Indian Institute of Technology Madras
End Semester Exam
ID5004W: AI in Predictive Maintenance,
Reliability and Warranty
Points: 25, Date: August 5, 2023

Questions

Q1 Consider the deterministic system, where

$$x_k = A_{k|k-1}x_{k-1} + B_{k-1}u_{k-1},$$

where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

It is desired to take the state from $\boldsymbol{X_o}$ to $\boldsymbol{X_f}$, where

$$X_0 = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \quad X_f = \begin{pmatrix} 6 \\ -8 \\ 2 \end{pmatrix},$$

If the input sequence is begun at **step 0**, and the system is completely controllable, then ;

(a) How many steps are required to move the system to the **desired *state***? (*1 mark*)

(b) Obtain the equation, governing the input state, ***x(0)*** and the inputs, ***u(i)*** *(1 mark)*

[Hint: Write down the discrete steps for controllability]

(c) Calculate the ***inputs u(i),*** required to move the system to the ***desired state***. (*1 mark*)

Q2 (a) Considering an overdetermined set of equations, represented by

$$y = Hx + v$$

Fill in the blanks, for the expressions (*1 mark*)

$\varepsilon_y =$	
$J =$	

$\frac{dJ}{d\hat{x}}$	
$\hat{x} =$	

(b) Using the expression, obtained above, find ***x***, **f**or the set of equations, given below *(1 mark)*

$3x_1 - x_2 = -4$

$2x_1 + x_2 = 1$

$x_1 - 2x_2 = -5$

Q3 Consider the System Dynamics, with uncertainties, given by

$$x_k = A_{k|k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}$$

Starting with the expression:

$$P_k = E[\tilde{x_k}\tilde{x_k}^T]$$ w.r.t. covariance minimization

(a) Obtain the expression for the a posteriori Covariance, by completing the in-between steps, in the blanks provided : *(1 mark)*

$$P_k = E\{[(I-K_kH_k)A_{k|k-1}\tilde{x}_{k-1}+K_kv_k][(I-K_kH_k)A_{k|k-1}\tilde{x}_{k-1}+K_kv_k]^T\}$$

$$P_k = P_k^- - K_kH_kP_k^- - P_k^-H_k^TK_k^T + K_k(H_kP_k^-H_k^T+R_k)K_k^T$$

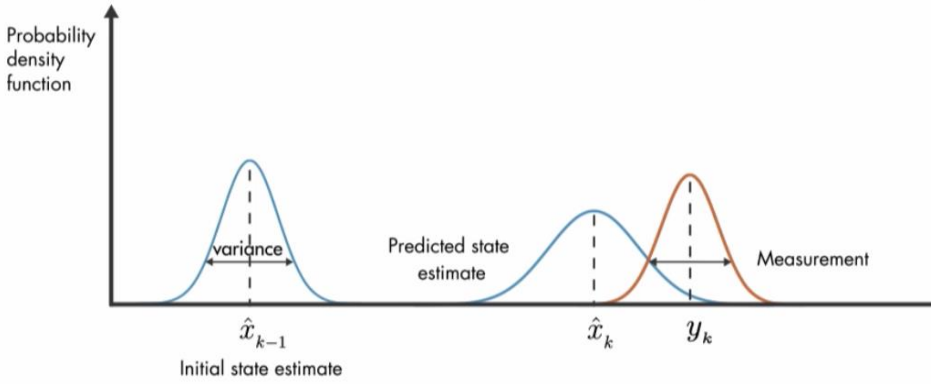
(b) And obtain thereon the expression for Kalman Gain : (*1 mark*)

$$K_k = P_k'H^T\left(HP_k'H^T + R\right)^{-1}$$

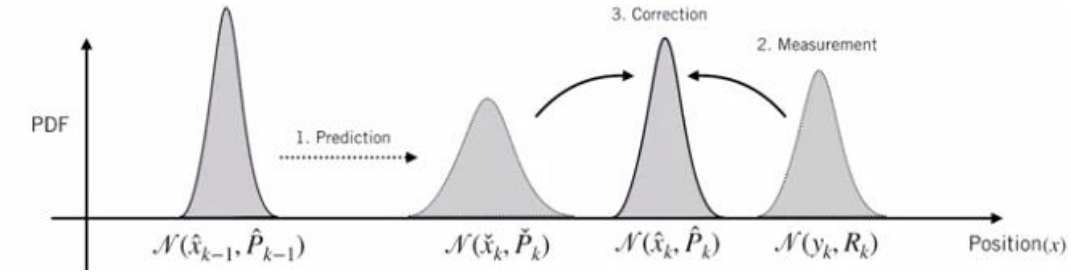
Stating all the assumptions, w.r.t. the ***statistical properties of the noise***

Q4 (a) Explain the *statistical concept* (*proof is not required*) resulting in an *increase in apriori-covariance*, in the prediction step, as shown below : *(1 mark)*

$$x_k = A_{k|k-1}x_{k-1} + w_{k-1}$$



(b) In the diagram given below, plug in the Kalman filter steps, corresponding to each of the ***PDFs*** shown *(1 mark)*



Steps	***Equations***	***Comment***

Q5 Considering the Bayesian Posterior, as indicated below,

Explain the steps to establish this relation, using the fundamental postulates of Baye's :

(1 mark)

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{z}_{1:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{1:k-1})}$$

Steps	**Formulae /Arguments**

Q6 Find parameters using the design matrix $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 4 & 8 \\ 1 & 9 & 17 \\ 1 & 14 & 64 \end{bmatrix}$ and data $[6 \quad 6.3 \quad 7.6 \quad 10.5 \quad 14]^T$. (*0.5 mark)*

Q7 Use the dataset labelled corresponding to the last two digits of your roll number (sample_classification_rollno in Q1.zip file) for the following analysis:

a. Calculate T^2 statistic for the given dataset for a new observation **x.** *(0.5 Mark)*
b. Compute the T^2 threshold for 90% significant level and provide the ellipsoid formula. *(0.5 Mark)*
c. Let $x_1 = [12.91 \quad 11.16 \quad 4.590 \quad 13.17 \quad 5.166]$ and $x_2 = [2.826 \quad 7.106 \quad 47.29 \quad 9.46 \quad 1.548]$ are the new observations recorded. Determine whether they are an outlier. *(1 Marks)*
d. Perform PCA analysis to obtain T^2 statistic with the principal components covering 90% of the variance. Compute the T^2 threshold for 90% significant level and provide the ellipsoid formula using PCA based approach and determine the in-control and out-of-control status of observations x_1 and x_2. *(3 Marks)*
e. Calculate the Q statistics for the given data *(0.5 Marks)*
f. Comment on the calculated Q statistic calculated for the new observations x_1 and x_2. (*1* Marks)
g. Calculate the contribution of the variable which is min(roll no)+1, For example: Roll no = 51, calculate the contribution of the variable 1+1=2. *(2 Marks)*

Q8 A study focusing on the time taken for various medical data until publication was conducted for 120 months. Conventional status code was used to indicate censored and un-censored data

a. Plot the survival curve for the dataset *(0.5 mark)*
b. Perform log-rank test and report the p-value *(0.5marks)*
c. Fit the data to the Cox's proportional hazards model with linear and nonlinear terms for the hazard function $h(t/x_i)$. The nonlinear terms should be x_ix_j form, for example, $x_1^2, x_2^2, \ldots, x_1x_3, x_2x_3, \ldots$ *(2 marks)*
d. Determine the significant contribution of the variables *(1 Marks)*

Use Q2_even or Q2_odd datasets depending on the even or odd roll number to answer the questions.

Q9 A manufacturing process, a failure process times is given by the following distribution.

$$p(t) = \lambda k\ (\lambda t)^{k-1} e^{-(\lambda t)k}, t\ \geq 0$$

Determine the hazard function form. Determine the average failure rate over an interval [2, t+3], where t is the last digit of your roll no. (*2 marks*)