Technion – Israel Institute of Technology

HW1

Autonomous Navigation and Perception

086762

Alon Spinner	305184335	alonspinner@gmail.com
Dan Hazzan	308553601	danhazzan@campus.technion.ac.il

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Question 1 – filtering and smoothing

Consider models:

$$p(x_{k+1}|x_k, a_k), p(z_k|x_k)$$

And prior:

$$p(x_0)$$

a. Recursive formulation for posterior over x_k (filtering)

$$p(x_k|a_{0:k-1}, z_{1:k}) \underbrace{=}_{bayes} \frac{p(z_k|x_k, a_{0:k-1}, z_{1:k-1}) \cdot p(x_k|a_{0:k-1}, z_{1:k-1})}{p(z_k|a_{0:k-1}, z_{1:k-1})} =$$

$$\{denoting\ \eta_k = p(z_k|a_{0:k-1}, z_{1:k-1})\}$$

$$\underbrace{=}_{indep.} \eta_k p(z_k|x_k) \cdot p(x_k|a_{0:k-1}, z_{1:k-1}) =$$

$$\underbrace{=}_{MR} \eta_k\, p(z_k|x_k) \int p(x_k|x_{k-1}, a_{0:k-1}, z_{1:k-1}) \cdot (x_{k-1}|a_{0:k-1}, z_{1:k-1})\, dx_{k-1} =$$

$$\underbrace{=}_{indep.} \eta_k \underbrace{p(z_k|x_k)}_{observation\ model} \underbrace{\int \underbrace{p(x_k|x_{k-1}, a_{k-1})}_{motion\ model} \cdot \underbrace{p(x_{k-1}|a_{0:k-1}, z_{1:k-1})}_{posterior\ on\ x_{k-1}} dx_{k-1}}_{\sim weighted\ integration\ for\ motion\ to\ x_k}$$

b. Smoothing - recursive

We developed in the previous section:

$$p(x_{0:k}|a_{0:k-1}, z_{1:k-1}) = \eta_k p(z_k|x_k) \cdot p(x_{0:k}|a_{0:k-1}, z_{1:k-1})$$

Instead of marginalizing, we will apply the chain rule:

$$p(x_{0:k}|a_{0:k-1}, z_{1:k-1}) = \eta_k p(z_k|x_k) \cdot p(x_k|a_{0:k-1}, z_{1:k-1}, x_{0:k-1}) \cdot p(x_{0:k-1}|a_{0:k-1}, z_{1:k-1}) =$$

$$\underbrace{=}_{indep.} \eta_k p(z_k|x_k) \cdot p(x_k|x_{k-1}, a_{k-1}) \cdot p(x_{0:k-1}|a_{0:k-2}, z_{1:k-2})$$

c. Smoothing – full solution

From the recursive formulation in the previous section, we can deduce the following:

$$p(x_{0:k}|a_{0:k-1}, z_{1:k-1}) = p(x_0) \prod_k \eta_k p(z_k|x_k) \cdot p(x_k|x_{k-1}, a_{k-1})$$

Question 2 – Occupancy grid and mapping

Given the map:

$$M = \{m_i\}, m \in \{free, occ\}$$

We denote:

$$p(m) = p(m = free)$$

We also assume an inverse observation model: $p(m|z, x)$

a. Find posterior $p(m|z_{1:k}, x_{1:k})$

$$p(m|z_{1:k}, x_{1:k}) \underbrace{=}_{bayes} \frac{p(z_k|x_{1:k}, z_{1:k-1}, m) \cdot p(m|x_{1:k}, z_{1:k-1})}{p(z_k|x_{1:k}, z_{1:k-1})} =$$

$$\underbrace{=}_{indep.} \frac{P(z_k|x_k, m) \cdot P(m|x_{1:k-1}, z_{1:k-1})}{P(z_k|x_{1:k}, z_{1:k-1})} =$$

$$\underbrace{=}_{bayes} \frac{\frac{p(m|x_k, z_k) \cdot p(z_k|x_k)}{p(m|x_k)} \cdot p(m|x_{1:k-1}, z_{1:k-1})}{p(z_k|x_{1:k}, z_{1:k-1})} =$$

$$= \frac{p(m|x_k, z_k) \cdot p(z_k|x_k) \cdot p(m|x_{1:k-1}, z_{1:k-1})}{p(z_k|x_{1:k}, z_{1:k-1}) P(m|x_k)} =$$

