Technion – Israel Institute of Technology



HW5

Vision Aided Navigation

086761

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## Question 1: Factor graph, variable elimination and Bayes net. Consider a SAM problem where a robot travels through an unknown environment and captures observations using its onboard sensors. Assume the robot starts at time , with a known prior and consider motion and observation models and , respectively, where denotes the landmark. The robot moves according to given controls and observes a single landmark at time instances and .

### a: Write the joint pdf corresponding to the above scenario until time

### b: Draw the corresponding factor graph.

Factor graphs are a graphical way of representing some function where can be written as a product of functions (factors).

where   
This manipulation enables us to encode the independence relationships.

Factor graphs are a bipartite graph: contain two types of nodes, and one type of edge.  
Big nodes represent variables, and the smaller ones are represented factors.  
Each factor node is connected to one or more variable nodes through edges according to the variables it acts upon.

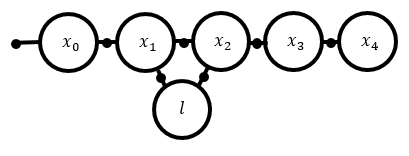
In our case, the big nodes will represent poses and landmarks (states).  
The small nodes will represent factors born from motion and measurement models.

For example, given some motional model , where represents gaussian noise with variance , each motion’s model conditional probability pdf has two states involved, and with the assumption of a gaussian distribution, corresponds to the following factor:

Note that the function itself is not explicit in the factor’s arguments.

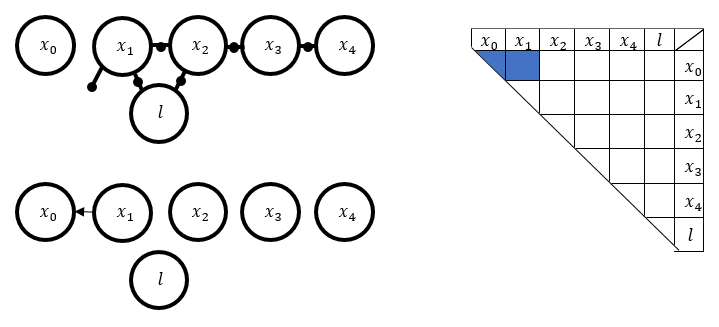
In our SLAM problem, a joint distribution over a state (poses and landmarks) is proportional to a multiplication of factors.  
Below is the factor graph representation for our SLAM problem.

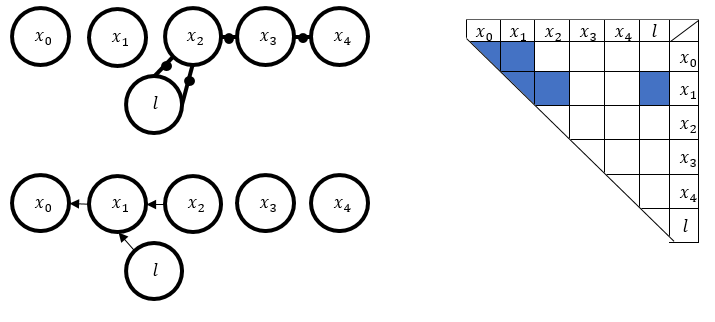
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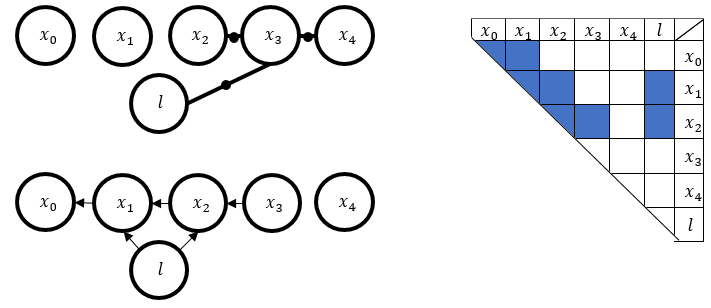


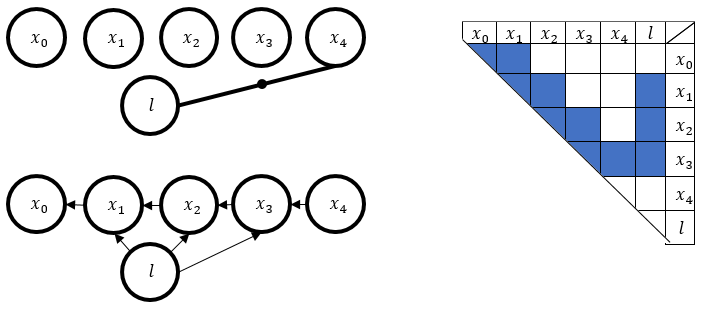
We would like to note that the advantage of factor graphs over Bayes Nets is that they can specify any factored function, and not just probability densities. This makes them better suited for inference (or so the documentation for GTSAM says…)

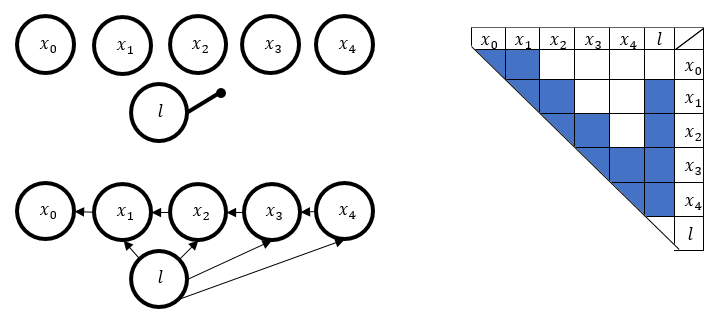
### c: Eliminate the factor graph into a Bayes net, assuming elimination order:

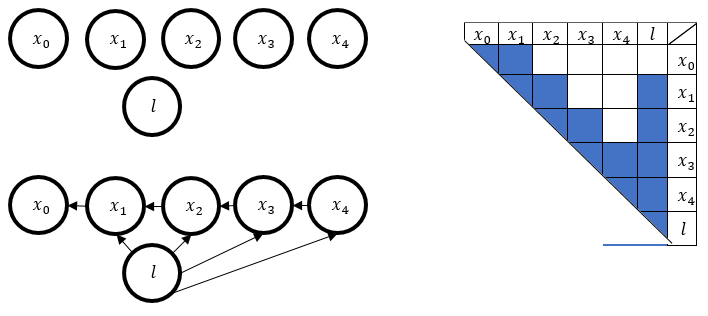




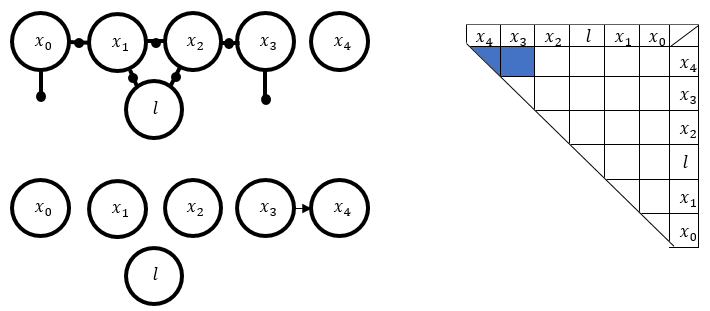


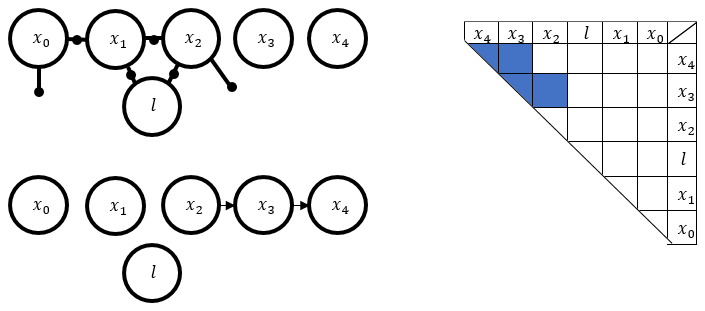


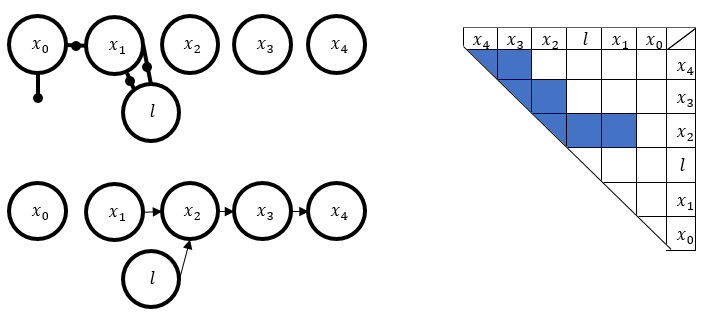


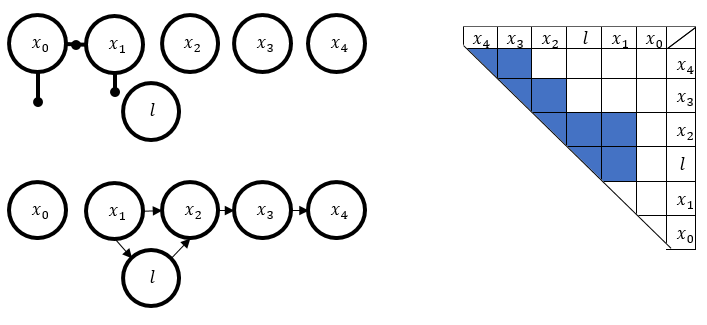


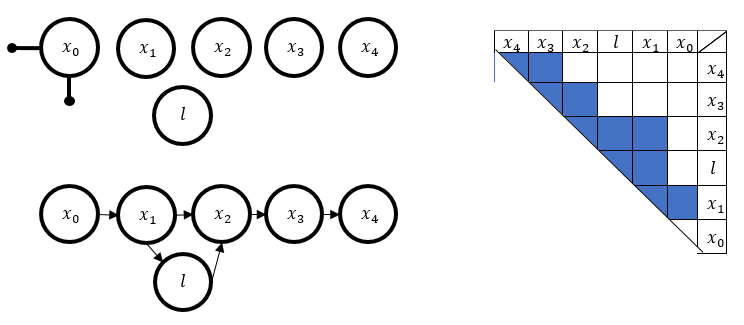
### d: Repeat the previous clause using a different variable elimination order:

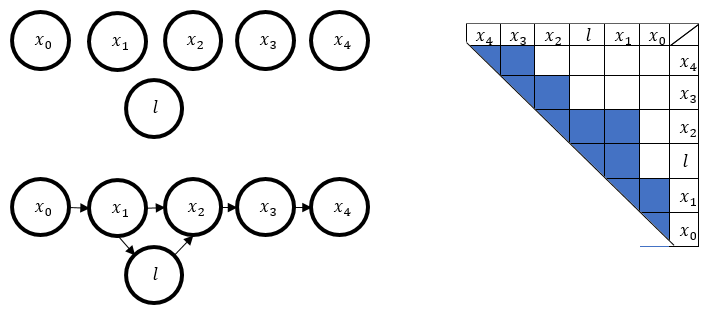












### e: Which of the two elimination orders you would prefer in terms of estimation accuracy and computational aspects?

We show the elimination order and resulting matrix for each section below.

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| --- | --- |
| {c}  Elimination Order: | {d}  Elimination Order: |
|  |  |

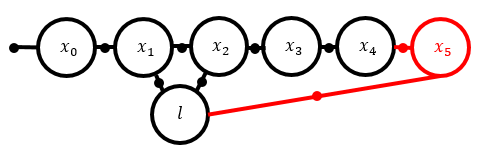
Regarding computation efficiency: We would prefer the elimination order in {d}, as it produces a sparser matrix (12 non-zero elements vs 14), and with more structure – all rows but one contains two elements at the start of the row.

Regarding Accuracy: Both matrices contain the same information. As such, the solution of the LMS problem is independent of the elimination order, or which matrix we choose to use.

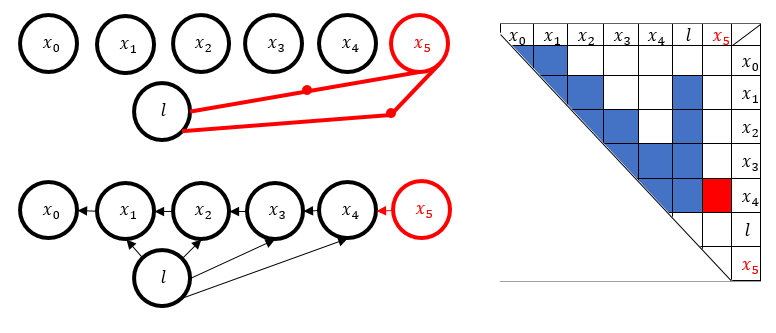
## Question 2: Incremental factorization. Consider now the robot, from question 1, executes command and moves to a new location; denote its new pose by . Assume the robot observes again the landmark from the new location.

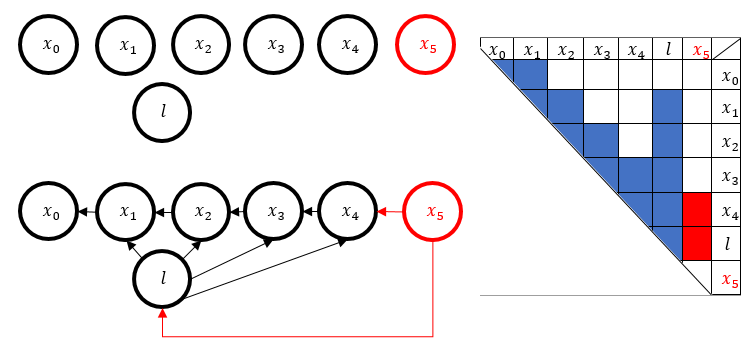
### a: Draw the factor graph of the problem and indicate the new factors and variable nodes.

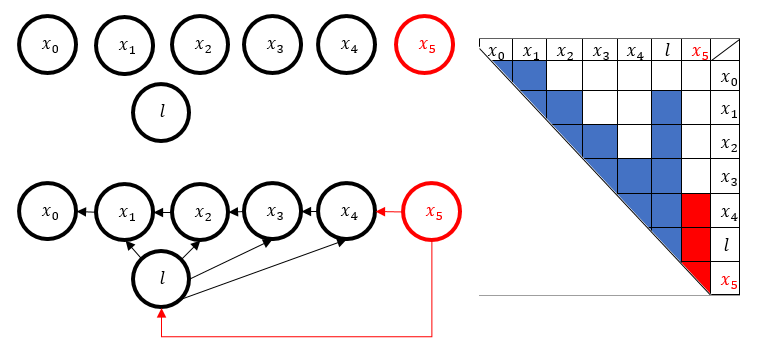
We add one node for pose , and two additional nodes for the factors that need to be computed: one for the motion model, and one for the measurement model.



### b: Consider the Bayes net from question 1(c) with elimination order Perform incremental factorization by updating this Bayes net with the new information using the elimination order:







### c: Show the corresponding updated square root information matrix

The new non-zero elements, the boxes colored red in the new matrix, describe states that depend on in the bayes-net graph.   
In general, values in the red triangle are in the “update impact zone” and are subject to change when computing the new matrix.

