ID-5004W: AI in Predictive Maintenance, Reliability, and Warranty
April – July 2023

Home Assignment - 01

Due: Monday, May 15th

Instructions

- Make any additional assumptions if needed and justify your assumption.
- Please submit your own work.

Problem - 1: Homogenous equation with constant coefficients

(a) Real roots of the characteristic equation: Find the solution to the initial value problem $y'' + 5y' + 6y = 0$ with $y(0) = 2, y'(0) = 3$

(b) Complex roots of the characteristic equation: Find the solution to the initial value problem $16y'' - 8y' + 145y = 0$ with $y(0) = -2, y'(0) = 1$

(c) Repeated roots of the characteristic equation: Find the solution to the initial value problem $y'' - y' + 0.25y = 0$ with $y(0) = 2, y'(0) = 1/3$

Problem - 2: Non-homogenous equation with constant coefficients

Using the method of undetermined coefficients, find a particular solution of

(a) $y'' - 3y' - 4y = 10$

(b) $y'' - 3y' - 4y = 2\ \sin wt$, (you may consider $w = 1$).

Problem - 3: Mechanical Systems

Suppose that a mass weighing 4.5 kg stretches a spring 40 mm. If the mass is displaced an additional 25 mm. and is then set in motion with an initial upward velocity of 0.5 m/s, determine the position of the mass at any later time. Also, determine the period, amplitude, and phase of the motion.

Problem - 4: Electrical Circuits/Systems

A series circuit has a capacitor of 10^3 micro Farad, a resistor of 3 X 10^2 ohms, and an inductor of 0.2 micro Henry. If the initial charge on the capacitor is 1 micro coulomb and there is no initial current, find the charge Q on the capacitor at any time t.

Problem - 5: Laplace transformation

Using the Laplace transform, find solutions for the following equations

(a) $5y' = e^{-3t}$ with $y(0) = 4, y'(0) = 0$

(b) $y'' = 1 - t$ with $y(0) = 0, y'(0) = 0$

(c) $y'' + 2y' + 2y = 0$ with $y(0) = 1, y'(0) = -1$

(d) $y'' + 4y = \cos(t)$ with $y(0) = a, y'(0) = b$

(e) $y'' + 16\, y = 16\, u(t-3) - 16$ with $y(0) = 0, y'(0) = 0$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n, \quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}}, \quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at}, \quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t), \quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Table Notes

1. This list is not a complete listing of Laplace transforms and only contains some of the more commonly used Laplace transforms and formulas.

2. Recall the definition of hyperbolic functions.

$$\cosh\left(t\right) = \frac{\mathbf{e}^{t} + \mathbf{e}^{-t}}{2} \qquad\qquad \sinh\left(t\right) = \frac{\mathbf{e}^{t} - \mathbf{e}^{-t}}{2}$$

3. Be careful when using "normal" trig function vs. hyperbolic functions. The only difference in the formulas is the "+ a^2" for the "normal" trig functions becomes a "- a^2" for the hyperbolic functions!

4. Formula #4 uses the Gamma function which is defined as

$$\Gamma\left(t\right) = \int_0^{\infty} \mathbf{e}^{-x} x^{t-1}\, dx$$

If n is a positive integer then,

$$\Gamma\left(n+1\right) = n!$$

The Gamma function is an extension of the normal factorial function. Here are a couple of quick facts for the Gamma function

$$\Gamma\left(p+1\right) = p\Gamma\left(p\right)$$

$$p\left(p+1\right)\left(p+2\right)\cdots\left(p+n-1\right) = \frac{\Gamma\left(p+n\right)}{\Gamma\left(p\right)}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$