Assignment 1: ID5004W **-** AI in Predictive Maintenance, Reliability and Warranty

Author: Aloy Banerjee

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Problem - 1: Homogenous equation with constant coefficients

(a) Real roots of the characteristic equation: Find the solution to the initial value problem $y'' + 5y' + 6y = 0$ with $y(0) = 2, y'(0) = 3$

(b) Complex roots of the characteristic equation: Find the solution to the initial value problem $16y'' - 8y' + 145y = 0$ with $y(0) = -2, y'(0) = 1$

(c) Repeated roots of the characteristic equation: Find the solution to the initial value problem $y'' - y' + 0.25y = 0$ with $y(0) = 2, y'(0) = 1/3$

Solution:

a) **Using Laplace method:**

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Problem 1 :- Homogenous equation with constant coefficients.

a) $y'' + 5y' + 6y = 0$ with $y(0) = 2$ & $y'(0) = 3$

Ans→

$y'' + 5y' + 6y = 0$

taking laplace transform,

$\Rightarrow s^2 Y(s) - sY(0) - Y'(0) + 5sY(s) - 5Y(0) + 6Y(s) = 0$

where, $L[Y(t)] = Y(s)$

substituting initial values,

$\Rightarrow s^2 Y(s) - 2s - 3 + 5sY(s) - 10 + 6Y(s) = 0$

$\Rightarrow [s^2 + 5s + 6]\,Y(s) = (2s+13)$

$\Rightarrow Y(s) = \frac{2s+13}{s^2+5s+6} = \frac{2s+13}{s^2+3s+2s+6} = \frac{2s+13}{(s+3)(s+2)}$

Use partial fraction decomposition,

$\Rightarrow Y(s) = \frac{2s+13}{(s+3)(s+2)} = \frac{A}{(s+3)} + \frac{B}{(s+2)}$

Solving for A & B,

$\Rightarrow (2s+13) = A(s+2) + B(s+3) = (A+B)s + (2A+3B)$

comparing we got,

$A + B = 2$ & $2A + 3B = 13$

$\Rightarrow A = 2 - B$ ...(i)

replacing $A = 2 - B$ we got,

$\Rightarrow 2(2-B) + 3B = 13$

$\Rightarrow 4 - 2B + 3B = 13 \Rightarrow B = 9$ ...(ii)

Replacing $B = 9$, we got,

$\Rightarrow A = 2 - 9 = -7$

Replacing the value of A & B in Y(s) we got,

$Y(s) = \frac{A}{(s+3)} + \frac{B}{(s+2)} = \frac{-7}{(s+3)} + \frac{9}{(s+2)}$

taking inverse laplace we got,

$L^{-1}[Y(s)] = L^{-1}\left[\frac{-7}{s+3} + \frac{9}{s+2}\right]$

$y(t) = L^{-1}\left[\frac{-7}{s+3}\right] + L^{-1}\left[\frac{9}{s+2}\right]$

$= -7e^{-3t} + 9e^{-2t}$

$\therefore\ y(t) = 9e^{-2t} - 7e^{-3t}$. Ans

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Using Root based method:

a) $y''+5y'+6y=0$ with $y(0)=2$ & $y'(0)=3$

Ans→ At the starting point we have to find the characteristic equation associated with the given differential equation.

The characteristic equation can be obtained by substituting $y=e^{rx}$ into the differential equation where r is the constant.

$$y''+5y'+6y=0 \quad \text{--- (i)}$$

if $y=e^{rx}$ where r is constant then equation (i) can be rewritten as,

$$\Rightarrow (r^2+5r+6)e^{rx}=0$$

$$\Rightarrow (r^2+5r+6)=0 \quad [\text{As } e^{rx}\neq 0 \text{ we can divide both side by } e^{rx}]$$

Now lets solve the quadratic equation,

$$\Rightarrow r^2+3r+2r+6=0$$

$$\Rightarrow (r+3)(r+2)=0$$

So the roots are $r=-3$ & -2, both are distinct & real valued.

Since both are distinct & real valued, the general solution can be written as,

$$y(x)=Ae^{-2x}+Be^{-3x} \quad \text{--- (ii)}$$

Where A & B are the constant & can be obtained using the initial value of $y(0)=2$ & $y'(0)=3$

from (ii) equation putting $x=0$ we got,

$$y(0)=Ae^{0}+Be^{0}$$

$$2=A+B \quad \text{--- (iii)}$$

taking derivative of the general equation (ii),

$$y'(x)=-2Ae^{-2x}-3Be^{-3x}$$

now putting $x=0$

$$y'(0)=3=-2A-3B \Rightarrow 2A+3B=-3 \quad \text{--- (iv)}$$

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$2A + 3B = -3$ & $A + B = 2$

