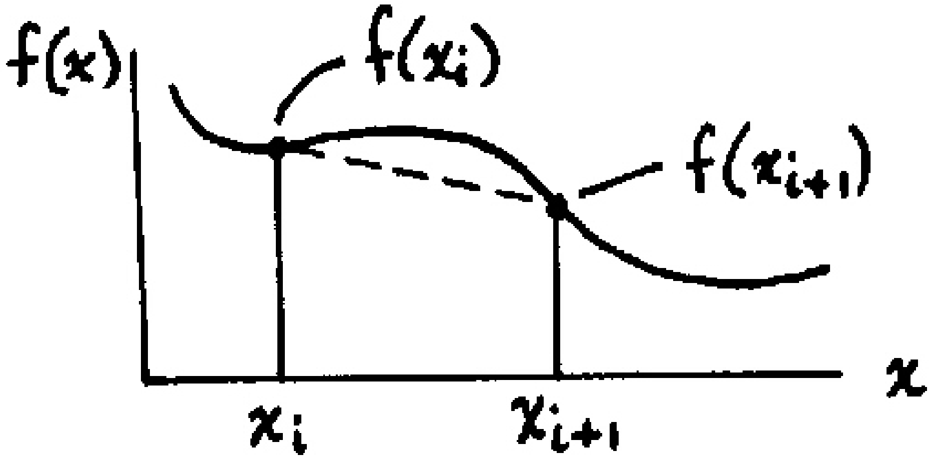
ME 203
Introduction to MATLAB

Differentiation

Differentiation is the calculation of the slope of a function. The easiest way to calculate the slope is to take the difference between two points.

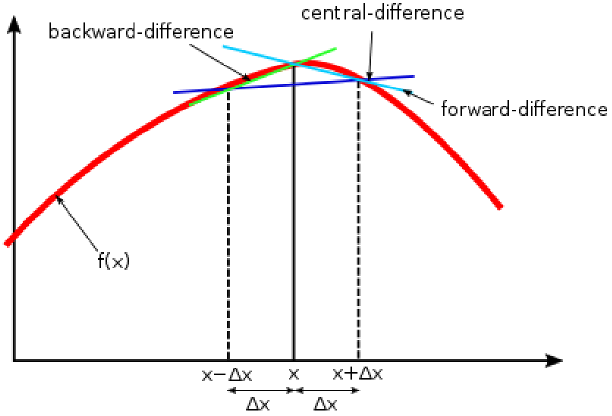
$$f'(x_k) = \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$$



1^{st} order Finite Difference Equations

Backward difference

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$



Forward difference

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$$

Central difference

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_{k-1})}{x_{k+1} - x_{k-1}}$$

1st order Finite Difference Equations

Backward difference

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

$$f'(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Undefined values

$$f'(x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f'(x_n) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

1^{st} order Finite Difference Equations

Forward difference

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k}$$

$$f'(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f'(x_{n-1}) = \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Undefined values

$$f'(x_n) = \frac{f(x_{n+1}) - f(x_n)}{x_{n+1} - x_n}$$

1^{st} order Finite Difference Equations

x = [0   1   2]
y = [5   4   2]

Find the slope at x = 1, using backward, forward and central difference method.

Backward difference

$$f'(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Forward difference

$$f'(x_1) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Central difference

$$f'(x_1) = \frac{f(x_2) - f(x_0)}{x_2 - x_0}$$

Consider:

$X_0 = 0; y_0=5$

$X_1 = 1; y_1=4$

$X_2 = 2; y_2=2$

Diff(x)

The diff function computes differences between adjacent values in a vector, generating a new vector with one less value.

`diff(x)`	For vector `x`, returns a vector of differences between adjacent values in `x`: `[x(2)-x(1) x(3)-x(2) ... x(n)-x(n-1)]`, where `n` is `length(x)`. For a matrix `x`, returns the matrix of column differences, `[x(2:n,:) - x(1:n-1,:)]`.

Differentiation Example 1

Calculate the derivative for the following set of data using both **forward and backward difference** methods. Plot the data and the slope for each method.

x = [	0	1	2	3	4	5	6	7	8	9	10]
y = [	5	4	2	3	4	7	9	7	6	5	4]

```
%Differentiation example 1
clc; clear all; format compact

%given
x = 0:10;
y = [5, 4, 2, 3, 4, 7, 9, 7, 6, 5, 4];

%calculate difference
dy = diff(y)
dx = diff(x)
```

Result for Differences

```
dy =
  -1  -2   1   1   3   2  -2  -1  -1  -1
dx =
   1   1   1   1   1   1   1   1   1   1
```

Differentiation Example 1

```
%calculate slope
dy_dx = dy./dx
```

Result for slope
dy_dx =
-1 -2 1 1 3 2 -2 -1 -1 -1

```
%calculate x values for forward difference plot
xf = x(1:end-1)
```

xf =
0 1 2 3 4 5 6 7 8 9

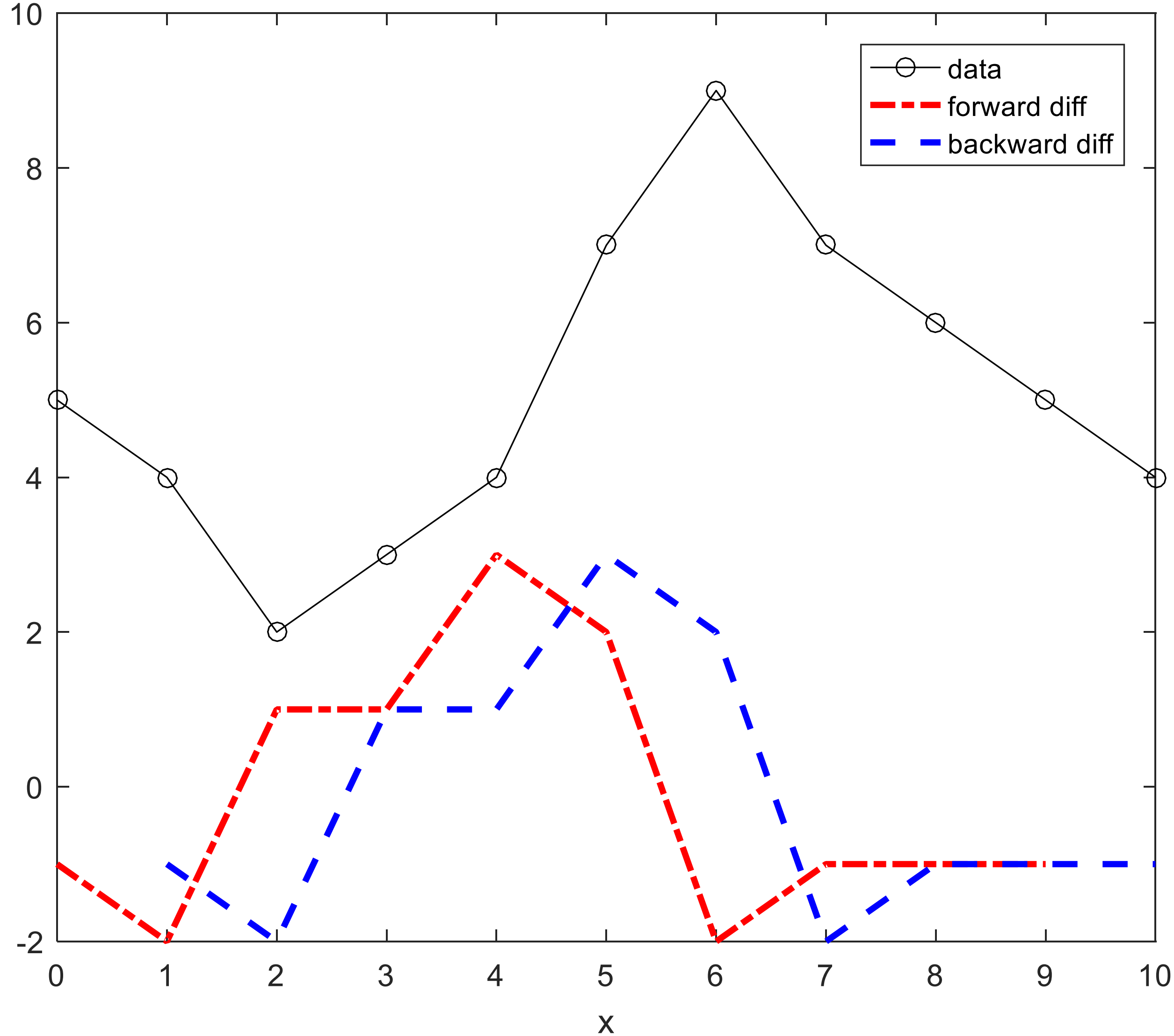
```
%calculate x values for backward difference plot
xb = x(2:end)
```

