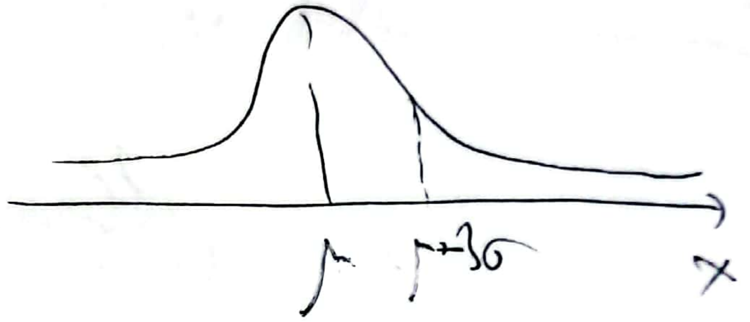
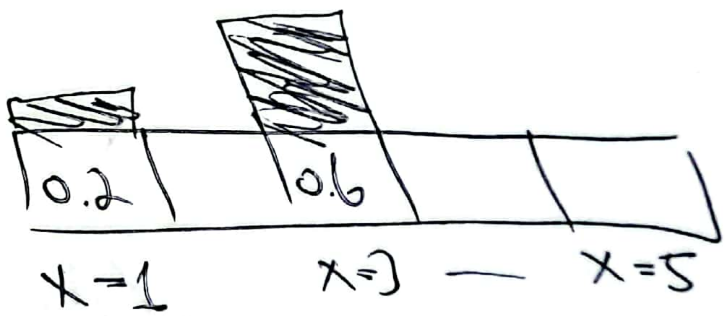
תרגול רביעי:

מטרה:
- להבין על מה שמנסה Kalman Filter
- להבין מה זה Smoothing
- איפה שתלויים ← פרובביליסטיקה.

$p(X=x)$



$\int p(x=x)dx=?$



$p(X=x) \sim p(x)$

$p(X=x, Y=y)$:

x=1	x=2	x=3	x=4	x \ y
4/32	2/32	1/32	1/32	y=5
2/32	4/32	4/32	1/32	y=10
2/32	2/32	2/32	4/32	y=15
8/32	0	0	0	y=20

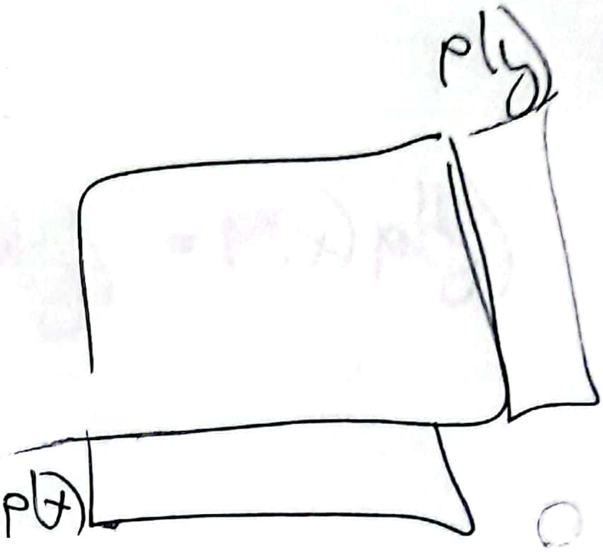
חוקים:

marginilization:

$$\sum p(x,y) = 1$$

$$p(x) = \sum_{y} p(x,y)$$

$$p(y) = \sum_{x} p(x,y)$$



$$p(x) = \int_{y} p(x,y)\,dy$$

chain rule:

$$p(x,y) = p(x|y)\,p(y) =$$
$$= p(y|x)\,p(x)$$

$\Rightarrow$ <u>bayes:</u> $p(x|y)\,p(y) = p(y|x)\,p(x)$

$$p(x|y) = \frac{p(y|x)\,p(x)}{p(y)}$$

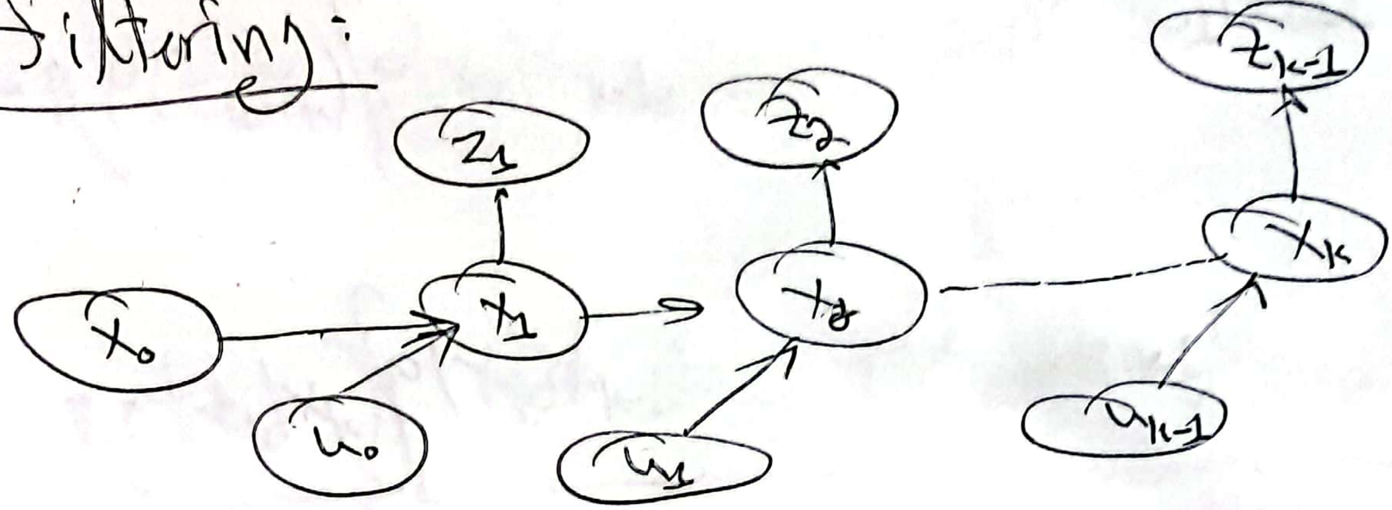
$$p(x,y|z) = p(x|y,z)\,p(y|z)$$

independance :

$$p(x|y) = p(x)$$

$$p(x,y) = p(x|y)p(y) = p(x)p(y)$$

filtering:



$$p(x_k | z_1, z_2, z_3 \ldots z_k, u_0, u_1 \ldots u_{k-1}) =$$

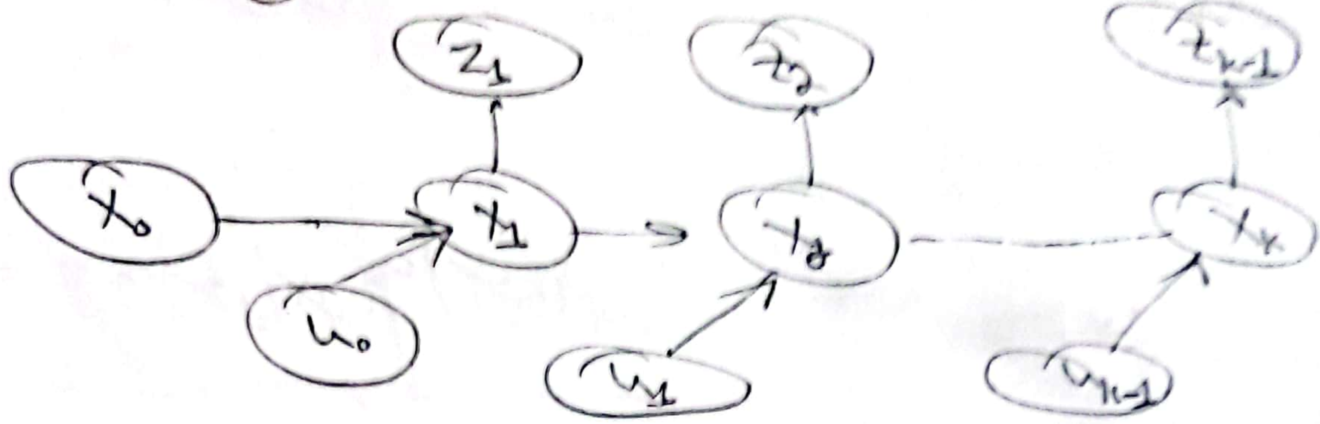
$$= p(x_k | z_{1:k}, u_{0:k-1}) =$$

$$= \frac{p(z_k | x_k, z_{1:k-1}, u_{0:k-1})\, p(x_k | z_{1:k-1}, u_{0:k-1})}{p(z_k | z_{1:k-1}, u_{0:k-1})} =$$

$$= \eta\, p(z_k | x_k)\, p(x_k | z_{1:k-1}, u_{0:k-1}) =$$

$$= \eta\, p(z_k | x_k) \int p(x_k, x_{k-1} | z_{1:k-1}, u_{0:k-1})\, dx_k =$$

filtering:



$$p(x_k|z_1, z_2, z_3 \ldots z_k, u_0, u_1 \ldots u_{k-1}) =$$

$$= p(x_k|z_{1:k}, u_{0:k-1}) =$$

$$= \frac{p(z_k|x_k, z_{1:k-1}, u_{0:k-1})\, p(x_k|z_{1:k-1}, u_{0:k-1})}{p(z_k|z_{1:k-1}, u_{0:k-1})} =$$

$$= \eta\, p(z_k|x_k)\, p(x_k|z_{1:k-1}, u_{0:k-1}) =$$

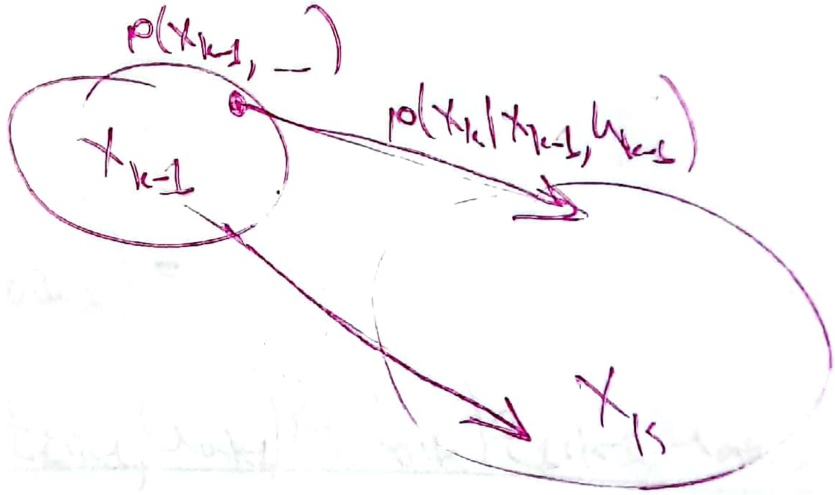
$$= \eta\, p(z_k|x_k) \int p(x_k, x_{k-1}|z_{1:k-1}, u_{0:k-1})\, dx_k =$$

$$= \eta\, p(z_k|x_k) \int p(x_k|x_{k-1}, z_{1:k-1}, u_{0:k-1})\, p(x_{k-1}|z_{1:k-1}, u_{0:k-1})$$

$$= \underset{\substack{\uparrow \\ \text{normalizer}}}{\eta}\, \underbrace{p(z_k|x_k)}_{\text{measurement model}} \underbrace{\int \underbrace{p(x_k|x_{k-1}, u_{k-1})}_{\text{Transition model}}\, \underbrace{p(x_{k-1}|z_{1:k-1}, u_{0:k-2})}_{\text{הלכד קודם}}\, dx_{k-1}}$$

כדי

שסכום יהיה 1



$$p(z_k|x_k) = \frac{1}{\sqrt{(2\pi)^k |\Sigma_v|}} \cdot e^{-(z_k - h(x_k))^T \Sigma_v^{-1} (z_k - h(x_k))}$$

$$p(x_k|x_{k-1}, u_{k-1}) = \frac{1}{\sqrt{(2\pi)^k |\Sigma_v|}} e^{-(x_k - g(x_{k-1}, u_{k-1}))^T \Sigma_w^{-1} (x_k - g(x_{k-1}, u_{k-1}))}$$

אם מניחים ש- המשתנים גאוסים
ואפשר לבנות אלגוריתם מלא להוכחה
אזי kalman filter.

אבל: gaussian · gaussian $\rightarrow$ gaussian

$\int$ gaussian $\rightarrow$ gaussian

Smoothing:

$$p(x_{0:k}|z_{1:k},u_{0:k-1}) =$$

$$= \frac{p(z_k|x_{0:k},z_{1:k-1},u_{0:k-1})\,p(x_{0:k}|z_{1:k-1},u_{0:k-1})}{p(z_k|z_{1:k-1},u_{0:k-1})} =$$

$$= \eta_k\, p(z_k|x_k)\,p(x_k|x_{0:k-1},z_{1:k-1},u_{0:k-1}) \cdot$$

$$\cdot\, p(x_{0:k-1}|z_{1:k-1},u_{0:k-1}) =$$

$$= \eta_k\, p(z_k|x_k)\,p(x_k|x_{k-1},u_{k-1})\,p(x_{0:k-1}|z_{1:k-1},u_{0:k-1})$$

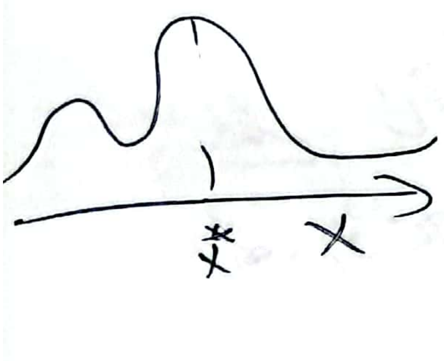
$$= \ldots = \prod \eta_{1:k} \prod_{i=1}^{k} p(z_i|x_i) \prod_{i=1}^{k} p(x_i|x_{i-1},u_{i-1}) \cdot p(x_0)$$

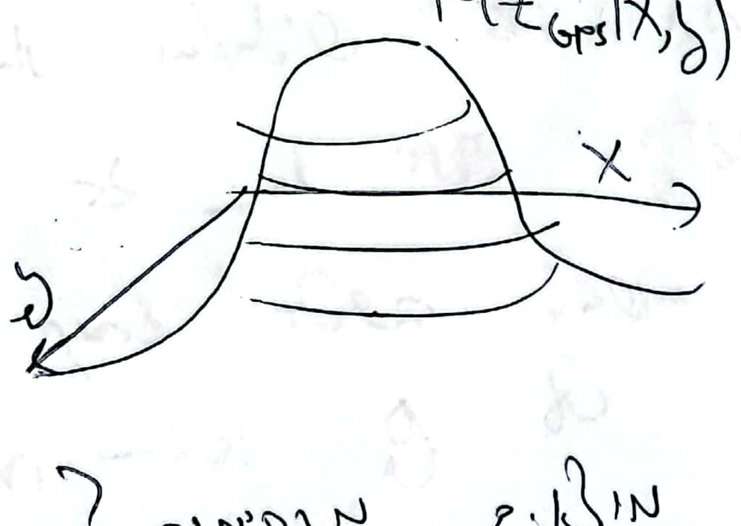
$$p(z_i|x_i) = \frac{1}{\sqrt{(2\pi)^k|\Sigma_v|}} e^{-(z_i-h(x_i))^T \Sigma_v^{-1} (z_i-h(x_i))}$$

$$p(x_i|x_{i-1},u_{i-1}) = \frac{1}{\sqrt{(2\pi)^k|\Sigma_w|}} e^{-(x_{i-1}-g(x_i,u_{i-1})) \Sigma_w^{-1} (x_{i-1}-g(x_i,u_{i-1}))}$$

$$p(x_0) = \frac{1}{\sqrt{(2\pi)^k|\Sigma_0|}} e^{-(x_0-\tilde{x}_0)^T \Sigma_0 (x-\tilde{x}_0)}$$

מכפלה של גאוסיאנים ← גאוסיין!





איך מוצאים מקסימום?

$$x^* = \arg\max_x (p(x)) = \arg\min_x (-\log(p(x)))$$

$$a^T \Sigma^{-1} a = \Sigma^{-\frac{1}{2}} a$$

$$\underset{x_{0:k}}{\arg\max}\, p(x_{0:k}, z_{1:k}, u_{0:k-1}) =$$

$$= \sum_{i=1}^{k} \left\| \Sigma_v^{-\frac{1}{2}} (h(x_i) - z_i) \right\|^2 +$$

$$+ \sum_{i=1}^{k} \left\| \Sigma_w^{-\frac{1}{2}} (x_i - g(x_{i-1}, u_{i-1})) \right\|^2 +$$

$$+ \left\| \Sigma_0^{-\frac{1}{2}} (x_0 - \tilde{x}_0) \right\|^2$$

$$\Downarrow$$

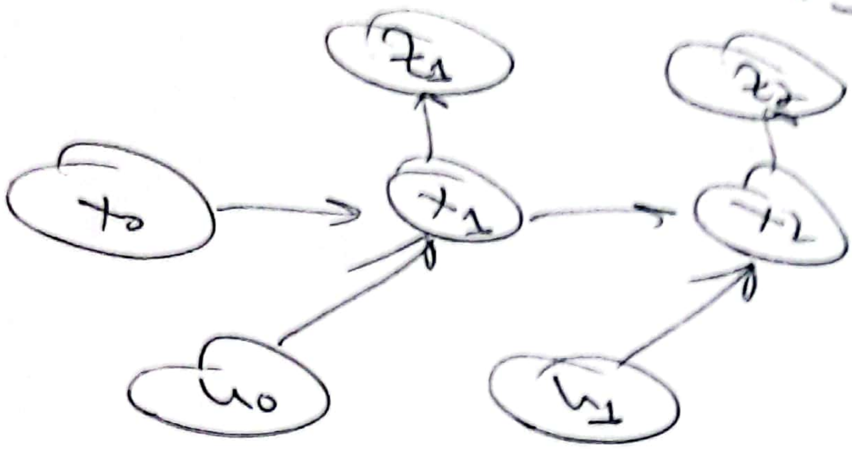
למצוא x_i שממזער את השגיאה.

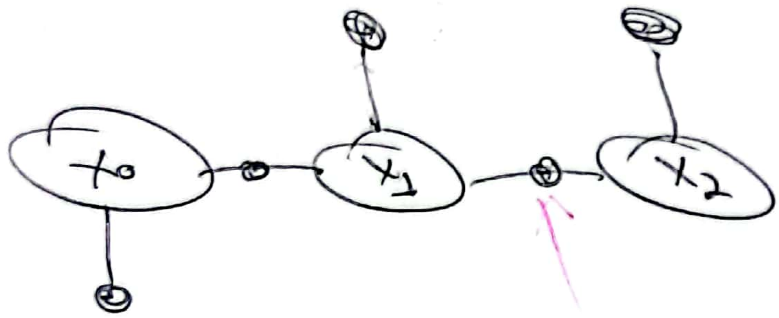
gtsam עושה את זה בשבילנו נומרית

מספיק להגדיר בחוץ יעקוביאנים

של g ושל h

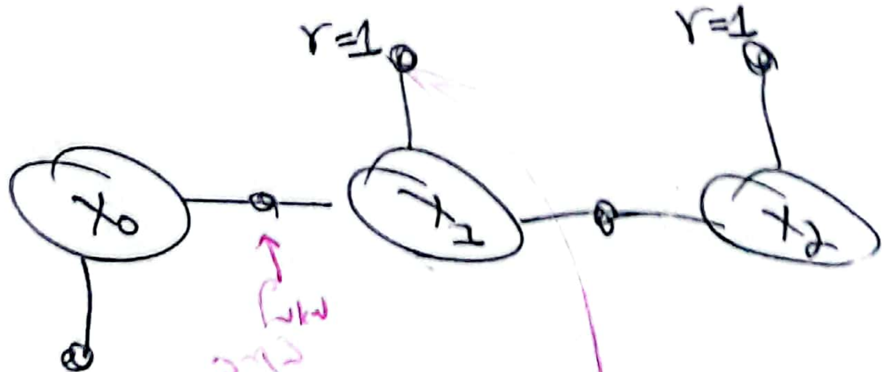
גרף רגיל:





פקטור גרף

פקטור, מעבר
אחר כמות מפלג
$p(x_2|x_1,u_1)$



$$\|\Sigma_r^{-\frac{1}{2}}(h(x_i)-r)\|^2$$