$\Rightarrow 2(2-B) + 3B = -3$

$\Rightarrow 4 - 2B + 3B = -3$

$\Rightarrow B = -3 - 4 = -7.$

$\Rightarrow A = (2-B)$

$A = 2 - (-7)$

$= 9$

$\therefore B = -7$ & $A = 9$

substituting the value of A & B in general differential equation,

$y(t) = Ae^{-2t} + Be^{-3t}$

$\therefore y(t) = 9e^{-2t} - 7e^{-3t}.$

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b) Using Laplace method:

(b) $16y'' - 8y' + 145y = 0$ with $y(0) = -2$ & $y'(0) = 1$

Ans

$16y'' - 8y' + 145y = 0$

taking laplace transform,

$\Rightarrow 16[s^2Y(s) - sy(0) - y'(0)] - 8[sY(s) - y(0)] + 145\,Y(s) = 0$

where $L[y(t)] = Y(s)$.

substituting initial value,

$\Rightarrow 16[s^2Y(s) + 2s - 1] - 8[sY(s) + 2] + 145\,Y(s) = 0$

$\Rightarrow 16s^2Y(s) + 32s - 16 - 8sY(s) - 16 + 145Y(s) = 0$

$\Rightarrow [16s^2 - 8s + 145]\,Y(s) = 32 - 32s$

$\Rightarrow Y(s) = \dfrac{32(1-s)}{(16s^2 - 8s + 145)}$

taking inverse laplace on both end we got,

$\Rightarrow L^{-1}(Y(s)) = y(t) = L^{-1}\left[\dfrac{32(1-s)}{(16s^2 - 8s + 145)}\right]$

$= L^{-1}\left[-2\dfrac{(s-1/4)}{(s-1/4)^2+9} + 3/2 \cdot \dfrac{1}{(s-1/4)^2+9}\right]$

$= -2\,L^{-1}\left[\dfrac{(s-1/4)}{(s-1/4)^2+9}\right] + \dfrac{3}{2}\,L^{-1}\left[\dfrac{1}{(s-1/4)^2+9}\right]$

$\Rightarrow y(t) = -2e^{t/4}\cos(3t) + \cancel{3}/2\, e^{t/4} \cdot 1/\cancel{3} \cdot \sin(3t)$

$= -2e^{t/4}\cos 3t + \frac{1}{2}e^{t/4}\sin 3t$.

$\therefore y(t) = -2e^{t/4}\cos 3t + \dfrac{e^{t/4}\sin 3t}{2}$. Ans

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Using Root based method:

1.b) $16y'' - 8y' + 145y = 0$ with $y(0) = -2$ & $y'(0) = 1$

Ans → At the starting point we have to find the characteristics equation associated with the given differential equation.

The characteristics equation can be obtained by substituting $y = e^{rx}$ into the differential equation where r is the constant.

$$16y'' - 8y' + 145y = 0 \quad \text{--- (i)}$$

if $y = e^{rx}$ where r is constant then equation (i) can be rewritten as,

$$\Rightarrow 16r^2 - 8r + 145 = 0 \quad \text{--- (ii)}$$

As $e^{rx} \neq 0$ we are rewriting the equation by simplifying the equation & obtained equation (ii)

∴ Solving the quadratic equation we got

$$r = \frac{1}{4} + 3i \;\&\; \frac{1}{4} - 3i \quad \left(\text{Applying } r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$$

So the roots are distinct but imaginary hence the general solution of the differential equation can be rewritten as,

$$y(t) = e^{\alpha t}(A\cos\beta t + B\sin\beta t) \quad \text{--- (iii)}$$

where α is the real part of the roots & β is the imaginary part of roots.

$\alpha = \frac{1}{4}$ & $\beta = 3$ (comparing the two roots value we got)

substituting the value of α & β in equation (iii) we got,

$$y(t) = e^{t/4}(A\cos 3t + B\sin 3t) \quad \text{--- (iv)}$$

Now we have to calculate the value of constant term A & B.

Let's say $t = 0$ in eq (iv) we got,

$$y(0) = -2 = A \quad \text{--- (v)}$$

differentiating the equation (iv) w.r.t t we got,

$$y'(t) = \frac{1}{4}e^{t/4}[A\cos 3t + B\sin 3t] + e^{t/4}[-3A\sin 3t + 3B\cos 3t] \quad \text{--- (vi)}$$

by putting $t = 0$ at equation (vi) we got

$y'(0) = 1 = \frac{1}{4}(A) + 3B$, replacing $A = -2$ we got

$B = 0.5$ or $1/2$

substituting the value of A & B we got,

$$y(t) = e^{t/4}\left(-2\cos(3t) + \tfrac{1}{2}\sin(3t)\right).$$

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c) Using Laplace method:

Q) $y'' - y' + 0.25y = 0$ with $y(0) = 2$ & $y'(0) = 1/3$

Ans→

$$y'' - y' + 0.25y = 0$$

taking laplace transform,

$$\Rightarrow [s^2Y(s) - sY(0) - Y'(0)] - [sY(s) - Y(0)] + 0.25\,Y(s) = 0$$

substituting the initial value,

$$\Rightarrow s^2Y(s) - 2s - 1/3 - sY(s) + 2 + 0.25\,Y(s) = 0$$

$$\Rightarrow [s^2 - s + 0.25]\,Y(s) = (2s - 5/3)$$

$$\Rightarrow Y(s) = \frac{(2s - 5/3)}{(s^2 - s + 0.25)} = \frac{6s-5}{3(s^2 - s + 1/4)} = \frac{6s-5}{3(s-1/2)^2}$$

$$\Rightarrow Y(s) = \frac{2s - 5/3}{(s - 1/2)^2}$$

Use partial fraction decomposition to represent Y(s),

$$\Rightarrow Y(s) = \frac{2s-5/3}{(s-1/2)^2} = \frac{A}{(s-1/2)} + \frac{B}{(s-1/2)^2}$$

$$\Rightarrow (2s - 5/3)(s-1/2)^2 = A(s-1/2)^2 + B(s-1/2)$$

now simplify the equation,

$$\Rightarrow 2s - 5/3 = A(s-1/2) + B \quad \text{--(i)}$$

we have denominator root on 1/2, putting the value in eq (i) we got, $s = 1/2$,

$$2s - 5/3 = B \Rightarrow B = -2/3$$

plugging the value of $B = -2/3$ in eq (i) we got,

$$(2s - 5/3) = A(s - 1/2) + (-2/3) = As - (A/2 + 2/3)$$

comparing, $As = 2s \Rightarrow A = 2$

putting value of $A = 2$ & $B = -2/3$ in Y(s) we got,

$$Y(s) = \frac{2}{(s-1/2)} + \frac{(-2/3)}{(s-1/2)^2}$$

taking inverse laplace on both and we got,

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$$y(t) = L^{-1}\left[\frac{2}{(s-1/2)} + \frac{(-2/3)}{(s-1/2)^2}\right]$$

$$= L^{-1}\left[\frac{2}{(s-1/2)}\right] + L^{-1}\left[\frac{(-2/3)}{(s-1/2)^2}\right]$$

$$= L^{-1}\left[\frac{4}{2s-1}\right] + L^{-1}\left[\frac{(-8/3)}{(2s-1)^2}\right]$$

$$= 2e^{t/2} - 2/3\, e^{t/2}\, t$$

$$= 2\left[e^{t/2} - 1/3\, e^{t/2}\, t\right]$$

$$\therefore y(t) = 2\left[e^{t/2} - \frac{e^{t/2}t}{3}\right]$$

Using Root based method:

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(c) $y'' - y' + 0.25y = 0$ with $y(0) = 2$ & $y'(0) = 1/3$

Ans → At the starting point find the characteristics equation associated with the given differential equation.

The characteristic equation can be obtained by substituting $y = e^{rx}$ into the differential equation where r is the constant.

$$y'' - y' + 0.25y = 0 \quad \text{--- (i)}$$

if $y = e^{rx}$ where r = constant term

equation (i) can be rewritten as,

$$\rightarrow r^2 - r + 0.25 = 0 \Rightarrow (r - 0.5)^2 = 0 \Rightarrow r = 0.5 \text{ (repeated)}$$

So looking at the characteristics equation we can see that the repeated roots are exists which value of 0.5.

Since it has repeated roots, the general solution of the differential equation will be,

$$y(x) = (A + Bx)\,e^{0.5x} \quad \text{--- (ii)}$$

where A & B are the constant value which can be solved by using the initial value given in the problem

$\therefore$ if $x = 0$ in equation (ii) we got,

$$y(0) = (A)\,e^{0} = A$$

$\therefore A = 2$

$\therefore$ take differentiation of equation (ii) we got,

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taking the differentiation of the general solution we got,

$$y(x) = (A+Bx)e^{0.5x}$$

$$\Rightarrow y'(x) = \frac{(A+Bx)\,e^{0.5x}}{2} + B\,e^{0.5x}$$

$$\Rightarrow y'(x) = \frac{(Bx+2B+A)}{2}\,e^{0.5x}$$

Now placing $x=0$ we got,

$$\Rightarrow y'(0) = B \quad \Rightarrow B = 1/3$$

so the constant values are,

$$A = 2 \;\&\; B = 1/3$$

substituting the A & B value in the general equation we got,

$$y(x) = (A+Bx)\,e^{0.5x}$$

$$= (2 + 1/3\,x)\,e^{1/2\,x}$$

Both the Laplace base method and root-based method are giving same result, so kept both.

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Problem - 2: Non-homogenous equation with constant coefficients

Using the method of undetermined coefficients, find a particular solution of

(a) $y'' - 3y' - 4y = 10$

(b) $y'' - 3y' - 4y = 2\ \sin wt$, (you may consider $w = 1$).

Solution:

a)

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Problem 2 :- Non-homogenous equation with constant coefficient. Find particular solution using method of undetermined coefficient.

a) $y'' - 3y' - 4y = 10$

Ans ⇒ Solution of any differential equation is the sum of the complementory (Homogenous) solution along with particular solution. we have to calculate only the particular solution for the given problem statement

$y'' - 3y' - 4y = 10$ --- (i)

roots of the homogenous equation will be,

let's say, $y'' = k^2$ & $y' = k$ then homogeneous equivalent of equation (i) will be,

$k^2 - 3k - 4 = 0$

$\Rightarrow k^2 - 4k + k - 4 = 0$

$\Rightarrow (k-4)(k+1) = 0$

so roots are, $k = 4$ & -1.

Now let consider that the particular solution of the given differential equation is y_p. & $y_p = A$ where $A \in \mathbb{R}$ (it's a real constant number)

$\therefore y_p = A$ then $y_p' = 0$ & $y_p'' = 0$

Replacing the value in the initial equation we got,

$-4A = 10 \quad \therefore A = -10/4 = -5/2 = -2.5$

$\therefore$ So, the particular solution of differential equation is $-5/2$ or -2.5

b)

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b)

$$y'' - 3y' - 4y = 2\sin\omega t$$

Ans→ solution of any differential equation is the sum of the compumentory (Homogenous) solution along with parti-cular solution. We have to calculate only the particular solution for the given problem statement.

$$y'' - 3y' - 4y = 2\sin\omega t \quad \text{--- (i)}$$

roots of the homogenous equation will be,

let say $y'' = k^2$ & $y' = k$ then homogenous equivalent of equation (i) will be,

$$k^2 - 3k - 4 = 0$$

$$\Rightarrow k^2 - 4k + k - 4 = 0$$

$$\Rightarrow (k-4)(k+1) = 0 \quad \therefore \text{ so roots are } k = 4 \text{ \& } -1.$$

Now lets consider that the particular solution of the given differential equation is y_p.

By observing eq(i) we can see the RHS of the equation contain $2\sin\omega t$ term hence, we consider,

$$y_p = A\cos\omega t + B\sin\omega t$$

taking derivative on both end we got,

$$\Rightarrow y_p' = \frac{d}{dt}\left[A\cos\omega t + B\sin\omega t\right]$$

$$= A\frac{d}{dt}(\cos\omega t) + B\frac{d}{dt}(\sin\omega t)$$

$$\Rightarrow y_p' = -A\omega\sin\omega t + B\omega\cos\omega t \quad \text{--- (ii)}$$

taking again derivative we got,

$$\Rightarrow y_p'' = \frac{d}{dt}(-A\omega\sin\omega t + B\omega\cos\omega t)$$

$$= \frac{d}{dt}(-A\omega\sin\omega t) + \frac{d}{dt}(B\omega\cos\omega t)$$

$$= -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

Now lets substitute the value of y_p' & y_p'' in the eq(i) we got,

$$y_p'' - 3y_p' - 4y_p = 2\sin\omega t$$

$$\Rightarrow (-A\omega^2\cos\omega t - B\omega^2\sin\omega t) - 3(-A\omega\sin\omega t + B\omega\cos\omega t) - 4(A\cos\omega t + B\sin\omega t) = 2\sin\omega t$$

$$\Rightarrow -A\omega^2\cos\omega t - B\omega^2\sin\omega t + 3A\omega\sin\omega t - 3B\omega\cos\omega t - 4A\cos\omega t - 4B\sin\omega t = 2\sin\omega t \quad \text{--- (iii)}$$

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→ Firstly make $\sin\omega t = 0$ in equation (iii) & solve for other term. We got,

$$-A\omega^2\cos\omega t - 3B\omega\cos\omega t - 4A\cos\omega t = 0 \quad \text{---- (iv)}$$

→ Now make $\cos\omega t = 0$ in equation (iii) & solve for other term, we got,

$$-B\omega^2\sin\omega t + 3A\omega\sin\omega t - 4B\sin\omega t = 2\sin\omega t \quad \text{--- (v)}$$

