Technion – Israel Institute of Technology



HW7

Numerical Methods

019003

|  |  |  |
| --- | --- | --- |
| Alon Spinner | 305184335 | alonspinner@gmail.com |
| Oren Elmakis | 311265516 | orenelmakis@gmail.com |

January 23, 2022

# Question 1: Gauss-Quadrature with 3 sampled points

Given points to sample from, we will construct a polynomial of degree .

Given an integral in bracket , one can introduce a variable change such that after, values in it will range from

As such, we will only practice on the integral in bracket .

We assume the integral could be written as a sum of weights:

Integrating over the polynomial (left side):

Substituting the polynomial values in the sum (right side):

Comparing left side evaluation to the right-side evaluation:

Comparing Coefficients:

We have a system of equations – 6 equations and 6 variables .

Using MATLAB’s symbolic solver to write a general script for points.  
The results for are as follows:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

# Question 2: Solving

In this question we will consider the bracket , computing the integral over with points.

## Analytical Solution:

We used MATLAB to solve the symbolic integral:

## Closed Newton-Cotes with 3 points

Closed Newton-Cotes is the Simpson method

## Open Newton-Cotes with 3 points

## Gauss-Quadrature with 3 points

From Question we know that:

Computing from :

To avoid losing accuracy to digit precession, we use the long format of double for presentation.

## Comparing the methods

Comparing the results, we produced the following table:

Graphical user interface, application

Description automatically generated

The first thing that’s visible right off the bat, is that GQ has the smallest error by a large margin, following NC-open

Having NC-open provide better result than NC-closed is not assured, but also not surprising, as both methods have their errors proportional to (assuming 3 points), but the step size in NC-open is smaller.

Additionally, we have seen in the lectures that the GQ method can integrate a polynomial of degree exactly, while the methods can do the same for a polynomial of degree . As such, it is of no surprise, that it outperforms them.

We decided to plot the function and the integration points for further insight, presenting the figure below:

Chart, line chart

Description automatically generated

The integration bracket, the function’s derivative is monotonically increasing. As such, having well placed integration points near the higher end of the bracket will result in a better integration estimation.  
Unfortunately, we weren’t able to obtain any further intuition from the figure.

One final note:  
We decided against evaluating and comparing the computation time for each method, as they are all . In the lecture we were presented with the fact that the GQ method is more efficient.

# Question 3: Adaptive Gauss Quadrature

We were asked to implement the function *AdaptQuad(fun,a,b,n,epsilon)* which returns the integral approximation *I*, of function *fun* (sometimes denoted *‘f’*), over bracket *[a,b]* given *n*, number of sample points per section, and *epsilon*, the error tolerance for section subdivision.

In “Numerical Methods for Engineers” 6th addition, we are given the error term for the GQ method in equation . It shows as it appears below:

A picture containing diagram

Description automatically generated

Where

Denoting:

One can write

We would like to find an error term such that it is a function of the difference between our integral estimates: .

Going back to the error term computation, we know that

As such, we can write:

We can also write the following:

Adding and subtracting equations as follows:

Denoting:

The same can be written as:

Given that

We are required to compute the integral with a parameter *epsilon* that is “the error tolerance for section subdivision”. In other words, for a middle value , it is required that:

Adding and subtracting to the left side of the equation and using the triangle inequality on the term:

## Solving

## Solving