Technion – Israel Institute of Technology



HW5

Numerical Methods

019003

|  |  |  |
| --- | --- | --- |
| Alon Spinner | 305184335 | alonspinner@gmail.com |
| Oren Elmakis | 311265516 | orenelmakis@gmail.com |

December 27, 2021

# Question 1 –

The control volume approach is based on assumption of steady state and equilibrium between the fluxes

## Section a

We were asked to use the volume control approach for calculating the points boundary points. We assume the plate is found in steady state, and first we will write the heat balance as follows:

### Point 1 –

We created a control volume of size . The flux direction of the convection is into the control volume, with intersection . Accordingly, the conduction flux in the direction is outward from the cell to cell , and the conduction flux in is moving into the control volume from downward, to .

According to the given grid , after the reduction, we get

Now, we will gather all the variables together and move the variable to the right side

### Point 2 –

We created a control volume of size . The flux direction of the convection is into the control volume, with intersection . Accordingly, the conduction flux in the direction is outward from the cell to cell , and the conduction flux in is moving into the control volume from the left side , to .

After the reduction, according to the second material grid and *,* we get

Now, we will gather all the variables together and move the variable to the right side

### Point 3 –

We created a control volume of size . The insulated edge preventing the heat flux from outward. Accordingly, the conduction flux in the direction is outward from the cell to two cells cell , and the conduction flux in is moving into the control volume from the material 1 in the left side , to .

The grid is after the reduction, we get

Now, we will gather all the variables together and move the variable to the right side

### Point 4 –

We created a control volume of size . The insulated edge preventing the heat flux from outward. Accordingly, the conduction flux in the direction is outward from the cell to two cells cell , and the conduction flux in is moving into the control volume from the material 1 in the left side , to .

According to the second material given grid and *,* after the reduction, we get

Now, we will gather all the variables together and move the variable to the right side

## Section b

We were asked to formulate the equations of points 5,6,7:

### Point 5 –

We created a control volume of size . We assume the flux is flows from left to right in direction , and from downward to upward in direction

The grid size is after the reduction, we get

Now, we will gather all the variables together and move the variable to the right side

### Point 6 –

According to the figure of the plate

### Point 7 –

We created a control volume of size . We assume the flux is flows from left to right in direction , and from downward to upward in direction

The grid size is after the reduction, we get

Now, we will gather all the variables together and move the variable to the right side

## Section c

We will open the equation using the finite difference approach using second order central difference.

The steps of and are equal, thus

We can arrange the equation as follows (keep remember that in material 1(

Multiply by , and we get the control volume scheme

# 



# Question 2 –

## Section a

The equation is given as

The time first derivative is based on forward differentiation

Accordingly, the explicit form of the partial second derivative is a upward triangle, with time propagation on the top, for index

The implicit representation form is based on inverted triangle,

The Crank Nicolson method uses dummy point that is in the middle of the time step, and represented as a combination of the time propagation and the former state:

## Section b

We were asked to solve the heat transfer function using the explicit method.

First, we have to formulate the boundary conditions of both sides:

In according to the conservation of heat equation:

From Fourier’s law

In , the outward flux is known

And the central nodes equations for temperature distribution

The convergence and stability of the explicit method is

As a result using and causing to diverge.

The following figure demonstrate the time-location distribution of the 1D heated rod with and :

Rectangle

Description automatically generated

## Section c

The boundary conditions in the same manner as in section a:

However now we don’t have the

And the boundary condition of the end of the rod:

The central nodes equation is:

The following figure demonstrate the time-location distribution of the 1D heated rod with and :

Rectangle

Description automatically generated

## Section d

In this question we were asked to implement the methods of explicit and crank Nicolson for solving the 1D heated rod pde. The main difference between the methods are the performing complexity and the stability. In compare to the Crank Nicolson which requires tridiagonal algebraic solution the explicit method implementation is straight forward. However, in contrast to the Crank Nicolson method that has a unconditionally stability the explicit approach is restricted to stability term of .

# Question 3 –

## Section a

Given that we have to prove the ADI scheme for inner points, the PDE equation for plate

The first step is forward differentiation of

time propagation of in the direction

And using second order central difference OF explicitly

Consequently, with and define the results

after some order

The second step from

time propagation of in the direction

With remaining the same.

Consequently, with and define the results

And after order

## Section b

The initial condition is

With the initial condition we got three trivial boundary conditions

And another Neumann condition

We will use dummy boundary condition, outside the domain (to keep the accuracy of the method, using central derivative)

Now, we can substitute to the ADI equation

## Section d

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |