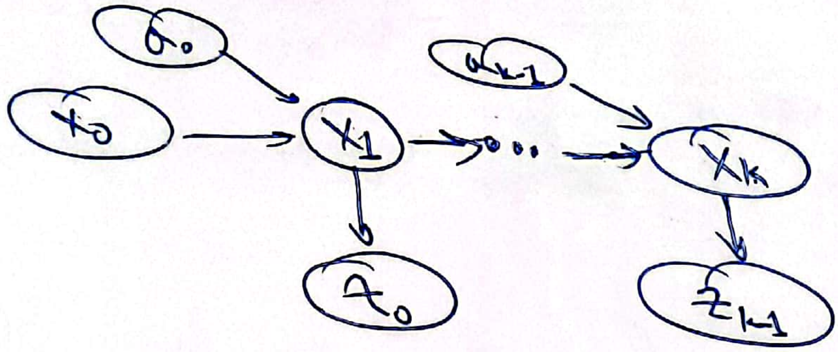
$p(x_{k+1}|x_k, a_k)$; $p(z_k|x_k)$ 1

(a) $p(x_k|a_{0:k-1}, z_{1:k-1}) = ?$



$p(x_k|a_{0:k-1}, z_{1:k-1}) =$

$$\underset{\text{bayes}}{=} \frac{p(z_k|x_k, a_{0:k-1}, z_{1:k-1})\, p(x_k|a_{0:k-1}, z_{1:k-1})}{p(z_k|a_{0:k-1}, z_{1:k-1})} =$$

$$\underset{\text{indep.}}{=} \eta_k\, p(z_k|x_k)\, p(x_k|a_{0:k-1}, z_{1:k-1}) =$$

$$\overset{\text{MR}}{=} \eta_k\, p(z_k|x_k) \int p(x_k|x_{k-1}, a_{0:k-1}, z_{1:k-1}) \cdot$$

$$\cdot\, p(x_{k-1}|a_{0:k-1}, z_{1:k-1})\, dx_{k-1} =$$

$$\overset{MA}{=} \underbrace{\eta_k\, p(z_k|x_k)}_{\text{meas. model}} \overbrace{\int \underbrace{p(x_k|x_{k-1}, a_{k-1})}_{\text{motion model}}\, \underbrace{p(x_{k-1}|a_{0:k-1}, z_{1:k-1})}_{\text{posterior on } x_{k-1}}\, dx_{k-1}}^{}$$

Expectancy on reaching x_k

$$\eta_k = p(z_k|a_{0:k-1}, z_{1:k-1}) = \int p(z_k|x_k)\, p(x_k|a_{0:k-1}, z_{1:k-1})\, dx_k$$

(not useful)

(b) $p(x_{0:k}|a_{0:k-1}, z_{1:k-1}) =$

$$\overset{\text{known from (a)}}{=} \eta_k\, p(z_k|x_k)\, p(x_{0:k}|a_{0:k-1}, z_{1:k-1}) =$$

$$\overset{CR}{=} \eta_k\, p(z_k|x_k)\, p(x_k|a_{0:k-1}, z_{1:k-1}, x_{0:k-1}) \cdot p(x_{0:k-1}|a_{0:k-1}, z_{1:k-1}) =$$

$$\overset{\text{indep.}}{=} \eta_k\, p(z_k|x_k)\, p(x_k|x_{k-1}, a_{k-1})\, p(x_{0:k-1}, a_{0:k-2}, z_{1:k-2})$$

(c) $p(x_{0:k}|a_{0:k-1}|z_{1:k-1}) = \Big\{$ recursive on result from (b)

$$= p(x_0) \prod_k \eta_k\, p(z_k|x_k)\, p(x_k|x_{k-1}, a_{k-1})$$

2. Occupancy grid (prob. robotics 4.2)

$$M = \{m_i\} \; ; \; m \in \{occ, free\}$$

given inverse measurement model $p(m|z,x)$

we denote $p(m = free) = p(m)$

$p(m = occ) = p(\neg m)$

(a) $p(m|z_{1:k}, x_{1:k}) =$

$$\overset{\text{bayes}}{=} \frac{p(z_k|x_{1:k}, z_{1:k-1}, m) \cdot p(m|z_{1:k-1}, x_{1:k})}{p(z_k|z_{1:k-1}, x_{1:k})} =$$

