Author: Aloy Banerjee

Roll. No.: CH22M503

Problem - 1: Derive the covariance relationship

Let x and y be jointly Gaussian n and m vectors. A and B are known p by n and p by m matrices, respectively. Then the random p vector z is defined as z = Ax + By is Gaussian characterized by:

Mean $\mu_z = A\mu_x + B\mu_z$ and Covariance $P_{zz} = AP_{xx}A^T + AP_{xy}B^T + BP_{yx}A^T + BP_{yy}B^T$

Derive the above covariance relationship.

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Robblem 1: - Derive the covariance relationship:
Answer:- Below information is given in the
problem statement,
         1) x & y be jointly Gaussian n & m rector
         i) or is an in-dimensional quessian rector por with mean My & warriance matrix Par
        iii) y is an n-dimensional Gaussian vector with mean My & warriance matrix Pyy
        (1) A & B are known maisix & there
         dimension are on follows,
               B > PXM
 Vector Z i's defined by
                   Z= AX+BX
of calculate the Mean of Z:-
    Let consider, My is the mean of vectorZ
      " My = E[Z]
             = E[ANTBY] : SUBSTITUTING
= AE[N] + BE[Y] Z= ANTBY
             = AMx + BMy
                              · · E[N]=Mx
                                       = sneem of x
                                  ELY] = MY
                                         2 Mean of 7
    " MZ = AM + BMY
> columne the covariance mutix of Z:-
  Let consider, UZ is the mean & PZZ is the covariance maistix of I
   ": PZZ = E[(Z-MZ)(Z-MZ)]
         ": substituting Z=Ax+By wegot)
   =>PZZ=E[(An+BY-ALX-BLY)]
(An+BY-ALX+BLY)]
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=> PZZ = E[(AM-ALX)(AM-ALX)]
           + E [ (BY-BMY) (BY-BMY) ] ]
            + E [ (AX-ALY) (BY-ELY) ]
            + ET (BY-BMY) (AX-AMX)TT
 =>P2z=A·AT. E[ (n-1/x) (n-1/x)]]
            + B.BT. E[(y-My) (y-My)]]
           + A.BT. EF (21-Mx) (Y-X14)]
           + B.AT EF (Y-MY) (X-MX) ]
   substituting the covariance matrix value of 28 y yeater in trevious equation we got,
=> P2Z = A.AT. Pxx + B.BT. Pyy + A.BT E[(x-Mx) (y-My)]
                    + BATE[ (Y-MY) (x-Mx)]
   Rearanging the equation based on marrix multiplication format we got,
-> PZZ = A.Pxx. AT + B. Pyy. BT + A. E[(2-Lx) (Y-Hy)] ET
+ B. E[(Y-Ly) (x-Lx)] ]. AT.
Aspertue forbum starement of 8 y are jointly gaussian vector, hence their coramance and be defined by,
       CON [2, y] = E [ (2-Mx)(Y-My)]
 substituting the value in PZZ regnation, Ne got,
=> PZZ = A. Pxx . AT + B. Pyy. BT + A. COY [M, Y]. BT
                        + B. CON [Y, 27. AT
    : covarrance maising can be pritten on,
               CON [M, 7] = Pxy
substituting the symbolic representation in P22 equation we got,
 PZZ = A.Pxx. AT + B. Pyy. BT + A. Pxy BT + B Pyx AT
time the covariance relation of gaussian vector Z

Hence the covariance relation of gaussian vector Z

15 PZZ = APXXAT+BPYYBT+APXYBT+BPYXAT.
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Problem - 2: Derivation

To determine matrix K_k (Kalman Gain), we used the following relationship:

$$E[\tilde{x_k}^T \tilde{x_k}] = \text{Trace } E[\tilde{x_k} \tilde{x_k}^T]$$

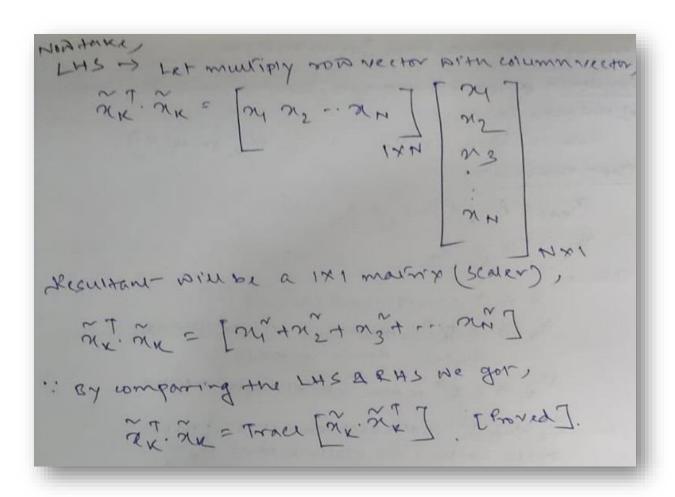
Derive the above relationship

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Problems
Assumption Daily a success Gain (K. Die
below, Assumption while deriving karman gain (Kx) is as
Amewer of Ler consider, The consider, The consider defined on, The consider of the consideration of the considerati
~ Let consider,
of is a column vector defined on,
then ~ +
LXN NXI
then at Diube a now vector m,
2.7
NOW. 2 = [24 x2 x3 21 N] 1 XN
NOW, L Z JIXH
PHS -> 1, F
RHS -> Let multiply the column & som vector,
α
$\frac{x}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \cdot \begin{bmatrix} x_1 x_2 - x_N \end{bmatrix}$
23
[XM] MXI
fecultant will be a NXN marsign,
7 - 7
1 d 45 w 1 what col
- 12 12
= [x1x4 ; x4] WXV = [x1x5 x5 , x4x]
so now if we take trace of above marry negot,
Trace [] = [21 + 2 + 2 + 2 + 2 + 2 + 7]

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Problem - 3: Kalman Gain Derivation

Derive the expression for Kalman Gain (K_k).

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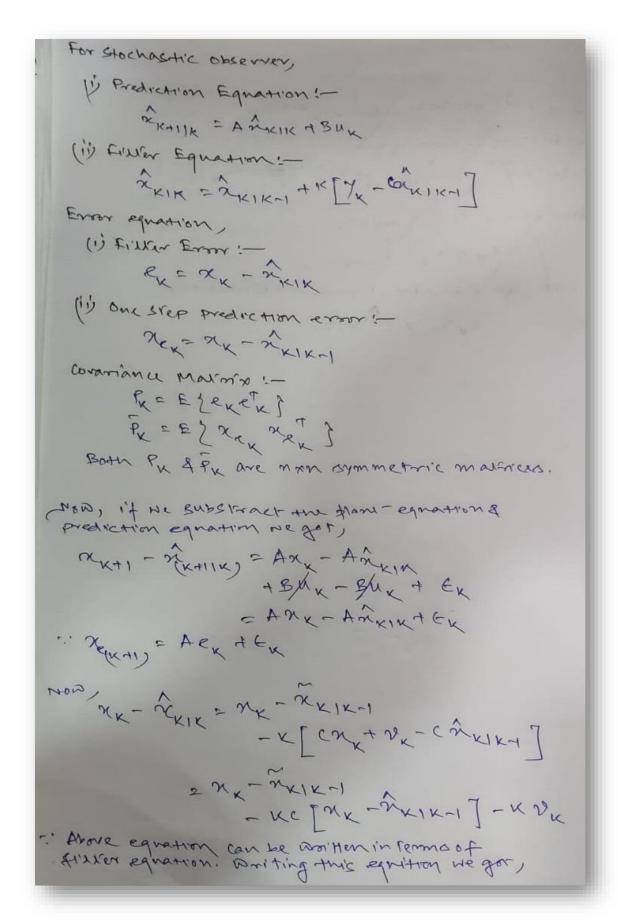
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Pooblem 3: Kalman Gain Derivation: Amower: - we are following the Least square derivation of Kalman Gain Kalman gain is defined by kx or k(x). Plant equation 1-2x+1 = Anx + Bux + Ex Tx = Cxx + Px Drive, K > Discuelle time instant xx → ER "is the stare vector at the diserere Hime instant K Ux -> ERM i's the control input wester Ex > ER" is the disturbance or process noise vector, We assume that Exis wear & uncorrelated arththe covarrance maising. given by E[ExE]=RI AERNAM ABERNAM are the atomb ginput mainices CE Bux 1,2 the online march to y E R" is the owight vector (observed measurement) re EIR" i's the measurement motise rector, we assumed that they is also Dhite in nature DIHM Reso mean & uncorrelated with concernance maising defined by E[NxNI] = R, For Deriministic

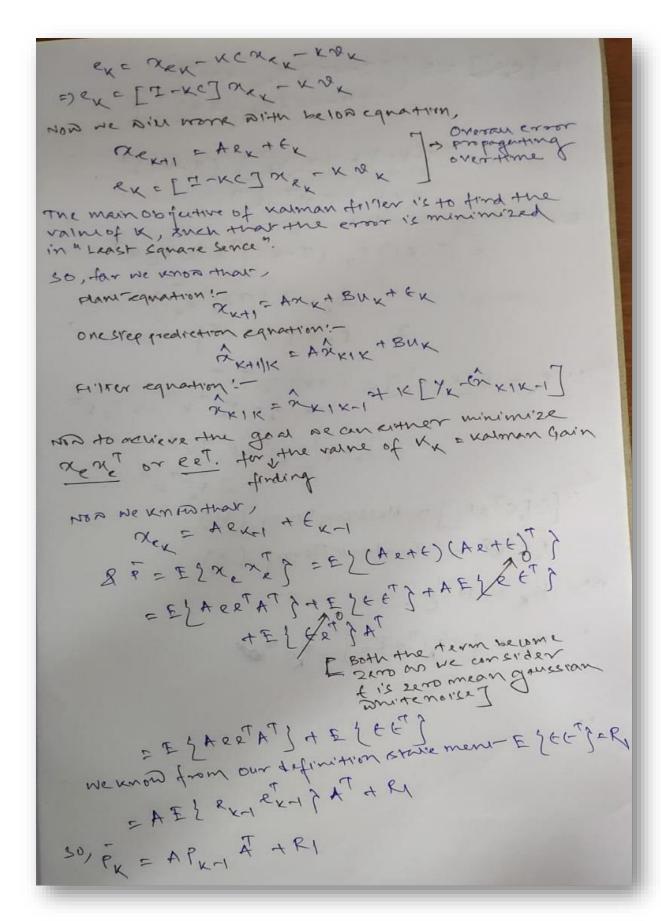
Open loop prediction regnation

Observer 8- Open loop prediction regnation 2x+1 = Anx + Bux+ LIJx - cnx]

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$$E[eet] = E[I-KC] xex - KN]$$

$$= (I-KC) E[xexex - Kn]$$

$$= E[xexex - Kn] = E[xexex - Kn]$$

$$= E[xexex - Kn]$$

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