שיטות נומריות למהנדסים (019003)

Preparation Homework #8: Integration

1. Use Trapezoidal rule and Simpson's 1/3 to calculate the integral $\int_0^1 e^x dx$ with the step size $h = 0.1$. Calculate the error bounds for both methods.

2. Use Trapezoidal rule to calculate the integral $\int_0^1 \int_0^1 (x + y) dx dy$ with the step size $h = 0.5$.

Alan Spinnw

305184335

Trapz

$$I = \int_0^1 e^x \, dx = \{h = 0.1, \ f(x) = e^x\}$$

$$= \frac{h}{2}\left(f(x_0) + f(x_n) + 2\sum_{i=1}^{n-1} f(x_i)\right) =$$

$$= \frac{0.1}{2}\left(e^0 + e^1 + 2\left(e^{0.1} + e^{0.2} - \cdots e^{0.9}\right)\right) =$$

$$= 1.7197$$

$$\epsilon = -\frac{1}{12}\frac{(x_n - x_0)^3}{h^2} \cdot f''(c) =$$

$$= \{c = 1 \cdot \text{ for max error}\} = -0.0023$$

Simpson

$$I = \frac{h}{3}\left(f(x_0) + f(x_n) + 4\sum_{i=0}^{\frac{n}{2}-1} f(x_{2i+1}) + \right.$$

$$\left. + 2\sum_{i=1}^{\frac{n}{2}-1} f(x_{2i}) \right) =$$

$$= \frac{0.1}{3}\left(e^0 + e^1 + 4\left(e^{0.1} + e^{3} + \sim e^{0.9}\right) + \right.$$

$$\left. 2\left(e^{0.2} + \sim e^{0.8}\right)\right) = 1.7183$$

$$\epsilon = -\frac{(x_n - x_0)^5}{140 \cdot n^4} \overline{f^{(4)}(c)} = -1.5102 e^{-6}$$

$$I = \int_0^1 \int_0^1 (x+y)\,dx\,dy = F(x,y)$$

$h = 0.5$

$$F(x,) = \frac{0.5}{2}\left(0+1+2\cdot0.5\right) = 0.5$$

$$F(x,\tfrac{1}{2}) = \frac{1}{2}\left(0.5+1.5+2\cdot1\right) = 1$$

$$F(x,1) = \frac{1}{2}\left(1+2+2\cdot1.5\right) = 1.5$$

$$F(x,y) = \frac{0.5}{2}\cdot\left(\tfrac{1}{2}+1.5+2\cdot1\right) = 1$$