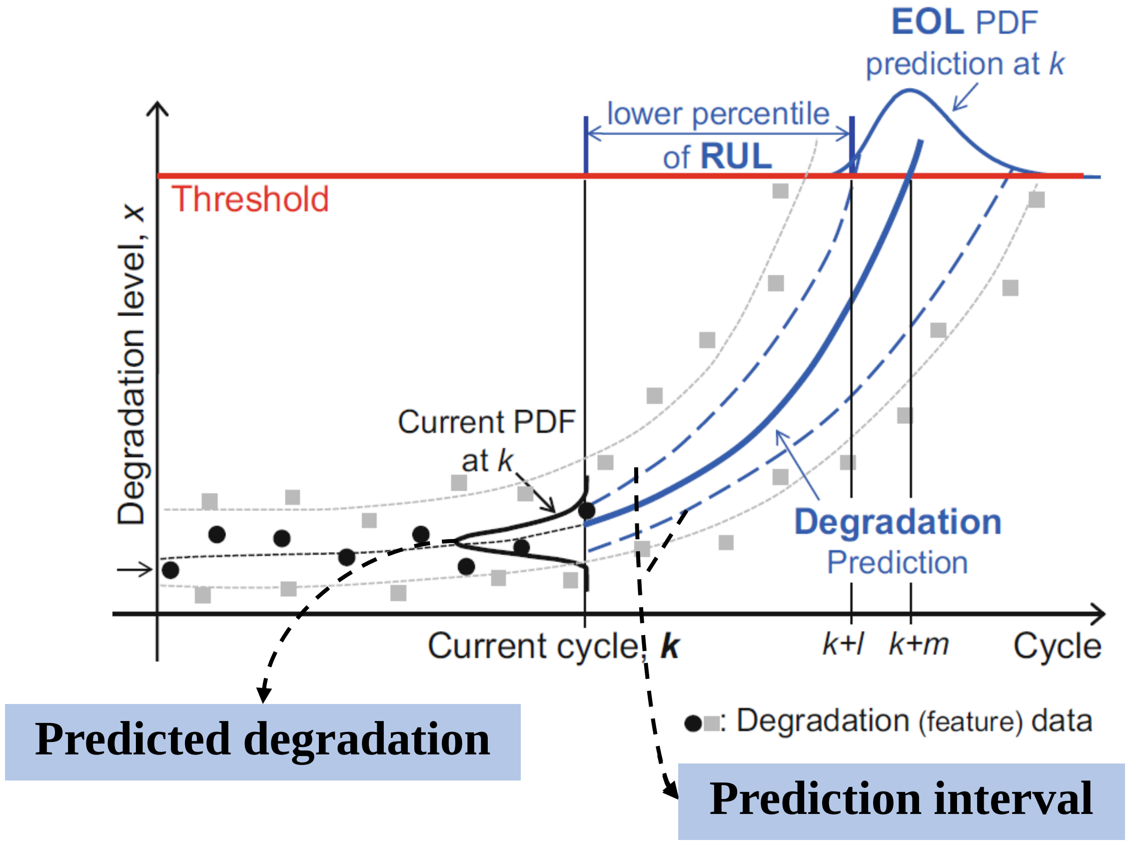
Prognostics

Degradation and RUL

Prognostics

Objective :

Predict the remaining useful life (RUL) before the damage grows beyond the threshold



- Due to uncertainty, predicted degradation and End of Life (EOL) is represented as distribution
- For a given time, the area under the EOL is considered as failure probability
- EOL signals the time for the system maintenance
- RUL, the remaining time to the maintenance from the current time is given by

$$t_{RUL} = t_{EOL} - t_k$$

- Lower bound of RUL is considered as the maintenance time

Kim, N. H., An, D., & Choi, J. H. (2017). Prognostics and health management of engineering systems. Switzerland: Springer International Publishing.

Prognostics methods

Categorized based on the availability of

Physical model describing the behaviour of damage

Field operating conditions

Damage degradation data from similar systems

Yes → **Physics-based**

- Used when usage conditions are available additionally
- Physics-based algorithm can be considered as parameter estimation method

No → **Data-driven**

- Used when failure phenomenon is too complex to model
- Requires several sets of run-to-failure data (training data)

Prediction of degradation behavior

Least squares method - Parameter estimation

- Used to find unknown parameter or coefficients by minimizing the sum of square errors (SS_E) between measured (y_k) and simulation (z_k ,from model/function) data
- Their relationship is given by

$$\mathbf{y_k = z_k + \varepsilon_k}$$

k is the time index

- The error ε_k can represent measurement error in y_k as well as in z_k
- Assumptions
  - Error comes from measurement only
  - Measurement error does not include bias but unbiased noise
  - Simulation model z (t;θ) is a linear function of input variable t (time) and parameter (θ)

$$\mathbf{Z(t;\theta) = \theta_1 + \theta_2 t, \qquad \theta = \{\theta_1\ \theta_2\}^T}$$

Prediction of degradation behavior

Sum of square errors (SS_E)

- The pair of input variable and measured degradation at data points is denoted as

$$(t_k, y_k), \qquad k = 1,\ldots\ldots,n_y \text{ and } y = \{y_1\ y_2 \ldots y_{n_y}\}^T$$

n_y is measured degradation

- The simulation model can be evaluated at data points and corresponding error is given as

$$z = \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{n_y} \end{Bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_{n_y} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = X\theta$$

$$e = \{\varepsilon_1 \quad \varepsilon_2 \quad \ldots \quad \varepsilon_{n_y}\}^T = y - z$$

X is the design matrix

$$SS_E = e^T e = \{y - z\}^T\{y - z\} = \{y - X\theta\}^T\{y - X\theta\}$$

$$\frac{d(SS_E)}{d\theta} = 2\left[\frac{de}{d\theta}\right]^T e = 2X^T\{y - X\theta\} = 0$$

$$\textit{The estimated parameter} = \hat{\theta} = [X^T X]^{-1}\{X^T y\}$$

Data-driven approach

- Functional relationship between input variables and output degradation has to be built from the data
- The quality of prediction depends on
  - Selection of mathematical function
  - Number of data
  - Level of noise in the measurement

Quality of fitting

- Coefficient of determination, R^2 , the ratio between the variation of function prediction to the data

$$\boldsymbol{R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}}$$

- SS_T (total sum of squares) = $\sum_{k=1}^{n_y}(y_k - \bar{y})^2$
- SS_R (regression sum of squares) = $\sum_{k=1}^{n_y}(z_k - \bar{y})^2$
- SS_E (residual sum of squares) = $\sum_{k=1}^{n_y}(y_k - z_k)^2$

Data-driven approach

Quality of fitting

- SS_T is the variation of data with respect to the mean of data $\bar{y}$
- SS_R is the variation of the function prediction z_k with respect to the mean of data
- SS_E is the sum of square of errors remaining after the fit
- When the sum of y_k is equal to the sum of z_k

$$\boldsymbol{SS_T = SS_R + SS_E}$$

- R^2 close to 1 is considered an accurate function
- However, R^2 only measures accuracy in data points which can be unrelated to the true accuracy of the function prediction
- Therefore adjusted R^2 denoted as $\bar{R}^2$ is used by penalizing the number of coefficients as

$$\bar{R}^2 = 1 - (1 - R^2)\frac{(n_y - 1)}{(n_y - n_p)}$$

n_y and n_p are the number of data and coefficients respectively

Data-driven approach

Overfitting

When the number of unknown coefficients is larger than the number of data, the least square method tends to fit the noise rather than the trend

- Overfitting is a modeling error occurring when function is overly complex and when the function has no conformability with the data shape
- Techniques available to avoid overfitting
  - Cross-validation
  - Regularization
  - Early stopping
  - Pruning
- Other approaches
  - Behavior of degradation is expressed with a simple function
  - More training data and usage conditions are used

RUL prediction

- Remaining time until the degradation grows to a threshold
- The threshold of degradation is determined so that the system is still safe but needs maintenance
- To find RUL, it is necessary to find the time cycle when the level of degradation reached a threshold
- The non-linear equation has to be solved to find the time cycle t_{EOL}

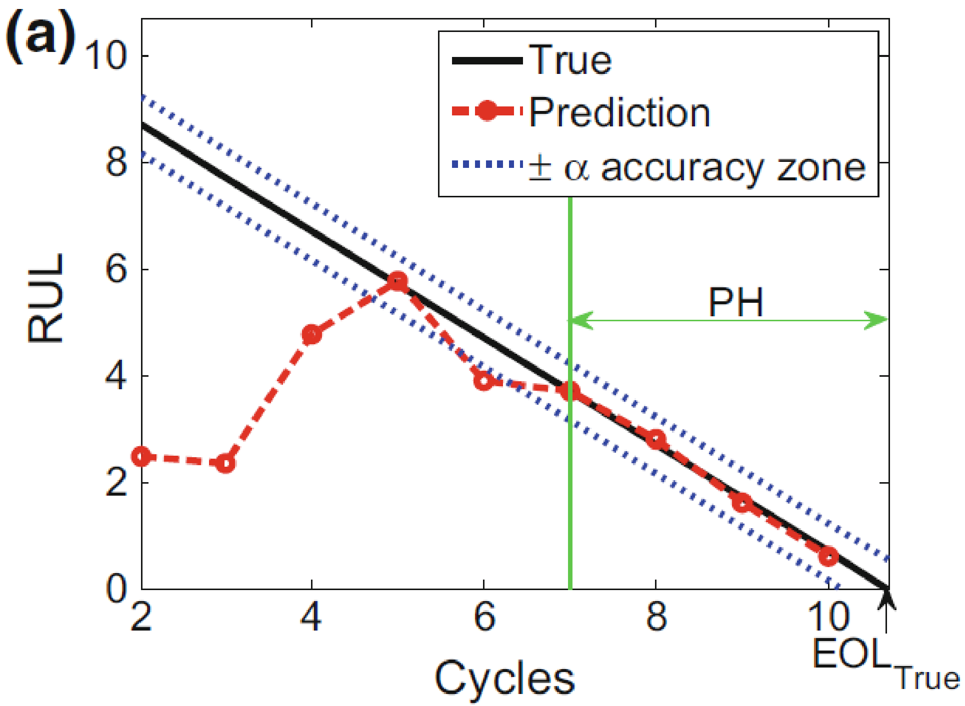
$$\boldsymbol{y_{threshold} - z\left(t_{EOL}; \widehat{\theta}\right) = 0}$$

- The above equation is solved numerically using Newton-Raphson iterative method
- Maintenance has to be ordered when the RUL becomes zero
- The performance of different method in predicting RUL is compared using metrics such as
  - Prognostic horizon (PH)
  - α-λ accuracy
  - Relative accuracy (RA)
  - Cumulative relative accuracy (CRA)
  - Convergence

Prognostics metrics

Prognostic Horizon (PH)

Difference between the EOL and the first time when the prediction result continuously resides in the pha accuracy zone

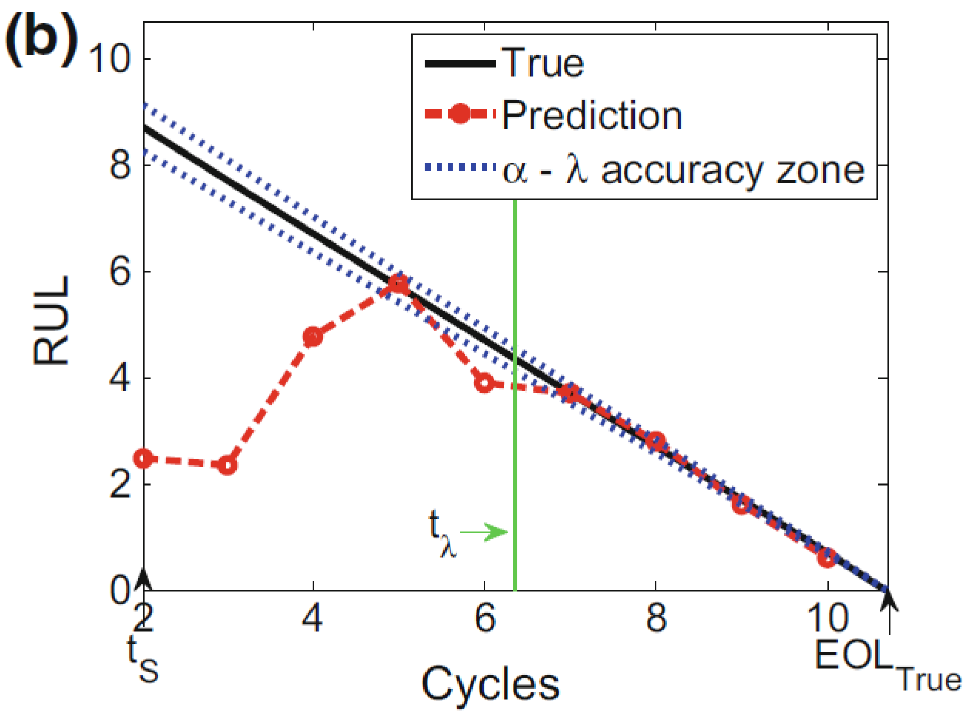


- The two parallel dotted lines indicates the constant bound of the accuracy zone with a magnitude of ±pha error with respect to true EOL.
- Here pha = 5%
- The prognostics method with a larger PH indicates the better performance

Prognostics metrics

α-λ accuracy

Determines whether a prediction result falls within the α accuracy zone at a specific cycle t_λ



- The accuracy zone varies with ±α ratio to the true RUL
- The dotted line shows the accuracy zone when α = 0.05
- The accuracy zone shrinks with more data suggesting an increase of prediction accuracy
- Specific cycle t_λ is expressed with a fraction of λ between 0 (starting cycle of RUL prediction) and 1 (true EOL)

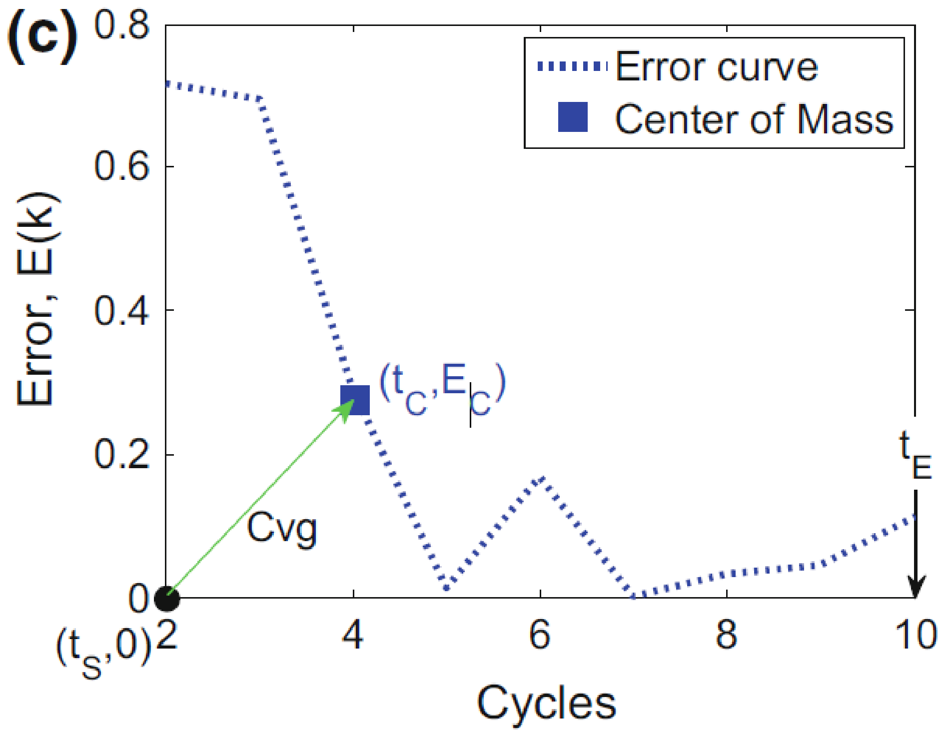
$$t_\lambda = t_s + \lambda(EOL_{True} - t_s)$$

Prognostics metrics

(Cumulative) Relative accuracy (RA, CRA)

RA is the relative accuracy between the true and prediction RUL at t_λ

CRA is same as the average of RA values accumulated at every cycle from t_s to t_E



