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הטכניון - מכון טכנולוגי לישראל

הפקולטה להנדסה אזרחית וסביבתית

Numerical Methods for Engineering – Graduate Course 019003

Prep HW 4: Ordinary differential equations with the initial conditions

Due date: next recitation (in class)

1. Solve the ODE with initial condition $y' = \frac{2}{t}y + t^2 e^t$, $t \in [1,2]$, $y(1) = 0$ using the following methods:
   1) Euler
   2) Runge-Kutta order 2.

   Use the step size h=0.1. the exact solution is $y(t) = t^2(e^t - e)$.

$$y' = \frac{2}{t} y + t^2 e^{t} \quad t \in [1, 2]$$

$$y(1) = 0$$

$$h = 0.1$$

$$y_{sol} = y = t^2 (e^{t} - e)$$

Euler:

$$h = t_i - t_{i-1}$$

$$y'(t_i) = y'_i$$

$$y''(t_i) = y''_i$$

$$y(t_i) = y(t_{i-1}) + h \cdot y'(t_{i-1}) + O(h^2)$$

$$y(1.1) = 0.2718$$

$$y(1.2) = 0.6848$$

$$y(1.3) = 1.2770$$

$$y(1.4) = 2.0935$$

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$y(1.5) = 3.1874$

$y(1.6) = 4.6208$

$y(1.7) = 6.4664$

$y(1.8) = 8.8091$

$y(1.9) = 11.7480$

$y(2) = 15.3982$

2

Runge-kutta 2:

$\dot{y} = f(t, y)$

in general: $y_{k+1} = y_k + \alpha_0 k_0 + \alpha_1 k_1 + \ldots$

$k_0 = h \cdot f(t_k, y_k)$

$k_1 = h f(t_k + \lambda_1 h, y_k + \beta_{10} k_0)$

$k_2 = h f(t_k + \lambda_2 h, y_k + \beta_{20} k_0 + \beta_{21} k_1)$

$\vdots$

$order = 2 \Rightarrow y_{k+1} = y_k + \alpha_0 k_0 + \alpha_1 k_1$

אם משווים עם טור טיילור של פונקציה:

$$\alpha_0 + \alpha_1 = 1$$

$$\alpha_1 \cdot \lambda = \frac{1}{2} \qquad \Rightarrow \quad 3 \text{ משוואות}$$

$$\alpha_1 \cdot \beta_{1,0} = \frac{1}{2} \qquad \text{עם 4 נעלמים}$$

נבחר $\alpha_1 = c \neq 0$ ונקבל:

$$\alpha_0 = 1 - c$$

$$\lambda = \frac{1}{2c}$$

$$\beta = \frac{1}{2c}$$

נבחר $c = \frac{1}{2}$ ונקבל לפתור:

$y(1.1) = 0.34$

$y(1.2) = 0.85$

$y(1.3) = 1.584$

$y(1.4) = 2.584$

$y(1.5) = 3.916$

$y(1.6) = 5.65$

$y(1.7) = 7.78$

$y(1.8) = 10.67$

$y(1.9) = 14.17$

$y(2) = 18.49$