Technion – Israel Institute of Technology



Vadim Indelman, Luca Carlone, and Frank Dellaert

International Journal of Robotics Research *-* 2015

|  |  |  |
| --- | --- | --- |
| Alon Spinner | 305184335 | alonspinner@gmail.com |
| Dan Hazan | 308553601 | Danhazzan4@gmail.com |

June 19, 2022

# Introduction and Motivation

This work investigates the problem of planning under uncertainty with application to mobile robotics. It proposes a general framework where the robot bases its decisions on probabilistic description of its own state and of external variables of interest - on the general belief space (GBS).  
Common applications for such work include path planning, active exploration and "next best view".

|  |  |  |
| --- | --- | --- |
| A picture containing chart  Description automatically generated | Chart  Description automatically generated with medium confidence | Shape, rectangle  Description automatically generated |
| Next best view | Active Exploration | Path Planning |

There are three limitations that are common in the related work that the authors do not partake in: discretization, maximum likelihood assumption and the availability of prior knowledge. The proposed method relays on a dual-layer architecture: an inner layer that infers a cost of some future GBS for a given set of actions, and an outer layer which optimizes those set of actions.

The authors research their proposed method using a simulation of a UAV that hovers above an unknown environment and is required to reach goals without passing some localization uncertainty limit.

A picture containing text, sky, outdoor, shore

Description automatically generated

# General Belief Space Inference (SLAM)

To understand how one performs inference on GBS in the future, we first focus on GBS inference in the present.  
We focus on the researched example, of a UAV hovering over unknown terrain, the GBS is over a state of UAV poses and landmarks.

Diagram

Description automatically generated

Text

Description automatically generated

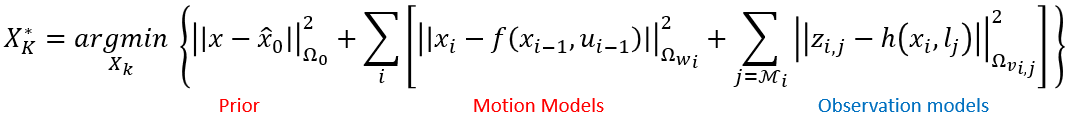
Given a collection of measurements and actions (data history) and motion and observation models, one can factor the inference problem as follows:

Diagram

Description automatically generated

Assuming the priors, motion model and observation models are all gaussian in nature, the posterior distribution over the state is also gaussian. The maximum a-posterior can then be found via expectation maximization, an approach that leads to a solution by least mean squares.

Diagram, schematic

Description automatically generated

As this is a non-linear least squares problem, the popular approach for a solution follows an iterative Gauss–Newton or Levenberg–Marquardt. In those, one is required to linearize the cost function on some point and optimizes around it in optimization step.

Text

Description automatically generatedText

Description automatically generated



A picture containing diagram

Description automatically generated

A key observation of which the authors take advantage, is that for the first optimization iteration, the matrix is not a function of the measurements.

# Planning in the General Belief Space

The authors suggest an algorithm that use the GBS to obtain the best set of actions given an objective function .

A picture containing text

Description automatically generated

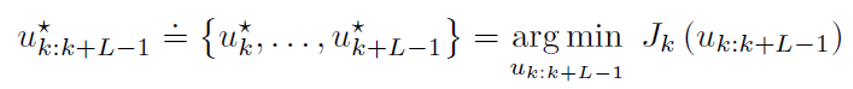
This algorithm use a dual-layer architecture that called inner and outer layers.

A picture containing diagram

Description automatically generated

Text

Description automatically generated



## Outer Layer

The outer layer is responsible for updating the set of actions given the objective function . We want to minimize and in order to do that the authors use gradient decent method.

Text

Description automatically generatedA picture containing diagram

Description automatically generated

Diagram

Description automatically generated with medium confidence

Text

Description automatically generated

Diagram, schematic

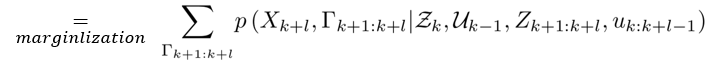
Description automatically generated

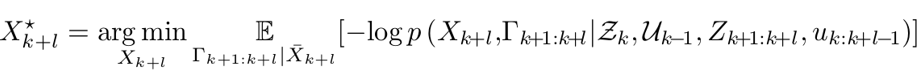
Text

Description automatically generated

## Inner Layer







# Diagram, schematic Description automatically generated

A picture containing text

Description automatically generated

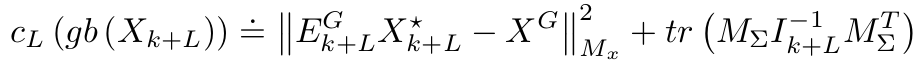
Diagram

Description automatically generated

# Formulating the Objective Function

Given the scenario at hand, the following cost functions were written to follow three principles: Minimizing control effort to create smooth trajectories, minimizing the robot's pose uncertainty, and getting as close as possible to the goal.



