Part 1
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Part 2

Q1 Consider the deterministic system, where

$$x_k = A_{k|k-1}x_{k-1} + B_{k-1}u_{k-1},$$
 where $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

It is desired to take the state from X_0 to X_f , where

$$X_0 = \left[\begin{array}{c} 0 \\ -1 \\ 3 \end{array} \right] \qquad X_f = \left[\begin{array}{c} 6 \\ -8 \\ 2 \end{array} \right]$$

If the input sequence is begun at step 0, and the system is completely controllable, then;

- (a) How many steps are required to move the system to the desired state? (1 mark)
- (b) Obtain the equation, governing the input state, x(0) and the inputs, u(i) (1 mark)
 [Hint: Write down the discrete steps for controllability]
- (c) Calculate the *inputs u(i)*, required to move the system to the *desired state*. (1 mark)

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maximum absolute change in any alment of 12
   1. N2 max 6, 19, 1-11 =
 so steps is of required
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For all i tem Lto F.

Q2 (a) Considering an overdetermined set of equations, represented by

$$y = Hx + v$$

Fill in the blanks, for the expressions (1 mark)

(b) Using the expression, obtained above, find x, for the set of equations, given below (1 mark)

$$3x_1 - x_2 = -4$$

$$2x_1 + x_2 = 1$$

$$x_1 - 2x_2 = -5$$

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Recursive Least square estimation &
That us considerative following linear system defined by,

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                         At a farticular instance K, above equation (

can be written as,

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the least equane certificate based on (k+1) measurements is,

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P tuni) = [H'(k) H(k)] | => H'(k+1) H(k+1) = H'(k) H(k) | (k+1) h'(k+1) |

Then equation (iv) can be | => H'(k) H(k) + h(k+1) h'(k+1) |

Then equation (iv) can be | => [H'(k) H(k) + h(k+1) h'(k+1)] |

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                Defining, S(k+1) = [HT(K)H(K)] -1

Defining, S(k+1) = 1+hT(k+1)P(K)N(k+1) - cvi)

[New York) = 1+hT(k+1)P(K)N(k+1) - cvi)
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using the defined value, update equation with become, $P(\kappa+1) = P(\kappa) - P(\kappa) N(\kappa+1) 8^{-1} (\kappa+1) N^{-1} (\kappa+1) P(\kappa)$ $= \left[\frac{\pi}{2} - \mathbf{K} (\kappa+1) N^{-1} (\kappa+1) \prod P(\kappa) - CViii) \right]$ since $S(\kappa) = \left[\left(H^{T}(\kappa) + (\kappa) \right) \right] + \left[\frac{\pi}{2} \times 2(\kappa) \right]$ thon agration of step (x41) MU brows,

2(x41) = P(x41) H(x41) Z(x41) [Z(x)]

= P(x41) [H(x) | h(x+1)] [Z(x)]

= P(x41) [H(x) | h(x+1)] [Z(x)]

NOW applying P(x41) value we got,

A(x41) = P(x41) [H(x) Z(x) + h(x41) Z(x41)]

= [P(x) - P(x) h(x41) 8 (x41) h(x41) Z(x41)]

= [P(x) - P(x) h(x41) 8 (x41) X(x41) Z(x41)]

- X[H(x) Z(x) + h(x41) Z(x41)] = P(K) HT(K) Z(K) A P(K) N (K+1) ZK+1 T (K) Z(K)

= P(K) H(K+1) 8 T (K+1) NT (K+1) P(K) H (K+1) Z (K)

- P(K) N (K+1) 8 (K+1) NT (K+1) P(K) A (K+1) Z (K+1). Now 2(K+1) = 2(K) +8 (K+1) P(K) N(K+1) ZK+1 -P(K) N(K+1) P(K)

+aking common on PHS 00801)

=12(K+1) = 2(K) + P(K) N(K+1) 8 (K+1) [ZK+1 - N(K+1) N(K)]

=12(K+1) = 2(K) + P(K) N(K+1) 8 (K+1) [ZK+1 - N(K+1) N(K)] 1: 2(KH) = 2(K) + K (KH) [2KH - N (KH) 2 (K)]

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Q3 Consider the System Dynamics, with uncertainties, given by

$$x_k = A_{k|k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1}$$

Starting with the expression:

$$P_k = E[\tilde{x_k}\tilde{x_k}^T]$$
 w.r.t. covariance minimization

(a) Obtain the expression for the a posteriori Covariance, by completing the in-between steps, in the blanks provided: (1 mark)

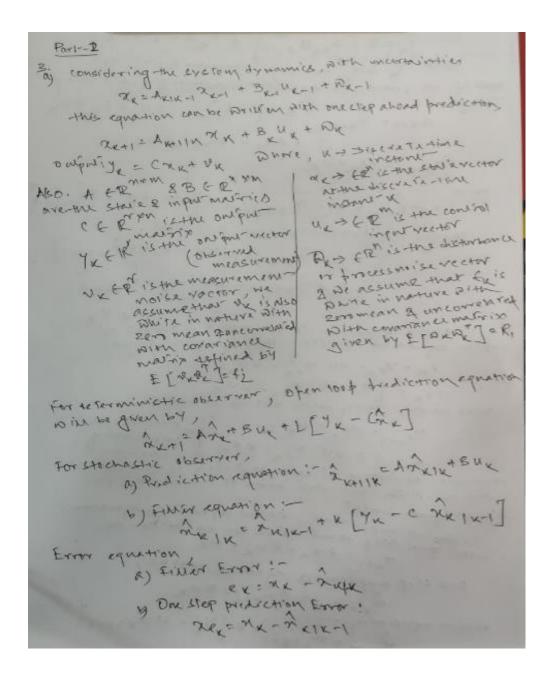
$$P_k = E\{[(I - K_k H_k) A_{k|k-1} \tilde{x}_{k-1} + K_k v_k][(I - K_k H_k) A_{k|k-1} \tilde{x}_{k-1} + K_k v_k]^T\}$$

$$P_{k} = P_{k}^{-} - K_{k} H_{k} P_{k}^{-} - P_{k}^{-} H_{k}^{T} K_{k}^{T} + K_{k} (H_{k} P_{k}^{-} H_{k}^{T} + R_{k}) K_{k}^{T}$$

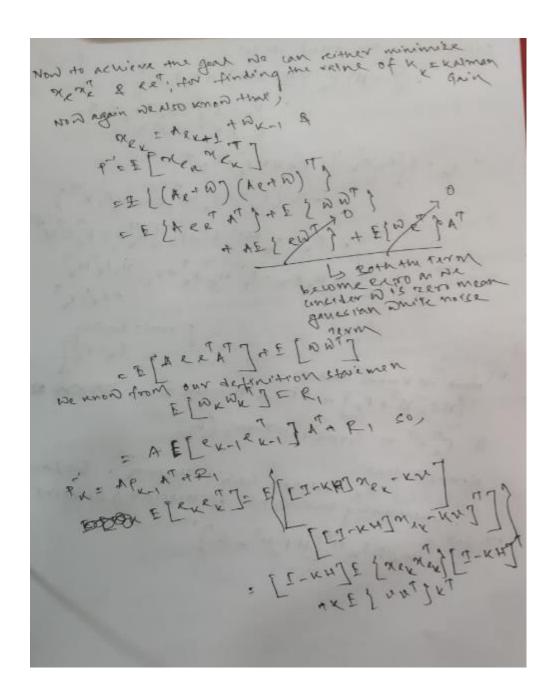
(b) And obtain thereon the expression for Kalman Gain: (1 mark)

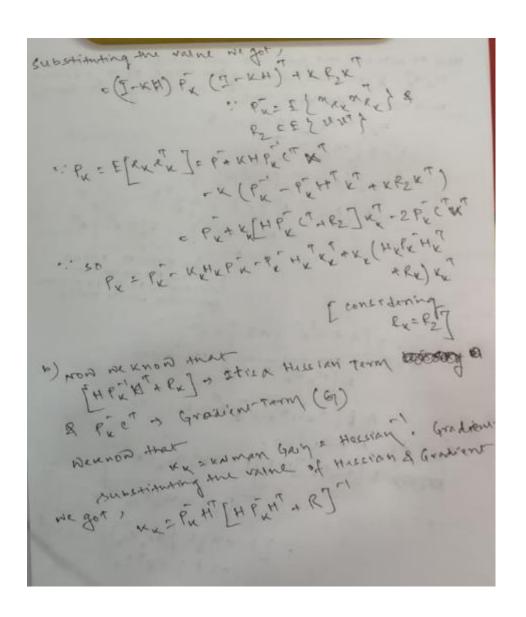
$$K_k = P_k'H^T (HP_k'H^T + R)^{-1}$$

Stating all the assumptions, w.r.t. the statistical properties of the noise



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Q5 Considering the Bayesian Posterior, as indicated below,

Explain the steps to establish this relation, using the fundamental postulates of Baye's:

(1 mark)

$$p(\mathbf{x}_k|\mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{z}_{1:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{1:k-1})}$$

According the problem chainment , eages

fordamental postularia is given p (2x | 21 × m)

P(2x | 21 × m) = P(2x | 22 × m)

P(2x | 21 × m)

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Q.6 7, 8, 9 are done in the jupyter notebook.