Final Project: Optimization of Anti-Reflective Coating for Enhanced Solar Panel Efficiency

ELG 3106

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Executive summary

The anti-reflective coating serves to enhance the efficiency of solar cells by reducing light reflection, enabling these cells to receive optimal light transmission. Broadband antireflective coatings, employing a design approach involving double-layer and triple-layer coatings, play a pivotal role in power production. The design strategy aims at reducing the reflection of incident electromagnetic waves, particularly at a central wavelength (λc = 650nm), to enhance power generation from solar cells. Notably, it was discovered that the maximization of power transfer into solar cells occurs when minimizing reflectance across the frequency spectrum, rather than solely focusing on reducing reflectance at the central wavelength, resulting in the most effective power production.

Introduction

Solar cells have emerged as a dependable and cost-effective source of renewable energy, playing a pivotal role in the global transition to cleaner energy sources. The increasing efficiency of solar cells in converting sunlight into electricity has contributed significantly to the growing economic viability of solar energy. An effective method in enhancing solar cell efficiency is the application of anti-reflective coating, which minimizes light reflection, ensuring maximum light transmission to the solar cells.

This design study aims to illustrate the impact of design restrictions on system response, specifically focusing on a solar cell equipped with either two or three layers of antireflection coating. The design approach determines the layers, refractive indices, and thicknesses, considering the idealized solar spectrum irradiance to provide the wavelength distribution of incident sunlight on the solar cells.

$$I(\lambda) = \frac{6.16 \times 10^{15}}{\lambda^5 \left(e^{2484/\lambda} - 1\right)}$$

Equation 1: Wavelength distribution of the incident sunlight on Solar Cells

The solar cell's response is characterized by electrical power production, expressed by an equation that accounts for the transmissivity of the anti-reflection coating. Despite the solar cell transforming all incident sunlight into electricity at 100% efficiency across all wavelengths, the presence of the antireflection coating introduces a wavelength-dependent transmissivity ($T(\lambda)$), affecting the electrical power production.

$$P = \int_{\lambda_1}^{\lambda_2} T(\lambda) I(\lambda) d\lambda$$

Equation 2: wavelength-dependent transmissivity

The research primarily aims to elucidate why the seemingly straightforward approach to antireflection coating design does not necessarily maximize solar cell power generation. The power is calculated by numerically integrating the given equation, employing the wavelength-dependent transmissivity obtained through the transfer matrix method (TMM) in a MATLAB script. This design study concentrates on double and triple-layer antireflection coatings, utilizing uniform plane waves to represent the impinging solar radiation on the solar cell system.

The study's overarching goal is to develop a multilayer antireflection coating with the highest achievable transmissivity over an extensive wavelength range. This pursuit results in a reflectivity versus wavelength curve that is both deep and wide, indicative of the broadest possible bandwidth and, consequently, optimal power generation.

Theory

Transfer Matrix Method (TMM)

The Image illustrates an arbitrary multilayer configuration, showcasing the electric field components at each interface with annotations. The orientation of the graphic is designed for convenience, with the incident wave moving to the right. Subscripts are used to represent the layer, while the + and – signs indicate forward and backward waves, respectively. Additionally, the prime notation signifies the difference between waves on the right and left sides of an interface.

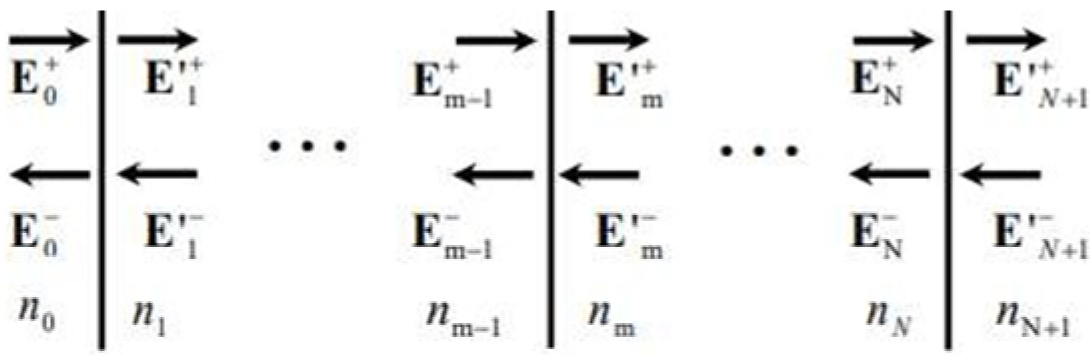


Image 1: Arbitrary Multilayer TMM Theory

Comprising N layers, excluding the unbounded media to the left (typically air), the multilayer structure is characterized by boundary conditions on the electric field vectors E on each side of an interface. This facilitates a simple representation of the mth interface through a 2x2 matrix which can be seen in Equation 3.

$$\begin{pmatrix} \mathbf{E}_{m-1}^{+} \\ \mathbf{E}_{m-1}^{-} \end{pmatrix} = Q_{m-1,m} \begin{pmatrix} \mathbf{E}_{m}^{\prime +} \\ \mathbf{E}_{m}^{\prime -} \end{pmatrix}$$

Equation 3: Boundary Conditions on the Electric Fields

Furthermore, $Q_{m-1,m}$, is the dynamical matrix in which relationships are denoted by the reflection and transmission coefficients. This can be seen below in Equation 4.

$$Q_{m-1,m} = \frac{1}{\tau_{m-1,m}} \begin{bmatrix} 1 & \Gamma_{m-1,m} \\ \Gamma_{m-1,m} & 1 \end{bmatrix}$$

Equation 4: Dynamical Matrix Equation

The propagation matrix serves to connect the field components on the left and right sides of the mth layer.

$$\begin{pmatrix} \mathbf{E}_m'^{+} \\ \mathbf{E}_m'^{-} \end{pmatrix} = P_m \begin{pmatrix} \mathbf{E}_m^{+} \\ \mathbf{E}_m^{-} \end{pmatrix}$$

Equation 5: Propagation Matrix Equation

P_m in the matrix is given by the following equation in Equation 5

$$P_m = \begin{bmatrix} \exp(j\delta_m) & 0 \\ 0 & \exp(-j\delta_m) \end{bmatrix}, \delta_m = \frac{2\pi}{\lambda} n_m d_m$$

Equation 6: P_m Equations Used

The aforementioned transformations are iteratively applied for the N layers and N+1 interfaces, yielding a product of (N+1) 2x2 matrices. This product effectively connects the total field in the left-hand unbounded media to that of the right-hand unbounded medium. Furthermore, below there are the reflection and transmission coefficients as well as the equation for lossless nonmagnetic mediums.Finally the reflective ad transmissivity can also be found below:

$$\begin{pmatrix} \mathbf{E}_0^{+} \\ \mathbf{E}_0^{-} \end{pmatrix} = \mathbf{T} \begin{pmatrix} \mathbf{E}_{\mathrm{N}+1}'^{+} \\ \mathbf{E}_{\mathrm{N}+1}'^{-} \end{pmatrix}$$

Equation 7: N+1 Boundary Conditions

$$\mathbf{T} = \begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix} = Q_{0,1} \prod_{\mathrm{m}=1}^{\mathrm{N}} P_{\mathrm{m}} Q_{\mathrm{m,m+1}}$$

Equation 8: T Variable for the 2 x 2 matrice

$$\Gamma = \frac{\mathbf{E}_{\mathrm{r}}}{\mathbf{E}_{\mathrm{i}}} = \frac{T_{2,1}}{T_{1,1}} \text{ and } \tau = \frac{\mathbf{E}_{\mathrm{t}}}{\mathbf{E}_{\mathrm{i}}} = \frac{1}{T_{1,1}}$$

Equation 9:Reflection and Transmission Coefficents

$$|\Gamma|^2 + |\tau|^2 \left(\frac{n_{N+1}}{n_0} \right) = 1,$$

Equation 10: Lossless, Nonmagnetic Medium

$$R = |\Gamma|^2 \quad \text{and} \quad T = |\tau|^2 \left(\frac{n_{N+1}}{n_0} \right).$$

Equation 11: Reflectivity and Transmissivity

Computational Theory

This section is written for the logical steps for the algorithms used in this project. The fully implemented code can be found in the Appendix as well as in the submission folder in an object-orient C file (.m) with the respective titles (Part2.m and Part4.m)

Code Flow Bullet Points for Part 2

- Begin the program.

Part 1: Reflection Spectrum Calculation:

- Initialize refractive indices and layer properties.
- Calculate reflection coefficients (RI_1, RI_2, RI_3).
- Set the central wavelength and calculate layer wavelengths (WL_1, WL_2, d_1, d_2).
- Initialize loop parameters and data array.
- Enter a loop for each wavelength point:
    - Calculate phase shifts (delta_1, delta_2).
    - Compute transfer matrices (M_11, M_12, M_21, M_22).
    - Calculate transmission and reflection coefficients (tao_21, tao_11, gamma_sys, R).
    - Store reflection percentage in the data array.
    - Increment loop variables.

- Plot the reflection spectrum.

Part 2: Power Calculation and Plotting:
- Initialize arrays for wavelength, reflectivity, transmissivity, power, and total power.
- Define material properties and coefficients.
- Enter a loop for each wavelength point:
    - Calculate transmission and reflection coefficients.
    - Calculate power and accumulate total power.
    - Increment loop variables.
- Plot the power spectrum.

Part 3: Refractive Index Variation and Power Calculation:
- Set central wavelength and wavelength range.
- Initialize variables for reflection and transmission coefficients.
- Enter a loop for each refractive index value:
    - Calculate transmission and reflection coefficients for each layer.
    - Enter a nested loop for each wavelength point:
        - Calculate phase shifts and transmission matrix.
        - Calculate reflection coefficient and power.
        - Accumulate power.
        - Increment loop variables.
    - Store total power for the current refractive index.
    - Increment refractive index.
- Plot power vs. refractive indices.
- Compare Reflectivity for Different Refractive Indices:
- Initialize arrays for reflectivity.
- Enter a loop for each wavelength point:
    - Calculate reflection coefficient for different refractive indices.
    - Store reflectivity values.
    - Increment loop variables.
- Plot reflectivity vs. wavelength for different refractive indices.
- End the program.

Code Flow Bullet Points for Part 4

- Start
- Calculate reflection coefficients (RI_1, RI_2, RI_3)
- Set central wavelength and calculate layer wavelengths (WL_1, WL_2, d_1, d_2)

Reflection Spectrum
- Enter a loop for each wavelength point
    - Calculate phase shifts (delta_1, delta_2)
    - Compute transfer matrices (M_11, M_12, M_21, M_22)
    - Calculate transmission and reflection coefficients (tao_21, tao_11, gamma_sys, R)
    - Store reflection percentage in the data array

    - Increment loop variables
- Plot the reflection spectrum

Power Calculation and Plotting
- Enter a loop for each wavelength point
    - Calculate transmission and reflection coefficients
    - Calculate power and accumulate total power
    - Increment loop variables
- Plot the power spectrum

Refractive Index Variation and Power Calculation
- Enter a loop for each refractive index value
    - Calculate transmission and reflection coefficients for each layer
    - Enter a nested loop for each wavelength point
        - Calculate phase shifts and transmission matrix
        - Calculate reflection coefficient and power
        - Accumulate power
        - Increment loop variables
    - Store total power for the current refractive index
    - Increment refractive index
- Plot Power vs. Refractive Indices

Compare Reflectivity for Different Refractive Indices
- Enter a loop for each wavelength point
    - Calculate reflection coefficient for different refractive indices
    - Store reflectivity values
    - Increment loop variables
- Plot reflectivity vs. wavelength for different refractive indices
- End

Computation Part 1

Calculating Center Wavelength Reflectivity and Power Transmission Without Anti-Reflective Layer.

Basic Information to Note: $n_0 = 1$, $n_{cell} = 3.5$, λ_c= 650 nm with no anti-reflective coating.

Initially, Begin by solving for the reflective coefficient Γ:

$$\Gamma = \frac{n_{\text{cell}} - n_o}{n_{\text{cell}} + n_o}$$

$$\Gamma = \frac{3.5 - 1}{3.5 + 1}$$

$$\Gamma = 0.56$$

$R=|\Gamma|^2 = 0.3086$

The formula for power transmitted is provided in equation 1, where I(λ) is defined by equation 2. For this specific inquiry, the task is to determine only the reflectivity at the center wavelength. Therefore, substituting 650 into the irradiance and finding the transmissivity and center wavelength is sufficient, and there's no need for the integral since the power transmitted is evaluated at a singular point, leading to its cancellation.

$$P = \frac{(1 - 0.31)\left(6.16 \times 10^{15}\right)}{(650)^5\left(e^{\frac{2484}{650}} - 1\right)}$$

$$P = 0.82\ \mathrm{W/m^2}$$

Double Layer Coating TTM Approach

The chosen method for analysis is the Transfer Matrix Method (TTM), an analytical approach. This method utilizes the refractive indices of the anti-reflective coating to calculate the transmissivity and reflectivity of multiple-layer anti-reflective coating systems.

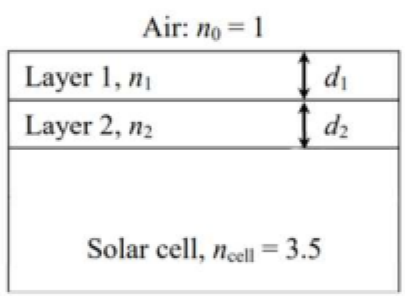


Figure 1: Double Layer Anti Reflective Coating

Some information to note from the Figure 1 d_n=¼ λm is the physical thickness of the material. $\lambda_n = \frac{\lambda o}{\lambda n}$ which is the wavelength, and finally $\delta n = \frac{2\xi}{\lambda o} = \frac{\pi}{2}$

For solving T, there are 3 dynamical matrices assuming the variable Q which is equation 4 and 2 Propagation Matricies P which is equation 6.

This gets the transfer matrix to be $T=Q_{01}P_1Q_{12}P_2Q_{23}$

The Algorithm for Part 2 is as Follows

Analytical Approach of Double Layer at Central Wavelength

$0= \Gamma 01 - \Gamma 12 + (1 - \Gamma 01 \Gamma 12)\Gamma 23$

$\Gamma 01=\frac{n0-n1}{n0+n1}$

$\Gamma 12=\frac{n1-n2}{n1+n2}$

$\Gamma 23=\frac{n2-n3}{n2+n3}$

Simplifies down to $non2^2 = n1^2 n3$

n3=ncell=3.5

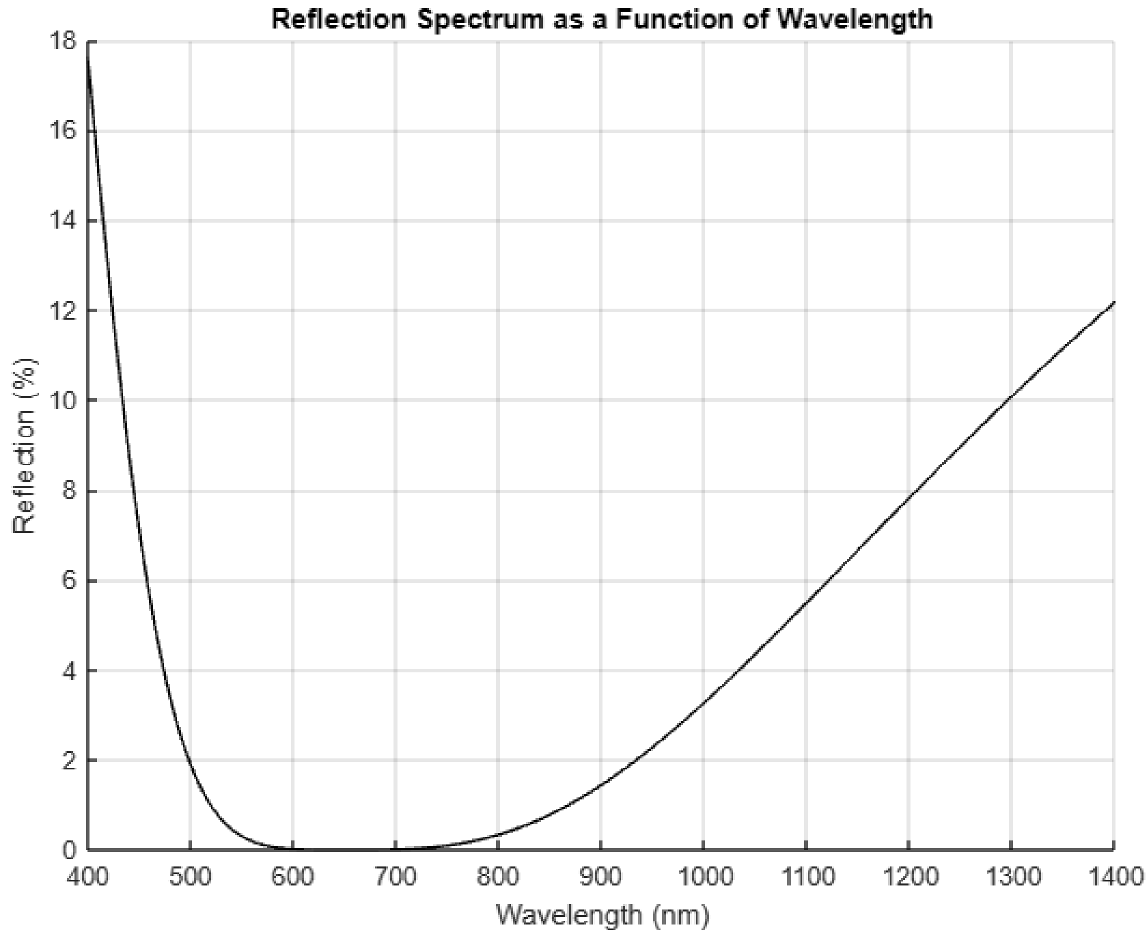
n1=1.4

n0=1

$n2=\sqrt{\frac{n1^2 n3}{n0}} = 2.62$

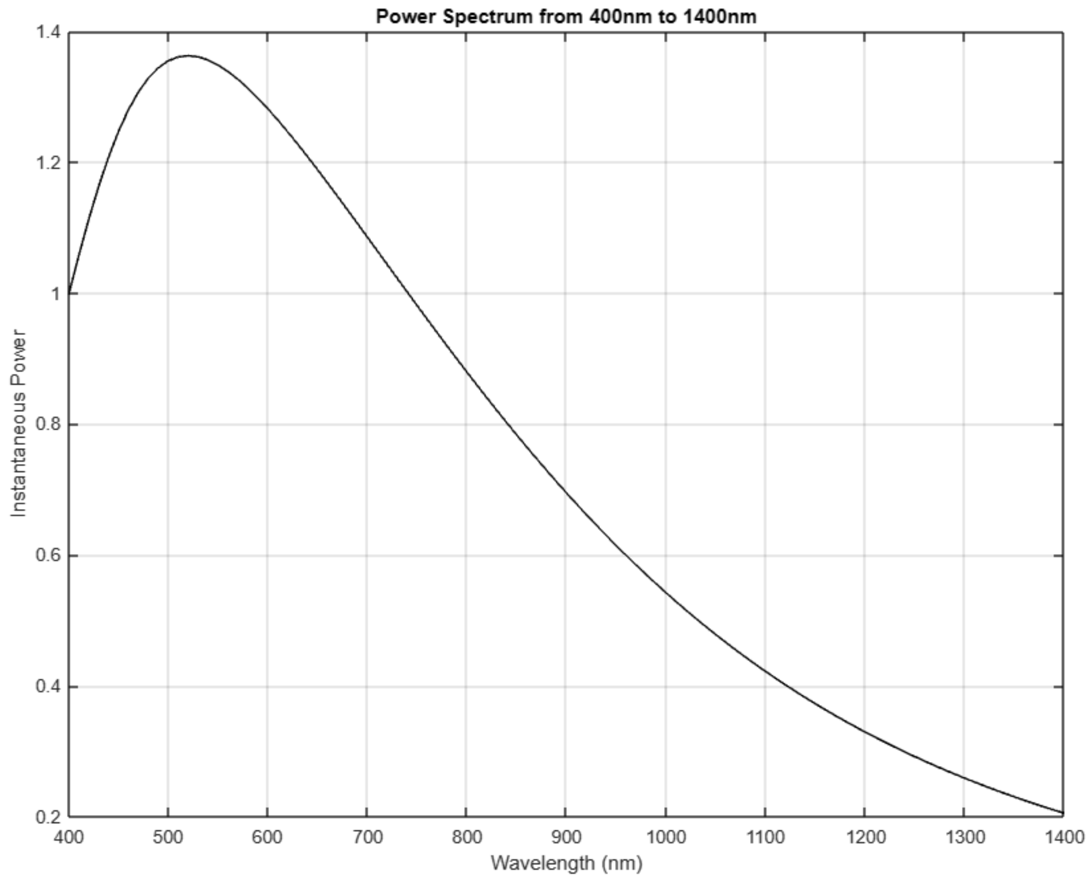
Computation Part 2

Reflectivity as a function of wavelength



Graph 1: Reflectivity as Function Wavelength, The Centre Wavelength the Reflectivity is 0.

Power is transmitted to a solar cell in various wavelengths



Graph 2: Power Spectrum for λ=400 and λ=1400

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The total found power from 400nm to 1400nm is 756.5623
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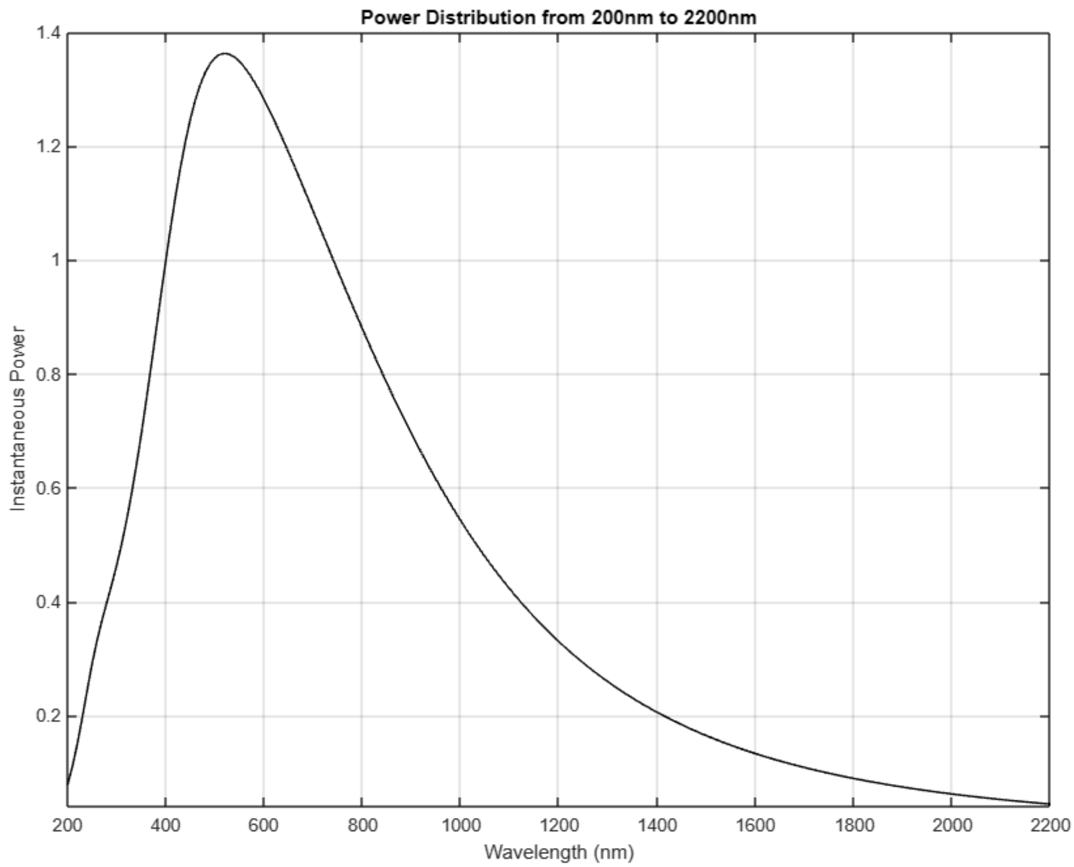
Figure 3: Total Power from 400 nm to 1400nm



Graph 3: Power Spectrum for λ=200 and λ=2200

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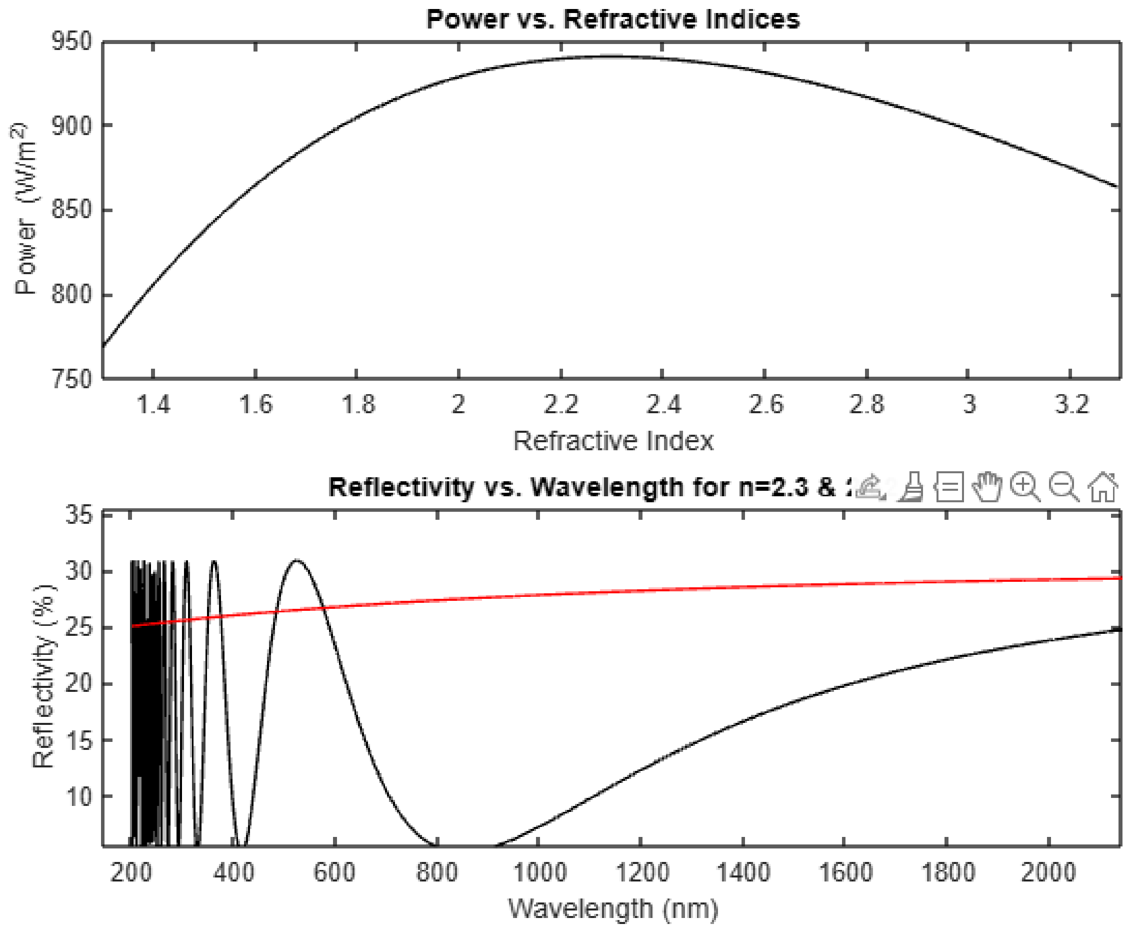
The total found power from 200nm to 2200nm is 930.1121
```

Figure 4: Total Power from 200nm to 2200nm

Power in Various Refractive Indices

A plot was generated depicting the total power generated at various refractive indices for n2. It was determined that a value of 2.3 yielded the optimal power output. This value resulted in approximately 939.673 W/m^2, surpassing the output at a refractive index of 2.62, which measured around 928.71 W/m^2.



Graphs 4 and 5: Total Power For Refractive Indices and Reflectivity Compared to Wavelength

The above graph clearly illustrates why a refractive index of n2 = 2.3 yields a higher output (The Red Line). The overall reflectivity percentage is lower for this refractive index compared to when n2 is equal to 2.62. The only instance where the reflectivity percentage is higher consistently in the smaller wavelengths until around 800 nm where it begins to slowly rise.

Computation Part 3

Analyzation of a three-layer anti-reflective coating

$$Q = \frac{1}{\tau_{01}\tau_{12}\tau_{23}\tau_{34}} \begin{bmatrix} 1 & \Gamma_{01} \\ \Gamma_{01} & 1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{12} \\ \Gamma_{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{23} \\ \Gamma_{23} & 1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{34} \\ \Gamma_{34} & 1 \end{bmatrix}$$

$$P_m = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$T = \frac{(1)(1)(1)}{\tau_{01}\tau_{12}\tau_{23}\tau_{34}} \begin{bmatrix} 1 & \Gamma_{01} \\ \Gamma_{01} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{12} \\ \Gamma_{12} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{23} \\ \Gamma_{23} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{34} \\ \Gamma_{34} & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & -\Gamma_{01} \\ \Gamma_{01} & -1 \end{bmatrix} \begin{bmatrix} 1 & -\Gamma_{12} \\ \Gamma_{12} & -1 \end{bmatrix} \begin{bmatrix} 1 & -\Gamma_{23} \\ \Gamma_{23} & -1 \end{bmatrix} \begin{bmatrix} 1 & \Gamma_{34} \\ \Gamma_{34} & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} (1-\Gamma_{01}\Gamma_{12}) & (\Gamma_{01}-\Gamma_{12}) \\ (\Gamma_{01}-\Gamma_{12}) & (1-\Gamma_{01}\Gamma_{12}) \end{bmatrix} \begin{bmatrix} (1-\Gamma_{23}\Gamma_{34}) & (\Gamma_{34}-\Gamma_{23}) \\ (\Gamma_{23}-\Gamma_{34}) & (-1+\Gamma_{23}\Gamma_{34}) \end{bmatrix}$$

$$T = \begin{cases} T_{11} = [(1-\Gamma_{01}\Gamma_{12})(1-\Gamma_{23}\Gamma_{34}) + (\Gamma_{01}-\Gamma_{12})(\Gamma_{23}-\Gamma_{34})] \\ T_{12} = [(1-\Gamma_{01}\Gamma_{12})(\Gamma_{34}-\Gamma_{23}) + (\Gamma_{01}-\Gamma_{12})(-1+\Gamma_{23}\Gamma_{34})] \\ T_{21} = [(\Gamma_{01}-\Gamma_{12})(1-\Gamma_{23}\Gamma_{34}) + (1-\Gamma_{01}\Gamma_{12})(\Gamma_{23}-\Gamma_{34})] \\ T_{22} = [(\Gamma_{01}-\Gamma_{12})(\Gamma_{34}-\Gamma_{23}) + (1-\Gamma_{01}\Gamma_{12})(-1+\Gamma_{23}\Gamma_{34})] \end{cases}$$

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \qquad \Gamma_{sys} = \frac{T_{21}}{T_{11}} \qquad \text{Reflectivity} = |\Gamma_{sys}|^2$$

$$|\Gamma_{sys}|^2 = \left| \frac{(\Gamma_{01} - \Gamma_{01}\Gamma_{23}\Gamma_{34} - \Gamma_{12} + \Gamma_{12}\Gamma_{23}\Gamma_{34}) + (\Gamma_{23} - \Gamma_{34} - \Gamma_{01}\Gamma_{12}\Gamma_{23} + \Gamma_{01}\Gamma_{12}\Gamma_{34})}{(1 - \Gamma_{23}\Gamma_{34} - \Gamma_{01}\Gamma_{12} + \Gamma_{01}\Gamma_{12}\Gamma_{23}\Gamma_{34}) + (\Gamma_{01}\Gamma_{23} - \Gamma_{01}\Gamma_{34} - \Gamma_{12}\Gamma_{23} + \Gamma_{12}\Gamma_{34})} \right|^2$$

Refractive indices and minimization of the reflectivity

As $T=Q_{01}P_1Q_{12}P_2Q_{23}P_3Q_{34}$ assume its equivalence, below hand written

Minimal Reflectivity $\Gamma_{sys}=0$

$$0=\Gamma_{01}$$

$$0=\Gamma_{01}-\Gamma_{01}\Gamma_{23}\Gamma_{34}-\Gamma_{12}+\Gamma_{12}\Gamma_{23}\Gamma_{34}+\Gamma_{23}-\Gamma_{34}-\Gamma_{01}\Gamma_{12}\Gamma_{23}+\Gamma_{01}\Gamma_{12}\Gamma_{34}$$

$$0=\left(\frac{n_0-n_1}{n_{01}+n_1}\right)-\left[\left(\frac{n_0-n_1}{n_0+n_1}\right)\left(\frac{n_2-n_3}{n_2+n_3}\right)\left(\frac{n_3-n_4}{n_3+n_4}\right)\right]-\frac{n_1-n_2}{n_1+n_2}+\left(\frac{n_1-n_2}{n_1+n_2}\right)\left(\frac{n_2-n_3}{n_2+n_3}\right)\left(\frac{n_3-n_4}{n_3+n_4}\right)$$

$$+\frac{n_2-n_3}{n_2+n_3}-\frac{n_3-n_4}{n_3+n_4}-\left[\left(\frac{n_0-n_1}{n_0+n_1}\right)\left(\frac{n_1-n_2}{n_1+n_2}\right)\left(\frac{n_2-n_3}{n_2+n_3}\right)\right]+\left[\left(\frac{n_{01}-n_2}{n_0+n_{02}}\right)\left(\frac{n_1-n_2}{n_1+n_2}\right)\left(\frac{n_3-n_4}{n_3+n_4}\right)\right]$$

$$0=(n_0-n_1)(n_1+n_2)(n_2+n_3)(n_3+n_4)-(n_0-n_1)(n_1+n_2)(n_2-n_3)(n_3-n_4)-(n_0+n_1)(n_1-n_2)(n_2+n_3)(n_3+n_4)$$
$$+(n_0+n_1)(n_1-n_2)(n_2-n_3)(n_3-n_4)+(n_0+n_1)(n_1+n_2)(n_2-n_3)(n_3+n_4)-(n_0+n_1)(n_1+n_2)(n_2+n_3)(n_3-n_4)$$
$$-(n_0-n_1)(n_1-n_2)(n_2-n_3)(n_3+n_4)+(n_0-n_1)(n_1-n_2)(n_2+n_3)(n_3-n_4)$$

$$0=2(n_0n_2-n_1^2)(n_2n_3+n_2n_4+n_3^2+n_3n_4)-2(n_0n_2-n_1^2)(n_2n_3-n_2n_4-n_3+n_3n_4)+2(n_0n_2+n_1^2)(n_2n_3+n_2n_4-n_3^2-n_3n_4)-$$
$$2(n_0n_2+n_1^2)(n_2n_3-n_2n_4+n_3^2-n_3n_4)$$

$$0=4(n_0n_2-n_1^2)(n_2n_4+n_3^2)+\left(4(n_0n_2+n_1^2)(n_2n_4-n_3^2)\right)$$

$$0=2n_0n_2^2-2n_1^2n_3^2$$

$$n_4n_2^2n_0=n_1^2n_3^2$$

$$\boxed{n_2=\frac{n_1n_3}{\sqrt{n_0n_4}}}$$

Therefore $N_2=\frac{n1n3}{\sqrt{n0n4}}$

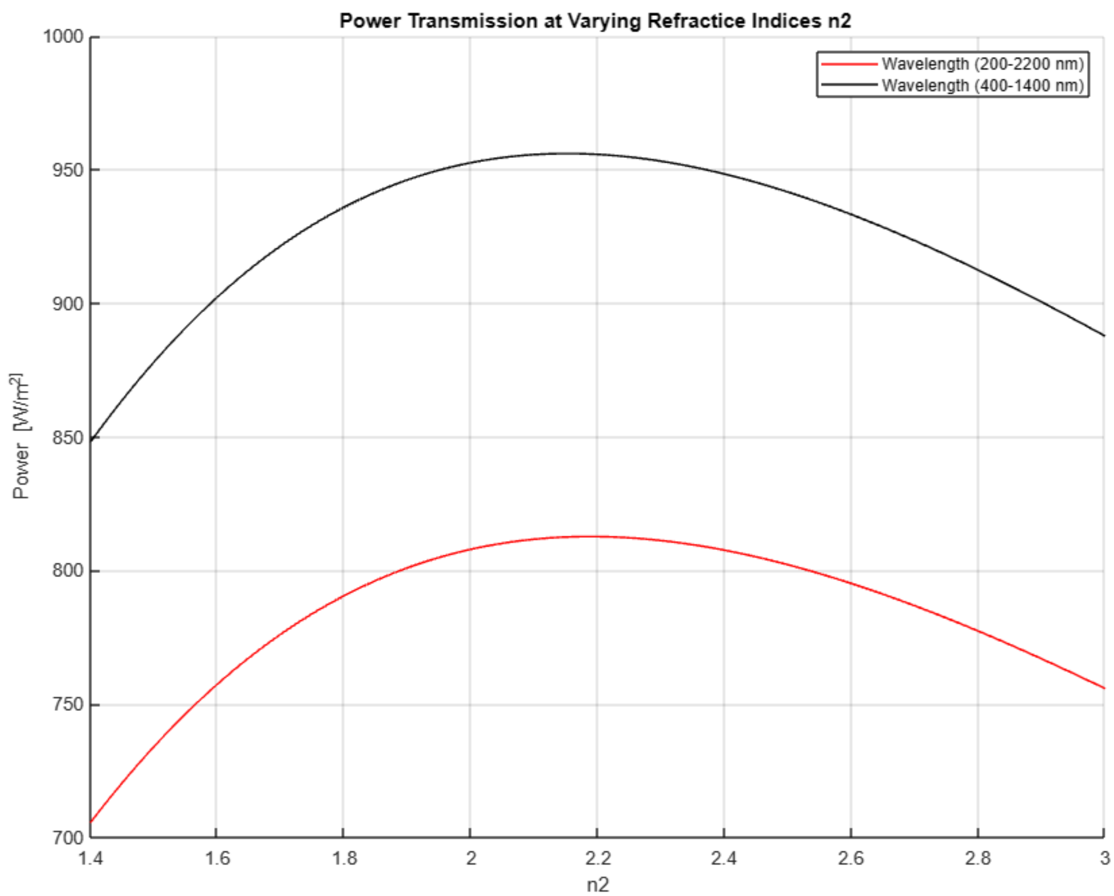
If n1 = 1.4 and n3 = 3.15, what is the value of n2

Since n0=1n1=1.4 n2=3.15 n3=3.5

$N_2=\frac{n1n3}{\sqrt{n0n4}}=\frac{(1.4)(3.15)}{\sqrt{(1)(3.5)}}=2.36$ meaning it will be the the most optimal refractive index to minimize reflectivity

Computation Part 4

Analyzation of a three-layer anti-reflective coating



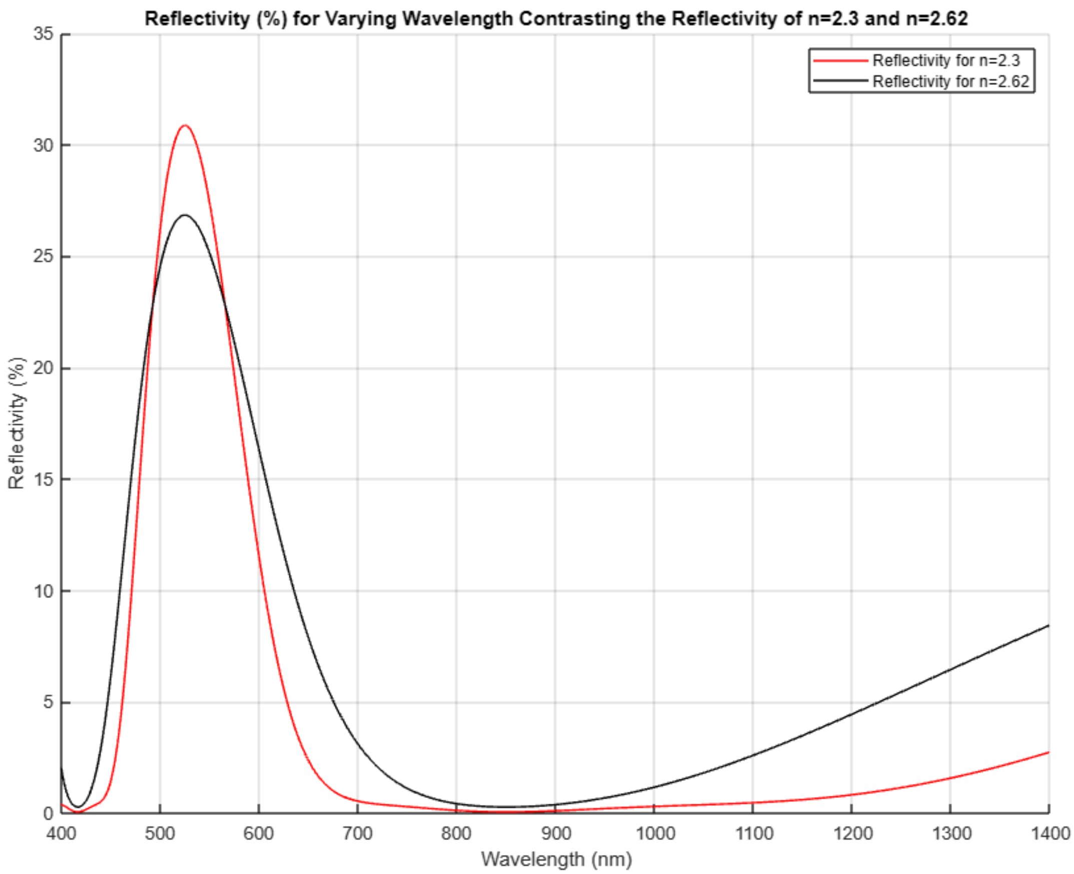
Graph 6: To Optimise Power, Plotting Power as a Function of Refractive Index

Maximum Power generated from a refractive index

The refractive index of $n2 = 2.15$ has been identified as optimal for maximizing the transmitted power to the solar cell, as depicted in Graph 6. In this context, it is observed that this specific refractive index, rather than the refractive index at the center wavelength, facilitates maximum power transmission to the solar cell.

- Centre Wavelength Refractive Index
  - P = $750W/m2$ 400nm < λ < 1400nm
  - P = $950W/m$ 2200nm < λ < 2200nm
- Maximum Transmission Refractive Index
  - P = $766W/m2$ 400nm < λ < 1400n
  - P = $966W/m2$ 200nm < λ < 2200nm

Comparison



Graph 7: Reflectivity as Function Wavelength at Different Refractive Indices

The analysis of the Total Power for Refractive Index $n2 = 2.36$ reveals that a refractive index of $n2 = 2.15$ achieves greater power transmission compared to a refractive index at the center wavelength of $n2 = 2.4$. As illustrated in graph 6, $n2 = 2.15$ transmits more power than $n2 = 2.4$ at the center wavelength. Graph 7 further illustrates this phenomenon, showing that the curve for $n2 = 2.4$ is deep (i.e., low reflectivity) at the center wavelength, while the plot for $n2 = 2.15$ has a larger bandwidth, encompassing the full spectrum. Additionally, with $n2 = 2.15$, reflectivity remains consistently lower across the bandwidth (i.e., higher transmission) of the frequency, except at the center wavelength.

This observation explains why $n2 = 2.15$ produces more power. The reflectance of the entire solar spectrum is lower at this index of refraction, resulting in a higher overall transmissivity and, consequently, greater power output in the solar cell. It emphasizes that minimizing reflectivity at the center wavelength ($\Gamma = 0$) does not always guarantee maximum power; instead, considering the entire solar spectrum proves crucial.

Conclusion

The study focused on double-layer and triple-layer anti-reflective coatings, with an emphasis on the center wavelength where reflectivity is zero ($\Gamma = 0$), and refractive indices were determined from this center wavelength. The Transfer Matrix Method (TMM) was employed to ascertain the refractive index of the second layer needed to achieve zero reflection at the center wavelength (λc = 650nm). The transmitted power was then plotted against the refractive index of the second layer to identify the value maximizing power. Interestingly, it was observed that the refractive index for maximum power differed from that obtained by minimizing center wavelength reflectivity. This finding aligns with the prediction that optimizing reflectivity for a single frequency is insufficient, considering solar cell power derives from a variety of light frequencies.

Additionally, the study revealed that increasing bandwidth enhances transmitted power across the spectrum with each added layer. A single anti-reflection layer, having a limited bandwidth, would only transmit power over a narrow frequency range. In contrast, double-layer and triple-layer anti-reflective coatings maximize transmissivity over a broader spectrum.

In summary, fixing the refractive index at the center wavelength is not a guarantee for achieving maximum power in solar cells, although it serves as a close approximation. The reflectivity graph must exhibit a wide bandwidth and depth across the entire frequency spectrum to achieve optimal power transfer into cells.

Appendix

Part2.m

```
% Part 1: Reflection Spectrum Calculation
n_air = 1;
n_layer1 = 1.4;
n_layer2 = 2.62;
n_layer3 = 3.5;
RI_1 = (n_air - n_layer1) / (n_air + n_layer1);
RI_2 = (n_layer1 - n_layer2) / (n_layer1 + n_layer2);
RI_3 = (n_layer2 - n_layer3) / (n_layer2 + n_layer3);
wavelength_c = 650;
WL_1 = wavelength_c / n_layer1;
WL_2 = wavelength_c / n_layer2;
d_1 = WL_1 / 4;
d_2 = WL_2 / 4;

n_points = 1001;
```

```
data = zeros(1, n_points);
L_val = 400;

for k_val = 1:n_points
    delta_1 = (2 * pi * n_layer1 * d_1) / L_val;
    delta_2 = (2 * pi * n_layer2 * d_2) / L_val;

    M_11 = exp(1i * delta_1);
    M_12 = exp(1i * delta_2);
    M_21 = exp(-1i * delta_1);
    M_22 = exp(-1i * delta_2);

    tao_21 = M_12 * (M_11 * RI_1 + M_21 * RI_2) + M_22 * RI_3 * (M_11 *
RI_1 * RI_2 + M_21);
    tao_11 = M_12 * (M_11 + M_21 * RI_1 * RI_2) + M_22 * RI_3 * (M_11 *
RI_2 + M_21 * RI_1);
    gamma_sys = tao_21 / tao_11;
    R = 100 * (abs(gamma_sys) * abs(gamma_sys));
    data(1, k_val) = R;
    L_val = L_val + 1;
end

% Plotting the Reflection Spectrum
figure;
hold on;
title('Reflection Spectrum as a Function of Wavelength');
xlabel('Wavelength (nm)');
ylabel('Reflection (%)');
grid on;
plot(400:1:1400, data,'k');

% Part 2: Power Calculation and Plotting
wavelength_array = 400:10:1400;
reflectivity_array = zeros(1, length(wavelength_array));
transmissivity_array = zeros(1, length(wavelength_array));
power_array = zeros(1, length(wavelength_array));
total_power = 0;

% Material properties
ref_coeff_air_layer1 = ref_coefficient(n_air, n_layer1);
```

```
ref_coeff_layer1_layer2 = ref_coefficient(n_layer1, n_layer2);
ref_coeff_layer2_cell = ref_coefficient(n_layer2, n_cell);

trans_coeff_air_layer1 = trans_coefficient(n_air, n_layer1);
trans_coeff_layer1_layer2 = trans_coefficient(n_layer1, n_layer2);
trans_coeff_layer2_cell = trans_coefficient(n_layer2, n_cell);

q_air_layer1 = (1/trans_coeff_air_layer1) * [1, ref_coeff_air_layer1;
ref_coeff_air_layer1, 1];
q_layer1_layer2 = (1/trans_coeff_layer1_layer2) * [1,
ref_coeff_layer1_layer2; ref_coeff_layer1_layer2, 1];
q_layer2_cell = (1/trans_coeff_layer2_cell) * [1, ref_coeff_layer2_cell;
ref_coeff_layer2_cell, 1];

wavelength_air = 650;
d_layer1 = 0.25 * wavelength_air / n_layer1;
d_layer2 = 0.25 * wavelength_air / n_layer2;

i_val = 1;
for wavelength_val = wavelength_array
    p_layer1 = prop_matrix(n_layer1, d_layer1, wavelength_val);
    p_layer2 = prop_matrix(n_layer2, d_layer2, wavelength_val);
    t_matrix = q_air_layer1 * p_layer1 * q_layer1_layer2 * p_layer2 *
q_layer2_cell;
    reflection_coefficient = abs(t_matrix(2, 1) / t_matrix(1, 1))^2;
    reflectivity_array(i_val) = reflection_coefficient * 100;
    transmission_coefficient = 1 - reflection_coefficient;
    transmissivity_array(i_val) = transmission_coefficient;
    intensity_val = calc_intensity(wavelength_val);
    power_val = transmission_coefficient * intensity_val;
    power_array(i_val) = power_val;
    total_power = total_power + power_val * 10;
    i_val = i_val + 1;
end

% Plotting Power vs. Wavelength
figure;
plot(wavelength_array, power_array, 'k');
ylabel('Instantaneous Power');
xlabel('Wavelength (nm)');
```

```
title('Power Spectrum from 400nm to 1400nm');
disp(['The total found power from 400nm to 1400nm is '
num2str(total_power)]);
grid on;
xlim([400, 1400]);
ylim([0.2, 1.4]);

wavelength_array_new = 200:10:2200;

reflectivity_array_new = zeros(1, length(wavelength_array_new));
transmissivity_array_new = zeros(1, length(wavelength_array_new));
power_array_new = zeros(1, length(wavelength_array_new));
total_power_new = 0;

ref_coeff_air_layer1 = ref_coefficient(n_air, n_layer1);
ref_coeff_layer1_layer2 = ref_coefficient(n_layer1, n_layer2);
ref_coeff_layer2_cell = ref_coefficient(n_layer2, n_cell);

trans_coeff_air_layer1 = trans_coefficient(n_air, n_layer1);
trans_coeff_layer1_layer2 = trans_coefficient(n_layer1, n_layer2);
trans_coeff_layer2_cell = trans_coefficient(n_layer2, n_cell);

q_air_layer1 = (1/trans_coeff_air_layer1) * [1, ref_coeff_air_layer1;
ref_coeff_air_layer1, 1];
q_layer1_layer2 = (1/trans_coeff_layer1_layer2) * [1,
ref_coeff_layer1_layer2; ref_coeff_layer1_layer2, 1];
q_layer2_cell = (1/trans_coeff_layer2_cell) * [1, ref_coeff_layer2_cell;
ref_coeff_layer2_cell, 1];

wavelength_air = 650;
d_layer1 = 0.25 * wavelength_air / n_layer1;
d_layer2 = 0.25 * wavelength_air / n_layer2;

i_val = 1;
for wavelength_val = wavelength_array_new
    p_layer1 = prop_matrix(n_layer1, d_layer1, wavelength_val);
    p_layer2 = prop_matrix(n_layer2, d_layer2, wavelength_val);
    t_matrix = q_air_layer1 * p_layer1 * q_layer1_layer2 * p_layer2 *
q_layer2_cell;
    reflection_coefficient = abs(t_matrix(2, 1) / t_matrix(1, 1))^2;
```

```
    reflectivity_array_new(i_val) = reflection_coefficient * 100;
    transmission_coefficient = 1 - reflection_coefficient;
    transmissivity_array_new(i_val) = transmission_coefficient;
    intensity_val = calc_intensity(wavelength_val);
    power_val = transmission_coefficient * intensity_val;
    power_array_new(i_val) = power_val;
    total_power_new = total_power_new + power_val * 10;
    i_val = i_val + 1;
end

% Plot
figure;
plot(wavelength_array_new, power_array_new, 'k');
ylabel('Instantaneous Power');
xlabel('Wavelength (nm)');
title('Power Distribution from 200nm to 2200nm');
disp(['The total found power from 200nm to 2200nm is '
num2str(total_power_new)]);
grid on;
xlim([200, 2200]);
ylim([0.04, 1.4]);

% Part 3: Refractive Index Variation and Power Calculation
lambda_c = 650;
wavelength_vals = 200:1:2199;
g01 = (n_air - n_layer1) / (n_air + n_layer1);  % reflection coefficient
between air and first layer
t01 = 2 * n_air / (n_air + n_layer1); % transmission coefficient between
air and first layer
q01 = (1 / t01) * [1, g01; g01, 1];

Power_vals = zeros(1, length(wavelength_vals));
PowerC_vals = zeros(1, 200);
indice_array_vals = 1.3:0.01:3.29;
n2_val = 1.3;

for n_val = 1:length(indice_array_vals)
    t12 = 2 * n_layer1 / (n_layer1 + n2_val);
    t23 = 2 * n2_val / (n2_val + n_layer3);
    g12 = (n_layer1 - n2_val) / (n_layer1 + n2_val);
```

```
    g23 = (n2_val - n_layer3) / (n2_val + n_layer3);
    q12 = (1 / t12) * [1, g12; g12, 1];
    q23 = (1 / t23) * [1, g23; g23, 1];
    L_val = 200;
    for k_val = 1:length(wavelength_vals)
        delta_m = (pi / 2) * (lambda_c / L_val);
        p1 = [exp(1i * delta_m), 0; 0, exp(-1i * delta_m)];
        T = q01 * p1 * q12 * p1 * q23;
        g_val = T(2, 1) / T(1, 1);
        r_val = abs(g_val)^2;
        pow_val = (((1 - r_val) * (6.16 * 10^15)) / ((L_val^5) * (exp(2484
/ L_val) - 1)));
        Power_vals(1, k_val) = pow_val;
        L_val = L_val + 1;
    end
    PowerC_vals(1, n_val) = sum(Power_vals);
    n2_val = n2_val + 0.01;
end

% Plotting Power vs. Refractive Indices
figure;
subplot(2, 1, 1);
plot(indice_array_vals, PowerC_vals, 'k');
title('Power vs. Refractive Indices');
ylabel('Power (W/m^2)');
xlabel('Refractive Index');
xlim([1.3, 3.3]);

% Comparing Reflectivity for Different Refractive Indices
Reflectivity_n23 = zeros(1, length(wavelength_vals));
Reflectivity_n262 = zeros(1, length(wavelength_vals));
n2_val = 2.3;

for k_val = 1:length(wavelength_vals)
    delta_m = (pi / 2) * (lambda_c / L_val);
    p1 = [exp(1i * delta_m), 0; 0, exp(-1i * delta_m)];
    T = q01 * p1 * q12 * p1 * q23;
    g_val = T(2, 1) / T(1, 1);
    r_val = abs(g_val)^2;
    Reflectivity_n23(1, k_val) = r_val;
```

```
    L_val = L_val + 1;
end

n2_val = 2.62;
for k_val = 1:length(wavelength_vals)
    delta_m = (pi / 2) * (lambda_c / L_val);
    p1 = [exp(1i * delta_m), 0; 0, exp(-1i * delta_m)];
    T = q01 * p1 * q12 * p1 * q23;
    g_val = T(2, 1) / T(1, 1);
    r_val = abs(g_val)^2;
    Reflectivity_n262(1, k_val) = r_val;
    L_val = L_val + 1;
end

% Plotting Reflectivity vs. Wavelength for Different Refractive Indices
subplot(2, 1, 2);
plot(wavelength_vals, Reflectivity_n23 * 100, 'k');
hold on;
plot(wavelength_vals, Reflectivity_n262 * 100, 'r');
title('Reflectivity vs. Wavelength for n=2.3 & 2.62');
xlabel('Wavelength (nm)');
ylabel('Reflectivity (%)');
xlim([200, 2200]);

function calc_ref_coefficient = ref_coefficient(n1, n2)
    calc_ref_coefficient = (n1 - n2) / (n2 + n1);
end

function calc_trans_coefficient = trans_coefficient(n1, n2)
    calc_trans_coefficient = (2 * n1) / (n1 + n2);
end

function calc_propagation_matrix = prop_matrix(n, d, wavelength)
    calc_propagation_matrix = [exp(1i * 2 * pi * n * d / wavelength), 0;
0, exp(-(1i * 2 * pi * n * d / wavelength))];
end

function calc_intensity = calc_intensity(wavelength)
    calc_intensity = (6.16 * 10^15) / ((wavelength^5) * (exp(2484 /
wavelength) - 1));
```

```
end
```

Part4.m

```
% Constants and initial values
refractive_index_air = 1;
refractive_index_1 = 1.4;
refractive_index_2 = 1.4;
refractive_index_3 = 3.15;
refractive_index_4 = 3.5;
wavelength_c = 650;
transmission_1 = 2 * refractive_index_air / (refractive_index_air +
refractive_index_1);
transmission_4 = 2 * refractive_index_3 / (refractive_index_3 +
refractive_index_4);
Q_1 = calculateQ(refractive_index_air, refractive_index_1,
transmission_1);
Q_4 = calculateQ(refractive_index_3, refractive_index_4, transmission_4);
wavelength_1 = wavelength_c / refractive_index_1;
thickness_1 = wavelength_1 / 4;
wavelength_3 = wavelength_c / refractive_index_3;
thickness_3 = wavelength_3 / 4;

% Initialize variables
num_steps_1 = 2001;
Range_power_1 = 200:1:2200;
Range_n_2 = 1.4:0.01:3.0;
data_power_1 = zeros(1, num_steps_1);
data_n_2_2k = zeros(1, length(Range_n_2));

% Loop for 200-2200 nm
data_n_2_2k = calculatePowerTransmission(Range_n_2, refractive_index_1,
refractive_index_2, refractive_index_3, num_steps_1, data_power_1,
data_n_2_2k, Q_1, Q_4, wavelength_c);

% Initialize variables for the second loop
num_steps_2 = 1001;
Range_power_2 = 400:1:1400;
Range_n_2 = 1.4:0.01:3.0;
data_power_2 = zeros(1, num_steps_2);
data_n_2_1k = zeros(1, length(Range_n_2));
```

```
% Loop for 400-1400 nm
data_n_2_1k = calculatePowerTransmission(Range_n_2, refractive_index_1,
refractive_index_2, refractive_index_3, num_steps_2, data_power_2,
data_n_2_1k, Q_1, Q_4, wavelength_c);

% Plotting
plotPowerTransmission(Range_n_2, data_n_2_1k, data_n_2_2k);

% Reflectivity calculations and plotting
plotReflectivity(wavelength_c, refractive_index_1, refractive_index_2,
refractive_index_3, refractive_index_4, Q_1);

function Q = calculateQ(n1, n2, t)
    % Calculate the Q matrix for a given layer
    reflection_coefficient = (n1 - n2) / (n1 + n2);
    Q = (1 / t) * [1 reflection_coefficient; reflection_coefficient 1];
end

function data_n_2 = calculatePowerTransmission(Range_n_2, n_1, n_2, n_3,
t, data_power, data_n_2, Q_1, Q_4, wavelength_c)
    % Calculate power transmission for a range of refractive indices
    for x = 1:length(Range_n_2)
        L = 200;
        t_2 = 2 * n_1 / (n_1 + n_2);
        t_3 = 2 * n_2 / (n_3 + n_2);
        reflection_2 = (n_1 - n_2) / (n_1 + n_2);
        reflection_3 = (n_2 - n_3) / (n_2 + n_3);
        Q_2 = calculateQ(n_1, n_2, t_2);
        Q_3 = calculateQ(n_2, n_3, t_3);

        % Rest of the loop
        for k = 1:t
            WL_2 = wavelength_c / n_2;
            d_2 = WL_2 / 4;
            delta_m = (pi / 2) * (wavelength_c / L);

            Prop_M = [exp(1i * delta_m) 0; 0 exp(-1i * delta_m)];
            T_Mat = (Q_1 * Prop_M * Q_2 * Prop_M * Q_3 * Prop_M * Q_4);
            gamma_sys = T_Mat(2, 1) / T_Mat(1, 1);
            R = abs(gamma_sys) * abs(gamma_sys);
```

```
            T = 1 - R;

            Power = T * (6.16e15 / (power(L, 5) * (exp(2484 / L) - 1)));
            data_power(1, k) = Power;
            L = L + 1;
        end

        data_n_2(1, x) = sum(data_power);
        n_2 = n_2 + 0.01;
    end
end

function plotPowerTransmission(Range_n_2, data_n_2_1k, data_n_2_2k)
    % Plot power transmission for different wavelength ranges
    figure
    hold on
    title('Power Transmission at Varying Refractive Indices n2 ');
    xlabel('n2');
    ylabel('Power [W/m^2]');
    plot(Range_n_2, data_n_2_1k, 'r');
    plot(Range_n_2, data_n_2_2k, 'k');
    legend('Wavelength (200-2200 nm)', 'Wavelength (400-1400 nm)');
    grid on;
end

function plotReflectivity(wavelength_c, n1, n2, n3, n4, Q_1)
    % Plot reflectivity for different refractive indices
    wavelength_range = 400:1399;
    Reflectivity_n2_19 = zeros(1, length(wavelength_range));
    L = 200;
    n2 = 2.3;
    t12 = 2 * n1 / (n1 + n2);
    t23 = 2 * n2 / (n2 + n3);
    t34 = 2 * n4 / (n3 + n4);
    g12 = (n1 - n2) / (n1 + n2);
    g23 = (n2 - n3) / (n2 + n3);
    g34 = (n3 - n4) / (n3 + n4);
    q12 = (1 / t12) * [1 g12; g12 1];
    q23 = (1 / t23) * [1 g23; g23 1];
    q34 = (1 / t34) * [1 g34; g34 1];
```

```
    for k = 1:length(wavelength_range)
        delta_m = (pi / 2) * (wavelength_c / L);
        p1 = [exp(1i * delta_m) 0; 0 exp(-1i * delta_m)];
        T = (Q_1 * p1 * q12 * p1 * q23 * p1 * q34);
        g = T(2, 1) / T(1, 1);
        r = abs(g)^2;
        Reflectivity_n2_19(1, k) = r;
        L = L + 1;
    end

    % Plot Reflectivity
    figure
    hold on
    title('Reflectivity (%) for Varying Wavelength Contrasting the
Reflectivity of n=2.3 and n=2.62');
    xlabel('Wavelength (nm)');
    ylabel('Reflectivity (%)');
    plot(wavelength_range, Reflectivity_n2_19 * 100, 'r');

    % Reflectivity calculations and plotting for n=2.62
    Reflectivity_n2_62 = zeros(1, length(wavelength_range));
    L = 200;
    n2 = 2.62;
    t12 = 2 * n1 / (n1 + n2);
    t23 = 2 * n2 / (n2 + n3);
    g12 = (n1 - n2) / (n1 + n2);
    g23 = (n2 - n3) / (n2 + n3);
    q12 = (1 / t12) * [1 g12; g12 1];
    q23 = (1 / t23) * [1 g23; g23 1];

    for k = 1:length(wavelength_range)
        delta_m = (pi / 2) * (wavelength_c / L);
        p1 = [exp(1i * delta_m) 0; 0 exp(-1i * delta_m)];
        T = (Q_1 * p1 * q12 * p1 * q23);
        g = T(2, 1) / T(1, 1);
        r = abs(g)^2;
        Reflectivity_n2_62(1, k) = r;
        L = L + 1;
    end
```

```
    % Plot Reflectivity
    plot(wavelength_range, Reflectivity_n2_62 * 100, 'k');
    legend('Reflectivity for n=2.3', 'Reflectivity for n=2.62');
    grid on;
    xlim([400, 1400]);
end
```