# שיטות נומריות למהנדסים (019003)

## Homework 5- Partial Differential Equations

General remarks:

- Many of the functionalities requested here appear in the recitation material. You may use and alter them as you wish.
- All plots must include grid lines (use *grid on*)
- Dec. .30th Update highlighted

1) The temperature distribution on the plate depicted in Figure 1 can be calculated using Laplace equation $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$

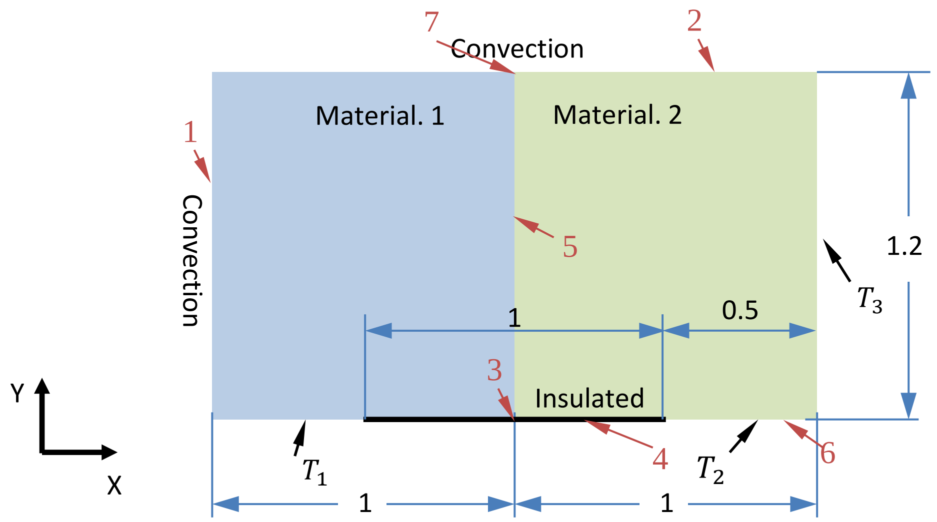



*Figure 1 A plate consisting of two materials with different properties*

Thermal conductivity values are $k_1$ for material 1 and $k_2$ for material 2. The heat flux due to convection through the boundaries (marked convection) is $q=h_c\left(T_a-T_{boundary}\right)$, where $T_a$ is the ambient temperature.

| **Material** | **1** | **2** |
|---|---|---|
| **Thermal conductivity $(k)$** | $k_1$ | $k_2$ |
| **Horizontal step size** | $\Delta x_1=\Delta x$ | $\Delta x_2=\Delta x_1=\Delta x$ |
| **Vertical step size** | $\Delta y_1=\Delta x$ | $\Delta y_2=2\Delta x$ |

a) Show in detail that the approximated pde equation for the nodes at the specified locations on the plate marked 1,2,3,4 are as follows. Use the control volume approach. $i$ and $j$ are in the direction of $x$ and $y$ respectively.

| Location | Equation |
|---|---|
| 1 | $T_{i,j}(h_c\Delta y_1 + 2k_1) = T_a h_c \Delta y_1 + \frac{k_1}{2}T_{i,j-1} + \frac{k_1}{2}T_{i,j+1} + k_1 T_{i+1,j}$ |
| 2 | $T_{i,j}\left(h_c\Delta x_2 + \frac{5}{2}k_2\right) = T_a h_c \Delta x_2 + \frac{k_2}{2}T_{i,j-1} + k_2 T_{i+1,j} + k_2 T_{i-1,j}$ |
| 3 | $T_{i,j}(k_1 + k_2) = \frac{k_1}{2}T_{i-1,j} + \frac{k_2}{2}T_{i+1,j} + \frac{k_1+k_2}{2}T_{i,j+1}$, where $i,j$ are indices of material 1 |
| 4 | $T_{i,j} = \frac{2T_{i+1,j} + 2T_{i-1,j} + T_{i,j+1}}{5}$ |

(Point #3 is the interface of both materials and the insulation).

b) Formulate the pde equation for locations 5, 6 and 7 on the plate. ~~using finite-difference.~~

c) For material 1, show that for the inner points (not on the boundary), the finite difference approach and the control volume approach will result in the same equation.

d) On the Moodle you will find the Matlab function *question1_function.p*. This function solves the PDE for the given plate. (the function is locked, and you will not be able to view or edit the code).
To call the function use the following syntax,

```
[T1,T2]=question1_function(k1,k2,hc,plot_flag,animation_flag)
```

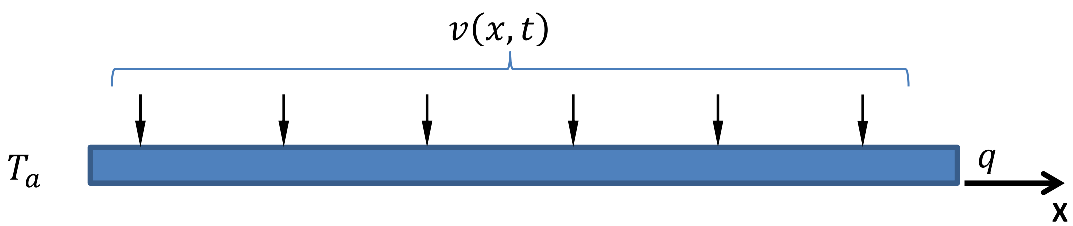
The function uses the values $\Delta x = 0.01, T_1 = 10, T_2 = 50, T_3 = 80, T_a = 100$.
`animation_flag=1` shows an animation of the plate at each iteration.
`plot_flag=1` shows a graph of the final state.
Run the function with the following variables:

| $k_1$ | $k_2$ | $h_c$ |
|---|---|---|
| 1 | 100 | 2 |
| 100 | 1 | 2 |
| 1 | 1 | 100 |

Plot the state for each scenario and discuss the figures.

2) A one-dimensional rod of length 1 is exposed to the following conditions.

- The left end is exposed to ambient temperature $T_a = 0.5$ and undergoes convection with coefficient $h_c = 1$.
- The right end has constant flux $q = 1$.
- The rod is heated by a heat source described by the function $v(x,t)$.
- The initial temperature of the rod is 0.



The heat transfer is described by the PDE $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + v(x,t)$, where $k = 1$. The function $v(x,t)$ is given on the course site HW4_2_v.

a) Write the finite difference equations for the rod using the explicit, implicit and Crank Nicolson methods.
b) Solve the heat transfer problem using the explicit method. Find a pair of Δx and Δt that cause the solution to diverge or oscillate. Draw the temperature (u) as a function of location (x) and time (t).
c) Solve the heat transfer problem using Crank Nicolson method. Draw the temperature (u) as a function of location (x) and time (t).
d) Discuss the results.

3) The heat transfer inside a rectangular plate 2x2 is described by the following PDE

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f(x,y)u,$$

$$\text{where } f(x,y) = \left(\frac{1}{(x-1)^2+(y-1)^2+0.1}\right)$$

Use $k = 2$. The initial and the boundary conditions of the plate are following:

$$u(x,y,0) = 0$$

$$u(x,0,t) = x(2-x)$$

$$\frac{\partial u(0,y,t)}{\partial x} = 0 \qquad\qquad u(2,y,t) = 2-y$$

$$u(x,2,t) = 1$$

a) Given that $dx = dy$. Prove that the ADI equations for the inner points of the plate can be written as:

$$-\lambda T_{i,j-1}^{l+0.5} + 2(1+\lambda)T_{i,j}^{l+0.5} - \lambda T_{i,j+1}^{l+0.5} = \lambda T_{i-1,j}^{l} + 2(1-\lambda)T_{i,j}^{l} + \lambda T_{i+1,j}^{l} + \Delta t f(x_i,y_j)T_{i,j}^{l}$$

$$-\lambda T_{i-1,j}^{l+1} + 2(1+\lambda)T_{i,j}^{l+1} - \lambda T_{i+1,j}^{l+1} = \lambda T_{i,j-1}^{l+0.5} + 2(1-\lambda)T_{i,j}^{l+0.5} + \lambda T_{i,j+1}^{l+0.5} + \Delta t f(x_i,y_j)T_{i,j}^{l+0.5}$$

b) Write $f(x,y)$ as a Matlab function $fval = f(x,y)$. The function will be called in every calculation step.
c) Use the ADI method to find the temperature distribution inside the plate with $\lambda = \frac{1}{6}$ and $\Delta x = \Delta y = 0.1$. Use the appropriate time step.
d) Draw the solution surface on the XY plane 5 times, starting from initial state to steady state. Discuss the graphs.
e) Bonus 5 points: use the PDE toolbox of Matlab to solve the problem. Compare the solution to your manual solution.