vt} \mid \mathsf{DT},\ \mathsf{JJ}) = \lambda_1 \times \frac{\mathsf{Count}(\mathsf{Dt},\ \mathsf{JJ},\ \mathsf{Vt})}{\mathsf{Count}(\mathsf{Dt},\ \mathsf{JJ})} + \lambda_2 \times \frac{\mathsf{Count}(\mathsf{JJ},\ \mathsf{Vt})}{\mathsf{Count}(\mathsf{JJ})} + \lambda_3 \times \frac{\mathsf{Count}(\mathsf{Vt})}{\mathsf{Count}()}$$

The transition parameters can be smoothed just like n-gram models

$$\lambda_1 + \lambda_2 + \lambda_3 = 1, \quad \text{and for all } i,\ \lambda_i \geq 0$$

For the emission probabilities we usually use pseudowords

$$e(\mathsf{base} \mid \mathsf{Vt}) = \frac{\mathsf{Count}(\mathsf{Vt},\ \mathsf{base})}{\mathsf{Count}(\mathsf{Vt})}$$

HMM: Smoothing with Pseudowords

Pseudowords use our existing information about the task to categorize words that appeared very few times, or not at all, in the training data

A common method is as follows:

- **Step 1**: Split vocabulary into two sets

  Frequent words = words occurring ≥ 5 times in training
  Low frequency words = all other words

- **Step 2**: Map low frequency words into a small, finite set, depending on prefixes, suffixes etc.

HMM: Smoothing with Pseudowords

[Bikel et. al 1999] **(named-entity recognition)**

Word class	Example	Intuition
twoDigitNum	90	Two digit year
fourDigitNum	1990	Four digit year
containsDigitAndAlpha	A8956-67	Product code
containsDigitAndDash	09-96	Date
containsDigitAndSlash	11/9/89	Date
containsDigitAndComma	23,000.00	Monetary amount
containsDigitAndPeriod	1.00	Monetary amount, percentage
othernum	456789	Other number
allCaps	BBN	Organization
capPeriod	M.	Person name initial
firstWord	first word of sentence	no useful capitalization information
initCap	Sally	Capitalized word
lowercase	can	Uncapitalized word
other	,	Punctuation marks, all other words

HMM: Smoothing with Pseudowords

Profits/NA soared/NA at/NA Boeing/SC Co./CC ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

$$\Downarrow$$

`firstword`/NA soared/NA at/NA `initCap`/SC Co./CC ,/NA easily/NA `lowercase`/NA forecasts/NA on/NA `initCap`/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP `initCap`/CP announced/NA first/NA quarter/NA results/NA ./NA

NA	=	No entity
SC	=	Start Company
CC	=	Continue Company
SL	=	Start Location
CL	=	Continue Location

HMM: Inference

The Viterbi Algorithm

Problem: for an input $x_1 \ldots x_n$, find

$$\arg \max_{y_1 \ldots y_{n+1}} p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

where the $\arg\max$ is taken over all sequences $y_1 \ldots y_{n+1}$ such that $y_i \in \mathcal{S}$ for $i = 1 \ldots n$, and $y_{n+1} = \text{STOP}$.

We assume that p again takes the form

$$p(x_1 \ldots x_n, y_1 \ldots y_{n+1}) = \prod_{i=1}^{n+1} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{n} e(x_i | y_i)$$

Recall that we have assumed in this definition that $y_0 = y_{-1} = *$, and $y_{n+1} = \text{STOP}$.

HMM: Inference

Brute Force Search is Hopelessly Inefficient

Problem: for an input $x_1 \ldots x_n$, find

$$\arg \max_{y_1 \ldots y_{n+1}} p(x_1 \ldots x_n, y_1 \ldots y_{n+1})$$

where the $\arg\max$ is taken over all sequences $y_1 \ldots y_{n+1}$ such that $y_i \in \mathcal{S}$ for $i = 1 \ldots n$, and $y_{n+1} =$ STOP.

HMM: Inference

The Viterbi Algorithm

- Define n to be the length of the sentence
- Define S_k for $k = -1 \ldots n$ to be the set of possible tags at position k:

$$S_{-1} = S_0 = \{*\}$$
$$S_k = S \quad \text{for } k \in \{1 \ldots n\}$$

- Define

$$r(y_{-1}, y_0, y_1, \ldots, y_k) = \prod_{i=1}^{k} q(y_i | y_{i-2}, y_{i-1}) \prod_{i=1}^{k} e(x_i | y_i)$$

- Define a dynamic programming table

$$\pi(k, u, v) = \text{maximum probability of a tag sequence ending in tags } u, v \text{ at position } k$$

that is,
$\pi(k, u, v) = \max_{\langle y_{-1}, y_0, y_1, \ldots, y_k \rangle : y_{k-1} = u, y_k = v} r(y_{-1}, y_0, y_1 \ldots y_k)$

HMM: Inference (example)

An Example

$$\pi(k, u, v) = \text{maximum probability of a tag sequence ending in tags } u, v \text{ at position } k$$

