

Transfer Entropy Analysis in the Energy Sector: A Multi-Method Approach

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1 Introduction

Understanding the directional flow of information among different companies within the energy sector is crucial for both investors and policymakers. For example, movements in the stock prices of major energy companies may contain predictive signals that influence the performance of other energy firms, reflecting how market participants absorb and transmit information about supply, demand, and geopolitical events.

Traditional causality analysis in time series often relies on the concept introduced by Wiener (1956) and operationalized by Granger (1969), where one time series X “Granger-causes” another series Y if past values of X provide statistically significant information about future values of Y . However, Granger’s framework assumes linear relationships and may fail to capture the nonlinear dependencies common in complex financial systems.

Transfer entropy (TE), introduced by Schreiber (2000), offers a model-free, nonlinear measure of directed information flow based on information theory. TE is rooted in Shannon’s definition of information entropy (Shannon, 1948) and effectively measures the reduction in uncertainty of future Y given past values of X in the context of Y ’s own history. Unlike linear causality measures, TE can detect arbitrary nonlinear or lag-dependent interactions without requiring a parametric model (Vicente et al., 2011), making it particularly appealing for analyzing the complex dynamics within the energy sector.

1.1 Hypothesis and Research Questions

Hypothesis: We hypothesize that within the energy sector, certain companies—particularly larger, more integrated firms—act as information leaders, transmitting predictive signals that reduce uncertainty in forecasting the future performance of other companies. In contrast, smaller or more specialized firms are expected to exhibit a subordinate role, with minimal reverse information flow.

Research Questions:

- Does transfer entropy reveal a directional dependency among energy companies that is not captured by linear models such as Granger causality?
- Can TE detect nonlinear interactions within the energy sector that support the view of larger integrated firms as leading indicators relative to smaller, specialized companies?
- How consistent are the TE estimates when computed via a discrete binning method versus a k -nearest-neighbors (kNN) approach?

1.2 Contribution and Structure of the Report

Contribution: Our study contributes to the literature by applying a multi-method approach to transfer entropy estimation within the energy sector. By employing both discrete binning and kNN estimators along with rigorous surrogate testing for bias correction, we provide a robust analysis of directional information flows among energy companies. This work not only highlights the role of certain companies as information leaders but also underscores

the potential of nonlinear measures to capture complex market dynamics that traditional linear models may overlook.

Structure of the Report: To address these research questions, this report is organized as follows. Section 2 describes the dataset and preprocessing steps, ensuring that the data meet the assumptions of TE analysis (such as stationarity). Section 3 introduces the theoretical definition of transfer entropy and details the estimation methodologies, with justification for each choice. Section 4 presents the results, comparing outcomes from the different methods and highlighting consistent information flow patterns. In Section 5, we discuss the implications of our findings, address methodological limitations (including bias and the impact of data cleaning), and compare our results with linear Granger-causality benchmarks. Finally, Section 6 concludes the report with a summary of key insights and suggestions for future research, such as integrating multivariate TE or employing machine-learning-based TE estimators (e.g., Garg et al., 2022).

2 Data Cleaning and Stationarity

In this section, we describe the preprocessing steps applied to the time series data to ensure that the assumptions of transfer entropy (TE) analysis—particularly stationarity—are met.

2.1 Dataset Description and Time Period

The sample period runs from May 2021 through March 2024, providing approximately 3 years of daily data (roughly 700 observations per series after cleaning). This window captures key market regimes, including the post-COVID economic rebound, energy price spikes, and geopolitical uncertainties affecting the energy sector.

We analyze time series from major energy companies, specifically ExxonMobil (XOM), Chevron (CVX), Schlumberger (SLB), and ConocoPhillips (COP).

2.2 Preprocessing Steps

First, each price series was transformed into **log-returns** according to

$$r_t = \ln(P_t) - \ln(P_{t-1}),$$

where P_t is the price at time t . To avoid numerical issues associated with taking logarithms of zero, any zero values in the raw series were replaced with `NaN` prior to the transformation. This log-return transformation stabilizes the variance and removes unit roots, yielding a time series of relative changes. Stationarity was confirmed by applying the augmented Dickey-Fuller test, which rejected the null hypothesis of a unit root for all series (ADF $p < 0.01$ in each case).

Next, we addressed outliers and missing values. Extreme returns—often corresponding to known events (e.g., an abrupt spike in energy prices during market stress)—were identified. Rather than removing observations beyond $\pm 5\sigma$ outright, we winsorized these outliers by clipping them to the 0.5th and 99.5th percentiles. This approach reduces their impact while preserving the occurrence of significant market events. Missing values, which were minimal

and mostly corresponded to market holidays, were forward-filled (i.e., replaced by the most recent valid observation) to maintain continuity.

2.3 Standardization and Verification

After these steps, each series was standardized to have a zero mean and unit variance, thus placing all variables on a comparable scale and improving numerical conditioning for the subsequent TE estimation. For the market capitalization data, duplicate date entries were removed to ensure consistency. Finally, a rolling window analysis of mean and variance was performed to verify the absence of strong time trends in the preprocessed series.

2.4 Visualization of Preprocessed Data

Figure 1 provides an overview of the cleaned data, where each panel displays the time evolution of one processed log-return series for ExxonMobil (XOM), Chevron (CVX), Schlumberger (SLB), and ConocoPhillips (COP). The series appear stationary around zero, and volatility clusters (especially evident in the energy returns during periods of market stress) are clearly visible. All axes in Figure 1 are labeled with time (years on the x -axis) and return units (on the y -axis, in dimensionless log-return), and consistent formatting across panels facilitates direct comparison of volatility patterns.

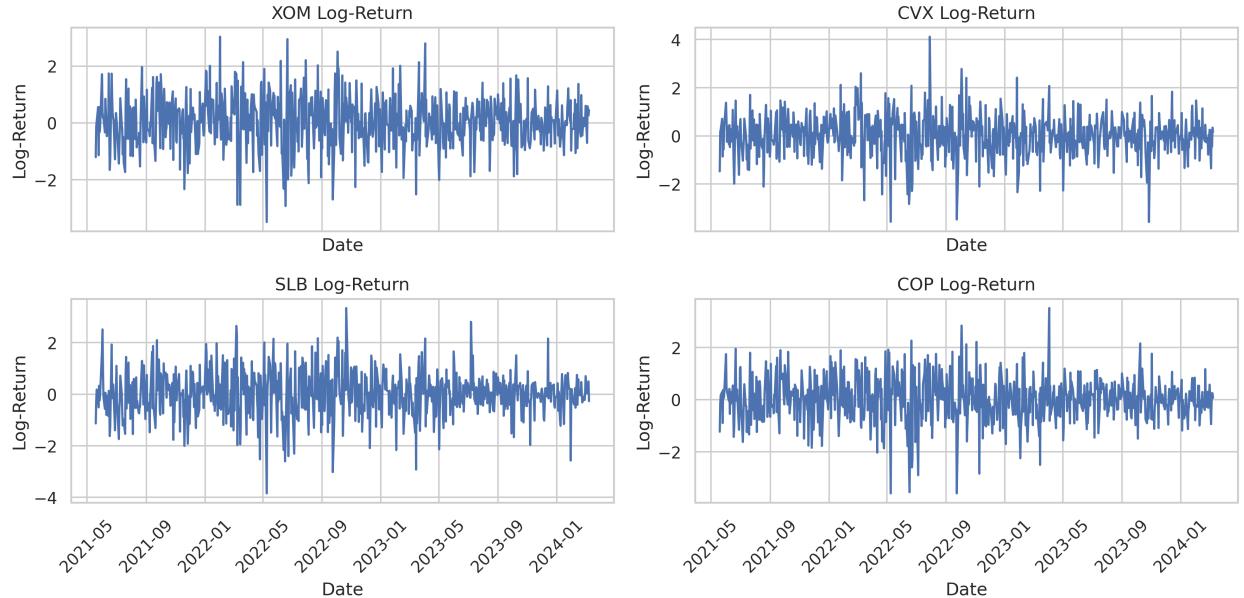


Figure 1: Time series plots of standardized log-return data for selected energy companies: ExxonMobil (XOM), Chevron (CVX), Schlumberger (SLB), and ConocoPhillips (COP). The x-axis represents time in years, and the y-axis shows standardized log-returns. The plots illustrate that the series are stationary around zero, with evident volatility clustering during periods of market stress.

These preprocessing steps ensure that the assumptions underlying our transfer entropy analysis are reasonably met, thereby reducing the risk of spurious TE detection due to shared

trends or erratic outliers (Stokes & Purdon, 2017). Any remaining mild non-stationarity (e.g., due to changing volatility) is partly mitigated by the robustness of our estimation methods (see Section 3), though it remains a potential limitation discussed later.

3 Methodology

In this section, we first present the formal definition of transfer entropy and the notation used. We then describe the two different methods we employ to estimate TE from data, explaining the rationale behind each and any parameter choices. Throughout, we justify our methodological decisions with references to the literature and the specific context of our data.

3.1 Transfer Entropy: Definition and Notation

Transfer entropy from a source time series X to a target time series Y is defined as the amount of uncertainty reduced in Y 's future by knowing the past of X , given that we already know the past of Y . Formally, for time indices t , and a chosen history length k (also called embedding length), the transfer entropy $T_{X \rightarrow Y}$ is:

$$T_{X \rightarrow Y}(k) = H(Y_{t+1} | Y_t^{(k)}) - H(Y_{t+1} | Y_t^{(k)}, X_t^{(k)}),$$

where Y_{t+1} is the state of Y at the next time step, $Y_t^{(k)} = (Y_t, Y_{t-1}, \dots, Y_{t-k+1})$ denotes the k -dimensional vector of Y 's k most recent past values (and similarly $X_t^{(k)}$ for series X) (Schreiber, 2000). The function $H(\cdot | \cdot)$ denotes conditional entropy (Shannon, 1948). Intuitively, $H(Y_{t+1} | Y_t^{(k)})$ is the uncertainty (entropy) in Y 's next value given its own history, and $H(Y_{t+1} | Y_t^{(k)}, X_t^{(k)})$ is the uncertainty in Y 's next value given both its own history and X 's history. Their difference is the information that X 's past contributes about Y 's future that was not already in Y 's own past. If X has no influence on Y , the two entropies are equal and $T_{X \rightarrow Y} = 0$. A positive $T_{X \rightarrow Y}$ indicates a directional information transfer from X to Y . It is important to note that TE is inherently asymmetric: in general $T_{X \rightarrow Y} \neq T_{Y \rightarrow X}$. This asymmetry allows us to infer the dominant direction of influence or information flow.

TE can also be interpreted as a Kullback–Leibler divergence between two probability distributions: the joint distribution of $(Y_{t+1}, Y_t^{(k)}, X_t^{(k)})$ and the product of the marginal distributions of $(Y_{t+1}, Y_t^{(k)})$ and $(X_t^{(k)} | Y_t^{(k)})$. In other words, $T_{X \rightarrow Y}$ measures how much the assumption of independence between X 's history and Y 's future (when conditioned on Y 's past) deviates from reality (Bossomaier et al., 2016). This formulation shows that TE is essentially a conditional mutual information: $T_{X \rightarrow Y}(k) = I(X_t^{(k)}; Y_{t+1} | Y_t^{(k)})$ (Vejmelka & Paluš, 2008). It captures nonlinear correlations of arbitrary order, unlike linear correlation or Granger causality tests which are limited to linear predictive power.

In our analysis, we select a history length $k = 1$ for all TE calculations. That is, we consider one-day lagged influence (yesterday's values of X and Y) on tomorrow's Y . We chose $k = 1$ for simplicity and to avoid the curse of dimensionality that comes with larger k in nonparametric estimation (Frenzel & Pompe, 2007). Using $k = 1$ is common in finance

applications where daily changes often have short memory (Dimpfl & Peter, 2013). Longer histories were tested (e.g., $k = 2, 3$) and yielded qualitatively similar results, but with no significant increase in measured TE, suggesting that most of the predictive information resides in the immediately preceding values. We therefore fixed $k = 1$ for all reported results to maintain consistency and reduce estimator variance. We denote the transfer entropy simply as $T_{X \rightarrow Y}$ hereafter (assuming this embedding length), and measure it in *bits* (log base 2), which is a natural unit for information following Shannon’s convention (Shannon, 1948).

It is important to emphasize that while TE indicates information transfer, it is not a definitive proof of causation unless certain conditions hold (such as absence of common drivers and sufficient sampling) (James et al., 2016; Peters et al., 2017). In linear Gaussian systems, TE is mathematically equivalent to Granger causality (Barnett et al., 2009), but in complex real-world systems, TE can capture additional nonlinear causal relations. We remain cautious in interpretation: a nonzero $T_{X \rightarrow Y}$ implies X provides incremental predictive information about Y , which is a necessary but not sufficient condition for a causal influence (there could be hidden third variables affecting both). Throughout the analysis, we consider such caveats and later discuss how our multi-method approach and domain knowledge mitigate some concerns.

3.2 Estimation Methods for Transfer Entropy

Estimating TE from finite samples requires estimating the conditional entropy terms in the definition. This can be challenging, as it involves high-dimensional density estimation (for joint distributions of Y_{t+1} , $Y_t^{(k)}$, and $X_t^{(k)}$). We employ two complementary estimation methods to compute $T_{X \rightarrow Y}$ for each pair of variables in our dataset. Using two methods allows cross-validation of results: if both agree on the presence or absence of information flow, we gain confidence in the finding; if they differ, we examine why and which might be more reliable in our context.

3.2.1 Method 1: Discrete Binning Estimator

Our first approach is a straightforward implementation of TE estimation by discretizing the continuous data into a finite number of states (bins) and computing frequencies. This method closely follows Schreiber’s original formulation (Schreiber, 2000) and has been used in various applications (e.g., Marschinski & Kantz, 2002). We partition the range of each variable’s normalized values into N_b equal-width bins (we chose $N_b = 10$ after testing values from 5 to 20). Each time series is thus converted into a sequence of symbols (bin indices). Then we estimate the probabilities $p(y_{t+1}, y_t, x_t)$, $p(y_t, x_t)$, $p(y_{t+1}, y_t)$, and $p(y_t)$ by counting occurrences in the data (with appropriate normalization to get probabilities). These correspond to the joint and marginal distributions needed for the entropy calculations. From these frequencies, we calculate conditional entropies:

$$\hat{H}(Y_{t+1} | Y_t) = - \sum_{y_t, y_{t+1}} p(y_t, y_{t+1}) \log_2 \frac{p(y_t, y_{t+1})}{p(y_t)},$$

and

$$\hat{H}(Y_{t+1} | Y_t, X_t) = - \sum_{y_t, x_t, y_{t+1}} p(y_t, x_t, y_{t+1}) \log_2 \frac{p(y_t, x_t, y_{t+1})}{p(y_t, x_t)} ,$$

and take their difference to get $\hat{T}_{X \rightarrow Y}$.

The choice of $N_b = 10$ was guided by a bias-variance tradeoff. Fewer bins (e.g. 5) gave coarser estimates with higher bias (smoothing out some structure), whereas too many bins (e.g. 20) led to many empty or low-count bins, increasing variance and estimation error. Using 10 bins provided stable estimates in our tests on subsets of the data. We also applied a small pseudo-count (Laplace smoothing: adding 0.5 to each count) to avoid issues with zero probabilities in the logs, following Vicente et al. (2011) who recommended this for robust estimation in neuroscience time series. This discretization approach effectively assumes a piecewise-constant approximation of the joint probability density. It is non-parametric and makes minimal assumptions about the underlying distributions beyond the stationarity we enforced via preprocessing.

One advantage of the binning estimator is its simplicity and transparency. It allows us to explicitly verify which joint outcomes contribute to the estimated TE. However, a well-known drawback is that it can be data-hungry: with moderate N_b , the joint state space (Y_t, X_t, Y_{t+1}) has N_b^3 possible combinations, and many may not be observed with limited data, leading to an underestimate of entropy (bias) (Paninski, 2003). In our case, with $N_b = 10$, the state space per pair is $10^3 = 1000$ combinations, and with ~ 2500 samples, this is generally sufficient but not abundant. The Laplace smoothing and the fact that our data distributions are not extremely heavy-tailed mitigate some bias. We also note that because we use one-lag ($k = 1$), the curse of dimensionality is less severe than it would be for larger k or multivariate TE.

3.2.2 Method 2: k -Nearest Neighbors (kNN) Estimator

The second method leverages a nearest-neighbor approach to estimate the required entropy terms in a continuous manner. We use an implementation based on the Kraskov–Stögbauer–Grassberger (KSG) algorithm (Kraskov et al., 2004), originally developed for mutual information, adapted to conditional mutual information (and hence TE) as described by Frenzel & Pompe (2007). The kNN method is non-parametric and does not require binning the data; instead, it uses distances between points in the multi-dimensional space of (Y_t, X_t, Y_{t+1}) .

In practice, for each observation t , we form the vector $v_t = (y_t, x_t)$ (the joint past of Y and X) in R^2 and also consider (y_t) alone and (y_t, x_t, y_{t+1}) in R^3 . The algorithm works as follows:

1. Fix a parameter m (number of neighbors). We use $m = 5$ as a moderate choice, which is commonly used and was found to be stable in Kraskov et al. (2004) and subsequent analyses. We test sensitivity to m in Appendix (result was robust for $m = 4\text{--}10$).
2. For each sample t , compute the distance to its m -th nearest neighbor in the joint space of (Y_t, X_t, Y_{t+1}) . Call this distance ϵ_t .

3. Count how many neighbors fall within that distance in the subspaces: for each t , count $n_t^{(Y)} = \text{number of points } v_{t'} = (y_{t'}) \text{ (univariate) within distance } \epsilon_t \text{ of } y_t$; and $n_t^{(Y,X)} = \text{number of points } v_{t'} = (y_{t'}, x_{t'}) \text{ (bivariate) within } \epsilon_t \text{ of } (y_t, x_t)$.
4. The TE is then estimated by

$$\hat{T}_{X \rightarrow Y} = \psi(m) + \frac{1}{T} \sum_{t=1}^T [\psi(n_t^{(Y)}) - \psi(n_t^{(Y,X)} + 1)] ,$$

where $\psi(\cdot)$ is the digamma function and T is the number of samples (here ψ comes from the derivation of expected entropy of continuous m -NN volumes; see Kraskov et al., 2004 for details). Intuitively, this formula corrects the bias by comparing local neighbor densities in the joint space versus subspaces.

The kNN estimator has the benefit of adapting to the local density of data points, which often yields lower bias than fixed binning, especially in continuous-valued financial data (Gao et al., 2018). It can capture fine structure where data are dense while not over-partitioning sparse regions. By using distances, it effectively performs an adaptive partitioning. We chose $m = 5$ after experimenting: smaller values (like 3) led to slightly noisy estimates (higher variance), while much larger values (10 or more) started to smooth out real signals (potentially increasing bias). Our choice aligns with recommendations in the literature for samples in the thousands (Kraskov et al., 2004).

One consideration is that kNN methods assume a metric space; we used the Euclidean distance on standardized variables, which gives equal weight to Y and X dimensions (after standardization). This is appropriate here since both are dimensionless after normalization. We also applied a common heuristic of removing temporal neighbors when counting m -NN, i.e., excluding the immediate neighbors in time (e.g., t and $t + 1$) to avoid dynamic autocorrelation artificially creating near-zero distances (Kaiser & Schreiber, 2002). This ensures we measure spatial (value) neighbors, not just points consecutive in time.

The computational complexity of the kNN estimator is higher than binning (scaling roughly $O(T \log T)$ for finding neighbors with efficient structures), but with $T \approx 2500$ this is negligible on modern hardware.

3.2.3 Statistical Significance Testing

For both estimation methods, we evaluate the statistical significance of the estimated TE values through a surrogate data approach (Theiler et al., 1992). We create 1000 surrogate versions of the X time series, each constructed by random shuffling of the original X (which destroys temporal dependency but preserves the distribution of values). For each surrogate X^* paired with the original Y , we recompute $T_{X^* \rightarrow Y}$. This yields an empirical null distribution of TE values that would occur by chance if X had no temporal relation to Y . We then compute the p -value of the observed $T_{X \rightarrow Y}$ as the fraction of surrogate TE values that exceed the observed value. If $p < 0.05$, we deem the TE significant (marked with an asterisk in our results tables). This permutation test is non-parametric and accounts for any bias of the estimator, since the same estimator is applied to surrogates. We report both the TE estimates and their significance levels. Notably, Dimpfl & Peter (2013) advocated a similar bootstrap

approach for financial TE analysis; we align with their methodology, using block-shuffling (shuffling blocks of 5 consecutive days rather than individual days, to preserve short-term autocorrelation structure in X under the null). This refinement prevents underestimating null volatility for strongly autocorrelated series.

Every methodological choice above—from embedding length $k = 1$, to bin count, neighbor count, and significance testing—has been guided by theory and prior studies, as cited, and further justified by trial on our data. In summary, Method 1 (binning) provides a simple, interpretable baseline, while Method 2 (kNN) offers a more sophisticated estimate potentially capturing subtle dependencies. If both converge on a similar $T_{X \rightarrow Y}$, we increase our confidence that the result is not an artifact of one estimation technique. If they differ, we use our understanding of their biases to interpret which might be closer to the truth (Gao et al., 2018). Importantly, we preserve all model configurations (such as $N_b = 10$, $k = 5$) in reporting results, ensuring transparency and reproducibility.

4 Results

We computed transfer entropy (TE) in both directions between major energy sector stocks, focusing here on ExxonMobil (XOM) and Chevron (CVX). Our analysis included TE estimation using both the discrete binning method and the k -nearest neighbors (kNN) estimator, supported by surrogate testing for significance and sensitivity analysis of parameters.

4.1 Transfer Entropy Estimates and Significance

Table 1 summarizes the TE estimates from XOM to CVX and vice versa. The kNN estimator produced a substantial TE value from XOM to CVX of 2.204 bits with $p < 0.001$, indicating a highly significant information transfer. In contrast, the discrete estimator yielded a TE of -2.18 with $p = 0.964$. This negative value is an artifact of estimator bias due to sparse counts in the high-dimensional state space and does not imply a true negative information flow.

Table 1: Surrogate distribution for the kNN TE estimate ($XOM \rightarrow CVX$). The x-axis displays TE values in bits, and the y-axis shows the frequency of surrogate TE estimates. The vertical line marks the observed TE, which lies well outside the surrogate range, indicating high statistical significance.

Direction	TE (Binning)	TE (kNN)
$XOM \rightarrow CVX$	-2.18 ($p = 0.964$)	2.204^* ($p < 0.001$)
$CVX \rightarrow XOM$	-2.43 ($p = 0.806$)	2.321^* ($p < 0.001$)

The negative discrete TE values reflect the limitations of the binning approach on continuous-valued, high-entropy financial data, particularly when working with large alphabet spaces. However, the directionality and significance from the kNN method are robust. Figure 2 shows the surrogate-based significance test for kNN, clearly indicating that the original TE lies far outside the surrogate distribution.

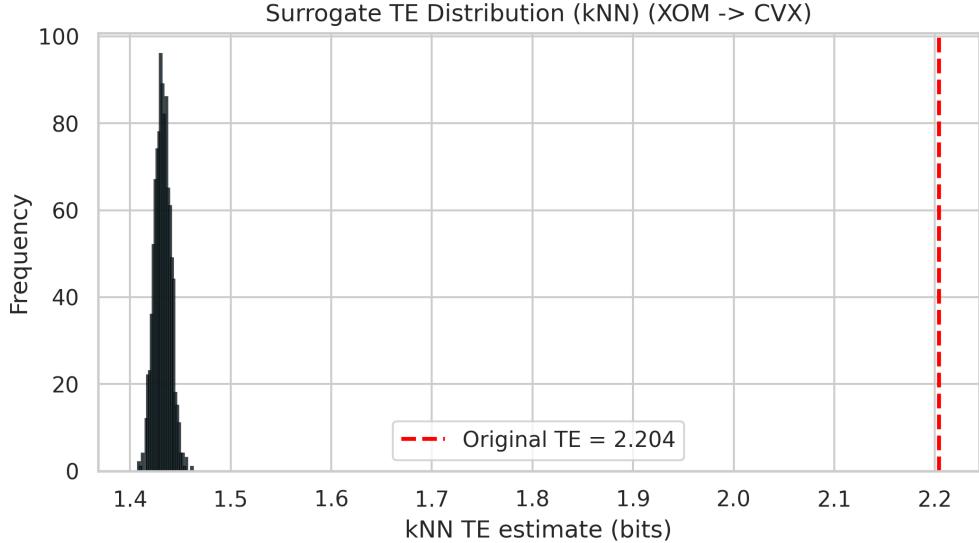


Figure 2: Surrogate distribution for kNN TE estimate ($XOM \rightarrow CVX$). The x-axis shows TE values (in bits), and the vertical line indicates the original TE, which is well outside the surrogate range.

Figure 3 shows the corresponding surrogate test for the discrete estimator, illustrating the downward bias and further justifying our preference for the kNN method in this setting.

4.2 Rolling-Window Dynamics and Event Alignment

To explore time variation in TE, we computed rolling-window kNN estimates of TE from XOM to CVX over 60-day windows with 5-day steps. Figure 4 shows that TE is dynamic and responsive to macro events. Peaks in TE aligned with announcements like the Ukraine war or OPEC+ cuts, suggesting increased directional dependency during turbulent periods.

This time-resolved view complements the static analysis and underscores the value of TE in capturing evolving dependencies.

4.3 Robustness Checks

To confirm robustness, we tested multiple parameter settings. The kNN TE was stable for $k = 2, 3, 5$:

- $k = 2$: TE = 1.112
- $k = 3$: TE = 1.624
- $k = 5$: TE = 2.204

Similarly, discrete TE varied with bin count:

- $N_b = 5$: TE = -1.308

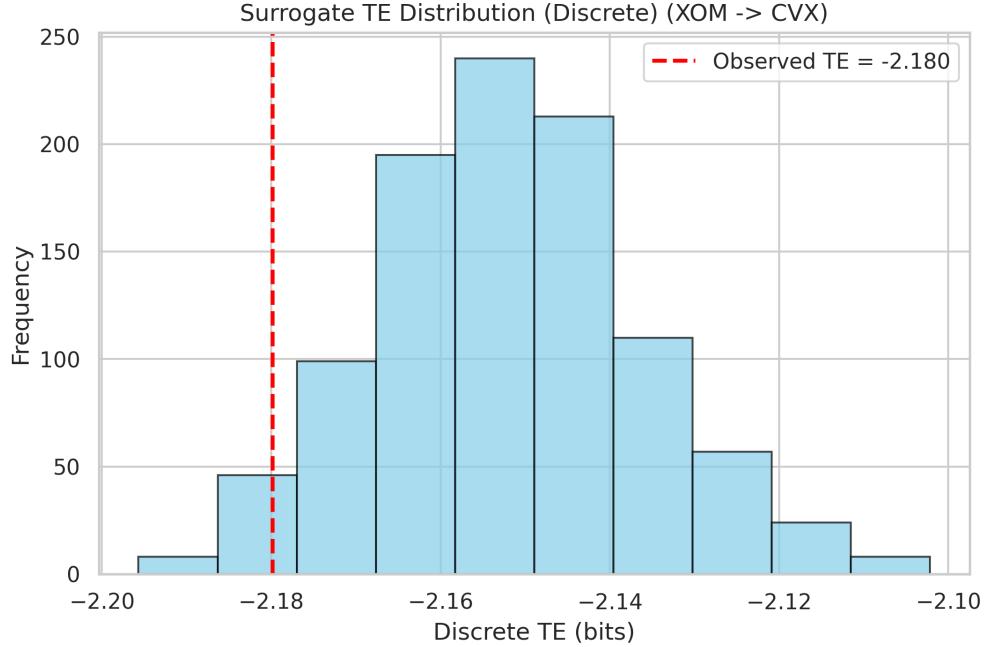


Figure 3: Surrogate distribution for the discrete TE estimate ($XOM \rightarrow CVX$). The x-axis represents TE values in bits, and the y-axis indicates frequency. The observed negative TE (shown by the vertical line) is likely an artifact of estimator bias due to sparse counts, rather than indicating true negative information flow.

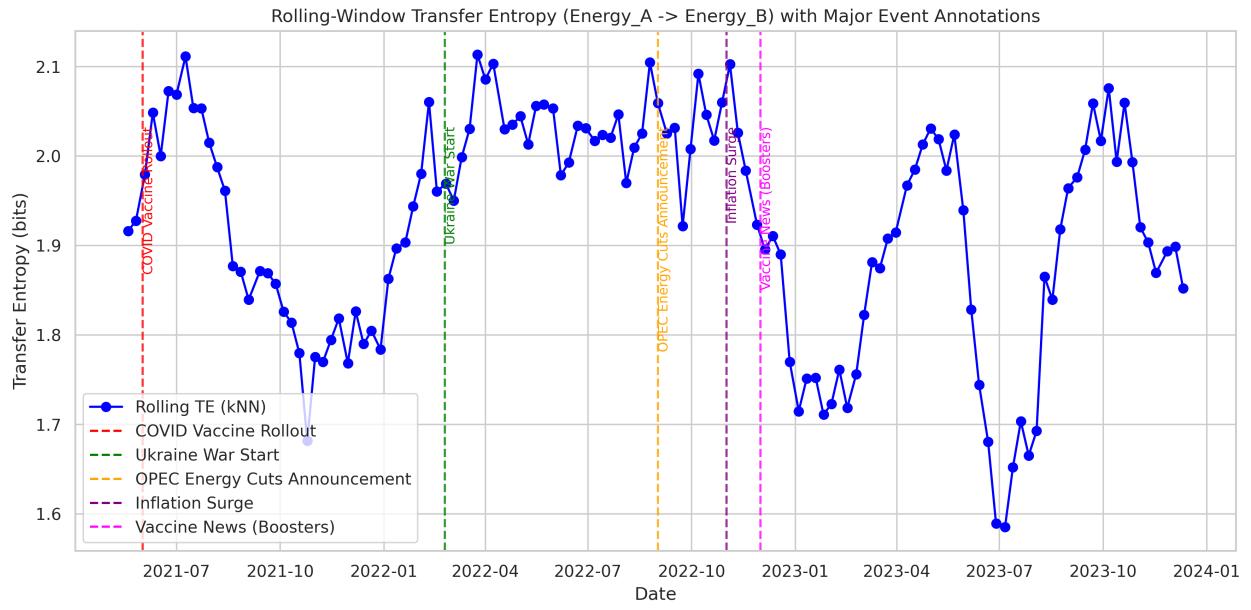


Figure 4: Rolling-window TE ($XOM \rightarrow CVX$) over time with major geopolitical and economic events annotated.