xb =
1 2 3 4 5 6 7 8 9 10

Differentiation Example 1

```
%plot data and slope
plot(x,y,'-ok')             %plot data

hold on

plot(xf,dy_dx,'-.r')        %plot forward diff

plot(xb,dy_dx,'--b')        %plot backward diff

xlabel('x')
legend('data','forward diff','backward diff')
```

Differentiation Example 1



1st order Finite Difference Equations

Central difference

$$f'(x_k) = \frac{f(x_{k+1}) - f(x_{k-1})}{x_{k+1} - x_{k-1}}$$

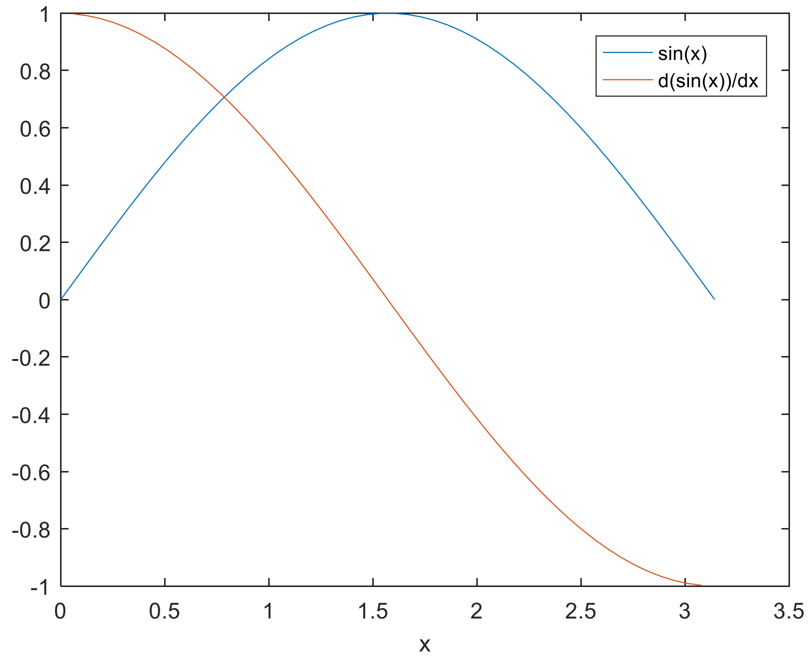
$$f'(x_2) = \frac{f(x_3) - f(x_1)}{x_3 - x_1}$$

$$f'(x_{n-1}) = \frac{f(x_n) - f(x_{n-2})}{x_n - x_{n-2}}$$

Differentiation Example 2

Calculate the derivative for y=sin(x) over the range x=[0 pi] using the **central difference** method. Plot the function and the slope.

```
%Differentiation example 1
clc; clear all; format compact

%given
x = linspace(0,pi,50);
y = sin(x);

n = length(x);

%calculate differences
dy = y(3:n) - y(1:n-2);
dx = x(3:n) - x(1:n-2);

%calculate slope
dy_dx = dy./dx;
plot(x,y,x(2:n-1),dy_dx)
legend('sin(x)','d(sin(x))/dx')
```



ODEs (Ordinary Differential Equations)

An ODE is an equation of the form

$$\frac{dy}{dx} = f(x, y)$$

where f(x,y) is any function of x and y

Typically, the f(x,y) is complicated so it is difficult or impossible to solve for y by rearranging the equation. You will learn the theory and the analytical way to solve this equation in another class. In this class we will use a built-in Matlab function to solve for y.