{Assuming: $P(m|x_k) = P(m)$ and denoting $\eta_k = \frac{P(z_k|x_k)}{P(z_k|x_{1:k}, z_{1:k-1})}$}

$$= \eta_k \frac{p(m|x_k, z_k) \cdot p(m|x_{1:k-1}, z_{1:k-1})}{p(m)}$$

As m has a binary state:

$$p(\neg m|z_{1:k}, x_{1:k}) = \eta_k \frac{p(\neg m|x_k, z_k) \cdot p(\neg m|x_{1:k-1}, z_{1:k-1})}{p(\neg m)}$$

b. Log-odds ratio $l(m)$

$$odds(m) = \frac{p(m)}{p(\neg m)} = \frac{p(m)}{1 - p(m)}$$

$$l(m) = \log(odds(m)) = \log\left(\frac{p(m)}{1 - p(m)}\right)$$

$$odds(m|z_{1:k}, x_{1:k}) = \frac{p(m|z_{1:k}, x_{1:k})}{p(\neg m|z_{1:k}, x_{1:k})} = \frac{\cancel{\eta_k} \frac{p(m|x_k, z_k) \cdot p(m|x_{1:k-1}, z_{1:k-1})}{p(m)}}{\cancel{\eta_k} \frac{p(\neg m|x_k, z_k) \cdot p(\neg m|x_{1:k-1}, z_{1:k-1})}{p(\neg m)}} =$$

$$= \frac{p(\neg m)}{p(m)} \cdot \frac{p(m|x_k, z_k)}{p(\neg m|x_k, z_k)} \cdot \frac{p(m|x_{1:k-1}, z_{1:k-1})}{p(\neg m|x_{1:k-1}, z_{1:k-1})} =$$

$$\underbrace{=}_{binary\ state} \frac{1 - p(m)}{p(m)} \cdot \frac{p(m|x_k, z_k)}{1 - p(m|x_k, z_k)} \cdot \frac{p(m|x_{1:k-1}, z_{1:k-1})}{1 - p(m|x_{1:k-1}, z_{1:k-1})}$$

Hence:

$$l_k(m|z_{1:k}, x_{1:k}) = \log(odds(m|z_{1:k}, x_{1:k})) =$$

$$= \log\left(\frac{1 - p(m)}{p(m)} \cdot \frac{p(m|x_k, z_k)}{1 - p(m|x_k, z_k)} \cdot \frac{p(m|x_{1:k-1}, z_{1:k-1})}{1 - p(m|x_{1:k-1}, z_{1:k-1})}\right) =$$

$$= \log\left(\frac{1-p(m)}{p(m)}\right) + \log\left(\frac{p(m|x_k,z_k)}{1-p(m|x_k,z_k)}\right) + \log\left(\frac{p(m|x_{1:k-1},z_{1:k-1})}{1-p(m|x_{1:k-1},z_{1:k-1})}\right)$$

$$= \log\left(\frac{p(m|x_k,z_k)}{1-p(m|x_k,z_k)}\right) - \log\left(\frac{p(m)}{1-p(m)}\right) + \; l_{k-1}(m|x_{1:k-1},z_{1:k-1})$$

c. Posterior over M_k

$$p(M_k|z_{1:k},x_{k:1}) = p(\{m_i\}|z_{1:k},x_{1:k}) =$$

{Assuming independence between cells $\{m_i\}$}

$$= \prod_i p(m_k^i|z_{1:k},x_{1:k}) =$$

{Using section a}

$$= \prod_i \eta_k \frac{p(m_k^i|x_k,z_k)\cdot p(m_k^i|x_{1:k-1},z_{1:k-1})}{p(m_k^i)}$$

Hands On

We successfully installed *Julia* with the required packages to run and debug the attached code, producing the following image: