Course Instructor: Arun Rajkumar.

Exam Date: 18-06-2023

Prepared By: Aloy Banerjee

Roll Number: CH22M503

(1) State the Doubling trick formally. Explain what problem it solves and show how. (5 points)

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9.1) Doubling Prick Algorithm: - Algorithm follows below steps, the 2m 2. Run a new instance of algorithm on dearning to 2m, - (2m+1-1) with optimal dearning rate. (for an algorithm that runs for 2 M STRPS) Posbum Doubling Trick Solved: - For randomize weighted For randomize weighted majority algorithm zero regret bound can be achieve if E* = Jund; Durve, T > No. of round d > NO. of expent Mathemetically it ether that for weighted majority algorithm, if Tox then RA(T) > 0

Ly Algorithm

Ly Mo. of

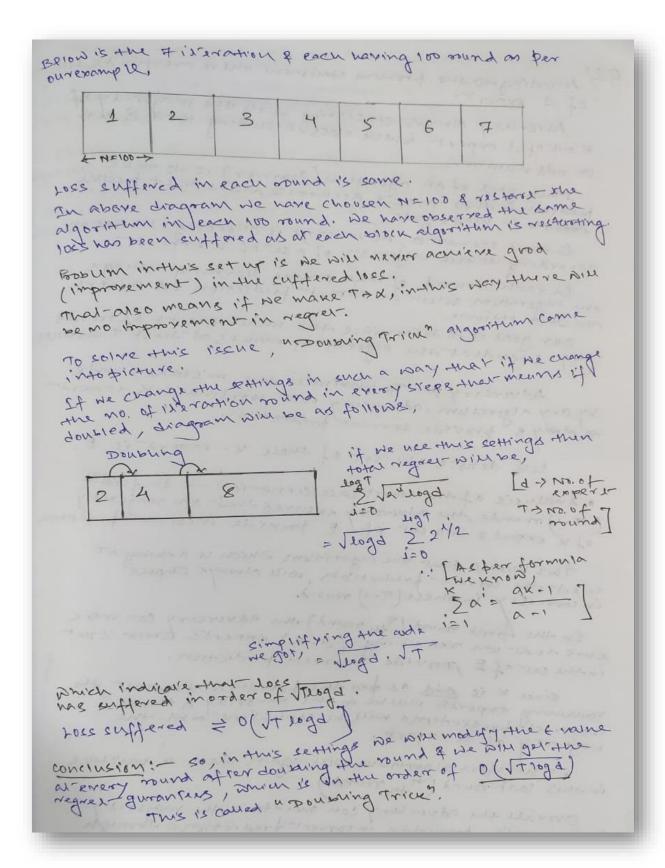
round THE ACTUALLY MEANS that on the number of round increase our algorithm can strack the best emperit & achieve the zero regret.

But the problem here is, in real life scenario
it is impossible to reach tox for any practical As the number of round in practical scenario is whays finise. TO OVERCOME THIS ISSUE ONE approach is to come up with some ufinite Horizon Regret Bound? To simulate the agarithm for Tox we can perform a) ext the algorithm fix's T as some finite value, Snack of the loss suffered. e) Restart the agostthagain & som i't for some N no of round.

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(2) Consider the usual online learning protocol with a finite set of d experts. Consider an adversary who is always consistent with the majority of k out of d experts in the class (assume k < d and k is odd). What is the worst case number of mistakes for any algorithm for this problem? Can you think of an algorithm which achieves the same? (10 points)

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9.2)
1 in take mem
a le of a expert.
finish cer of the concieting with coming
Adversery amongs in the class ass
of none of a subser ! per-
KLDAKIS an odd vice (hearner) is to
According to the problem start an emajority timize est of a expert in the class accuming of xour of a expert in the class accuming x < d & x is an odd number x < d & x is an odd number whain good of on algorithm (rearner) is to
Talain dead in man are us account.
make brediktion in wind sign
but bossible expert
make frediction that are as accurate on the best possible expert in wind sight.
Aspertus in broadly 2 mm)
this problem
make frediction in wind signed best fossible superior in wind signed the can think began the problem starsament, we can think the problem in broadly 2 way, this problem in broadly 2 way, a) Either K experis are consission-with
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1001/484
of 11 experts consistent with adversery
is consistent with
of 11 experts consistent with adversery of experts consistent with adversery to sider both cases mentioned above;
if we consider both cases mentioned above)
both cases mentioned above
IL WE CONSIDER
in majoristy volt and
of It he do with majorish now orderized ! Linch consiner
a) It he go I anderision I thank consins
2 - 6 4 11
diridia 69 11 Mistake
Jirided 64 11 0 1 Mistake Michael
Mistake O Mistake
916
we are half the appearst in each steps are removed.
we are half the appear in each steps are removed.
a was and the are consticted.
Since, we know that in they are the majority
Since, we know shar at least k are consissent. So we have to wait him they get the majority
-10.
none,
Now for the abbourse of
mistakes are

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```
similarly ,
                                     ND. of mystaus
I working at that take we can observe that, on the vetue of the x increases, no of mistakes decrease
since as per problem stratement we have to find
our the worst number of mistakes, k value herrobe
decrease actually & should be minimum.
b) As per the prosum starament we are open to
debate on any against on for worst case mistaves,
if we are stand picking our expend randomly & adversery for a us to make another our put no incorrect adversery for a us to make another maning (d-k) mistakes.
     And if we consider the worst case KEL then
 that will become (d-1) mistakes.
De can say that if we do random queeses for our owfour the we can achieve the worst unistance of
      NOW it we consider the next invertore textion,