The man saw the dog with the telescope

HMM: Inference

A Recursive Definition

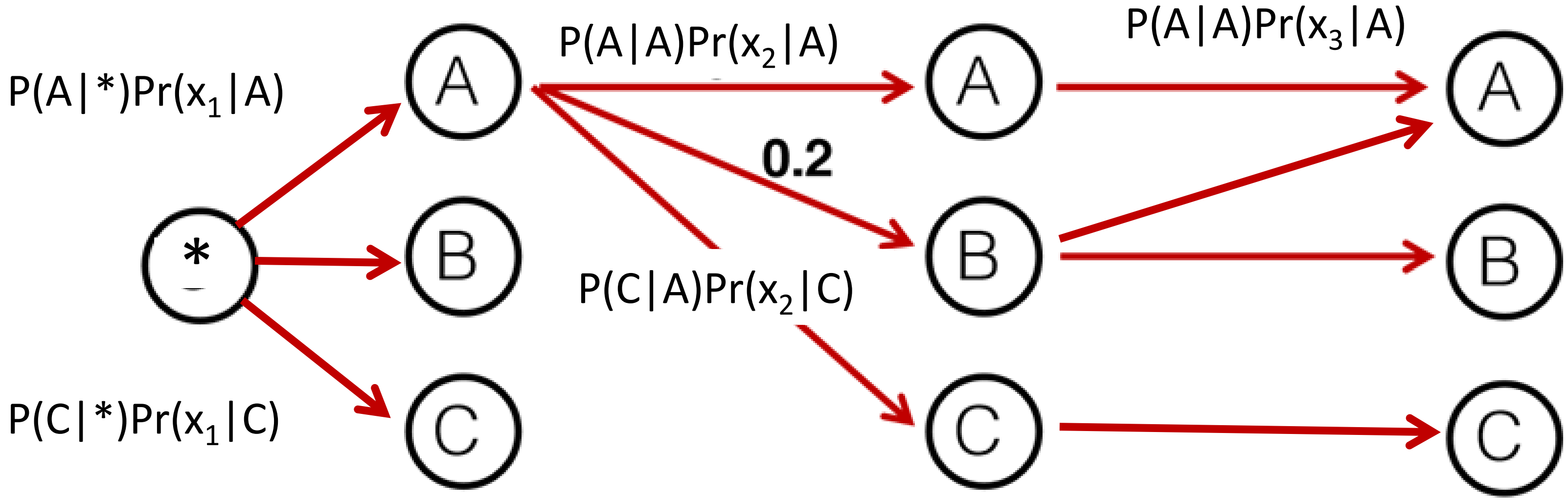
Base case:

$$\pi(0, *, *) = 1$$

Recursive definition:
For any $k \in \{1 \ldots n\}$, for any $u \in \mathcal{S}_{k-1}$ and $v \in \mathcal{S}_k$:

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1, w, u) \times q(v|w, u) \times e(x_k|v)\right)$$

HMM: Graphic Presentation of Viterbi Alg.



- Finding the most likely assignment for $x_1,\ldots,x_n$ that ends with a certain state *s* is equivalent to finding the maximum weighted path from * to the state *s* in the *n-th* layer
  - The weight of a path is here the product of the weights of its edges

→ a dynamic programming approach solves the inference problem

HMM: Inference with the Viterbi Algorithm

The Viterbi Algorithm

Input: a sentence $x_1 \ldots x_n$, parameters $q(s|u,v)$ and $e(x|s)$.

Initialization: Set $\pi(0, *, *) = 1$

Definition: $\mathcal{S}_{-1} = \mathcal{S}_0 = \{*\}$, $\mathcal{S}_k = \mathcal{S}$ for $k \in \{1 \ldots n\}$

Algorithm:

- For $k = 1 \ldots n$,
  - For $u \in \mathcal{S}_{k-1}$, $v \in \mathcal{S}_k$,

$$\pi(k, u, v) = \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1, w, u) \times q(v|w,u) \times e(x_k|v)\right)$$

- **Return** $\max_{u \in \mathcal{S}_{n-1}, v \in \mathcal{S}_n} \left(\pi(n, u, v) \times q(\mathsf{STOP}|u,v)\right)$

HMM: Inference with the Viterbi Algorithm

The Viterbi Algorithm with Backpointers

Input: a sentence $x_1 \dots x_n$, parameters $q(s|u,v)$ and $e(x|s)$.

Initialization: Set $\pi(0, *, *) = 1$

Definition: $\mathcal{S}_{-1} = \mathcal{S}_0 = \{*\}$, $\mathcal{S}_k = \mathcal{S}$ for $k \in \{1 \dots n\}$

Algorithm:

- For $k = 1 \dots n$,
  - For $u \in \mathcal{S}_{k-1}$, $v \in \mathcal{S}_k$,

$$\pi(k,u,v) = \max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1,w,u) \times q(v|w,u) \times e(x_k|v)\right)$$

$$bp(k,u,v) = \arg\max_{w \in \mathcal{S}_{k-2}} \left(\pi(k-1,w,u) \times q(v|w,u) \times e(x_k|v)\right)$$

- Set $(y_{n-1}, y_n) = \arg\max_{(u,v)} \left(\pi(n,u,v) \times q(\mathsf{STOP}|u,v)\right)$
- For $k = (n-2) \dots 1$, $y_k = bp(k+2, y_{k+1}, y_{k+2})$
- **Return** the tag sequence $y_1 \dots y_n$

HMM: Inference with the Viterbi Algorithm

The Viterbi Algorithm: Running Time

- $O(n|\mathcal{S}|^3)$ time to calculate $q(s|u,v) \times e(x_k|s)$ for all k, s, u, v.
- $n|\mathcal{S}|^2$ entries in π to be filled in.
- $O(|\mathcal{S}|)$ time to fill in one entry

HMM: Pros and Cons

- Pros:
  - A simple and widely used model with competitive performance in many sequence labeling tasks
  - Very simple to train
  - Inference is efficient

- Cons:
  - Strong independence assumption
  - Inability to use features to model the emission distribution