From equation (iv) we got,

$$(-A\omega^2 - 3B\omega - 4A)\cos\omega t = 0$$

$$\Rightarrow (-A\omega^2 - 3B\omega - 4A) = 0 \quad \text{--- (vi)}$$

Now equation (v) we got,

$$(-B\omega^2 + 3A\omega - 4B)\sin\omega t = 2\sin\omega t$$

$$\Rightarrow (-B\omega^2 + 3A\omega - 4B) = 2 \quad \text{--- (vii)}$$

As per the problem statement we can consider $\omega = 1$, replacing ω value in equation (vi) & (vii) we got,

$$-A - 3B - 4A = 0 \quad \& \quad -B + 3A - 4B = 2$$

$$\Rightarrow -5A - 3B = 0 \qquad \Rightarrow 3A - 5B = 2 \quad \text{--- (ix)}$$

$$\Rightarrow 5A + 3B = 0 \quad \text{--- (viii)}$$

Solving equation (viii) & (ix) for finding our constant term A & B we got,

$$15A + 9B = 0 \longrightarrow eq(viii) \times 3$$

$$\underset{(+)}{15A} \underset{(-)}{-} 25B = 10 \longrightarrow eq(ix) \times 5$$

$$34B = -10$$

$$\Rightarrow B = -10/34 = -5/17$$

Substituting $B = -5/17$ in equation (viii) we got,

$$5A + 3\cdot\left(-5/17\right) = 0$$

$$\Rightarrow 5A = 15/17$$

$$\Rightarrow A = 3/17$$

So, particular solution of the differential equation will be,

$$y_p = A\cos\omega t + B\sin\omega t$$

replacing $A = 3/17$, $B = -5/17$ & $\omega = 1$ we got

$$y_p = 3/17\cos\omega t - 5/17\sin\omega t$$

$$= 3/17\cos t - 5/17\sin t. \quad \text{Ans.}$$

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Problem - 3: Mechanical Systems

Suppose that a mass weighing 4.5 kg stretches a spring 40 mm. If the mass is displaced an additional 25 mm. and is then set in motion with an initial upward velocity of 0.5 m/s, determine the position of the mass at any later time. Also, determine the period, amplitude, and phase of the motion.

Solution:

Problem 3 :-

Ans→ The equation of motion for an undamped, unforced harmonic oscillator, (like a mass spring) is given by,

$m x'' + Kx = 0$ --(i) , where m = mass

x'' = Accelaration

K = spring constant

x = displacement

Firstly we need to calculate the spring constant (K) which can be obtained by,

$K = m \cdot g / x_0 = 4.5 \times 9.8 / 40 \times 10^{-3}$

$m = 4.5$ kg

$g = 9.8$ m/sec^2

$x_0 = 0.04$ meter

$K = 1102.5$ N/m

From the equation (i),

$m x'' + K x = 0$

$\Rightarrow x'' + \frac{1102.5}{4.5} x = 0$ --(ii) [substituting the value of different parameter]

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equation (ii) is in the form of,

$$x'' + \omega^2 x = 0$$ where ω = angular frequency

$$\therefore \omega^2 = \frac{1102.5}{4.5} \text{ rad/sec}$$

$$\Rightarrow \omega = \sqrt{\frac{1102.5}{4.5}} = 7\sqrt{5} \text{ rad/sec}$$

or 15.65 rad/sec

$\therefore$ angular frequency (ω) = 15.65 rad/sec

General solution of the motion can be written as,

$$x(t) = A\cos\omega t + B\sin\omega t \quad \text{---(iii)}$$

where A & B are some unknown constant.

Substituting angular frequency value in equation (iii) we got,

$$x(t) = A\cos(15.65)t + B\sin(15.65)t$$

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Mass was released from additional 25mm from the equlibriam position, so,

$$x(0) = 25\times10^{-3} \text{ m}$$

As per the problem statement, it has upward velocity of 0.5 m/s, hence $v_0 = x'(0) = -0.5$ m/s
(negative sign due to upward direction)

$$\therefore x(0) = A\cos 0 + B\sin 0$$

Substituting the value of $x(0)$ & calculating the value of $\sin 0 = 0$ & $\cos 0 = 1$ we got,

$$A = 25\times10^{-3}.$$

$$\because v_t = x'(t) = -\omega A\sin\omega t + \omega B\cos\omega t \quad \text{---(iv)}$$

$$\because x'(0) = \omega B = v_0$$

substituting the $t=0$ in equation (iv) we got the velosity value.