2 experts one consistent with adversery
  -> then, from the previous conclusion we can
                      d- (xx1) no. of mistakes will happen
     50, in this scenario k can have minimum value of
    for werst case scenarios
     3 such that worst case mistakes can be (d-2).
```

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- (3) Assume that you want to estimate the probability of a coin coming up heads. Let the true probability be μ .
 - Assume $\mu = 0.75$. If you want to make a statement with at least 99% confidence that the estimate $\hat{\mu}$ satisfies $|\mu \hat{\mu}| \leq 0.1$, how many times would you have to toss the coin? (4 points)
 - If the true μ changed from 0.75 to 0.5, how does your answer from the first part change? (2 points)
 - If the confidence changed from 99% to 90%, how does your answer from the first part change? (2 points)
 - If the tolerance change from 0.1 to 0.01, how does your answer from the first part change? (2 points)

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```
Assuming thout I want to retimate the probability of a coin coming up as heads.

Let the true probability is M.

Li the retimates Dinich entirety | M-M/2011
     According to the "Horffding's Inequality" fromides an upper bound on the probability that the sum of bounded independent vandom variables deviate from its expected value by more than a certain
                  P(111-1176) < 2 < xp (-2ne") ... vi)
      amount.
       then equation (i) can be written as,
                 taxe log in both Side of the equation,
                > 2nd < In [2 Rmp (-2ne")]
              = 1nd = 1n2 + (-2ne")
               57 Ind 5 In2 - 2ne"
PART 1:- 7 n 7 In (2/d) - - cii)

Non according the fromm statement,

for 991. confidence bound d = (10.99)

At = 011

Plotting the value in equation (1)

In (2/014)
                    n 7 2x(0.1)~
                7 m 7 5.29832
                =1 × 7/ 264.9 ≈ 265
    " He have to toes the coin for all least 265 times.
```

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PART 2:-Will not be having any impact.

Because if we consider "Hoeffding Inequality"

then it is clear that m value does not depend on the And frebability.

17 (2/d)

18 clear

19 10 (2/d)

2 cr ; no structure inthis

requation equation derived using " Horffeing Inequality"

Ne Know that, We Know that,

N > 10 (2/2)

2 t ~ As the confidence sevel changes from agis to goil. SO, putting this value in the formula wegot, we can prace, $n > \frac{\ln(2/d)}{2t^{2}}$ $9n7, \frac{\ln(2/01)}{2\pi(01)^{2}}$ =1 N7 219957 × 149.785 × 150 for alleast 150 times. PART 4:- In this settings tolarance change from 0:1 to oral in this case values well be vive as follows, X=0.01 8 € = 0.01 Ny In (2/d) =1 N7, 26491,586 ≈ 26492 ": so in this settings we have to flip the coin at least 26 492 +1mxs.

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- (4) An epsilon-greedy strategy for the stochastic multi-armed bandits set up exploits the current best arm with probability $(1-\epsilon)$ and explores with a small probability ϵ . Consider a problem instance with 10 arms where the reward for the *i*-th $(i=1,\ldots,10)$ arm is Beta distributed with parameters $\alpha_i=5, \beta_i=5*i$.
 - Implement the *epsilon*-greedy algorithm and compare it with the performance of the UCB and the EXP-3 algorithm. (5 points)
 - Comment on your observations about the regret plots obtained in the previous part. (2.5 points)
 - If you vary ϵ , how does the regret change? (2.5 points)

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Screenshot:

```
Part 1 - Implement the epsilon-greedy algorithm and compare it with the performance of the UCB and the EXP-3 algorithm. (5 points)
Importing Library
      1 import numpy as np
2 import matplotlib as mat
       3 import matplotlib.pyplot as plt
       4 import seaborn as sns
       5 import pandas as pd
Common Variable
      1 | # Setting seed to control randomness for reproducibility
       2 seed = 48
      4 # Initializing random number generator with the defined seed
       5 np.random.seed(seed)
      7 # Defining common variables for running the algorithm
8 num_arms = 10 # Number of arms/options in the multi-armed bandit problem
9 num_iterations = 1000 # Number of iterations to run the algorithm
     11 # Parameters of the Beta distributions for each arm's reward
     12 # Success parameters (alpha) for each arm's Beta distribution, all set to 5
     13 success_params = np.ones(num_arms) * 5
14 # Failure parameters (beta) for each arm's Beta distribution, increasing by 5 for each arm
     failure_params = np.arange(5, 5*(num_arms+1), 5)
     # Reshape failure_params to match the shape of success_params
# Ensuring the failure_params array has same shape as success_params
failure_params = np.resize(failure_params, success_params.shape)
```

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```
1 # Ensilon areedy Algorithm
 2 def epsilon greedy policy(epsilon val, num options, num iterations, success params, failure params):
        Implements the epsilon-greedy algorithm for a multi-armed bandit problem.
            epsilon_val (float): The probability of choosing a random option (exploration).
             num_options (int): The number of options, or "arms," of the bandit.
             num_iterations (int): The number of iterations to run the algorithm.
            success_params (array_like): Parameters of the Beta distribution for success for each option. failure_params (array_like): Parameters of the Beta distribution for failure for each option.
11
12
14
15
            np.array: The cumulative regret after each time step.
        This function operates over a specified number of iterations. At each time step, it chooses the option with the highest average reward with probability (1-epsilon),
18
        and chooses a random option with probability epsilon. The function calculates the "regret" of each option as the difference in success probability from the optimal
19
        option. The function returns the cumulative regret after each time step.
21
22
24
25
        \# Initialize cumulative rewards and number of times each option is chosen
        total rewards = np.zeros(num options)
26
        option_uses = np.zeros(num_options)
28
        # Initialize rewards and regret lists for tracking over time
29
        reward tracker = []
31
        # Determine the "best" option under current understanding
32
        optimal_option = np.argmax((success_params / (success_params + failure_params)))
34
35
        # Beain the main loop for the specified number of time steps
36
        for time_step in range(num_iterations):
             # Choose the option with highest average reward most of the time (probability 1-epsilon)
            if np.random.random() < (1 - epsilon_val):
    # Add small value to prevent division by zero</pre>
38
                 selected_option = np.argmax(total_rewards / (option_uses + 1e-6))
41
          else: # but choose randomly some of the time (probability epsilon)
                 selected option = np.random.randint(num options)
44
45
            # Generate reward based on a Beta distribution (specific to each option)
           current_reward = np.random.beta(success_params[selected_option], failure_params[selected_option])
46
            # Update rewards and usage for the selected option
48
            total_rewards[selected_option] += current_reward
49
            option_uses[selected_option] += 1
51
            # Record current reward and regret (difference in success probability from optimal option)
52
             reward tracker.append(current reward)
53
             regret_tracker.append(abs(success_params[optimal_option] / (success_params[optimal_option]
54
55
                                                                                    failure_params[optimal_option])
                                           - success_params[selected_option] / (success_params[selected_option]
56
                                                                                     + failure params[selected option])))
58
         # Return cumulative rearet over time
        return np.cumsum(regret tracker)
```

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```
62 # UCB algorithm
 63 def UCB_algorithm(num_options, num_iterations, success_params, failure_params):
 65
         Implements the Upper Confidence Bound (UCB) algorithm for a multi-armed bandit problem.
 66
 67
 68
             num_options (int): The number of options, or "arms," of the bandit.
 69
             num_iterations (int): The number of iterations to run the algorithm.
 70
             success_params (array_like): Parameters of the Beta distribution for success for each option.
 71
72
73
             failure_params (array_like): Parameters of the Beta distribution for failure for each option.
 74
75
76
             np.array: The cumulative regret after each time step.
         The function operates over a specified number of iterations. At each time step,
 77
78
         it calculates the UCB value for each option, and chooses the option with the
         highest UCB value. The UCB value for an option is its average reward plus an
 79
80
         uncertainty term, which decreases as that option is chosen more frequently.
 81
         The function calculates the "regret" of each option as the difference in success
 82
         probability from the optimal option. The function returns the cumulative regret
 83
         after each time step.
84
85
 86
         # Initialize cumulative rewards and number of times each option is chosen
 87
         total_rewards = np.zeros(num_options)
 88
         option_uses = np.ones(num_options)
 89
90
         # Initialize rewards and regret lists for tracking over time
 91
         reward_list = []
 92
         regret_list = []
 93
         # Determine the "best" option under current understanding
optimal_option = np.argmax((success_params / (success_params + failure_params)))
94
95
 96
         # Begin the main Loop for the specified number of time steps, starting from num_options # (as each option needs to be tried once)
 97
 98
99
         for time_step in range(num_options, num_iterations):
    # Calculate UCB values for each option
100
101
             ucb_values = total_rewards / option_uses + np.sqrt(2 * np.log(time_step) / option_uses)
102
              # Choose the option that currently has the highest UCB value
103
             selected_option = np.argmax(ucb_values)
104
105
             # Generate reward based on a Beta distribution (specific to each option)
106
             current_reward = np.random.beta(success_params[selected_option], failure_params[selected_option])
107
108
            # Update rewards and usage for the selected option
109
            total_rewards[selected_option] += current_reward
option_uses[selected_option] += 1
110
111
             # Record current reward and regret (difference in success probability from optimal option)
112
113
             reward_list.append(current_reward)
             regret_list.append(abs(success_params[optimal_option] / (success_params[optimal_option]
114
                                                                             failure_params[optimal_option])
115
                                      - success_params[selected_option] / (success_params[selected_option]
116
117
                                                                               + failure_params[selected_option])))
118
119
         # Return cumulative rearet over time
         return np.cumsum(regret_list)
```

