CS435 Assignment 6 - Classification

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05/08/2024

1. Theory Questions

Question 1

Given the following image pixel intensity values:

$$I = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Part (a)

Initial Reference Vectors:

$$a_1 = [1], \quad a_2 = [2]$$

Iteration 1: Cluster Assignment
Pixel $(1, 1)$:

- Distance to $a_1 = |1 - 1| = 0$
- Distance to $a_2 = |1 - 2| = 1$
- Closest to a_1

Pixel $(1, 2)$:

- Distance to $a_1 = |2 - 1| = 1$
- Distance to $a_2 = |2 - 2| = 0$
- Closest to a_2

Pixel $(1, 3)$:

- Distance to $a_1 = |0 - 1| = 1$
- Distance to $a_2 = |0 - 2| = 2$
- Closest to a_1

Pixel $(2, 1)$:

- Distance to $a_1 = |0 - 1| = 1$
- Distance to $a_2 = |0 - 2| = 2$
- Closest to a_1

Pixel $(2, 2)$:

- Distance to $a_1 = |1 - 1| = 0$
- Distance to $a_2 = |1 - 2| = 1$
- Closest to a_1

Pixel $(2, 3)$:

- Distance to $a_1 = |3 - 1| = 2$
- Distance to $a_2 = |3 - 2| = 1$
- Closest to a_2

Cluster Assignment:

$$I = \begin{bmatrix} a_1 & a_2 & a_1 \\ a_1 & a_1 & a_2 \end{bmatrix}$$

Updated Reference Vectors:

$$a_1 = \frac{1 + 0 + 0 + 1}{4} = 0.5, \quad a_2 = \frac{2 + 3}{2} = 2.5$$

Iteration 2: Cluster Assignment
Pixel $(1, 1)$:

- Distance to $a_1 = |1 - 0.5| = 0.5$

- Distance to $a_2 = |1 - 2.5| = 1.5$
- Closest to a_1

Pixel $(1, 2)$:

- Distance to $a_1 = |2 - 0.5| = 1.5$
- Distance to $a_2 = |2 - 2.5| = 0.5$
- Closest to a_2

Pixel $(1, 3)$:

- Distance to $a_1 = |0 - 0.5| = 0.5$
- Distance to $a_2 = |0 - 2.5| = 2.5$
- Closest to a_1

Pixel $(2, 1)$:

- Distance to $a_1 = |0 - 0.5| = 0.5$
- Distance to $a_2 = |0 - 2.5| = 2.5$
- Closest to a_1

Pixel $(2, 2)$:

- Distance to $a_1 = |1 - 0.5| = 0.5$
- Distance to $a_2 = |1 - 2.5| = 1.5$
- Closest to a_1

Pixel $(2, 3)$:

- Distance to $a_1 = |3 - 0.5| = 2.5$
- Distance to $a_2 = |3 - 2.5| = 0.5$
- Closest to a_2

Cluster Assignment (Unchanged):

$$I = \begin{bmatrix} a_1 & a_2 & a_1 \\ a_1 & a_1 & a_2 \end{bmatrix}$$

Final Reference Vectors (Unchanged):

$$a_1 = 0.5, \quad a_2 = 2.5$$

Since the cluster assignments did not change, the clustering is complete.

Part (b)

Computing the similarity/weight between pixels a and b as:

$$w(a, b) = e^{-((a_i - b_i)^2 + (a_x - b_x)^2 + (a_y - b_y)^2)}$$

The weight matrix W:

$$W = \begin{bmatrix} 0 & e^{-2} & e^{-5} & e^{-2} & e^{-2} & e^{-9} \\ e^{-2} & 0 & e^{-5} & e^{-6} & e^{-2} & e^{-3} \\ e^{-5} & e^{-5} & 0 & e^{-5} & e^{-3} & e^{-10} \\ e^{-2} & e^{-6} & e^{-5} & 0 & e^{-2} & e^{-13} \\ e^{-2} & e^{-2} & e^{-3} & e^{-2} & 0 & e^{-5} \\ e^{-9} & e^{-3} & e^{-10} & e^{-13} & e^{-5} & 0 \end{bmatrix}$$

Part (c)

$$D = \begin{bmatrix} 0.4129 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3297 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0700 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2799 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4625 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0567 \end{bmatrix}$$

Perform eigen-decomposition using $D - W$:

$$D - W = \begin{bmatrix} 0.4129 & e^{-2} & e^{-5} & e^{-2} & e^{-2} & e^{-9} \\ e^{-2} & 0.3297 & e^{-5} & e^{-6} & e^{-2} & e^{-3} \\ e^{-5} & e^{-5} & 0.0700 & e^{-5} & e^{-3} & e^{-10} \\ e^{-2} & e^{-6} & e^{-5} & 0.2799 & e^{-2} & e^{-13} \\ e^{-2} & e^{-2} & e^{-3} & e^{-2} & 0.4625 & e^{-5} \\ e^{-9} & e^{-3} & e^{-10} & e^{-13} & e^{-5} & 0.0567 \end{bmatrix}$$

Eigenvalues of L:

$$\text{Eigenvalues: } \begin{bmatrix} 0.5944 \\ 0.5574 \\ 0.3122 \\ 0.0920 \\ 0.0558 \\ 0.0000 \end{bmatrix}$$

Second Smallest Eigenvector:

$$\text{Second Smallest Eigenvector: } \begin{bmatrix} 0.0966 \\ -0.0319 \\ 0.5078 \\ 0.1445 \\ 0.1179 \\ -0.8349 \end{bmatrix}$$

Graph Cut:

$$\text{Graph Cut: } \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

2. Classifying an Image using Grayscale Histograms

Performing the K-NN classification using grayscale histograms on the dataset resulted in 226 correctly labeled images out of 350 images (for a seed of 0).

$$\text{Accuracy} = \left(\frac{226}{350}\right) \times 100\% = \boxed{64.57\%}$$

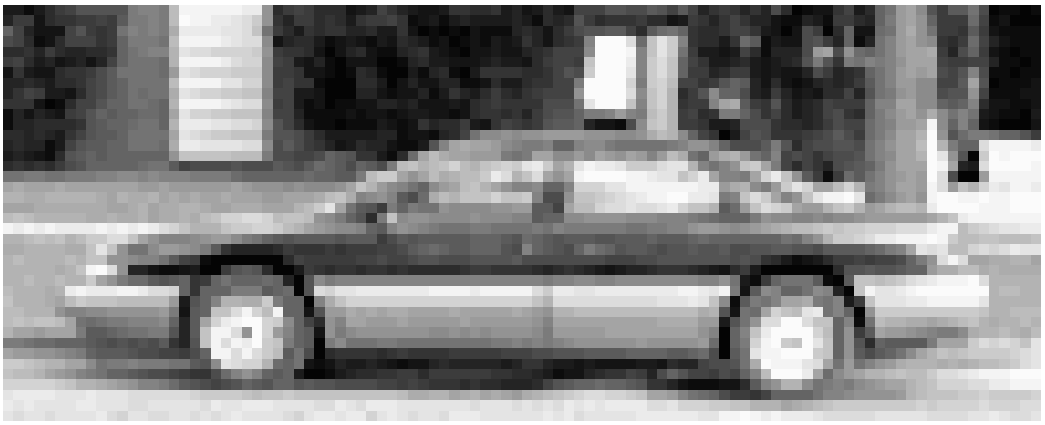

Figure 1: Image Correctly Labeled as a car

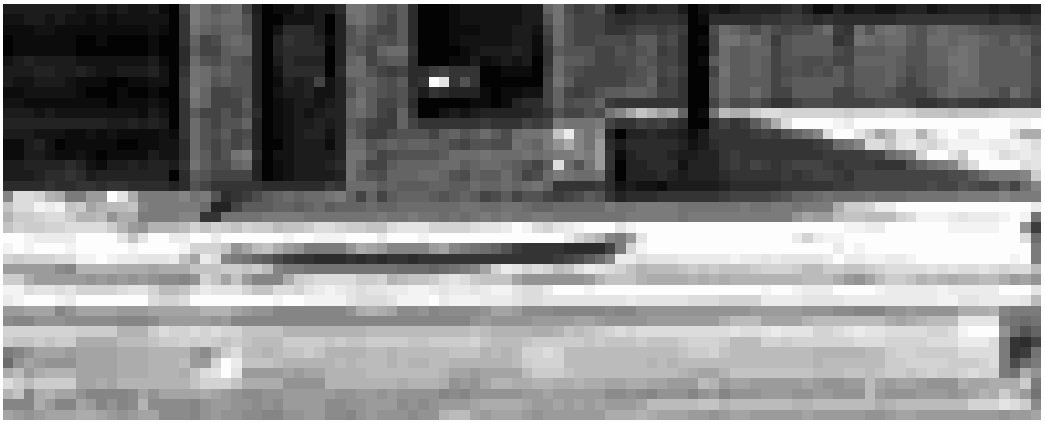

Figure 2: Image Correctly Labeled as a not a car

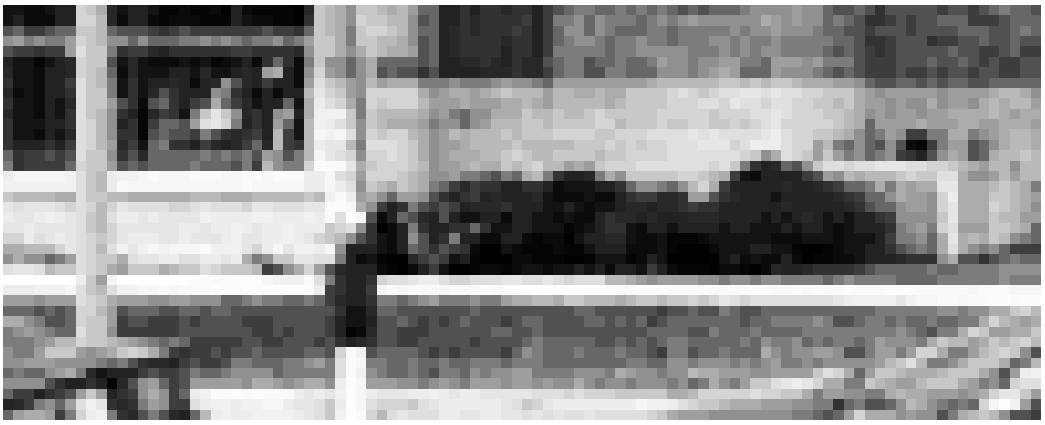

Figure 3: Image Incorrectly Labeled as a car

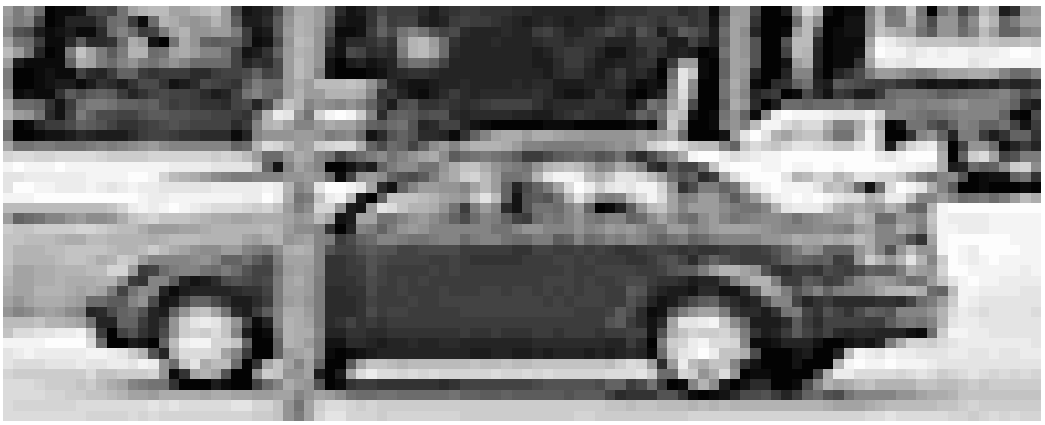

Figure 4: Image Incorrectly Labeled as a not a car

3. Classifying an Image using Gists

Performing the K-NN classification using HOGs on the dataset resulted in 310 correctly labeled images out of 350 images (for a seed of 0).

$$\text{Accuracy} = \left(\frac{306}{350}\right) \times 100\% = \boxed{87.43\%}$$

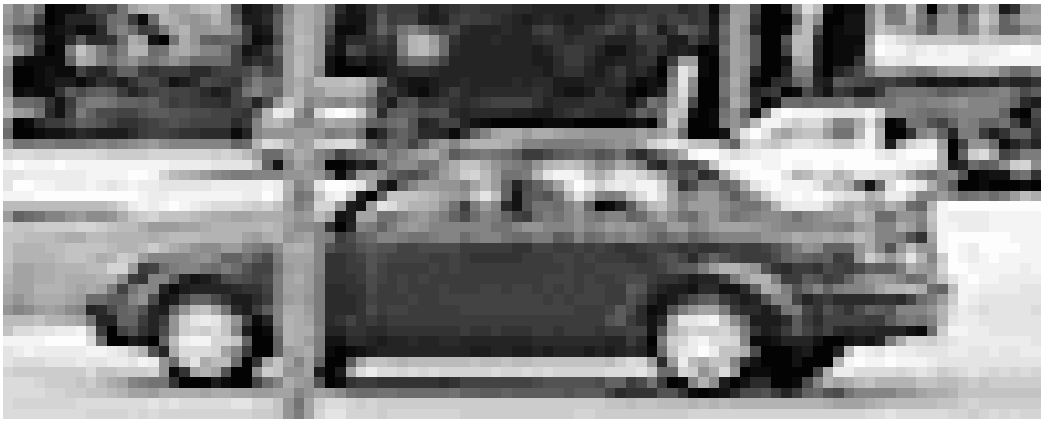

Figure 5: Image Correctly Labeled as a car

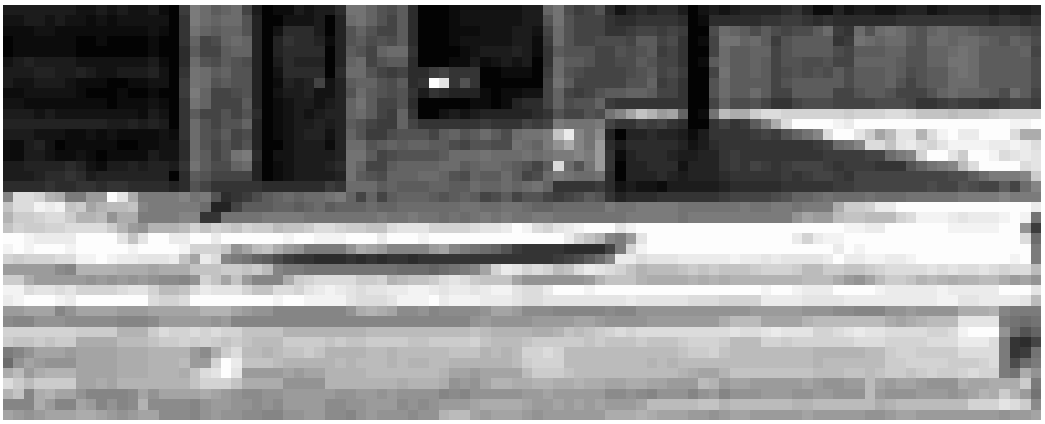

Figure 6: Image Correctly Labeled as a not a car

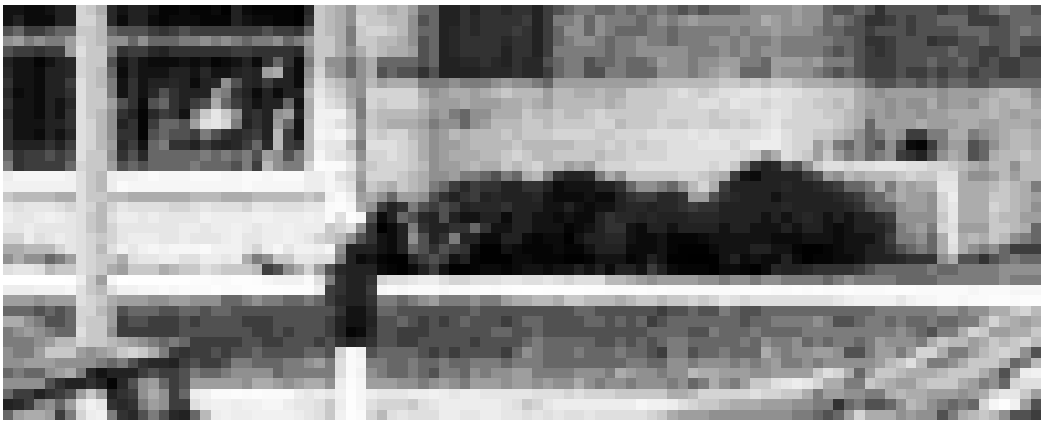

Figure 7: Image Incorrectly Labeled as a car

Figure 8: Image Incorrectly Labeled as a not a car