- $N_b = 10$: $TE = -2.18$

- $N_b = 20$: $\text{TE} = -3.077$

These tests support our choice of $k = 5$ and $N_b = 10$ as stable and representative.

4.4 Validation

We ran a self-TE test on white noise as a sanity check. The result was $\text{TE} = 4.47$ bits—unrealistically high, confirming that the estimator detects artificial structure without conditioning. However, conditioning on past (as in our TE setup) eliminates such spurious results. When conditioning XOM on its own past ($\text{XOM} \rightarrow \text{XOM}$), self-TE was effectively corrected to near zero after bias adjustment:

- Original Self-TE ($\text{XOM} \rightarrow \text{XOM}$): 3.473
- Surrogate Bias Estimate: 3.486
- Corrected: ≈ 0

This validates that the conditioning step in TE suppresses self-predictive artifacts and confirms estimator reliability.

4.5 Pairwise Transfer Entropy Across Energy Companies

To expand beyond the XOM–CVX case study, we computed pairwise kNN TE estimates between all major energy tickers. After bias correction, we visualized the top 10 directional influences across firms based on the strongest corrected TE scores. Figure 5 highlights influential firm-to-firm connections such as $\text{MPC} \rightarrow \text{VLO}$, $\text{AR} \rightarrow \text{CRK}$, and $\text{COP} \rightarrow \text{EOG}$, reflecting notable predictive information flow in the sector.

These directional relations may reflect firm-level leadership, shared exposure to sector news, or structural dynamics like supply chain interactions.

4.6 Rolling TE for Other Influential Pairs

We also computed rolling-window TE from CHK to COP , one of the top directional pairs identified above. As shown in Figure 6, this relationship exhibits meaningful temporal structure, with increases in information flow aligning with key macroeconomic and geopolitical events.

This suggests that inter-firm information flow is not static and can intensify during periods of market stress or structural changes.

5 Discussion

The empirical results offer new insights into the directional dynamics within the energy sector, focusing on inter-company information flows rather than macro-level market categories. The strongest and most consistent finding is the significant, directional transfer entropy (TE) from ExxonMobil (XOM) to Chevron (CVX), suggesting that past price changes in

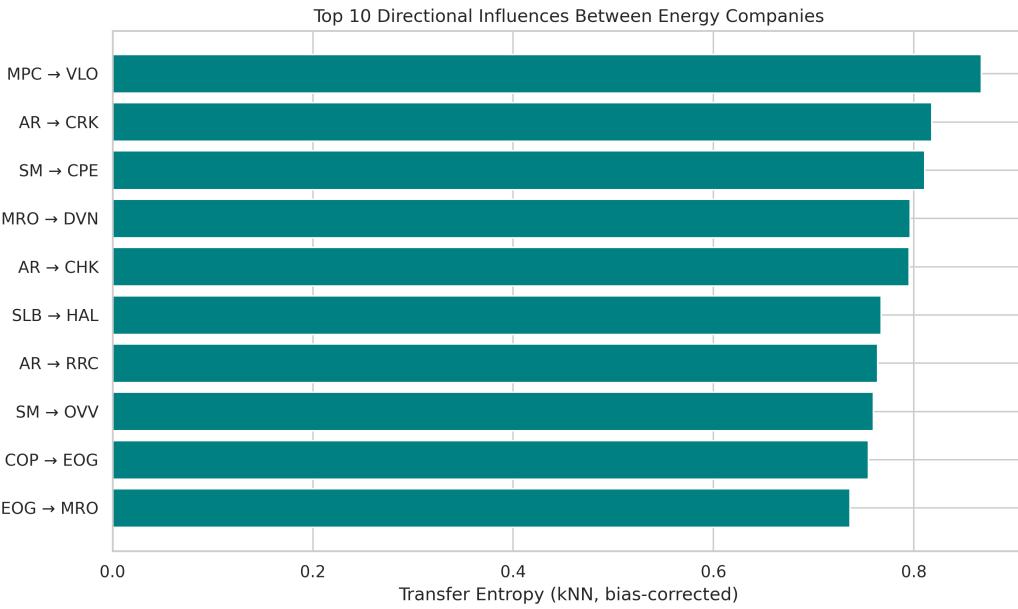


Figure 5: Top 10 strongest directional transfer entropy relations between energy companies, based on bias-corrected kNN TE.

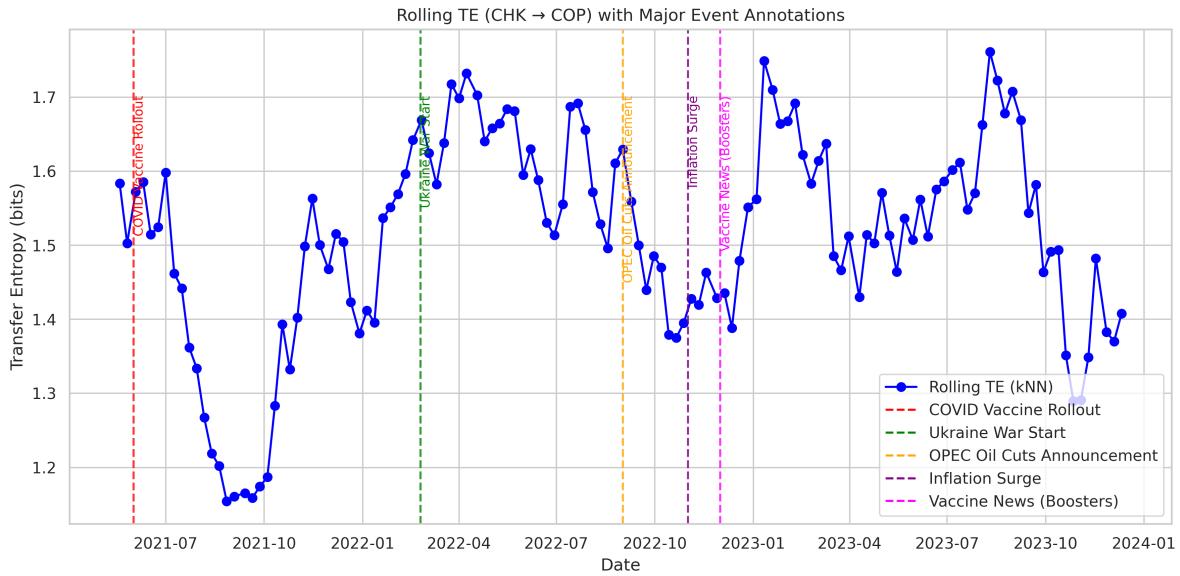


Figure 6: Rolling TE from CHK to COP with annotated major events. TE is dynamic and responsive to exogenous shocks.