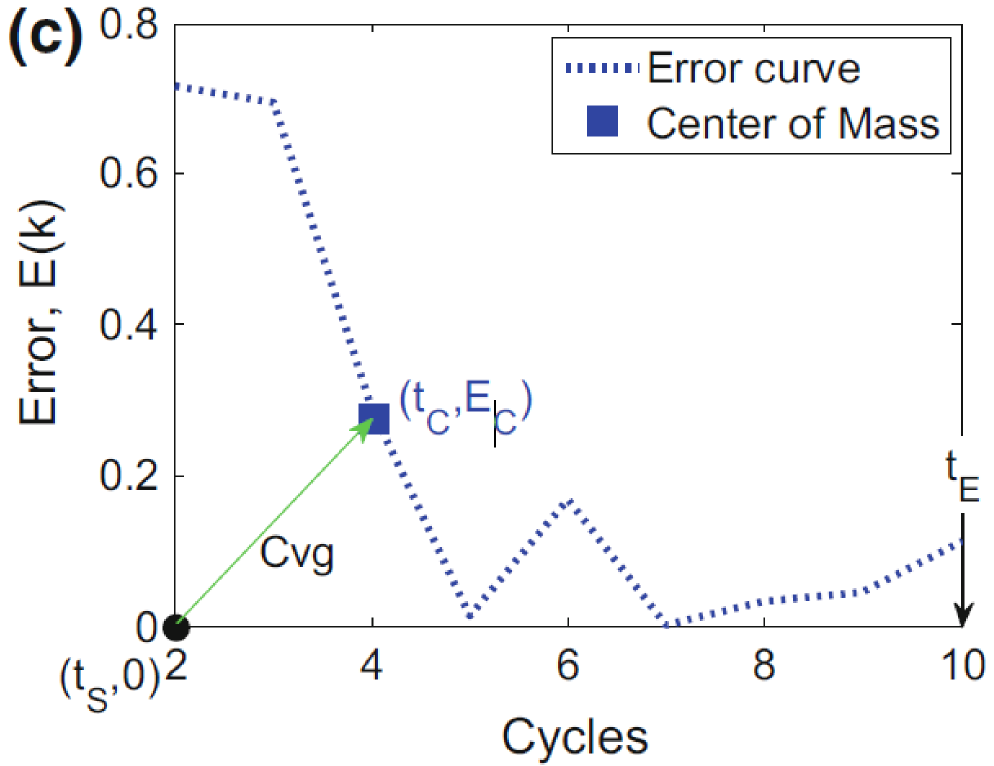
$$RA = 1 - \frac{|RUL_{True} - RUL|}{RUL_{True}} \; at \; t_\lambda$$

- The relative error shown in the plot (dotted curve) can be used to calculate RA (RA = 1- relative error)
- When RA and CRA are close to 1, prediction accuracy is high

Prognostics metrics

Convergence

Euclidean distance between the point (t_s,0) and the center of mass of the area under the error curve (t_C, E_C)



Center of mass = $CoM = \sqrt{(t_C - t_S)^2 + E_C^2}$

Where $t_C = \frac{1}{2}\frac{\sum_{k=S}^{E}(t_{k+1}^2 - t_k^2)(E(k))}{\sum_{k=S}^{E}(t_{k+1} - t_k)E(k)}$

$$E_C = \frac{1}{2}\frac{\sum_{k=S}^{E}(t_{k+1} - t_k)(E^2(k))}{\sum_{k=S}^{E}(t_{k+1} - t_k)E(k)}$$

- The lower the distance is, the faster the convergence is

Uncertainty

- Noise in measured data is caused by measurement variability, which represent uncertain measurement environment
- Noise is random in nature and thus it is assumed to be Gaussian, *i.e.*, statistical noise having a probability density function equal to that of the normal distribution

Objective: Estimate the level of uncertainty in estimated model parameters and remaining useful life when measured data have noise

Quantification using LS regression

- Assumption:
  - Noise is normally distributed with zero mean
  - They are independent and identically distributed (i.i.d)
- Variance of error is given by

$$\hat{\boldsymbol{\sigma}}^2 = \frac{\boldsymbol{SS_E}}{\boldsymbol{n_y} - \boldsymbol{n_p}}$$

$(\boldsymbol{n_y} - \boldsymbol{n_p})$ represents the degree of freedom. $\boldsymbol{n_y}$ and $\boldsymbol{n_p}$ are the number of data and unknown parameters respectively

Uncertainty (Model parameters)

Quantification using LS regression

- For the linear model with 2 parameters ($z(x) = \theta_1 + \theta_2 x$), the variance of parameters is derived using the estimated parameter and the variance of error equations

$$\textbf{Design matrix} = \mathbf{X} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ 1 & x_{n_y} \end{bmatrix}, \quad \textbf{Therefore,} \quad X^T X = \begin{bmatrix} n_y & \sum_{i=1}^{n_y} x_i \\ \sum_{i=1}^{n_y} x_i & \sum_{i=1}^{n_y} x_i^2 \end{bmatrix}, \quad X^T y = \begin{bmatrix} \sum_{i=1}^{n_y} y_i \\ \sum_{i=1}^{n_y} x_i\, y_i \end{bmatrix}$$

$$\hat{\theta} = \begin{Bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{Bmatrix} = \begin{bmatrix} n_y & \sum_{i=1}^{n_y} x_i \\ \sum_{i=1}^{n_y} x_i & \sum_{i=1}^{n_y} x_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n_y} y_i \\ \sum_{i=1}^{n_y} x_i\, y_i \end{bmatrix} = \begin{Bmatrix} \bar{y} - \hat{\theta}_2 \bar{x} \\ \dfrac{\sum_{i=1}^{n_y} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n_y} (x_i - \bar{x})^2} \end{Bmatrix}$$

$$\hat{\theta} = \begin{Bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{Bmatrix} = \begin{Bmatrix} \bar{y} - \bar{x} S_{xy}/S_{xx} \\ S_{xy}/S_{xx} \end{Bmatrix}$$

Uncertainty (Model parameters)

Quantification using LS regression

- By using a theorem for variance calculation, the variance of the two parameters is obtained as

$$\boldsymbol{Var(\hat{\theta}_2) = Var\left(\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{S_{xx}}\right) = \frac{\sum(x_i-\bar{x})^2\sigma^2}{S_{xx}^2} = \frac{S_{xx}\sigma^2}{S_{xx}^2} = \frac{\sigma^2}{S_{xx}}}$$

$$\boldsymbol{Var(\hat{\theta}_1) = Var(\bar{y}-\hat{\theta}_2\bar{x}) = \frac{\sigma^2}{n_y} + \bar{x}^2\frac{\sigma^2}{S_{xx}}}$$

- The terms related with x are constants because the source of uncertainty is from the noise in the data and the terms related with data y_i are random variables
- The variance of two parameters in a linear model is

$$\boldsymbol{Var(\hat{\theta}) = \begin{bmatrix} \frac{\sigma^2}{n_y} + \bar{x}^2\frac{\sigma^2}{S_{xx}} \\ \frac{\sigma^2}{S_{xx}} \end{bmatrix}, \qquad S_{xx} = \sum_{i=1}^{n_y}(x_i-\bar{x})^2}$$

Uncertainty (Model parameters)
Quantification using LS regression

- The variance in model parameters is linearly proportional to the variance of data
- Large number of data reduces uncertainty in model parameters and eventually makes them deterministic ($n_y \to \infty$, $S_{xx} \to \infty$)
- In case of many unknown parameters, the derivation is complicated and the following can be employed to describe the correlation between the parameters

$$\sum\nolimits_{\hat{\boldsymbol{\theta}}} = \boldsymbol{\sigma}^{\mathbf{2}}[\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}]^{-\mathbf{1}}$$

- Since the variance of error in data is unknown, usually estimated value can be used instead

Issues in practical prognostics

- In practical cases, simple polynomial functions and Gaussian noise does not suffice
- Bayesian-based approaches are employed instead of linear regression method
- The noise and bias have an effect on the prognostics results
- It is difficult to identify model parameters accurately when they are correlated
- Loading conditions can be correlated with the parameters too
- Physical degradation models are rare in practice
- In case of data-driven approach, it is not easy to obtain several sets of degradation data due to expensive time and costs
- Degradation data cannot be directly measured in most case and need to be extracted from sensor signals

THANK
YOU!