Assignment 3: ID5004W - AI in Predictive Maintenance, Reliability and Warranty

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Problem - 1: Differential equation to State-space representation

What will be the state-space representation for the system given by the following differential equation?

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 10y = 8u(t)$$

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1. Differential equation to state space representation:—

differential equation is,

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 10y = 8u(t) \quad --(i)$$

take laplace transformation on both side with all the initial condition are zero regor,

$$\Rightarrow s^3 Y(s) + 6s^2 Y(s) + 11 s Y(s) + 10 Y(s) = 8/s$$

$$\Rightarrow (s^3 + 6s^2 + 11s + 10) Y(s) = 8/s \quad [\because \mathcal{L}[u(t)] = U(s) = 1/s]$$

$$\Rightarrow Y(s) = \frac{8}{s(s^3+6s^2+11s+10)}$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{8}{(s^3+6s^2+11s+10)} = \text{transfer function}$$

Consider the state space system, state variable is defined as,

$x_1 = y$

$x_2 = dy/dt$

$x_3 = d^2y/dt^2$

$$y = x_1 \quad ---(i)$$

$$\frac{dx_1}{dt} = x_2 = \dot{y} \quad --(ii)$$

$$\frac{dx_2}{dt} = \frac{d^2y}{dt^2} = x_3 \quad --(iii)$$

$$\frac{dx_3}{dt} = \frac{d^3y}{dt^3} \quad --(iv)$$

generic equation,

$$\frac{dx}{dt} = \dot{x} = Ax + BU$$

$$Y = Cx + DU$$

replace the value in equation (i),

$$\frac{dx_3}{dt} + 6\frac{dx_2}{dt} + 11\frac{dx_1}{dt} + 10x_1 = 8u(t)$$

$$\therefore \frac{dx_3}{dt} = -(6x_3 + 11x_2 + 10x_1) + 8u(t)$$

standard state space model,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix} U$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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so,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -11 & -6 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

Problem - 2: Differential equation to Transfer function

What will be the transfer function for the system given by the following differential equation?

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = Px + Q\frac{dx}{dt}$$

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2. Differential equation to transfer function :-

Differential equation as per given problem is,

$$A\frac{d^2y}{dt^2} + B\frac{dy}{dt} + Cy = Px + Q\frac{dx}{dt}.$$

take laplace transform on both side considering all the initial conditions are zero we got,

$$As^2Y(s) + BsY(s) + CY(s) = P(X(s)) + QsX(s)$$

[Assuming, laplace transform of,
$L[y(t)] = Y(s)$
& $L[x(t)] = X(s)$]
Where $x \rightarrow$ input & $y \rightarrow$ output

$$\Rightarrow [As^2 + Bs + C]Y(s) = [P + Qs]X(s)$$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{P+Q(s)}{As^2+Bs+C} = \frac{P+Qs}{As^2+Bs+C}.$$

$\therefore$ transfer function of the given differential equation is,

$$\frac{P+Qs}{As^2+Bs+C}$$

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Problem - 3: Transfer function to State-space representation

Find the State-space representation of the following transfer function:

$$\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

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3. Transfer function to state space representation,

$$\frac{C(s)}{R(s)} = \frac{24}{s^3+9s^2+26s+24} \rightarrow \text{transfer function}$$

generic equation of state space model is

$\dot{x} = Ax + BU \rightarrow$ state equation

$Y = Cx + DU \rightarrow$ output equation

$$\frac{C(s)}{R(s)} = \frac{X(s)}{R(s)} \cdot \frac{C(s)}{X(s)}$$

[laplace transform of $X(s) \rightarrow$ state variable
$C(s) \rightarrow$ output variable
$R(s) \rightarrow$ input variable]

$$= \left[\frac{1}{s^3+9s^2+26s+24}\right] \cdot [24]$$

state equation ↑ output equation ↑

$\therefore$ state equation,

$$\frac{X(s)}{R(s)} = \frac{1}{s^3+9s^2+26s+24} \quad \ldots \text{(i)}$$

output equation

$$\frac{C(s)}{X(s)} = 24 \Rightarrow C(s) = 24(X(s))$$

from the equation(i) we got,

$R(s) = s^3X(s) + 9s^2X(s) + 26sX(s) + 24X(s)$

considering all the initial conditions are zero, if we take inverse laplace transformation we got,

$$R(t) = \frac{d^3x(t)}{dt^3} + 9\cdot\frac{d^2x(t)}{dt^2} + 26\frac{dx(t)}{dt} + 24x(t) \quad \text{-(vi)}$$

[$\rightarrow L^{-1}[R(s)] = R(t)$
$L^{-1}[X(s)] = x(t)$
inverse laplace]

consider state space system where state variable is defined as,

$x = x_1$ --- (ii).

$$\frac{dx_1}{dt} = x_2 = \dot{x} \text{ -- (iii)}$$

$$\frac{dx_2}{dt} = x_3 = \ddot{x} \text{ --- (iv)}$$

$$\frac{dx_3}{dt} = \dddot{x} \text{ ---- (v)}$$

comparing the value we got, equation (vi) can be rewritten,

$$R(t) = \frac{dx_3}{dt} + 9\frac{dx_2}{dt} + 26\frac{dx_1}{dt} + 24x_1$$

$$\Rightarrow dx_3/dt = R(t) - [9\underset{x_3}{\frac{dx_2}{dt}} + 26\underset{x_2}{\frac{dx_1}{dt}} + 24x_1]$$

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Problem 3 - continues,

standard state space model,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&1\\-24&-26&-9\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} R(t)$$

$$\frac{C(s)}{X(s)} = 24$$

$$\therefore C(s) = 24\,X(s)$$

$$\therefore C = \begin{bmatrix}24 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$\text{Output} = \begin{bmatrix}24 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

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Problem - 4: Transfer function to State-space representation

For a system with transfer function H(s), what is the matrix A in the state space from?

$$H(s) = \frac{3(s-2)}{s^3 + 4s^2 - 2s + 1}$$

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(4) Transfer function to state space equation

$$H(s) = \frac{3(s-2)}{s^3+4s^2-2s+1} = \text{transfer function}$$

generic equation of state space model is,

$\dot{x} = Ax + BU \rightarrow$ state equation

$Y = Cx + DU \rightarrow$ output equation

$$H(s) = \frac{C(s)}{R(s)} = \underbrace{\frac{X(s)}{R(s)}}_{\text{state equation}} \cdot \underbrace{\frac{C(s)}{X(s)}}_{\text{output equation}}$$

[where, laplace transform

$X(s) =$ state variable

$R(s) \rightarrow$ input "

$C(s) \rightarrow$ output "]

$\therefore$ state equation

$$\frac{X(s)}{R(s)} = \frac{1}{s^3+4s^2-2s+1} \quad \text{--- (i)}$$

output equation

$$\frac{C(s)}{X(s)} = 3(s-2) \quad \text{--- (ii)}$$

From equation (i) we got,

$$R(s) = (s^3+4s^2-2s+1)X(s)$$

$$\Rightarrow R(s) = s^3X(s) + 4s^2X(s) - 2sX(s) + X(s)$$

taking inverse laplace transform with all the initial condition as zero we got,

$$\Rightarrow R(t) = \frac{d^3x(t)}{dt^3} + 4\frac{d^2x(t)}{dt^2} - 2\frac{dx(t)}{dt} + x(t) \quad \text{--- (iii)}$$

[Inverse laplace of

$L^{-1}[R(s)] \rightarrow R(t)$

$L^{-1}[X(s)] \rightarrow x(t)$]

consider state space system where state variable defined as,

$x = x_1$ --- (iii)

$dx_1/dt = \dot{x}_1 = x_2 = \dot{x}$ --- (iv)

$dx_2/dt = \dot{x}_2 = x_3 = \ddot{x}$ --- (v)

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$\therefore \dddot{x} = \frac{dx_3}{dt} = \dot{x}_3$ --- (vi)

Replacing the equation in (vii) we got,

$$R(t) = \dddot{x} + 4\ddot{x} - 2\dot{x} + x$$

$$= \frac{dx_3}{dt} + 4\cdot x_3 - 2\cdot x_2 + x_1$$

$$\Rightarrow \boxed{\frac{dx_3}{dt} = R(t) - [4x_3 - 2x_2 + x_1]} \quad \text{--- (viii)}$$

from equation (ii) we got,

$$\frac{C(s)}{X(s)} = 3(s-2)$$

$$\Rightarrow C(s) = (3s-6)X(s) \quad \text{--- (ix)}$$

$$\Rightarrow C(s) = 3sX(s) - 6X(s).$$

taking inverse laplace on both side, considering all the initial condition as zero we got,

$$\Rightarrow \boxed{c(t) = 3\cdot\frac{dx(t)}{dt} - 6x(t)} \quad \text{--- (x)}$$

[Inverse laplace of $L^{-1}[X(s)] = x(t)$, $L^{-1}[C(s)] = c(t)$]

$x = x_1$ then

$$\Rightarrow \boxed{c(t) = 3\cdot x_2 - 6x_1}$$

∴ Standard state space model, from equation (viii)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} R(t)$$

From equation (x),

$$c(t) = \underbrace{\begin{bmatrix} -6 & 3 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \cancel{\begin{bmatrix} \ \end{bmatrix}} R(t)$$

(no relation with R(t) so cancel out)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\& \; C = \begin{bmatrix} -6 & 3 & 0 \end{bmatrix}$$

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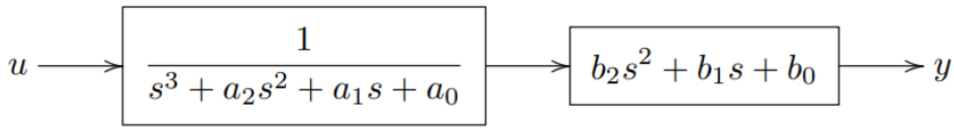
Problem - 5: Transfer function to State-space representation

Find the State-space representation of the following system:



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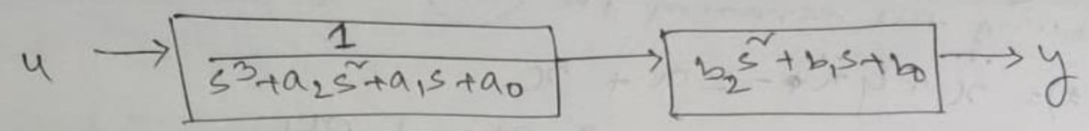
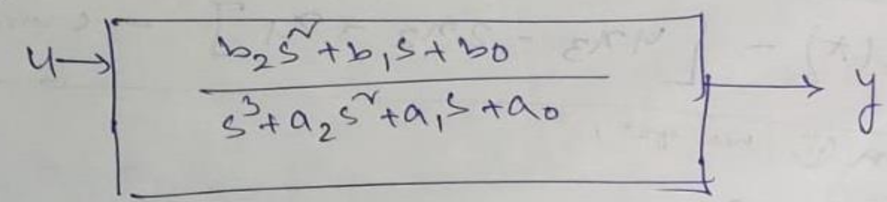
5. Transfer function to state space representation,



can be represented as,



Generic equation of state space model is,

$\dot{x} = Ax + Bu$ → state equation

$y = Cx + Du$ → output equation

$$\frac{Y(s)}{U(s)} = \text{transfer function} = \frac{b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}$$

$$\Rightarrow \frac{X(s)}{U(s)} \cdot \frac{Y(s)}{X(s)} = \frac{b_2s^2+b_1s+b_0}{s^3+a_2s^2+a_1s+a_0}$$

state equation ↑ output equation ↑

$$\frac{X(s)}{U(s)} = \frac{1}{s^3+a_2s^2+a_1s+a_0} \quad \text{--- (i)} \quad [\text{state equation}]$$

$$\frac{Y(s)}{X(s)} = b_2s^2+b_1s+b_0. \quad \text{--- (ii)} \quad [\text{output equation}]$$

from equation (i) we got,

$$U(s) = s^3X(s) + a_2s^2X(s) + a_1sX(s) + a_0X(s)$$

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take inverse laplace considering all the initial conditions are 0,

$$u(t) = \frac{d^3x(t)}{dt^3} + a_2\frac{d^2x(t)}{dt^2} + a_1\frac{dx(t)}{dt} + a_0x(t) \quad \text{--- (vii)}$$

[where, $u(t) = L^{-1}[U(s)]$ & $x(t) = L^{-1}[X(s)]$]

consider state space system where state variable defined as,

$$x = x_1 \quad \text{... (iii)}$$

$$\frac{dx_1}{dt} = x_2 = \dot{x} \quad \text{-- (iv)}$$

$$\frac{dx_2}{dt} = x_3 = \ddot{x} \quad \text{-- (v)}$$

$$\frac{dx_3}{dt} = x_4 = \dot{x}_3 = \dddot{x} \quad \text{--- (vi)}$$

replacing those values in equation (vii) we got,

$$u(t) = \dot{x}_3 + a_2x_3 + a_1x_2 + a_0x_1$$

$$\Rightarrow \dot{x}_3 = u(t) - [a_2x_3 + a_1x_2 + a_0x_1] \quad \text{--- (viii)}$$

$$\Rightarrow \boxed{\frac{dx_3}{dt} = u(t) - [a_2x_3 + a_1x_2 + a_0x_1] \quad \text{-- (viii)}}$$

from equation (ii) we got the,

$$\frac{Y(s)}{X(s)} = b_2s^2 + b_1s + b_0$$

$$\Rightarrow Y(s) = (b_2s^2 + b_1s + b_0)X(s) \quad \text{--- (ix)}$$

$$\Rightarrow Y(s) = b_2s^2X(s) + b_1sX(s) + b_0X(s)$$

taking inverse laplace on both side considering all the initial conditions are zero,

$$\Rightarrow y(t) = b_2\frac{d^2x(t)}{dt^2} + b_1\frac{dx(t)}{dt} + b_0x(t)$$

[where $L^{-1}[Y(s)] = y(t)$ & $L^{-1}[X(s)] = x(t)$]

$$\Rightarrow y(t) = b_2\ddot{x}(t) + b_1\dot{x}(t) + b_0x(t) \quad \text{-- (x)}$$

$$= b_2x_3 + b_1x_2 + b_0x_1 \quad \text{--- (xi)}$$

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standard state space model,
from equation (ii, iii) we got,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_0 & -a_1 & -a_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u(t)$$

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -a_0 & -a_1 & -a_2\end{bmatrix} \quad \& \quad B = \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}.$$

from equation (*) we got,

$$y(t) = \begin{bmatrix}b_0 & b_1 & b_2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$C = \begin{bmatrix}b_0 & b_1 & b_2\end{bmatrix} \qquad D = \begin{bmatrix}0\end{bmatrix}$$

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Problem - 6: State-space representation to transfer function

Find the transfer function of the system described by the state equation $X' = AX + BU$. The output is given by Y = CX for:

(a) $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

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6. State-space representation of transfer function:

According to the formulae we know that,

$$\text{Transfer function } (T(s)) = C(SI-A)^{-1}B$$

(a) $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Ans

$$(SI - A) = s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ +3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}$$

now, $(SI-A)^{-1} = \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}^{-1}$ (performing inversing)

$$= \begin{bmatrix} \frac{s+1}{s^2+5s+7} & -1/s^2+5s+7 \\ \frac{3}{s^2+5s+7} & \frac{s+4}{s^2+5s+7} \end{bmatrix}$$

So transfer function,

$$T(s) = \begin{bmatrix} 1 & 0 \end{bmatrix}_{1\times 2} \begin{bmatrix} \frac{s+1}{s^2+5s+7} & \frac{-1}{s^2+5s+7} \\ \frac{3}{s^2+5s+7} & \frac{s+4}{s^2+5s+7} \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= \begin{bmatrix} \frac{s+1}{s^2+5s+7} & \frac{-1}{s^2+5s+7} \end{bmatrix}_{1\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= \frac{s+1}{s^2+5s+7} + \frac{(-1)}{s^2+5s+7} = \frac{s}{s^2+5s+7}.$$

$$\text{Transfer function } (T(s)) = \frac{s}{s^2+5s+7}$$

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6. b) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Ans

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

taking inverse on both side we got,

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} s+3/s^2+3s+2 & 1/s^2+3s+2 \\ -2/s^2+3s+2 & s/s^2+3s+2 \end{bmatrix}$$

so the transfer function will be,

$$T(s) = \begin{bmatrix} 1 & 0 \end{bmatrix}_{1\times 2} \begin{bmatrix} s+3/s^2+3s+2 & 1/s^2+3s+2 \\ -2/s^2+3s+2 & s/s^2+3s+2 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= \begin{bmatrix} s+3/s^2+3s+2 & 1/s^2+3s+2 \end{bmatrix}_{1\times 2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= 1/s^2+3s+2$$

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so transfear function is ,

$1/s^2+3s+2.$

Problem - 7: State transition matrix

Find the state transition matrix for a system, $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

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7. State transition matrix :-

$$A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

According to the formula to find state transition matrix, we have to follow below,

Step (1) $(SI-A)$

Step (2) $(SI-A)^{-1} = \phi(s) = \dfrac{Adj[SI-A]}{det[SI-A]}$

Step (3) $\phi(t) = e^{At}$

where $L^{-1}[\phi(s)] = \phi(t)$

So let find the step 1,

$$(SI-A) = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix}$$

Lets take inverse of $(SI-A)$ which is nothing but $\phi(s)$,

$$\phi(s) = (SI-A)^{-1} = \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix}^{-1}$$

$$\Rightarrow \phi(s) = \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+3}{s^2+3s+2} & -2/s^2+3s+2 \\ 1/s^2+3s+2 & s/s^2+3s+2 \end{bmatrix}$$

$$\Rightarrow \phi(s) = \begin{bmatrix} s+3/s^2+3s+2 & -2/s^2+3s+2 \\ 1/s^2+3s+2 & s/s^2+3s+2 \end{bmatrix}$$

Now taking the inverse laplace transformation on both side we got,

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$$\Rightarrow L^{-1}[\phi(s)] = \begin{bmatrix} L^{-1}\left[\frac{s+3}{s^2+3s+2}\right] & L^{-1}\left[\frac{-2}{s^2+3s+2}\right] \\ L^{-1}\left[\frac{1}{s^2+3s+2}\right] & L^{-1}\left[\frac{s}{s^2+3s+2}\right] \end{bmatrix}$$

Now let's calculate the inverse laplace for each term we got,

1st Term

$$L^{-1}\left[\frac{s+3}{s^2+3s+2}\right]$$

$$\therefore \frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+1)(s+2)}$$

taking partial fraction we got,

$$= \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

Multiplying both side by denominator,

$$\Rightarrow \frac{(s+3)(s+1)(s+2)}{(s+1)(s+2)} = \frac{A(s+1)(s+2)}{(s+1)} + \frac{B(s+1)(s+2)}{(s+2)}$$

$$\Rightarrow s+3 = A(s+2) + B(s+1)$$

$$\Rightarrow s+3 = (As+Bs) + (2A+B)$$

comparing the coefficients,

$A+B=1$ & $2A+B=3$

$\Rightarrow A=(1-B)$ $\quad$ $\therefore$ $2(1-B)+B=3$

$\Rightarrow A=1-(-1)$ $\quad$ $\Rightarrow 2-2B+B=3$

$=2$ $\quad$ $\Rightarrow -B=1$

$\Rightarrow B=-1$

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$$\therefore \frac{s+3}{s^2+3s+2} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$= \frac{2}{s+1} - \frac{1}{s+2}$$

now taking laplace inverse we got,

$$L^{-1}\left[\frac{s+3}{s^2+3s+2}\right] = L^{-1}\left[\frac{2}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right]$$

$$= \left[2e^{-t} - e^{-2t}\right]$$

2nd Term

$$L^{-1}\left[\frac{1}{s^2+3s+2}\right]$$

taking partial fraction we got,

$$\therefore \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

multiplying both side by denominator,

$$\Rightarrow 1 = A(s+2) + B(s+1)$$

$$\Rightarrow 1 = (As + Bs) + (2A + B)$$

$\therefore$ comparing the coefficient,

$A + B = 0$ $\quad$ $2A + B = 1$

$A = -B$ $\quad$ $\Rightarrow -2B + B = 1$

$A = 1$ $\quad$ $\Rightarrow B = -1$

$$\therefore \frac{1}{s^2+3s+2} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{1}{s+1} - \frac{1}{s+2}$$

now taking inverse laplace we got,

$$L^{-1}\left[\frac{1}{s^2+3s+2}\right] = L^{-1}\left[\frac{1}{s+1}\right] - L^{-1}\left[\frac{1}{s+2}\right]$$

$$= \left[e^{-t} - e^{-2t}\right]$$

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3rd Term :-

$$L^{-1}\left[-2/s^2+3s+2\right]$$

$$= -2L^{-1}\left[1/s^2+3s+2\right]$$

we know that,

$L^{-1}\left[1/s^2+3s+2\right]$ has been calculated previously & the value of that is,

$$L^{-1}\left[1/s^2+3s+2\right] = e^{-t} - e^{-2t}$$

Now $L^{-1}\left[-2/s^2+3s+2\right] = -2\left[e^{-t} - e^{-2t}\right]$

$$= 2e^{-2t} - 2e^{-t}.$$

4th term ;-

$$L^{-1}\left[\frac{s}{s^2+3s+2}\right]$$

taking partial fraction we got,

$$\frac{s}{(s^2+3s+2)} = \frac{s}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

∴ Multiplying both side by denominator we got,

$$s = A(s+2) + B(s+1)$$

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solve the unknown parameter by plugging the real roots of the denominator we got,

$s = -1 \; \& \; -2$

so for $s=-1$ we got, $A = -1$

& for $s=-2$ we got $B = 2$

$\therefore$ so plugging back to main equation we got,

$$\frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

taking inverse laplace on both side we got,

$$\Rightarrow L^{-1}\left[\frac{s}{(s+1)(s+2)}\right] = L\left[\frac{-1}{s+1}\right] + 2L^{-1}\left[\frac{1}{s+2}\right]$$

$$= -e^{-t} + 2e^{-2t}$$

$\therefore$ now plugging back the inverse laplace term of each portion we got

$\phi(t) =$ State Transition Matrix

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & 2e^{-2t} - 2e^{-t} \\ e^{-t} - e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$