ID-5004W: AI in Predictive Maintenance, Reliability, and Warranty
April – July 2023

Mid Semester

Due: 5:30 PM, Sunday, June 4th

Marks distribution: Maximum Mark: 100

Ques. No.	Q1	Q2	Q3	Q4	Q5
Max. Marks	10	15	25	20	30

Instructions

- Make any additional assumptions if needed and justify your assumption.
- Please submit your own work.
- Write your results and explanations in a Word file and save it as "RollNumber_EndSem.doc (or *.pdf)."
- Make a zip or rar file including all your MATLAB (or any software) files and the report mentioned above, file name "RollNumber_Solutions.zip," and upload it in Moodle.

Problem - 1: Laplace transformation

Part A:

Find the transfer function of a second-order system given the differential equation:

$y'' + 3\,y' + 2y \ = 4u(t)$, (u(t) represents the unit step input)

Part B:

Solve the differential equation from Part A using the Laplace transform method, given that $y' = 0, y(0) = 1$.

Problem - 2: Steady-State Error

Part A:

Calculate the steady-state error for a unity feedback control system with the following transfer functions: Process Transfer Function (G(s)) and Control Transfer Function (H(s)). The input reference is a unit step.

$$G(s) = \frac{10}{s^2 + 5s + 10} \; and \; H(s) = 2s + 3$$

Part B:

(1) Determine the steady-state error for the same system from Part A, but now the input reference is a ramp with a slope of 2.

(2) Determine the steady-state error for the same system from Part A, but now the input reference is a sinusoidal signal with a frequency of 2 rad/s.

Problem - 3: State transition matrix

Part A:

Consider a system with the following state-space representation: $\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = C\,x(t)$. Calculate the state transition matrix $\Phi(t)$ for the system.

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, and\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Part B:

Consider a linear time-invariant system described by the following state-space representation:

$\dot{x}(t) = Ax(t) + Bu(t)$ and $y(t) = C\,x(t)$.

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 2 \\ 1 & -1 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, and\ C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Given that the initial state vector $x(0) = [1, 2, 3]^T$ and the input $u(t) = 2$ for $t \geq 0$, determine the state transition matrix $\Phi(t)$ for this system.

Problem - 4: Differential equation, Transfer function, and State-space representation

Part A:

Convert the following fourth-order differential equation to a transfer function and state space representation: $d^4y(t)/dt^4 + 2d^3y(t)/dt^3 + 3d^2y(t)/dt^2 + 4dy(t)/dt + 5y(t) = 6u(t)$

Part B:

Convert the state space representation given in Problem – 3 Part B to a third-order differential equation.

Problem - 5: System dynamics

Consider a second-order system with a transfer function given by G(s)

$$G(s) = \frac{5s + 2}{s^2 + 4s + 4}$$

Part A:

Describe the type of roots and the nature of the response of the system based on the transfer function.

Part B:

Given that the system is excited by a unit step input (U(s) = 1/s), determine the following:

1. The value of the steady-state error of the system.
2. The value of the peak time (tp) for the system response.
3. The value of the peak overshoot (Mp) for the system response.
4. The settling time (ts) for the system response with an acceptable error of ±0.01.