$\because$ substituting the value we got,

$$B = \frac{x'(0)}{\omega} = \frac{-0.5}{7\sqrt{5}} = -\sqrt{5}/70 = -0.032.$$

$\because$ In the general equation of $x(t)$ if we substitute the value of A & B constant we got,

$$x(t) = (25\times10^{-3})\cos(15.65)t + (-0.032)\sin(15.65)t$$

Now the desired value of different terms are as follows,

→ Period $(T) = \frac{2\pi}{\omega} = 0.4014$ sec

→ Amplitude $(A) = \sqrt{A^2+B^2} = 0.0405$ m

→ Phase $(\phi) = \tan^{-1}(A/B) = -0.664$ rad

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Problem - 4: Electrical Circuits/Systems

A series circuit has a capacitor of 10^3 micro Farad, a resistor of $3 \text{ X } 10^2$ ohms, and an inductor of 0.2 micro Henry. If the initial charge on the capacitor is 1 micro coulomb and there is no initial current, find the charge Q on the capacitor at any time t.

Solution:

Ans →

$C = 10^3 \times 10^{-6}$ Farad

$R = 3 \times 10^2 \ \Omega$

$L = 0.2 \times 10^{-6}$ Henry

∴ we know,

$$E(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q$$

Substituting the value of component of RLC circuit, we got,

$$0.2\times10^{-6}\frac{d^2q}{dt^2} + 300\frac{dq}{dt} + 10^3 q = 0$$

$$\Rightarrow \frac{d^2q}{dt^2} + (1500\times10^6)\frac{dq}{dt} + (5\times10^9)q = 0$$

Let say $K^2 = \frac{d^2q}{dt^2}$, $K = \frac{dq}{dt}$ then, the equation can be rewritten as,

$$K^2 + (1.5\times10^9)K + (5\times10^9) = 0$$

solving the roots we got below roots for the homogeneous equation,

$K = -3.33, -1.49\times10^9$, both are real valued & distinct.

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∴ considering general equation,

$q(t) = C_1 e^{k_1 t} + C_2 e^{k_2 t}$

$\Rightarrow q(t) = C_1 e^{-3.33t} + C_2 e^{(-1.49\times10^{9})t}$ (substituting the roots)

$q(0) = 10^{-6}$ & $i(0) = \frac{d}{dt} q(0) = 0$

$q(0) = C_1 + C_2 = 10^{-6}$ & $q'(0) = -3.33 C_1 - 1.499\times10^{9} C_2 = 0$

∴ by solving for C_1 & C_2 we got,

$C_1 = 1\times10^{-6}$ & $C_2 = -2.22\times10^{-15}$

C_2 is very small value & we can approximately consider it as 0, so, $C_2 \simeq 0$

$\therefore q(t) = e^{-3.33t}$ μ Coloumb.

$= e^{-3.33t}$ microcoloumb.

Problem - 5: Laplace transformation

Using the Laplace transform, find solutions for the following equations

(a) $5y' = e^{-3t}$ with $y(0) = 4, y'(0) = 0$

(b) $y'' = 1 - t$ with $y(0) = 0, y'(0) = 0$

(c) $y'' + 2y' + 2y = 0$ with $y(0) = 1, y'(0) = -1$

(d) $y'' + 4y = \cos(t)$ with $y(0) = a, y'(0) = b$

(e) $y'' + 16\,y = 16\,u(t-3) - 16$ with $y(0) = 0, y'(0) = 0$

Solution:

a)

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Problem 5 :- Laplace transformation:-

(a) $5y' = e^{-3t}$ with $y(0) = 4$ & $y'(0) = 0$

Ans→

$$5y' = e^{-3t}$$

taking laplace transformation on both end,

$$\Rightarrow 5\left(sY(s) - y(0)\right) = \frac{1}{s+3}$$

$$\Rightarrow 5sY(s) - 5y(0) = \frac{1}{s+3}$$

substituting the initial value we got,

$$\Rightarrow 5sY(s) - 20 = \frac{1}{s+3} \Rightarrow 5sY(s) = 20 + \frac{1}{s+3} = \frac{20s+61}{s+3}$$

$$\Rightarrow Y(s) = \frac{1}{5s}\left[\frac{20s+61}{s+3}\right] = \frac{20s+61}{5s(s+3)}$$

partial decomposition of $Y(s)$ give,

$$\Rightarrow Y(s) = \frac{A}{s} + \frac{B}{s+3}$$

multiply both side by denominator,

$$\Rightarrow \frac{5s(20s+61)(s+3)}{5s(s+3)} = \frac{5A\cancel{s}(s+3)}{\cancel{s}} + \frac{5Bs\cancel{(s+3)}}{\cancel{(s+3)}}$$

$$\Rightarrow 20s + 61 = 5A(s+3) + 5B(s)$$

for denominator root $s=0$, we got,

$$A = 61/15$$

& for denominator root $s=-3$, we got,

$$B = -1/15$$

putting value of A & B in $Y(s)$ equation,

$$Y(s) = \frac{61/15}{s} + \frac{(-1/15)}{(s+3)}$$

taking inverse laplace on both side we got,

$$\Rightarrow y(t) = L^{-1}\left[\frac{61/15}{s}\right] + L^{-1}\left[\frac{(-1/15)}{s+3}\right]$$

$$\Rightarrow y(t) = 61/15 - 1/15\, e^{-3t}.$$

$$\therefore y(t) = \boxed{\frac{1}{15}\left[61 - e^{-3t}\right].}$$ Ans

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b)

b

$y'' = (1-t)$ with $y(0) = 0$ & $y'(0) = 0$

Ans→

$y'' = (1-t)$

taking laplace transform on both side,

$\Rightarrow s^2 Y(s) - s\,y(0) - y'(0) = 1/s - 1/s^2$

substituting the initial value,

$\Rightarrow s^2 Y(s) = \dfrac{(s-1)}{s^2}$

$\Rightarrow Y(s) = \dfrac{(s-1)}{s^4} = \dfrac{1}{s^3} - 1/s^4$

taking inverse laplace on both end we got,

$\Rightarrow y(t) = L^{-1}(1/s^3) - L^{-1}(1/s^4)$

$= \boxed{t^2/2 - t^3/6}$. Ans

c)

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(c) $y'' + 2y' + 2y = 0$ with $y(0) = 1$ & $y'(0) = -1$

Ans) $y'' + 2y' + 2y = 0$

taking laplace transform on both side,

$$\Rightarrow [s^2 Y(s) - sY(0) - Y'(0)] + 2[sY(s) - Y(0)] + 2Y(s) = 0$$

$$\Rightarrow s^2 Y(s) - sY(0) - Y'(0) + 2sY(s) - 2Y(0) + 2Y(s) = 0$$

substituting the initial value,

$$\Rightarrow s^2 Y(s) - s + 1 + 2Y(s)\cdot s - 2 + 2Y(s) = 0$$

$$\Rightarrow [s^2 + 2s + 2]\, Y(s) = (1+s)$$

$$\Rightarrow [Y(s)] = \frac{(s+1)}{(s^2+2s+2)} = \frac{(s+1)}{(s+1)^2+1}$$

taking inverse laplace transform, $\because \left[L^{-1}[F(s-a)] = e^{at} f(t)\right]$

$$\Rightarrow y(t) = L^{-1}\left[\frac{(s+1)}{(s+1)^2+1^2}\right]$$

combing the formulae mentioned, $\because \left[L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at\right]$

we got,

$$\Rightarrow y(t) = \boxed{e^{-t}\cos(t)} \text{ Ans}$$

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d)

(d) $y''+4y=\cos t$ with $y(0)=a$ & $y'(0)=b$

Ans→

$y''+4y=\cos t$

taking laplace tramformation on both side,

$\Rightarrow [s^2Y(s)-sY(0)-Y'(0)]+4Y(s)=\frac{s}{s^2+1}$

substituting the initial value,

$\Rightarrow s^2Y(s)-as-b+4Y(s)=s/(s^2+1)$

$\Rightarrow (s^2+4)Y(s)=\frac{s}{(s^2+1)}+as+b$

$\Rightarrow Y(s)=\frac{s}{(s^2+1)(s^2+4)}+\frac{as}{(s^2+4)}+\frac{b}{(s^2+4)}$

taking inverse laplace tramformation on both side

$\Rightarrow y(t)=L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]+L^{-1}\left[\frac{as}{s^2+4}\right]+L^{-1}\left[\frac{b}{s^2+4}\right]$

$=L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]+aL^{-1}\left[\frac{s}{s^2+4}\right]+bL^{-1}\left[\frac{1}{s^2+4}\right]$

Inverse laplace Tramformation of each term,

$L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right]$

$\frac{s}{(s^2+1)(s^2+4)}$

taking partial fraction (decomposition),

$=\frac{As+B}{(s^2+1)}+\frac{Cs+D}{(s^2+4)}$

$\Rightarrow \frac{s(s^2+1)(s^2+4)}{(s^2+1)(s^2+4)}=\frac{(As+B)(s^2+1)(s^2+4)}{(s^2+1)}+\frac{(Cs+D)(s^2+1)(s^2+4)}{(s^2+4)}$

Simplifying the above eq we got,

$\Rightarrow s=(As+B)(s^2+4)+(Cs+D)(s^2+1)$

$\Rightarrow s=As^3+4As+Bs^2+4B+Cs^3+Cs+Ds^2+D$

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$\Rightarrow s = (A+C)s^3 + (B+D)s^2 + (4A+C)s + (4B+D)$

comparing the coefficient of s term, we got

$A+C=0 \quad --- (i) \rightarrow A = -C$

$B+D=0 \quad --- (ii) \rightarrow B = -D$

$4A+C=1 \quad --- (iii) \rightarrow C = -1/3$

$4B+D=0 \quad --- (iv) \rightarrow D = 0$

$\therefore A = 1/3 \;\&\; C = -1/3$

$B = D = 0.$

replacing the value,

$$L^{-1}\left[\frac{\frac{1}{3}s}{(s^2+1)} + \frac{(-\frac{1}{3})s}{(s^2+4)}\right]$$

$$= \frac{1}{3}L^{-1}\left[\frac{s}{s^2+1}\right] + \left(-\frac{1}{3}\right)L^{-1}\left[\frac{s}{s^2+2^2}\right]$$

$$= \frac{1}{3}\cos t - \frac{1}{3}\cos 2t \quad --- (i)$$

$$L^{-1}\left[\frac{as}{s^2+4}\right] = a\cos 2t \quad --- (ii)$$

$$L^{-1}\left[\frac{b}{s^2+4}\right] = \frac{b}{2}\sin 2t \quad --- (iii)$$

$$\therefore y(t) = L^{-1}\left[\frac{s}{(s^2+1)(s^2+4)}\right] + aL^{-1}\left[\frac{s}{s^2+4}\right] + bL^{-1}\left[\frac{1}{s^2+4}\right]$$

placing the value of individual inverse laplace term in final equation we got,

$$y(t) = \frac{1}{3}\cos t - \frac{1}{3}\cos 2t + a\cos 2t + \frac{b}{2}\sin 2t. \quad \text{Ans}$$

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e)

(e)

$y'' + 16y = 16u(t-3) - 16$ with $y(0) = 0$ & $y'(0) = 0$

Ans→

$y'' + 16y = 16u(t-2) - 16$

taking laplace transformation on both side,

$\Rightarrow [s^2 Y(s) - sY(0) - Y'(0)] + 16Y(s) = \frac{16}{s} e^{-3s} - 16/s$

substituting the initial value,

$\Rightarrow (s^2+16)Y(s) = \frac{16}{s}[e^{-3s} - 1]$

$\Rightarrow Y(s) = \frac{16}{s(s^2+16)} \cdot (e^{-3s} - 1)$

taking inverse laplace transformation we got,

$$y(t) = L^{-1}\left[\frac{16(e^{-3s}-1)}{s(s^2+16)}\right]$$

$$= 16 L^{-1}\left[\frac{e^{-3s}-1}{s(s^2+16)}\right]$$

for unit step signal $u(t)$,

$L^{-1}\{F(s)\} = f(t)$ then $L^{-1}[e^{-as}F(s)]$
$= f(t-a)u(t-a)$

let's, $F(s) = \frac{1}{s(s^2+16)}$

$f(t) = L^{-1}[F(s)] = L^{-1}\left[\frac{1}{s(s^2+16)}\right]$

taking partial ~~fra~~ decomposition,

$$\frac{1}{s(s^2+4^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+16}$$

multiplying both side,

$\Rightarrow 1 = A(s^2+16) + (Bs+C)\cdot s$ --- (i)

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for denominator we have two roots,

$s = 0$ & 4

for root $s=0$, we got,

$$A = 1/16$$

putting $A = 1/16$ in equation(i)

$$1 = \frac{1}{16}(s^2+16) + s(Bs+C)$$

$$\Rightarrow 1 = \frac{s^2}{16} + 1 + Bs^2 + sC$$

$$\Rightarrow 1 = s^2(B + 1/16) + Cs + 1$$

comparing the coefficient we got,

$$B + 1/16 = 0 \qquad C = 0 \qquad A = 1/16.$$

$$\Rightarrow B = -1/16$$

$\therefore A = 1/16$; $B = -1/16$ & $C = 0$.

substituting the value of A, B, & C we got,

$$L^{-1}\left[\frac{1}{s(s^2+16)}\right] = L^{-1}\left[\frac{1}{16s} - \frac{s}{16(s^2+16)}\right]$$

$$= L^{-1}\left[1/16s\right] - L^{-1}\left[\frac{s}{16(s^2+16)}\right]$$

$$= 1/16 - \frac{1}{16}L^{-1}\left[\frac{s}{s^2+4^2}\right]$$

$$= 1/16 - \frac{1}{16}\cos(4t).$$

Now we know,

$$y(t) = 16\, L^{-1}\left[\frac{e^{-3s}}{s(s^2+16)}\right] - 16\, L^{-1}\left[\frac{1}{s(s^2+16)}\right]$$

replacing the laplace transform value & 1st frequency shifting term we got,

$$y(t) = (1 - \cos 4(t-3))\,u(t-3) - (1 - \cos 4t)$$

$$= -1 + u(t-3) + \cos 4t - \cos(4t-12)\,u(t-3)$$

Ans.