The OpenLoop controller: We want to control position and velocity of the robots and deciding about force for reaching our desired position and velocity. So our input is going to be accelerator($F = ma$). If we show acceleration by a, velocity by v and position in x coordinate by P_x and in y coordinate by P_y, we have the following equations

$$\dot{P}_x = v_x \tag{1}$$

$$\dot{v}_x = a_x \tag{2}$$

$$\ddot{P}_x = a_x \tag{3}$$

Respectively we have the following equations for y axis:

$$\dot{P}_y = v_y \tag{4}$$

$$\dot{v}_y = a_y \tag{5}$$

$$\ddot{P}_y = a_y \tag{6}$$

The state-space representation of our OpenLoop controller is:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + We(t) \\ y &= Cx(t) + Du(t) \end{aligned} \tag{7}$$

where *x(t)* represents our states, *u(t)* is our input and *e(t)* represents noise in the system. We also have *y* as our output.
First we assume that we don't have noise in the system and we also have just one robot. As mentioned, *a* is our input and *x, y,* and their velocities are our states that we want to control.
We define our states as following:

$$X_1 = P_x \tag{8}$$

$$\dot{X}_1 = X_2 = \dot{P}_x = v_x \tag{9}$$

$$X_3 = P_y \tag{10}$$

$$\dot{X}_3 = X_4 = \dot{P}_y = v_y \tag{11}$$

So our state space representation is:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} a \tag{12}$$

We want to find number of states that we can control. we need to know *B, AB,*A^2B, A^3B, ...

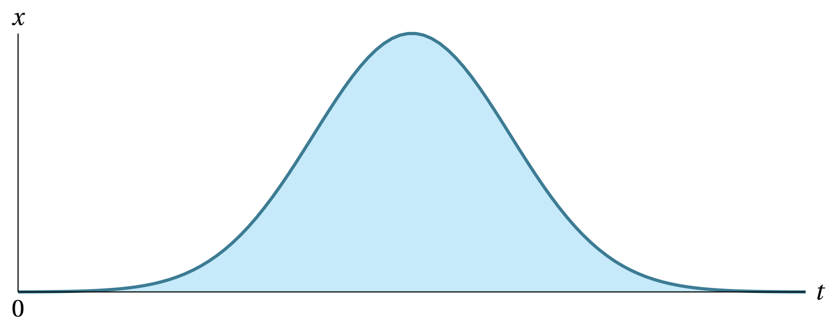
$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, AB = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, A^2B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \dots \tag{13}$$

So we have 2 controllable states. (One for the x axis and one for the y axis.)
we know that we can control velocity of our robot, we want to see what happens if we had more than one robot? As we saw here, v_x is completely independent of v_y, so if we wanted to have n robots, we had n states in x axis, and n states in y axis, which were completely independent from each other.
If we had n robots, we had exactly the same symmetry of what we had for 1 robot and we could just control 2 states. So it seems that we can control average position of the robots, and average linear velocity of them. Or we could just control one robot when trying to have global control.

Now, we assume that we don't have the symmetry by having a noise (e(t) in equation (7)). Assume that noise has a Gaussian distribution:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} \tag{14}$$

After a while, if we have lots of robots, we should have some bell shape of their position because of noise:



Probability of the position of the robots, if we have Gaussian noise.

If robots face a wall, we expect to have the following shape: