Computational Photography Assignment 3 - Canny Edge Detection

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1. Theory Question

1. Defining the 3×3 mean kernel

$$\text{Mean Kernel} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Now we slide the 3×3 kernel over the matrix and computing the mean for each window we get

$$\begin{bmatrix} 5.3 & 5.2 & 4.1 & 3.4 & 2.9 & 3.2 \\ 3.8 & 4.3 & 3.7 & 3.2 & 2.9 & 3.9 \\ 3.6 & 3.9 & 3.8 & 3.1 & 2.7 & 3.3 \\ 2.4 & 2.9 & 3.6 & 4.0 & 4.2 & 4.9 \\ 3.4 & 3.9 & 4.7 & 4.9 & 4.8 & 4.2 \\ 3.6 & 4.0 & 4.4 & 4.7 & 4.2 & 4.0 \end{bmatrix}$$

2. Generating a 5×5 Gaussian kernel with $\sigma = 2$ and normalize it so that its elements sum to one, we'll use the Gaussian function formula

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Where

- x and y are distances from the center of the kernel.
- σ is the standard deviation.

The normalized 5×5 Gaussian kernel for $\sigma = 2$ will be

$$\begin{bmatrix} 0.0232 & 0.0338 & 0.0383 & 0.0338 & 0.0232 \\ 0.0338 & 0.0492 & 0.0558 & 0.0492 & 0.0338 \\ 0.0383 & 0.0558 & 0.0632 & 0.0558 & 0.0383 \\ 0.0338 & 0.0492 & 0.0558 & 0.0492 & 0.0338 \\ 0.0232 & 0.0338 & 0.0383 & 0.0338 & 0.0232 \end{bmatrix}$$

3. Given

$$\frac{\partial}{\partial x} = \begin{bmatrix} -1/3 & 0 & 1/3 \\ -1/3 & 0 & 1/3 \\ -1/3 & 0 & 1/3 \end{bmatrix}, \quad \frac{\partial}{\partial y} = \begin{bmatrix} -1/3 & -1/3 & -1/3 \\ 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}, \quad I = \begin{bmatrix} 7 & 7 & 6 \\ 3 & 3 & 2 \\ 5 & 4 & 7 \end{bmatrix}$$

Convolving I with the kernels $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

Magnitude and direction of the gradient at the center pixel

$$G_x = 0$$

$$G_y = -\frac{4}{3}$$

$$|G| = \sqrt{{G_x}^2 + {G_y}^2} = \sqrt{-\frac{4}{3}^2} = \boxed{\frac{4}{3}}$$

$$\theta = arctan(\frac{-\frac{4}{3}}{0}) = \boxed{-\frac{\pi}{2}}$$

4. Given

$$|G| = \begin{bmatrix} 2 & 3 & 4 & 5 & 1 \\ 1 & 0 & 2 & 2 & 1 \\ 4 & 3 & 5 & 1 & 2 \\ 4 & 5 & 4 & 4 & 6 \\ 4 & 4 & 2 & 0 & 2 \\ 2 & 3 & 3 & 0 & 3 \end{bmatrix}$$

a) The binary image will be

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Where a value of 1 indicates an edge pixel, and a value of 0 indicates a non-edge pixel.

b) The binary images will be

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

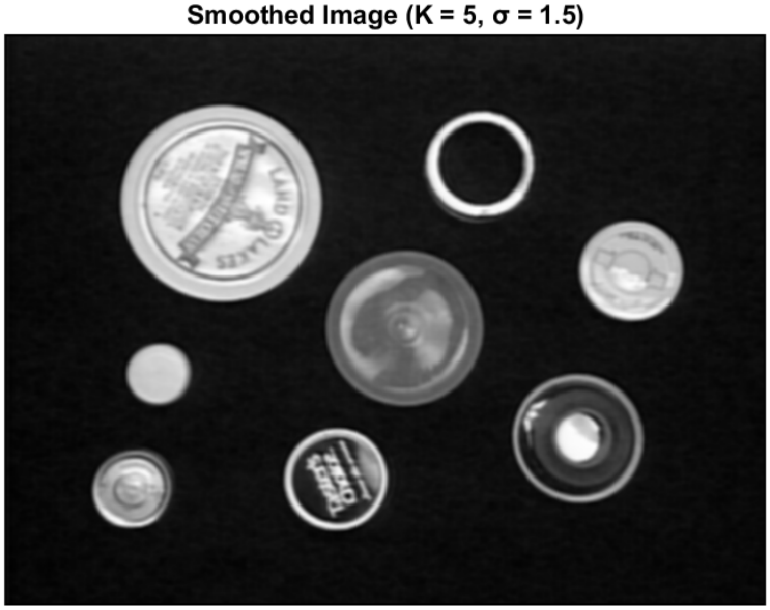
Where a value of 1 indicates an edge pixel, and a value of 0 indicates a non-edge pixel.

2. Guassian Smoothing

Original Grayscale Image

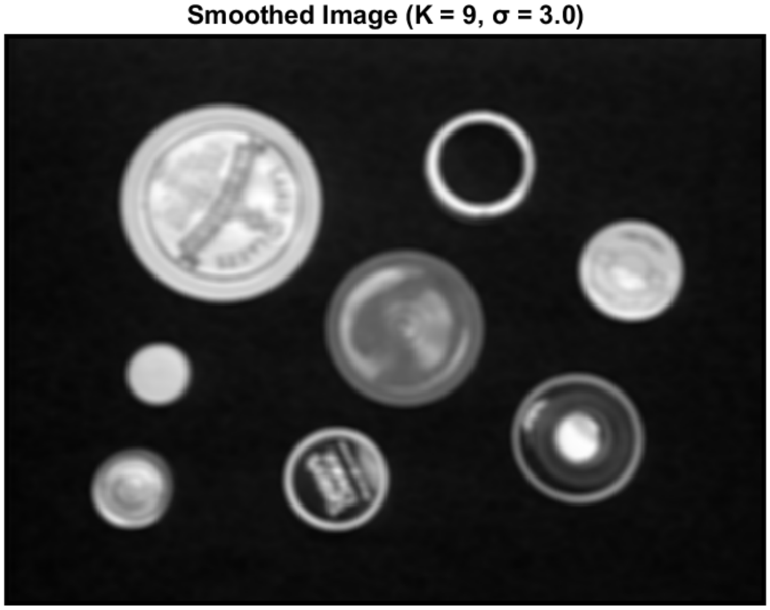
Smoothed Image (K = 3, σ = 1.0)

Smoothed Image (K = 5, σ = 1.5)



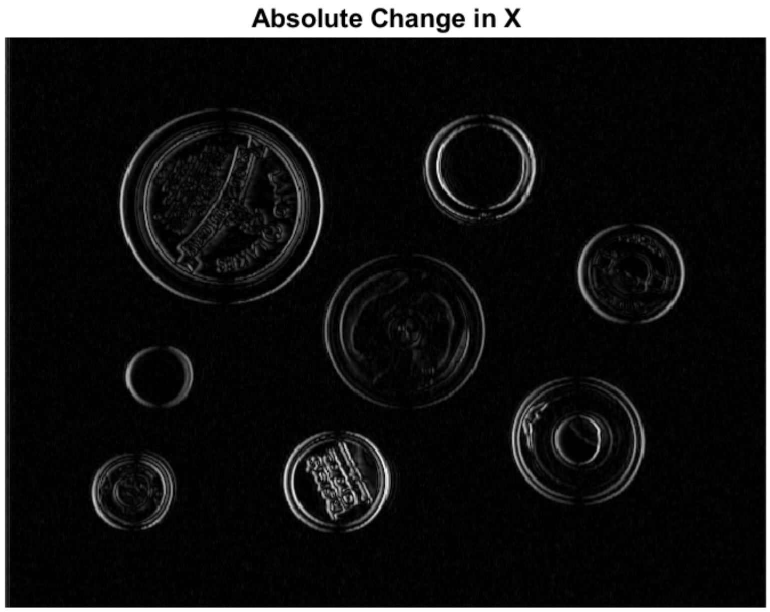
Smoothed Image (K = 7, σ = 2.0)

Smoothed Image (K = 9, σ = 3.0)

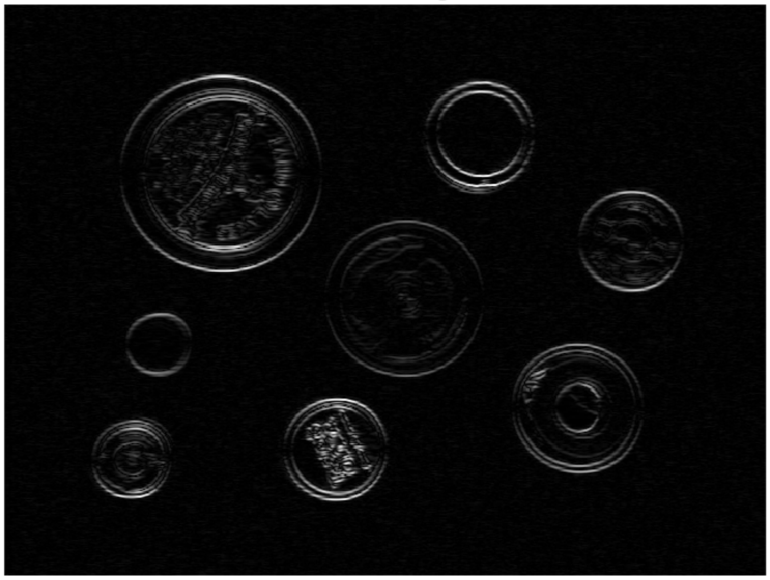


3. Gradients

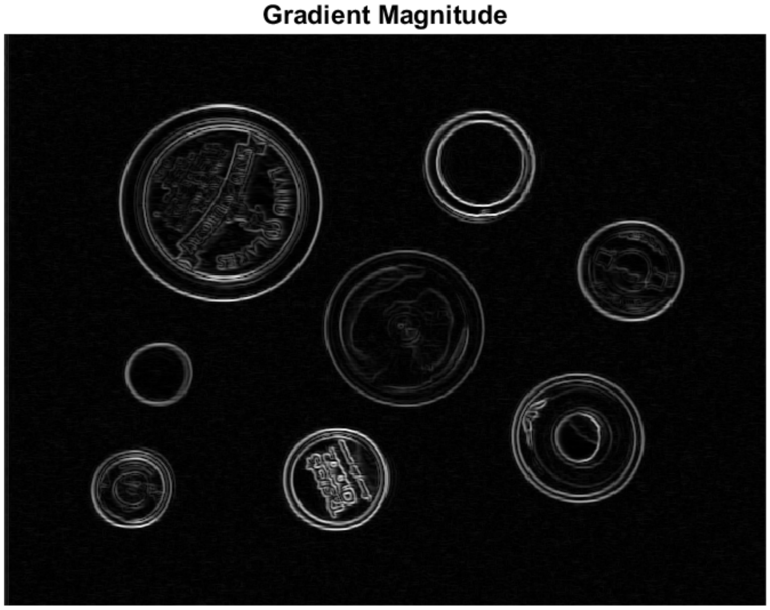
Kernel Used: Sobel Kernel



Absolute Change in Y

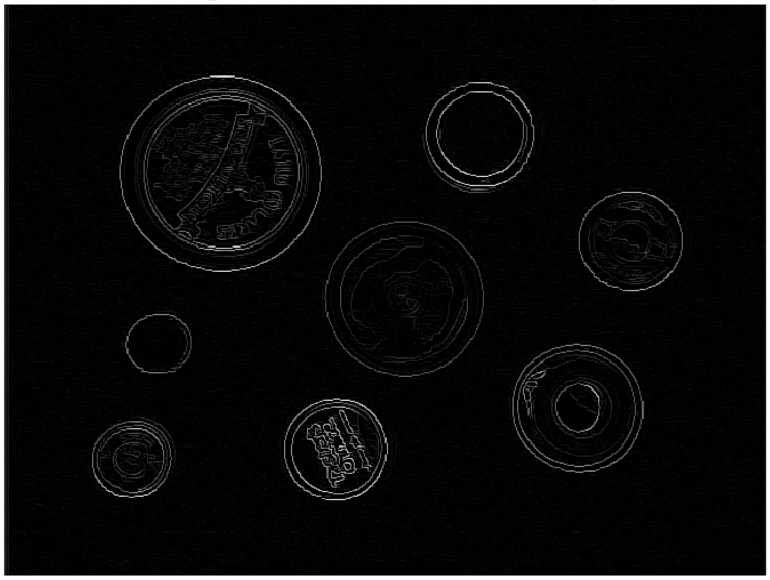


Gradient Magnitude



4. Non-maximum suppression

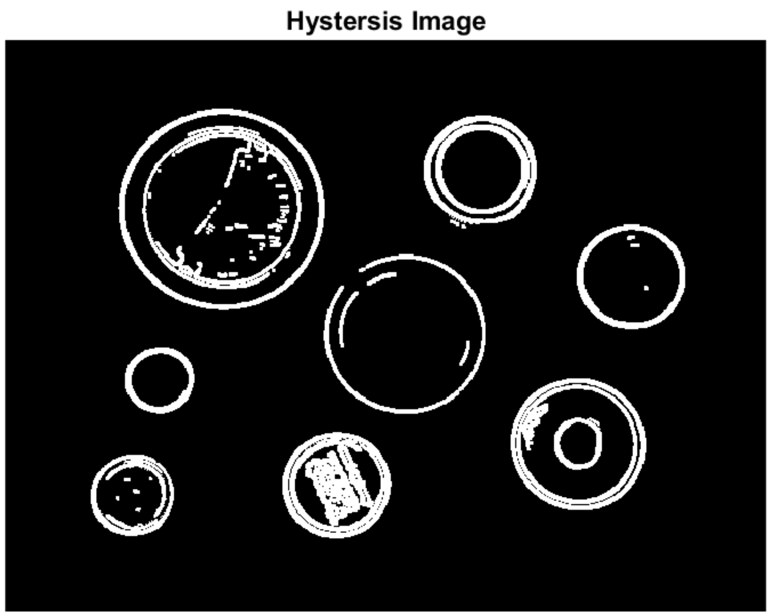
Gradient Magnitude after Non-Maximum Suppression



5. Hysterisis

Parameter Used:

- Low Threshold: 0.1 of Max Gradient Magnitude
- High Threshold: 0.3 of Max Gradient Magnitude
- Used standard division of 1 when creating Gaussian smoothing kernel

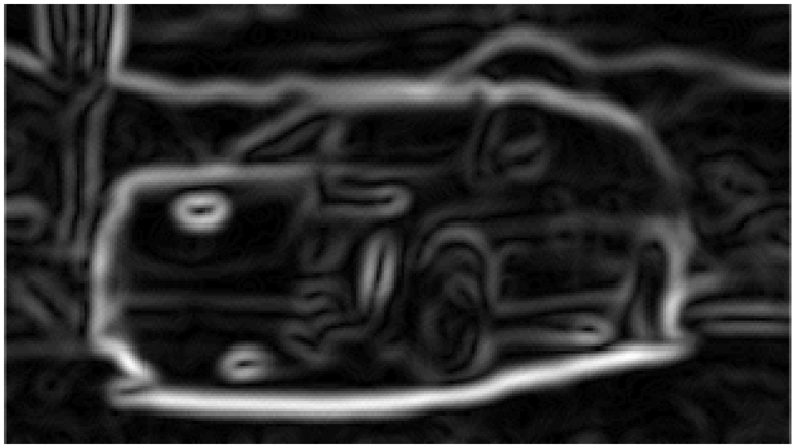


6. Test on another image

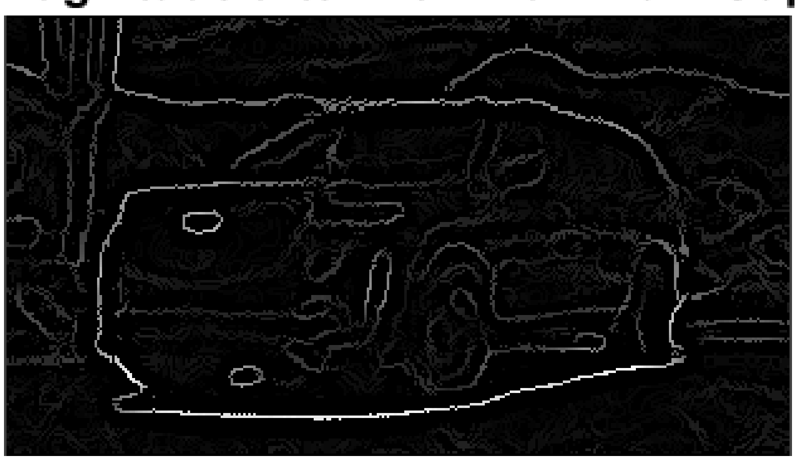
Grayscale Image

Smoothed Image

Gradient Magnitude



Gradient Magnitude after Non-Maximum Suppression



Edge Map after Double Thresholding

Final Edge Map