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```
122 # EXP-3 algorithm
123 def EXP3_algorithm(num_options, num_iterations, success_params, failure_params, learning_rate):
124
125
         Implements the Exponential-weight algorithm for Exploration and Exploitation (EXP3) for a multi-armed bandit problem.
             num_options (int): The number of options, or "arms," of the bandit.
128
             num_iterations (int): The number of iterations to run the algorithm.
success_params (array_like): Parameters of the Beta distribution for success for each option.
129
130
              failure_params (array_like): Parameters of the Beta distribution for failure for each option.
             learning_rate (float): Learning rate parameter for the EXP3 algorithm.
134
        Returns:
135
             np.array: The cumulative regret after each time step.
136
        The function operates over a specified number of iterations. At each time step, it calculates the probabilities for each option based on their weights and selects
138
139
        an option randomly according to the calculated probabilities.
141
         The function calculates the "regret" of each option as the difference in success
142
         probability from the optimal option. The function returns the cumulative regret
143
         after each time step.
144
        # Initialize cumulative rewards and weights for each option
145
146
         total_rewards = np.zeros(num_options)
147
        option_weights = np.ones(num_options)
148
149
        # Initialize rewards and regret lists for tracking over time
150
         reward_records = []
151
         regret_records = []
         # Determine the "best" option under current understanding
154
         optimal option = np.argmax((success params / (success params + failure params)))
155
156
         # Begin the main Loop for the specified number of time steps
157
         for time_step in range(num_iterations):
    # Calculate probabilities for each option
158
             option_probs = (1 - learning_rate) * (option_weights / np.sum(option_weights)) + learning_rate / num_options
159
              # Select an option randomly according to the calculated probabilities
160
161
             selected_option = np.random.choice(num_options, p=option_probs)
162
             # Generate reward based on a Beta distribution (specific to each option)
             current_reward = np.random.beta(success_params[selected_option]), failure_params[selected_option])
166
             # Update the total reward for the selected option
167
             total_rewards[selected_option] += current_reward
168
             # Estimate the reward and update the weight of the selected option
170
             estimated_reward = total_rewards[selected_option] / option_probs[selected_option]
171
             option_weights[selected_option] *= np.exp(learning_rate * estimated_reward / num_options)
              # Normalize weights to prevent underflow
             option_weights /= np.sum(option_weights)
175
176
             # Record current reward and regret (difference in success probability from optimal option)
             reward_records.append(current_reward)
178
             regret_records.append(abs(success_params[optimal_option] / (success_params[optimal_option] + failure_params[optimal
180
         # Return cumulative regret over time
181
         return np.cumsum(regret records)
```

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```
84 def plot_cumulative_regret(epsilon_greedy_cumulative_regret, ucb_cumulative_regret, exp3_cumulative_regret, epsilon_values,
        Plots the cumulative regret for Epsilon-Greedy, UCB, and EXP3 algorithms.
86
             epsilon_greedy_cumulative_regret (array_like): Cumulative regret values from the Epsilon-Greedy algorithm.
             ucb_cumulative_regret (array_like): Cumulative regret values from the UCB algorithm.
             exp3_cumulative_regret (array_like): Cumulative regret values from the EXP3 algorithm.
            epsilon_values (float): Epsilon value used in the Epsilon-Greedy algorithm. learning_params (float): Learning parameter used in the EXP3 algorithm.
        The function creates a line plot for each set of regret values, using different colors
       for each algorithm. It sets labels for the x-axis and y-axis, a title for the plot, and
       a legend. It also includes grid lines for easier viewing of the data points.
        # Create a colormap using the 'cool' colormap from the matplotlib library
       cmap = mat.colormaps['cool']
        # Generate three colors from the colormap. These colors are evenly spaced
        # from the beginning (0) to the end (1) of the colormap.
       colors = cmap(np.linspace(0, 1, 3))
        # Set the figure size for the plot
       plt.figure(figsize=(10,7))
       # Plot the cumulative regret for the Epsilon-Greedy algorithm.
       # The color for this line is the first color from the colormap.
plt.plot(epsilon_greedy_cumulative_regret, label="Epsilon-Greedy", color=colors[0])
       # Plot the cumulative regret for the UCB algorithm.
# The color for this line is the second color from the colormap.
       plt.plot(ucb_cumulative_regret, label="UCB", color=colors[1])
        # Plot the cumulative regret for the EXP-3 algorithm.
       # The color for this line is the third color from the colormap.
      plt.plot(exp3_cumulative_regret, label="EXP-3", color=colors[2])
# Set the Label for the x-axis
       plt.xlabel("No. of Rounds", fontsize=14)
        # Set the Label for the y-axis
       plt.ylabel("Cumulative Regret", fontsize=14)
       # Set the title for the plot plt.title(f"Comparison of Epsilon-Greedy, UCB, and EXP-3 algorithms for epsilon value {epsilon_values} & learning param
        # Display the Legend
        plt.legend(fontsize=12)
         # Display grid Lines
        plt.grid(True, linestyle='--', alpha=0.6)
         # Show the plot
        plt.show()
```

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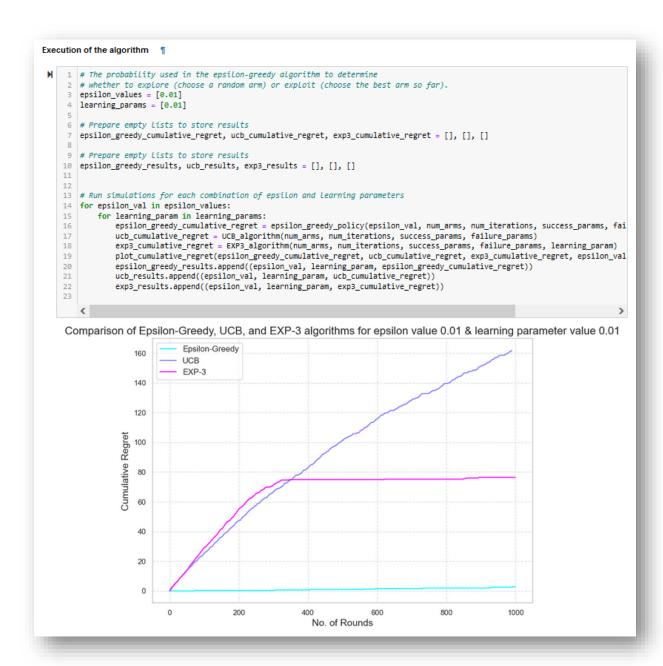
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```
def process_results(df, results, algorithm_name, epsilon_val, learning_param):
    Process the results of a multi-armed bandit algorithm, recording data for each set of parameters.
        df (pandas.DataFrame): The DataFrame to which the results will be appended.
        results (list): A list of tuples containing the parameters and regret values for each run of the algorithm.
        algorithm_name (str): The name of the algorithm used.
        Each tuple in `results` should contain three elements:
        1. epsilon_val (float): The epsilon value used in the algorithm run.
        2. learning_param (float): The learning parameter used in the algorithm run.
        3. regrets (list): A list of regret values from the algorithm run.
    The function identifies the maximum and minimum regret in each run and their respective iterations,
   creates a new DataFrame with these details, and appends it to the provided DataFrame `df`.
    df_analyser (pandas.DataFrame): The original DataFrame, updated with the new data.
    # Iterate through each tuple in the results list
    for epsilon_val, learning_param, regrets in results:
         # Find the maximum regret value and its corresponding iteration number
        max_regret_iter, max_regret = max(enumerate(regrets), key=lambda x: x[1])
# Find the minimum regret value and its corresponding iteration number
        min_regret_iter, min_regret = min(enumerate(regrets), key=lambda x: x[1])
        # Create a new DataFrame to store the results of the current tuple
        new_data = pd.DataFrame({
             'Algorithm': [algorithm_name], # The algorithm name
             'Epsilon': [epsilon_val], # The epsilon value used
             'Learning_Rate': [learning_param], # The Learning parameter used
             'Max_Regret': [max_regret], # The maximum regret value
'Max_Regret_Iter': [max_regret_iter + 1], # The iteration number for the maximum regret value
'Min_Regret': [min_regret], # The minimum regret value
             'Min_Regret_Iter': [min_regret_iter + 1] # The iteration number for the minimum regret value
        # Append the new data to the existing DataFrame
        df_analyser = pd.concat([df, new_data], ignore_index=True)
    # Return the updated DataFrame
    return df_analyser
```

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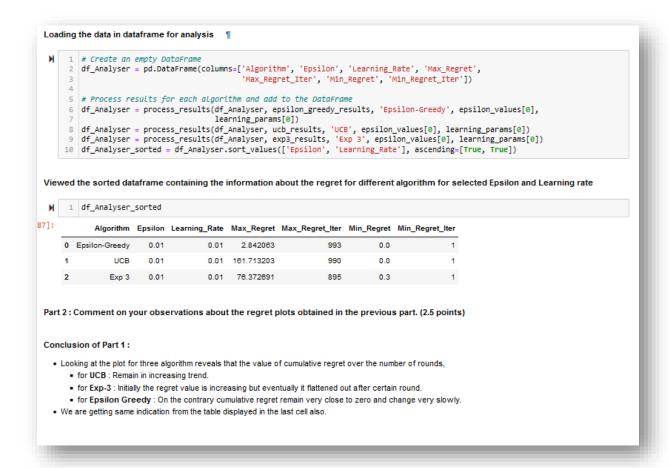
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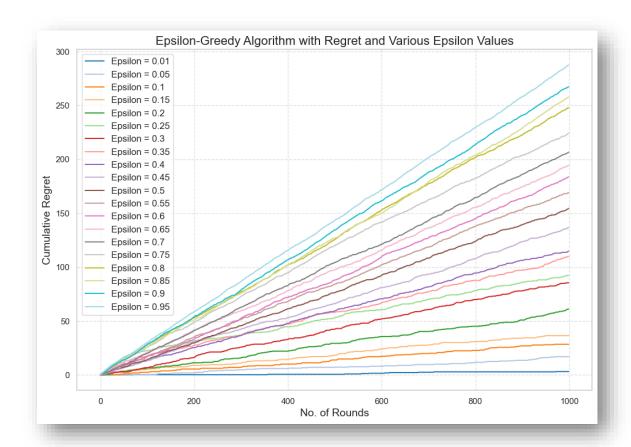
```
Part 3: Execution of the algorithm for different values of \epsilon
    1 seed = 20
      num_iterations = 1000
4  # Different epsilon values to explore
      epsilon_values_to_explore = [0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95]
      8 # List to store the cumulative regrets for each epsilon value
      9 cumulative_regrets_for_each_epsilon = []
     11 # Run the epsilon-greedy policy for each epsilon value and store the cumulative regrets
     for epsilon in epsilon_values_to_explore:
cumulative_regret = epsilon_greedy_policy(epsilon, num_arms, num_iterations, success_params, failure_params)
              cumulative_regrets_for_each_epsilon.append(cumulative_regret)
     16 # Set the style and context of the plot to make it look more attractive
     17 sns.set(style='whitegrid', context='notebook')
     19 # Set the color palette for the plot
     20 colors = [mat.colormaps['tab20'](iloop) for iloop in range(20)]
     22 # Create a Larger figure
     23 plt.figure(figsize=(12, 8))
     25  # Loop over epsilon values
26  for i, epsilon in enumerate(epsilon_values_to_explore):
           # Plot the cumulative regret for each epsilon, with a different color and line style for each one plt.plot(cumulative_regrets_for_each_epsilon[i], color=colors[i], linestyle='-', label=f"Epsilon = {epsilon}")
     # Set Labels and title with Larger fonts
plt.xlabel("No. of Rounds", fontsize=14)
plt.ylabel("Cumulative Regret", fontsize=14)
plt.title("Epsilon-Greedy Algorithm with Regret and Various Epsilon Values", fontsize=16)
     35 # Display the Legend and increase its font size
     36 plt.legend(fontsize=12)
     38 # Add a slight grid for better readability
     39 plt.grid(True, linestyle='--', alpha=0.6)
     41 # Show the plot
     42 plt.show()
```

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Conclusion Part 3:

Various epsilon values (0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95) demonstrate various regret
patterns in the Epsilon-Greedy algorithm. In terms of minimizing regrets and observing that the regret is growing its slope as and when we are raising the
epsilon value over the number of rounds, smaller epsilon values (for example, 0.01) tend to perform better than bigger epsilon values (for example, 0.95). This
suggests that a more experimental strategy (lower epsilon) can eventually result in greater overall performance.

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Code Text:

Part 1 - Implement the epsilon-greedy algorithm and compare it with the performance of the UCB and the EXP-3 algorithm. (5 points)

```
Importing Library
```

```
import numpy as np
import matplotlib as mat
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
```

```
Common Variable
# Setting seed to control randomness for reproducibility
seed = 48
# Initializing random number generator with the defined seed
np.random.seed(seed)
# Defining common variables for running the algorithm
num_arms = 10 # Number of arms/options in the multi-armed bandit problem
num iterations = 1000 # Number of iterations to run the algorithm
# Parameters of the Beta distributions for each arm's reward
# Success parameters (alpha) for each arm's Beta distribution, all set to
success params = np.ones(num arms) * 5
# Failure parameters (beta) for each arm's Beta distribution, increasing
by 5 for each arm
failure_params = np.arange(5, 5*(num_arms+1), 5)
# Reshape failure_params to match the shape of success_params
# Ensuring the failure params array has same shape as success params
failure params = np.resize(failure params, success params.shape)
```

Common Method

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```
# Epsilon greedy Algorithm
def epsilon greedy policy(epsilon_val, num options, num iterations,
success params, failure params):
    Implements the epsilon-greedy algorithm for a multi-armed bandit
problem.
   Args:
        epsilon_val (float): The probability of choosing a random option
(exploration).
       num_options (int): The number of options, or "arms," of the
bandit.
        num iterations (int): The number of iterations to run the
algorithm.
        success_params (array_like): Parameters of the Beta distribution
for success for each option.
        failure_params (array_like): Parameters of the Beta distribution
for failure for each option.
    Returns:
        np.array: The cumulative regret after each time step.
    This function operates over a specified number of iterations. At each
time step,
   it chooses the option with the highest average reward with probability
(1-epsilon),
    and chooses a random option with probability epsilon. The function
calculates the
    "regret" of each option as the difference in success probability from
the optimal
    option. The function returns the cumulative regret after each time
step.
   # Initialize cumulative rewards and number of times each option is
    total_rewards = np.zeros(num_options)
    option_uses = np.zeros(num_options)
    # Initialize rewards and regret lists for tracking over time
    reward tracker = []
```

regret_tracker = []

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```
# Determine the "best" option under current understandina
    optimal_option = np.argmax((success_params / (success_params +
failure params)))
    # Begin the main loop for the specified number of time steps
    for time_step in range(num_iterations):
        # Choose the option with highest average reward most of the time
(probability 1-epsilon)
        if np.random.random() < (1 - epsilon_val):</pre>
            # Add small value to prevent division by zero
            selected_option = np.argmax(total_rewards / (option_uses + 1e-
6))
        else: # but choose randomly some of the time (probability
epsilon)
            selected option = np.random.randint(num options)
        # Generate reward based on a Beta distribution (specific to each
option)
        current reward = np.random.beta(success params[selected option],
failure_params[selected_option])
        # Update rewards and usage for the selected option
        total_rewards[selected_option] += current_reward
        option_uses[selected_option] += 1
        # Record current reward and regret (difference in success
probability from optimal option)
        reward_tracker.append(current_reward)
        regret_tracker.append(abs(success_params[optimal_option] /
(success_params[optimal_option] + failure_params[optimal_option]) -
success_params[selected_option] / (success_params[selected_option] +
failure_params[selected_option])))
    # Return cumulative regret over time
    return np.cumsum(regret_tracker)
# UCB algorithm
def UCB_algorithm(num_options, num_iterations, success_params,
failure_params):
    Implements the Upper Confidence Bound (UCB) algorithm for a multi-
armed bandit problem.
```

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Args: num_options (int): The number of options, or "arms," of the bandit. num iterations (int): The number of iterations to run the algorithm. success params (array like): Parameters of the Beta distribution for success for each option. failure params (array like): Parameters of the Beta distribution for failure for each option. Returns: np.array: The cumulative regret after each time step. The function operates over a specified number of iterations. At each time step, it calculates the UCB value for each option, and chooses the option with the highest UCB value. The UCB value for an option is its average reward uncertainty term, which decreases as that option is chosen more frequently. The function calculates the "regret" of each option as the difference in success probability from the optimal option. The function returns the cumulative regret after each time step. # Initialize cumulative rewards and number of times each option is chosen total rewards = np.zeros(num options) option_uses = np.ones(num_options) # Initialize rewards and regret lists for tracking over time reward list = [] regret_list = [] # Determine the "best" option under current understanding optimal_option = np.argmax((success_params / (success_params + failure params)))

Begin the main loop for the specified number of time steps, starting

from num_options

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```
# (as each option needs to be tried once)
    for time_step in range(num_options, num_iterations):
        # Calculate UCB values for each option
        ucb_values = total_rewards / option_uses + np.sqrt(2 *
np.log(time_step) / option_uses)
        # Choose the option that currently has the highest UCB value
        selected_option = np.argmax(ucb_values)
        # Generate reward based on a Beta distribution (specific to each
option)
        current_reward = np.random.beta(success_params[selected_option],
failure_params[selected_option])
        # Update rewards and usage for the selected option
        total_rewards[selected_option] += current_reward
        option_uses[selected_option] += 1
        # Record current reward and regret (difference in success
probability from optimal option)
        reward list.append(current reward)
        regret list.append(abs(success params[optimal option] /
(success_params[optimal_option] + failure_params[optimal_option]) -
success_params[selected_option] / (success_params[selected_option] +
failure_params[selected_option])))
    # Return cumulative regret over time
    return np.cumsum(regret_list)
# EXP-3 algorithm
def EXP3_algorithm(num_options, num_iterations, success_params,
failure params, learning rate):
    Implements the Exponential-weight algorithm for Exploration and
Exploitation (EXP3) for a multi-armed bandit problem.
    Args:
       num options (int): The number of options, or "arms," of the
bandit.
        num_iterations (int): The number of iterations to run the
algorithm.
        success params (array like): Parameters of the Beta distribution
for success for each option.
        failure params (array like): Parameters of the Beta distribution
for failure for each option.
```

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```
learning_rate (float): Learning rate parameter for the EXP3
algorithm.
    Returns:
        np.array: The cumulative regret after each time step.
    The function operates over a specified number of iterations. At each
time step,
   it calculates the probabilities for each option based on their weights
and selects
    an option randomly according to the calculated probabilities.
    The function calculates the "regret" of each option as the difference
in success
   probability from the optimal option. The function returns the
cumulative regret
    after each time step.
    # Initialize cumulative rewards and weights for each option
    total rewards = np.zeros(num options)
    option_weights = np.ones(num_options)
    # Initialize rewards and regret lists for tracking over time
    reward records = []
    regret records = []
    # Determine the "best" option under current understanding
    optimal_option = np.argmax((success_params / (success_params +
failure_params)))
    # Begin the main loop for the specified number of time steps
    for time_step in range(num_iterations):
        # Calculate probabilities for each option
        option_probs = (1 - learning_rate) * (option_weights /
np.sum(option_weights)) + learning_rate / num_options
        # Select an option randomly according to the calculated
probabilities
        selected_option = np.random.choice(num_options, p=option_probs)
        # Generate reward based on a Beta distribution (specific to each
option)
        current reward = np.random.beta(success params[selected option],
failure params[selected option])
```

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```
# Update the total reward for the selected option
        total_rewards[selected_option] += current_reward
        # Estimate the reward and update the weight of the selected option
        estimated_reward = total_rewards[selected_option] /
option probs[selected option]
        option_weights[selected_option] *= np.exp(learning rate *
estimated_reward / num_options)
        # Normalize weights to prevent underflow
        option_weights /= np.sum(option_weights)
        # Record current reward and regret (difference in success
probability from optimal option)
        reward_records.append(current_reward)
        regret_records.append(abs(success_params[optimal_option] /
(success_params[optimal_option] + failure_params[optimal_option]) -
success_params[selected_option] / (success_params[selected_option] +
failure_params[selected_option])))
    # Return cumulative regret over time
    return np.cumsum(regret_records)
def plot_cumulative_regret(epsilon_greedy_cumulative_regret,
ucb_cumulative_regret, exp3_cumulative_regret, epsilon_values,
learning_params):
    Plots the cumulative regret for Epsilon-Greedy, UCB, and EXP3
algorithms.
    Args:
        epsilon_greedy_cumulative_regret (array_like): Cumulative regret
values from the Epsilon-Greedy algorithm.
        ucb cumulative regret (array like): Cumulative regret values from
the UCB algorithm.
        exp3_cumulative_regret (array_like): Cumulative regret values from
the EXP3 algorithm.
        epsilon_values (float): Epsilon value used in the Epsilon-Greedy
        learning params (float): Learning parameter used in the EXP3
algorithm.
```

The function creates a line plot for each set of regret values, using

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```
different colors
    for each algorithm. It sets labels for the x-axis and y-axis, a title
for the plot, and
    a legend. It also includes grid lines for easier viewing of the data
points.
    # Create a colormap using the 'cool' colormap from the matplotlib
Library
    cmap = mat.colormaps['cool']
    # Generate three colors from the colormap. These colors are evenly
    # from the beginning (0) to the end (1) of the colormap.
    colors = cmap(np.linspace(0, 1, 3))
    # Set the figure size for the plot
    plt.figure(figsize=(10,7))
    # Plot the cumulative regret for the Epsilon-Greedy algorithm.
    # The color for this line is the first color from the colormap.
    plt.plot(epsilon_greedy_cumulative_regret, label="Epsilon-Greedy",
color=colors[0])
    # Plot the cumulative regret for the UCB algorithm.
    # The color for this line is the second color from the colormap.
    plt.plot(ucb_cumulative_regret, label="UCB", color=colors[1])
    # Plot the cumulative regret for the EXP-3 algorithm.
    # The color for this line is the third color from the colormap.
    plt.plot(exp3 cumulative regret, label="EXP-3", color=colors[2])
    # Set the label for the x-axis
    plt.xlabel("No. of Rounds", fontsize=14)
    # Set the label for the y-axis
    plt.ylabel("Cumulative Regret", fontsize=14)
    # Set the title for the plot
    plt.title(f"Comparison of Epsilon-Greedy, UCB, and EXP-3 algorithms
for epsilon value {epsilon_values} & learning parameter value
{learning_params}", fontsize=16)
    # Display the Legend
    plt.legend(fontsize=12)
    # Display grid lines
    plt.grid(True, linestyle='--', alpha=0.6)
    # Show the plot
    plt.show()
def process results(df, results, algorithm name, epsilon val,
learning_param):
```

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Process the results of a multi-armed bandit algorithm, recording data for each set of parameters.

```
Args:
```

df (pandas.DataFrame): The DataFrame to which the results will be appended.

results (list): A list of tuples containing the parameters and regret values for each run of the algorithm.

algorithm name (str): The name of the algorithm used.

Each tuple in `results` should contain three elements:

- 1. epsilon_val (float): The epsilon value used in the algorithm run.
- 2. Learning_param (float): The Learning parameter used in the algorithm run.
 - 3. regrets (list): A list of regret values from the algorithm run.

The function identifies the maximum and minimum regret in each run and their respective iterations,

Returns:

 $\it df_analyser$ (pandas.DataFrame): The original DataFrame, updated with the new data.

Iterate through each tuple in the results list

for epsilon_val, learning_param, regrets in results:

Find the maximum regret value and its corresponding iteration number

max_regret_iter, max_regret = max(enumerate(regrets), key=lambda
x: x[1])

Find the minimum regret value and its corresponding iteration number

min_regret_iter, min_regret = min(enumerate(regrets), key=lambda
x: x[1])

Create a new DataFrame to store the results of the current tuple
new_data = pd.DataFrame({

'Algorithm': [algorithm_name], # The algorithm name

'Epsilon': [epsilon val], # The epsilon value used

'Learning_Rate': [learning_param], # The Learning parameter

used

'Max_Regret': [max_regret], # The maximum regret value

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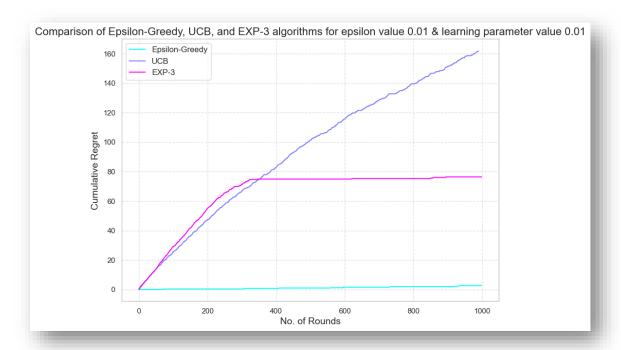
```
'Max_Regret_Iter': [max_regret_iter + 1], # The iteration
number for the maximum regret value
            'Min_Regret': [min_regret], # The minimum regret value
            'Min Regret Iter': [min regret iter + 1] # The iteration
number for the minimum regret value
        # Append the new data to the existing DataFrame
        df_analyser = pd.concat([df, new_data], ignore_index=True)
    # Return the updated DataFrame
    return df_analyser
Execution of the algorithm
# The probability used in the epsilon-greedy algorithm to determine
# whether to explore (choose a random arm) or exploit (choose the best arm
so far).
epsilon_values = [0.01]
learning params = [0.01]
# Prepare empty lists to store results
epsilon_greedy_cumulative_regret, ucb_cumulative_regret,
exp3_cumulative_regret = [], [], []
# Prepare empty lists to store results
epsilon_greedy_results, ucb_results, exp3_results = [], [], []
# Run simulations for each combination of epsilon and learning parameters
for epsilon val in epsilon values:
    for learning_param in learning_params:
        epsilon_greedy_cumulative_regret =
epsilon_greedy_policy(epsilon_val, num_arms, num_iterations,
success params, failure params)
        ucb_cumulative_regret = UCB_algorithm(num_arms, num_iterations,
success_params, failure_params)
        exp3_cumulative_regret = EXP3_algorithm(num_arms, num_iterations,
success_params, failure_params, learning_param)
        plot cumulative_regret(epsilon_greedy_cumulative_regret,
ucb_cumulative_regret, exp3_cumulative_regret, epsilon_val,
learning param)
        epsilon_greedy_results.append((epsilon_val, learning_param,
epsilon_greedy_cumulative_regret))
```

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Loading the data in dataframe for analysis

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```
df_Analyser_sorted = df_Analyser.sort_values(['Epsilon', 'Learning_Rate'],
ascending=[True, True])
```

Viewed the sorted dataframe containing the information about the regret for different algorithm for selected Epsilon and Learning rate

df_Analyser_sorted

```
Algorithm Epsilon Learning_Rate Max_Regret Max_Regret_Iter \
 Epsilon-Greedy
                    0.01
                                  0.01 2.842063
                                                              993
             UCB
                    0.01
                                  0.01 161.713203
                                                              990
1
2
           Exp 3
                    0.01
                                  0.01 76.372691
                                                              895
  Min_Regret Min_Regret_Iter
0
         0.0
         0.0
                          1
1
         0.3
                          1
```

Part 2: Comment on your observations about the regret plots obtained in the previous part. (2.5 points)

Conclusion of Part 1:

- Looking at the plot for three algorithm reveals that the value of cumulative regret over the number of rounds,
 - for UCB: Remain in increasing trend.
 - for Exp-3: Initially the regret value increased but eventually it flattened out after a certain round.
 - for Epsilon Greedy: On the contrary cumulative regret remains very close to zero and changes very slowly.
- We are getting the same indication from the table displayed in the last cell also.

Part 3: Execution of the algorithm for different values of ϵ

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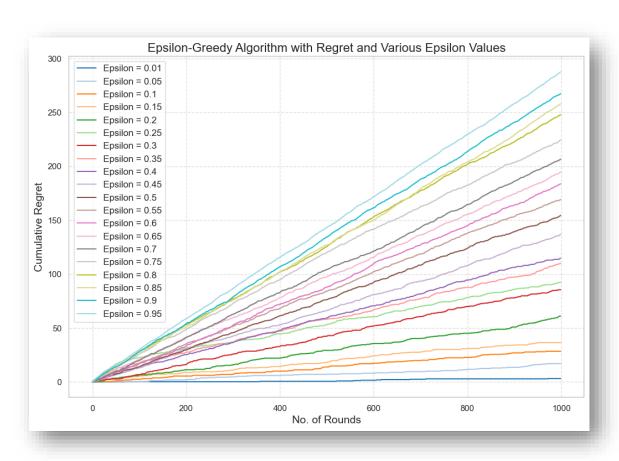
```
# Run the epsilon-greedy policy for each epsilon value and store the
cumulative regrets
for epsilon in epsilon values to explore:
    cumulative_regret = epsilon_greedy_policy(epsilon, num_arms,
num_iterations, success_params, failure_params)
   cumulative_regrets_for_each_epsilon.append(cumulative_regret)
# Set the style and context of the plot to make it look more attractive
sns.set(style='whitegrid', context='notebook')
# Set the color palette for the plot
colors = [mat.colormaps['tab20'](iloop) for iloop in range(20)]
# Create a larger figure
plt.figure(figsize=(12, 8))
# Loop over epsilon values
for i, epsilon in enumerate(epsilon values to explore):
    # Plot the cumulative regret for each epsilon, with a different color
and line style for each one
    plt.plot(cumulative_regrets_for_each_epsilon[i], color=colors[i],
linestyle='-', label=f"Epsilon = {epsilon}")
# Set labels and title with larger fonts
plt.xlabel("No. of Rounds", fontsize=14)
plt.ylabel("Cumulative Regret", fontsize=14)
plt.title("Epsilon-Greedy Algorithm with Regret and Various Epsilon
Values", fontsize=16)
# Display the legend and increase its font size
plt.legend(fontsize=12)
# Add a slight grid for better readability
plt.grid(True, linestyle='--', alpha=0.6)
# Show the plot
plt.show()
```

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Conclusion Part 3:

• Various epsilon values (0.01, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95) demonstrate various regret patterns in the Epsilon-Greedy algorithm. In terms of minimizing regrets and observing that the regret is growing its slope as and when we are raising the epsilon value over the number of rounds, smaller epsilon values (for example, 0.01) tend to perform better than bigger epsilon values (for example, 0.95). This suggests that a more experimental strategy (lower epsilon) can eventually result in greater overall performance.