שיטות נומריות למהנדסים (019003)

Homework 7 – interpolation and integration

1. In the last lecture we saw how to find the coefficients (weights) and sampling points for the Gauss-Quadrature method with one and two sampling points. You are tasked to develop and find, in detail, the coefficients and sampling points for 3 points.
2. For the following integral $\int_0^2 x^2 \tan^{-1}(x)\, dx$.
    a. Find the exact analytic solution. use any means you like.
    b. Find the approximated solution using **closed** Newton-Cotes method with 3 points.
    c. Find the approximated solution using **open** Newton-Cotes method with 3 points.
    d. Find the approximated solution using **Gauss - quadrature** method with 3 points.
    e. Compare the previous results and discuss.
3. Implements the adaptive gauss quadrature method in Matlab. The implementation will have the following form $I = AdaptQaud(fun, a, b, n, epsilon)$ where,

    I - the integral approximation (output)

    fun – function handle to the integrand function

    a – lower integration limit

    b – upper integration limit

    n – number of sample points per section

    epsilon – the error tolerance for section subdivision

For the weights and sample point either download a table from the web (give ref.) or use the provided function *lgwt.m* from Moodle. For the following integrals, $\int_0^{\pi/4} e^{3x}\, sin(2x)\, dx$ and $\int_0^{\pi} x\, sin(x^2) dx$. With $\epsilon = 10^{-6}$.

a. Investigate the influence of the number of sample points *n* on the performance of the method in term of accuracy and computation speed. Use the appropriate graphs where relevant.
b. Compare your implementation with Matlab's *integral* function, in terms of accuracy and computation time.