To solve for y you need an initial value of y and a range for x.

ode45

[X, Y]=ode45(@f, [X-range],[Y-initial value])

f: previously defined function whose input arguments are x and y

X-range: initial and final values of the independent variable

Y-initial value: initial value of the dependent variable

ODE Example 1

Solve the ODE

$$\frac{dy}{dx} + 10y = 0$$

Use: x=[0, 0.5], y(x=0)=2. Plot the solution.

x-range: 0 to 0.5

Initial value (i.e., when x=0) of y: y(0) = 2

$$\frac{dy}{dx} = -10y$$

Find: Plot y vs. x

ODE Example 1

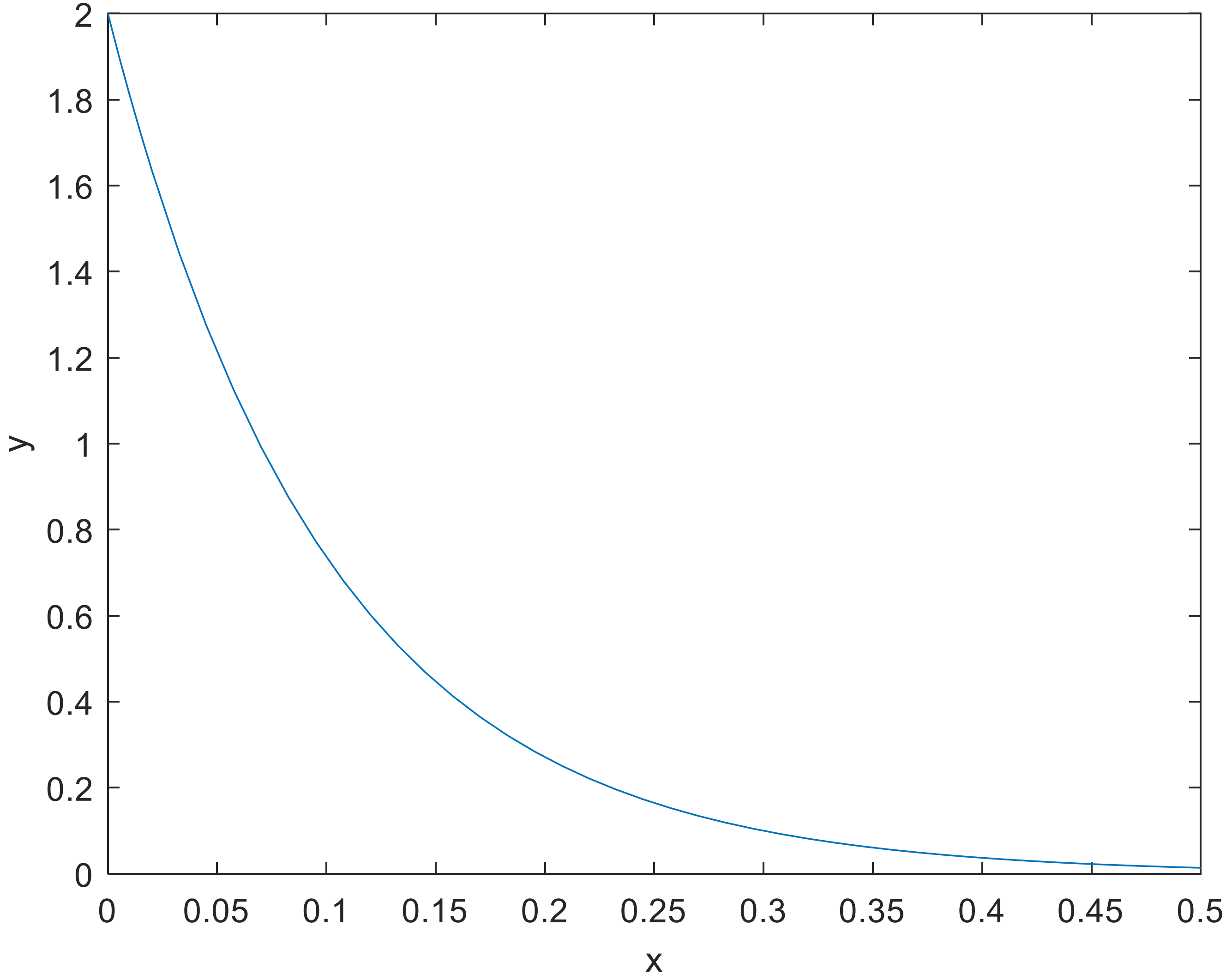
Define the function file:

```
function dy_dx = odeexample1 (x,y)
dy_dx = -10*y;
end
```