$$\overset{\text{indep.}}{=} \frac{p(z_k|x_k, m) \cdot p(m|z_{1:k-1}, x_{1:k-1})}{p(z_k|z_{1:k-1}, x_{1:k})} =$$

$$\overset{\text{bayes}}{=} \frac{\frac{p(m|z_k, x_k) \cdot p(z_k|x_k)}{p(m|x_k)} \cdot p(m|z_{1:k-1}, x_{1:k-1})}{p(z_k|z_{1:k-1}, x_{1:k})} =$$

$$= \frac{p(m|z_k, x_k)\, p(z_k|x_k)\, p(m|z_{1:k-1}, x_{1:k-1})}{p(m|x_k)\, p(z_k|z_{1:k-1}, x_{1:k})} =$$

$$\left\{ \text{assume} \quad \begin{aligned} p(z_k|x_{1:k}) &= p(z_k) \\ p(m|x_k) &= p(m) \end{aligned} \right\}$$

lighting in room is uniform

$$= \frac{p(m|z_k, x_k)\, p(z_k)\, p(m|z_{1:k-1}, x_{1:k-1})}{p(m)\, p(z_k|z_{1:k-1})} =$$

$$\left\{ \text{denoting} \quad \frac{p(z_k)}{p(z_k|z_{1:k-1})} = \eta_k \right\}$$

$$= \eta_k \frac{p(m|z_k, x_k)\, p(m|z_{1:k-1}, x_{1:k-1})}{p(m)}$$

as m has a binary state:

$$p(\neg m|z_{1:k}, x_{1:k}) = \eta_k \frac{p(\neg m|z_k, x_k)\, p(\neg m|z_{1:k-1}, x_{1:k-1})}{p(\neg m)}$$

binary state m

(b) $odds(m) = \frac{p(m)}{p(\neg m)} = \frac{p(m)}{1-p(m)}$

$$l(m) = \log(odds(m)) = \log\left(\frac{p(m)}{1-p(m)}\right)$$

$$odds(m|z_{1:k}, x_{1:k}) = \frac{p(m|z_{1:k}, x_{1:k})}{p(\neg m|z_{1:k}, x_{1:k})} =$$

$$= \frac{\frac{\cancel{\eta_k}\, p(m|z_k, x_k)\, p(m|z_{1:k-1}, x_{1:k-1})}{p(m)}}{\frac{\cancel{\eta_k}\, p(\neg m|z_k, x_k)\, p(\neg m|z_{1:k-1}, x_{1:k-1})}{p(\neg m)}} =$$

$$= \frac{p(\neg m)}{p(m)} \cdot \frac{p(m|z_k, x_k)}{p(\neg m|z_k, x_k)} \cdot \frac{p(m|z_{1:k-1}, x_{1:k-1})}{p(\neg m|z_{1:k-1}, x_{1:k-1})} =$$

binary state m

$$= \frac{1-p(m)}{p(m)} \cdot \frac{p(m|z_k, x_k)}{1-p(m|z_k, x_k)} \cdot \frac{p(m|z_{1:k-1}, x_{1:k-1})}{1-p(m|z_{1:k-1}, x_{1:k-1})} =$$

$$l_k(m|z_{1:k}, x_{1:k}) = \log(\text{odds}(m|z_{1:k}, x_{1:k})) = \quad \log_e(ab) = \log(a) + \log(b)$$

$$= \log\left(\frac{1-p(m)}{p(m)}\right) + \log\left(\frac{p(m|z_k, x_k)}{1-p(m|z_k, x_k)}\right) +$$

$$+ \log\left(\frac{p(m|z_{1:k-1}, x_{1:k-1})}{1-p(m|z_{1:k-1}, x_{1:k-1})}\right) =$$

$$= \log\left(\frac{p(m|z_k, x_k)}{1-p(m|z_k, x_k)}\right) - \log\left(\frac{p(m)}{1-p(m)}\right) + l_{k-1}(m|z_{1:k-1}, x_{1:k-1})$$

(c) Posterior over distribution over M_k

$$p(M_k|z_{1:k}, x_{1:k}) = p(\{m_k^i\}|z_{1:k}, x_{1:k}) =$$

$$= \{\text{assuming independance between cells}\} =$$

$$= \prod_i p(m_k^i|z_{1:k}, x_{1:k}) - \{2a\}$$

$$= \prod_i \eta_k \frac{p(m^i|z_k, x_k)\, p(m^i|z_{1:k-1}, x_{1:k-1})}{p(m^i)}$$