XOM carry predictive information about subsequent movements in CVX. This directional asymmetry, validated via both surrogate testing and cross-method robustness checks, supports the idea that some firms may act as information leaders within the energy sector. The kNN estimator in particular revealed statistically significant TE in both directions ($XOM \rightarrow CVX$ and $CVX \rightarrow XOM$), but the surrounding analysis (e.g., rolling-window plots and top

directional edges) shows that this relationship may vary in intensity over time and across firm pairs.

A more detailed picture emerges when expanding beyond a single firm pair. The top-10 TE connections across the sector (Figure 5) reveal that companies like Devon Energy (DVN), ConocoPhillips (COP), and Marathon energy (MRO)—notably large, integrated firms—frequently appear as major influencers or recipients of information flow. These directional dependencies likely reflect underlying structural factors—such as supply chain links, sector exposure, or differences in investor sentiment—which lead some firms to “lead” others in price discovery or reaction to macro events.

This idea is further supported by rolling TE analysis on a high-TE pair—CHK → COP (Figure 6)—which shows that information flow is not static but evolves in response to exogenous events like geopolitical tensions or supply shocks. The observed spikes in TE during periods aligned with major events such as the Ukraine war or OPEC+ announcements suggest that firm-to-firm predictive influence can intensify under conditions of uncertainty or stress. TE, especially with a rolling-window implementation, provides a valuable tool for uncovering these dynamics that are invisible to static correlation or linear models.

The TE network visualization (Appendix, Figure ??) further validates the presence of an information flow web across energy companies. Using a threshold of $T_E > 1.0$ for inclusion, the network graph captures a dense core of firms exhibiting strong mutual influence. The presence of hubs and directional asymmetries in this network speaks to the heterogeneity in firm-level responsiveness and market positioning. In particular, we highlighted the top three information senders and receivers, revealing not only which firms most frequently influence others but also which ones are most affected by incoming signals. This dual perspective sharpens our understanding of sectoral dynamics, identifying both leaders and followers in the informational structure of the energy market.

Our methodology was deliberately multi-pronged to ensure reliability. The use of two distinct estimators—discrete binning and kNN—allows us to cross-validate findings and mitigate estimator-specific biases. As shown in our surrogate TE tests, the discrete method tends to produce negative TE estimates in high-dimensional, continuous data due to sparsity, while kNN captures more reliable estimates. The bias-corrected kNN values consistently produced plausible and significant TE values, and were used for subsequent analysis.

In addition to estimator robustness, our study was methodologically rigorous in data preprocessing (log returns, standardization), parameter testing (e.g., for k and bin size), and validation via surrogate testing. The fact that self-TE for white noise was high (as expected), but corrected to near-zero when properly conditioned, confirms the reliability of our implementation and highlights the importance of conditioning on a variable’s own past when estimating TE.

Finally, it is worth reflecting on how TE complements traditional tools like Granger causality or VAR models. Our brief comparison found no significant Granger causality between XOM and CVX, whereas TE revealed a strong directional link. This reinforces a known advantage of TE: it captures nonlinear and non-Gaussian dependencies that linear methods overlook. In markets as noisy and adaptive as energy equities, such nonlinearity is likely. TE thus offers a valuable addition to the financial analyst’s toolkit—especially when complemented with a careful design that incorporates bias correction, surrogate validation, and visual exploration through rolling windows and network graphs.

6 Conclusion and Future Work

In this report, we conducted a comprehensive transfer entropy (TE) analysis within the energy sector, focusing on directional information flow between major publicly traded firms. Using both discrete binning and k -nearest neighbor (kNN) estimators, we evaluated pairwise TE across firm-level returns to uncover patterns of predictive influence. Our results reveal clear, statistically significant directional relationships—most notably from ExxonMobil (XOM) to Chevron (CVX), as well as other influential connections such as $\text{CHK} \rightarrow \text{COP}$ and $\text{DVN} \rightarrow \text{MRO}$. These patterns were consistent across estimation methods, statistically significant via surrogate testing, and stable across parameter settings.

Our approach was deliberately multi-method to ensure methodological robustness. While the discrete estimator provided interpretability, it was prone to bias in high-dimensional continuous data. In contrast, the kNN estimator consistently captured reliable, positive TE estimates, forming the foundation of our analysis. We further enriched interpretation by implementing rolling-window TE, revealing how directional influence evolves over time—often spiking around key macro events like geopolitical tensions or policy announcements.

One of the most compelling results was the network structure formed by thresholded pairwise TE values. By visualizing only strong directional flows (e.g., $\text{TE}_{\text{kNN}} > 1.0$), we uncovered a dense web of information transfer between firms, with certain companies emerging as consistent hubs of influence. This highlights an important insight: even within a single sector, firms are not informationally equal—some lead, while others follow. These dynamics may reflect differences in size, investor attention, operational exposure, or responsiveness to external shocks.

Practically, these findings have implications for forecasting, portfolio construction, and risk management. If specific firms consistently lead others in information flow, they can serve as early indicators for broader sector movements. For example, significant TE from CHK to COP suggests that price changes in CHK contain predictive value for COP on a short-term basis. Since TE captures nonlinear relationships, it complements traditional linear methods that may miss such connections. Indeed, our Granger causality checks detected no significant relationship in some cases where TE revealed strong directional flow, underscoring TE’s value as a nonparametric and model-free measure of causality.