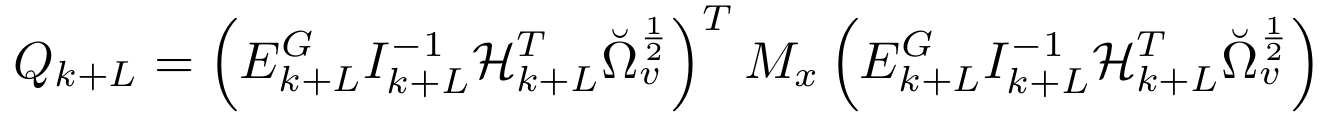


The terms are uncertainty matrices, and is a selection matrix.  
 is a function on the control, calculating the control effort.

Plugging the cost functions into the objective function, we obtain the following:

Diagram, schematic

Description automatically generated



Term is not a function of the measurements, and hence is just a sum over the control effort.   
Under the assumption that one iteration of the gauss-newton suffices for future estimation of the GBS, is also not a function of the measurements, hence the sum in term .  
Term is the outcome of the expectancy over . It splits to two terms, the first is the distance of the last pose of the linearization point from the goal, and the second is a complex term that would nullify under the assumption of maximum likelihood measurements.

## Choice of

To show the effect that each weight has on the planning algorithm, the authors provided an example of a "petal flower" planning problem.  
When is higher, the robot is seen to make bigger turns as changing the turning angle is expensive.

Chart

Description automatically generated

## Choice of and

In order to balance pose uncertainty against the distance to goal in the decision process, a parameter was introduced, which serves as soft upper limit of the pose's trace.  
From a variable determining the uncertainty balance can be derived, where high values of correlate to low uncertainty budget.  
This variable is than used to balance the weight matrices and , and is capped at a maximum of , to avoid negative weights.







# Experimental Evaluation

Beyond other evaluations tests of horizon length, and robustness to different noise models, the authors checked their work in three scenarios (presented below), and compared to 3 planning algorithms (4 in total):

* GBS – the presented method
* GBS-ML – the presented method with the assumption of Maximum likelihood (last term nullifies)
* CNU – the presented method with no weight given to uncertainty in planning (measurements not involved in planning).
* Discrete - method by Kim, A. and Eustice, R. (2013)

Chart, scatter chart

Description automatically generated

In all scenarios the results were consistent:

* CNU provides the largest estimation error
* GBS and GBS-ML are very similar in performance
* Planning time in GBS and GBS-ML increases when turns are required (more iterations in gradient decent)
* The discrete approach can produce a plan that satisfies soft uncertainty, but the 'zig zag' actions make sure that the control effort is high.

# Implementation

We implemented the proposed method via GTSAM in a python environment with some modifications. The robot moves at a constant velocity, and our control is only on the steering, hence, we decided to minimize the distance of any planned pose to goal (rather than the last).  
Additionally, we used middle derivative for the gradient decent, making the process more stable.  
Lastly, using isam2, we had not found a way to ensure only 1 gauss-newton iteration, forcing us to change the cost function to include a different term that encourages loop closures by punishing the distance from informative landmarks:

Chart, scatter chart

Description automatically generated

# Possible Extensions

The authors provide an extensive analysis of the weakness of their method.  
The first relates to the problematic nature of Quadratic costs, where the stimuli to achieve a goal is reduced as one gets closer to it.  
One potential offered solution is use norms, but these are not differentiable at , and the derivation of the cost function will be very difficult.

The second point is concerned with the soft uncertainty constraint built into the planning algorithm. The experimental results showed that the robot does not continue to chance the goal for long after the uncertainty budget is cleared, but perhaps one can ensure that the change to uncertainty reduction planning is immediate. For this the authors propose two solutions, the first is a logarithmic barrier, ensuring infinite objective function cost to break the uncertainty budget, and the second is solving a constraint optimization.

Furthermore, the authors address the problem of "out of range" goals, where the uncertainty budget would not allow the robot to reach the goal, having it go back and forth from it, creating "loopy solutions".  
To avoid this, one can devise an adaptive uncertainty budget, an adaptive , that increases when the goal is deemed to be too far away, and no loop closures for uncertainty reduction can help.

Lastly, the authors talk about increasing the efficiency of the gradient decent solution with smarter optimization solvers presented in recent years.

We would add to those the following:

* exact differentials with Lie Algebra for faster objective function derivative computation. The inference tools already possess those capabilities.
* Smart initialization of the action set for faster convergence. For example, after loop closure has occurred, a good first guess for the next set of actions would be those that go towards the goal. This will reduce the "turning cost" by a lot.
* We noticed that earlier actions in the action set are more significant than later ones due to nature of actions being the derivatives of states. We would think that optimizing for a shorter horizon and then increasing it can be beneficial.