Technion – Israel Institute of Technology



HW4

Numerical Methods

019003

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| Alon Spinner | 305184335 | alonspinner@gmail.com |
| Oren Elmakis | 311265516 | orenelmakis@gmail.com |

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# Question 1 – Implement your own ODE solver

## We were asked to implement Runge-Kutta-4

The RK-4 method can be written as such:

Where:

Our for-loop based implementation can be found in *MY\_RK4.m*

## Preform order reduction to the pendulum equations to create a state derivative function.

We utilized Matlab’s symbolic toolbox for the task.

First we defined the symbolic variabiles:

syms t th1 th2 dth1 dth2 ddth1 ddth2 real

First, we defined the state equations:

eq1 = (m1+m2)\*l1\*ddth1 + m2\*l2\*ddth2\*cos(th2-th1) == m2\*l2\*dth2^2\*sin(th2-th1)-(m1+m2)\*g\*sin(th1);

eq2 = l2\*ddth2 + l1\*ddth1\*cos(th2-th1) == -l1\*dth1^2\*sin(th2-th1) - g\*sin(th2);

Then we asked the symbolic engine to extract from the first equation:

ddth1\_updated = solve(eq1,ddth1);

We substituted our new term for into equation 2:

eq2 = subs(eq2,ddth1,ddth1\_updated)

And with a couple more operations we managed to extract our state derivative vector such that

ddth2\_updated = solve(eq2,ddth2);

ddth1\_updated2 = subs(ddth1\_updated,ddth2,ddth2\_updated);

y = [th1,th2,dth1,dth2];

dy = [dth1,dth2,ddth1\_updated2,ddth2\_updated]';

We than created the Matlab function *My\_DoublePendulum* from our symbolic expression as follows:

txt = {'This function was created automatically from funcbund.createMyDoublePendulum',...

'accepts column vectors and returns column vectors',...

't is entered to allow function to be called in ODEsolvers even though its not used'};

fdy = matlabFunction(dy,'File','My\_DoublePendulum',...

'vars',{t,y'},...

'comments',txt);

# Question 2 – Solving for initial condition

## Solve for different time steps: and draw the solutions at .

We plotted the pendulum’s state at for the given initial state, and different step sizes. The script can be found in *HW4.m* section *Q2*.

Chart, line chart

Description automatically generated

## Analyze the results

The most precise solution is the one with the smallest time step. Still, we can see that all solutions are really “close” (depending on the application). Extremely stable systems tend to make an easy life with increased time steps, such is our case here.

## Make a video

We made a cool video with flashy colors and all! Watch it in *h1e-2.avi*.  
Here’s a taste:

Chart

Description automatically generated

# Question 3 – Adding a wall to the simulation at

## Update the solver so it stops if one of the balls hits the wall (distance less than )

We expanded upon the function from Question 1 with the following adjustments:

1. turning the *for* loop into a *while* loop (unknown amount of time steps due to reducing time step when approaching obstacles)
2. Adding a *break* rule for collision

If , break out of the solver’s loop.

1. Adding a *continue* rule for reducing step size when approaching obstacle

If , go back to the previous time step and try to continue loop with time step cut in half.

You can view the function in *MY\_RK4\_event.m*

## Plot the solution with the wall

We add two plots below, and a movie called *h1e-1wall.avi*  
There you can verify that the simulation stopped after the second ball hit the wall, and that upon collision our solver reduced its time steps (more dots per at end of simulation, most visible in second plot).

Chart, line chart

Description automatically generated

Chart, scatter chart

Description automatically generated

You can re-create the plots via the third section in *HW4.m*

# Question 4 – Shooting method

## Finding

Simple geometry would show that

## Explain the shooting method in your own words:

The shooting method incorporates our intelligent function zero-crossing or function minimization solvers into our ODE solvers to relax two-sided boundary problems into one-sided. For example, if we are given an end state which we must achieve, and some condition on the initial state:

1. Start with a variable conditioned initial state
2. Evolve your ODE in time until you’ve reached the final time step
3. Calculate the distance between your solution and the wanted end state (You must invent some distance function)
4. Change the variable of your initial state such that it will minimize the distance between the solution at the final time step and the wanted end state, still ensuring that it keeps the condition.
5. Repeat steps 2-4 while choosing initial conditions intelligently with a function minimizing (on the distance function) or zero crossing solver.

## Explain the difficulties in using the shooting method in the case of 4th order differential equation

## Explain the difficulties in using the shooting method in case of the requested collision, regardless of the difficulty in section 3.