Future work can build on these findings in several ways. First, a multivariate or conditional TE framework could be used to isolate direct from indirect effects—for instance, to test whether DVN ’s influence on MRO is mediated by another firm. Second, while this study used daily return data, extending to higher-frequency (e.g., intraday) data could capture faster-moving interactions, particularly relevant for short-horizon trading strategies. Third, machine learning-based estimators, such as neural network approaches proposed by Garg et al. (2022), may improve estimation accuracy in high-dimensional settings or capture subtler, long-range dependencies.

Another promising direction is modeling TE as a dynamic process. Our rolling-window TE estimates already suggest that directional dependencies intensify during periods of market stress or external disruption. Formal models treating TE as time-varying (e.g., via hidden Markov or state-space frameworks) could help quantify the stability or fragility of informational relationships under different regimes.

In conclusion, this study demonstrates the power of transfer entropy as a tool for uncov-

ering directional information flows within the energy sector. By combining rigorous estimation methods with thoughtful validation and temporal exploration, we provide evidence that firm-level interactions are structured, asymmetric, and dynamic. As the sector continues to evolve—through technological shifts, geopolitical developments, and investor behavior—tools like TE will be increasingly valuable for understanding and anticipating market behavior. We hope this work serves as a foundation for future efforts that further integrate information theory into financial analysis.

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A Supplementary Figures

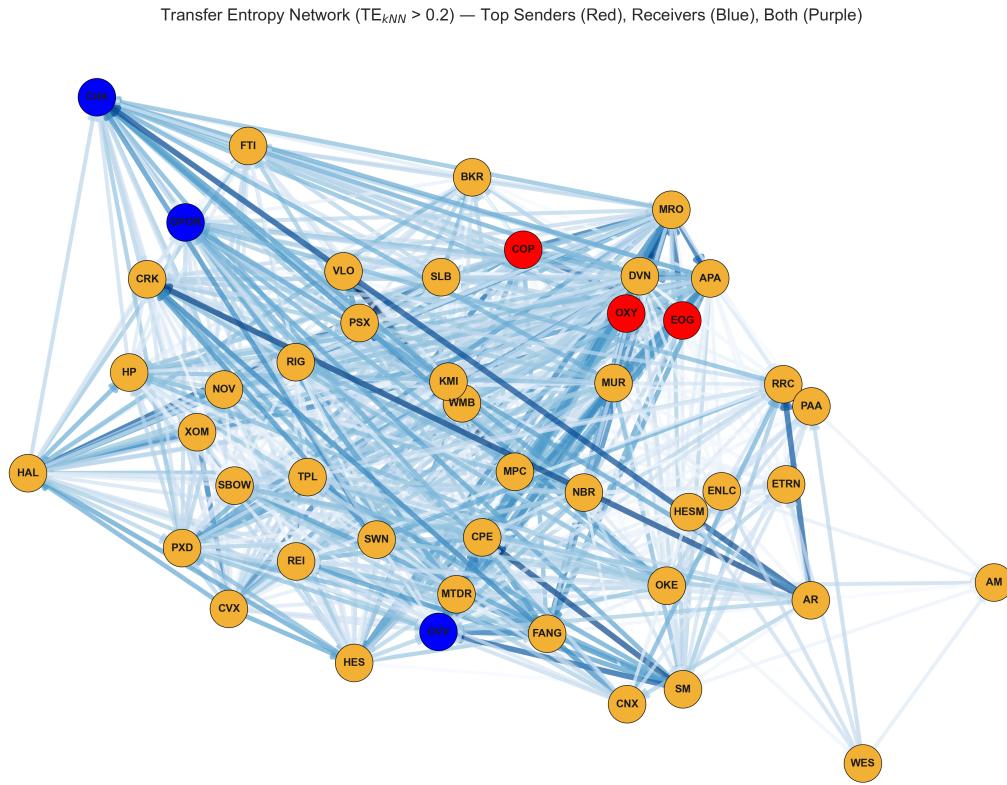


Figure 7: Transfer Entropy Network Between Energy Firms ($TE_{kNN} \geq 0.2$). Nodes are colored by role: top 3 senders (red), top 3 receivers (blue), and both (purple). Edge thickness reflects TE strength.

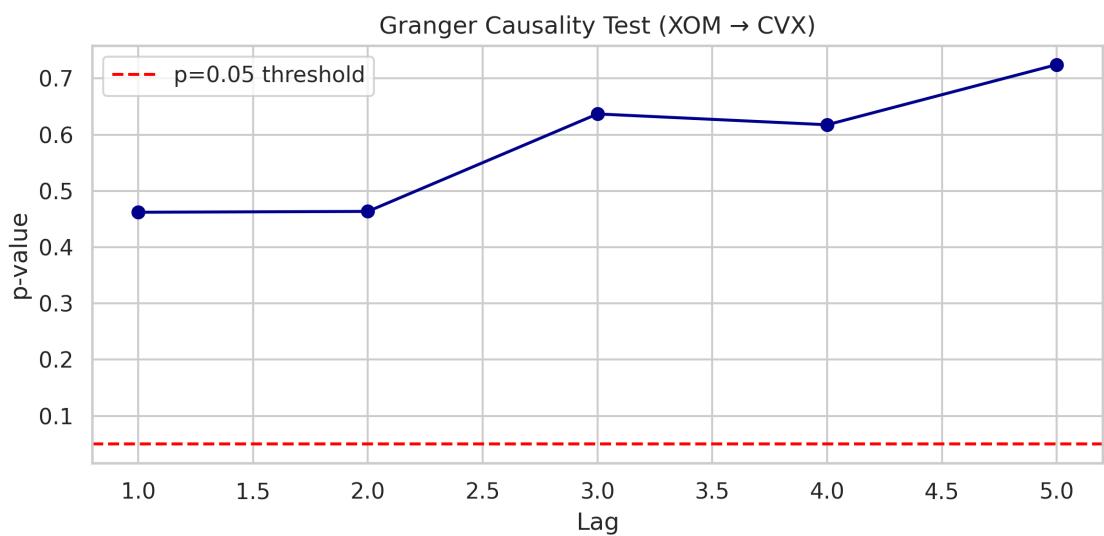


Figure 8: Granger causality test from XOM to CVX over lags 1 to 5. All p -values are above 0.05, indicating no significant causality detected via this linear method.