1. Two examples without tiling

No shape for tiles allow to interpret the identity:

$$(1)5^2 + (2)4^2 + (3)3^2 + (4)2^2 = 10^2$$

because the odd terms can no be grouped in such a way the remaining part has even measures. The same apply to the very short identity:

$$(9)3^2 + (10)2^2 = 11^2$$

which is a segment of the anti-partridge sum $(1)11^2 + (2)10^2 + \cdots + (11)1^2$.

2. Tails of given length

We consider the length-ℓ tails of the sum $(1)n^2 + .. + (n-1)2^2 + (n)1^2$.

The general formula for the value of the tail (as function of n and ℓ) is quite easy to find; however it seems very hard to establish when the result is a square.

Trying with small values of ℓ we will show that the result can be a square when ℓ is 2 or 6; while it can not be a square for ℓ =3, 4 and 5.

Strangely enough, also the whole anti-partridge sum gives a square when n =2 and 6, while it is not a square for n =3, 4, 5...

- $\ell = 2$.
  The value of the tail is $(n-1)4 + (n) = 5n - 4$. Such a quantity is a square if and only if $n = 5h^2 + 2h + 1$ (no assumtion on the sign of h).

  The "if" part is obvious (the tail value being the square of $|5h+1|$); the "only if" part follows from $(n-1)4 + (n) = 5(n-1) + 1$; this can be a square (say $5(n-1) + 1 = k^2$) only if $(k+1)(k-1)$ is a multiple of 5; and the claim follows. The corresponding tiling is quite trivial, both with square as well as with triangular shapes.
- $\ell = 3$.
  The value of the tail is $(n-2)9 + (n-1)4 + (n) = 14n - 22$; such a value is *never* a square. (look at the remainder modulo 14 of $x^2 + 22, 0 \leq x \leq 13$)
- $\ell = 4$.
  The value of the tail is $(n-3)16 + (n-2)9 + (n-1)4 + (n) = 30n - 70$, that again is never a square.
- $\ell = 5$.
  The value of the tail is $(n-4)25 + (n-3)16 + (n-2)9 + (n-1)4 + (n) = 55n - 170$, that again is never a square.
- $\ell = 6$.
  The value of the tail is $(n-5)36 + (n-4)25 + (n-3)16 + (n-2)9 + (n-1)4 + (n) = 91n - 350$; this value is a square if and only if $n = 91h^2 + 28h + 6$ (no assumption on the sign of h).

  The "if" part is obvious, the tail value being the square of $|91h + 14|$; the "only if" part follows from $91n - 350 = 91(n-6) + 196$ that is a square (say $91n - 350 = 91(n-6) + 196 = k^2$) if and only if $91(n-6) = k^2 - 14^2$; thus k is a multiple of 7, and the claim easely follows.

  The case $h = 0$ corresponding to the whole anti-partridge sum, the next smallest case corresponds to $h = -1, n = 69$; the 6-tail of the sum is:

  $$(64)6^2 + (65)5^2 + (66)4^2 + (67)3^2 + (68)2^2 + (69)1^2 = 77^2$$

  and the corresponding tiling is quite easy (see the picture).