Call the function from another script file or command window:

```
%ODE example 1
clc, clear all, format compact

%Given
x_range = [0, 0.5];
y_initial = 2;

[x, y] = ode45 (@odeexample1, x_range, y_initial);
plot(x,y)
xlabel('x')
ylabel('y')
```

ODE Example 1



ODE Example 1

Alternatively, one can use a function handle within the script file instead of defining a separate function file

```
%ODE example 1 - another method
clc, clear all, format compact

%function handle
fun = @(x,y) -10*y;

[x,y] = ode45 (fun, [0, 0.5], 2);
plot(x,y)
xlabel('x')
ylabel('y')
```

ODE Example 1

Alternatively, one can use a function handle within the script file instead of defining a separate function file

```
%ODE example 1 - another method
clc; clear all; format compact

[x,y] = ode45 (@(x,y) -10*y, [0, 0.5], 2);
plot(x,y)
xlabel('x')
ylabel('y')
```

ODE Example 2

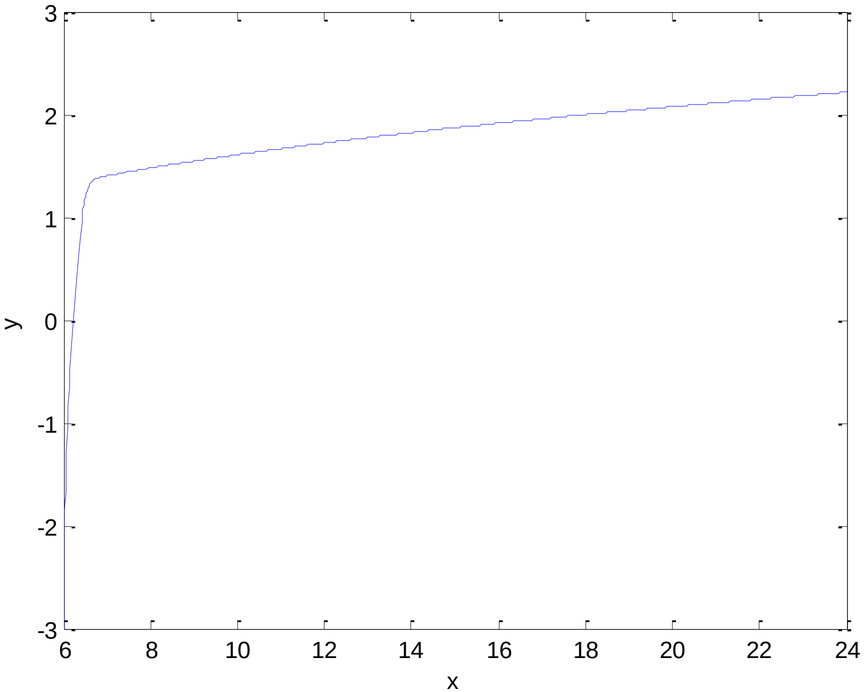
Solve the ODE: $\frac{dy}{dx} = -2y^3 + x - y$

Given: x=[6, 24], y(x=6)=-3, Find: Plot y vs. x

```
%ODE example 2
clc; clear all; format compact

%given
x_ran = linspace(6,24,100); %to divide the x range into
100 steps
y_ini = -3;
```

ODE Example 2

```
%function handle
fun2 = @(x,y) (-2*y.^3+x-y);

[t,y] = ode45 (fun2, x_ran, y_ini);
plot(t,y), xlabel('t'), ylabel('y')
```



Final Exam

11 December 2020, 12:10 pm to 2:10 pm

- **All topics covered in Matlab**
- **Similar to Exam 2**
- **Questions will be in two parts:**
- **Part (a): Short questions on WebCampus**
- **Part (b): Write Matlab code, 4-5 problems**