הפקולטה להנדסה אזרחית וסביבתית   הטכניון - מכון טכנולוגי לישראל

Numerical Methods for Engineering – Graduate Course 019003

Prep HW 5: Ordinary differential equations with boundary conditions

Due date: next recitation

1. Solve the heat transfer equation $\dfrac{d^2T}{dx^2}+(T_0-T)=0$ in the domain $x\in[0\ 1]$ with the boundary conditions $T(x=0)=0$, $T(x=1)=1$ using the shooting and Euler method with the step size $h=0.5$, $T_0=1$.

2. Solve the previous equation using the finite difference method.

$$\frac{\partial^2 T}{\partial x^2} + (T_0 - T) = 0 \qquad x \in [0, 1]$$

$$T(x=0) = 0 \qquad T_0 = 1$$

$$T(x=1) = 1 \qquad h = \frac{1}{2} \quad \leftarrow \text{Euler}$$
$$\text{shooting}$$

$$y(x_{i+1}) = y(x_i) + f(x_i, y_i) \cdot h$$

$$\frac{\partial T}{\partial x}(x=0) = 0 \quad \text{n.j.}$$

(1) $\frac{\partial^2 T}{\partial x^2}(0) = T(0) - T_0 = -1$

$$T'\left(\frac{1}{2}\right) = T'(0) + \frac{\partial^2 T}{\partial x^2}(0) \cdot h = T'(0) - \frac{1}{2}$$

$$-\frac{1}{2}$$

$$T\left(\frac{1}{2}\right) = T(0) + T'(0) \cdot h = T'(0) \cdot \frac{1}{2}$$

(2) $T''\left(\frac{1}{2}\right) = T\left(\frac{1}{2}\right) - T_0 = T'(0) \cdot \frac{1}{2} - 1$

$$T'(1) = T'(\tfrac{1}{2}) + T''(\tfrac{1}{2}) \cdot h =$$

$$= T'(0) - \tfrac{1}{2} + (T'(0) \cdot \tfrac{1}{2} - 1) \cdot \tfrac{1}{2} =$$

$$= \tfrac{5}{4} T'(0) - 1$$

$$T'(1) = T'(\tfrac{1}{2}) + T'(\tfrac{1}{2}) \cdot h =$$

$$= T'(0) \cdot \tfrac{1}{2} + (T'(0) - \tfrac{1}{2}) \cdot \tfrac{1}{2} =$$

$$= T'(0) - \tfrac{1}{4} = 1$$

$$\implies T'(0) = \tfrac{5}{4}$$

	X=0	X=½	X=1
T	0	2.5/4	1
T'	$\tfrac{5}{4}$	$\tfrac{2}{1}$	0.5625
T''	-1		

$$T'' = T - T_o.$$

$$T(0) = 0 \qquad T_o = 1$$

$$T(1) = 1 \qquad h = \frac{1}{2} \qquad \text{finite difference}$$

$$\delta_i'' = \frac{\delta_{i+1} - 2\delta_i + \delta_{i-1}}{h^2}$$

$$\Rightarrow \text{for } x = \frac{1}{2}:$$

$$\frac{\overset{1}{T(1)} - 2 \cdot T(\tfrac{1}{2}) + \overset{0}{T(0)}}{\underset{\tfrac{1}{4}}{} \, \underset{\curvearrowright}{h^2}} = T(\tfrac{1}{2}) - \overset{1}{T_o}$$

$$1 - 2T(\tfrac{1}{2}) = 4T(\tfrac{1}{2}) - 4$$

$$6T(\tfrac{1}{2}) = 5 \qquad \Rightarrow T(\tfrac{1}{2}) = \frac{5}{6}$$