This paper considers the problem of controlling a swarm of many robot. Rather than sending unique control signals to individual robots, each robot receives the same control signal. Based upon human user experiments, it is postulated that controlling the swarm using only the robots' mean position or mean position and variance as feedback is more effective than using full state feedback. Each robot is modeled as a two dimensional double integrator in the plane, with the robot's acceleration being the control input. It is shown that the mean position and mean velocity of the swarm satisfy a two dimensional linear system of differential equations, and it is shown that this system is controllable.

Next, a heuristic is given for increasing or decreasing the variance of the swarm. This heuristic relies on the presence of a flat wall in the swarm's environment. Then a hybrid controller is proposed for controlling both the mean and variance. Simulations are performed that demonstrate the algorithm's ability to control both the mean and the variance, and the controllers are used to complete a block pushing task.

Below are my comments and suggested changes:

Abstract:

In the 14^{th} line of the abstract, it is claimed that under certain conditions, the variance is controllable. You should be careful about making this claim, at least in the abstract and introduction, because the variance is not "controllable" in the normal sense, i.e., in the same way that the mean position is controllable. Controlling the variance relies on the presence of a flat obstacle, and this should be made explicit.

Section II:

In Section IIA, the meaning of the statement "this constraint limits the population size that is under active control" is not clear. What do you mean by active control?

In the last sentence of Section IIA, "Making progress requires…", making progress in what?

In the last sentence of Section IIB, it is not clear if "This paper" refers to the paper just referenced, or your paper. Maybe say "The current paper" instead.

In Section IIC, you compare your method of manipulation to compliant manipulators, but you point out that compliant manipulators have a natural restoring force, while you have to enforce an artificial restoring force in the swarm. Without a restoring force, the swarm "flows" around the object. Is there any literature regarding manipulation using fluids that would be relevant to your work?

Section III:

In equation (1), the variables p and v are not explicitly defined. Readers should be able to interpret their meaning, although using p for position is slightly confusing since it is often used to denote momentum.

After equation (3), add "the" after "We want to find".

In equation (5), the controllability matrix is usually written as a single matrix, and not a set of matrices. Also, to be consistent with equation (4), you should include A^2*B and A^3*B, even if they are zero.

In equation (7), it is not completely clear that the remaining columns of the matrix are zero. Say this explicitly or add 1 or 2 more columns of zeros to make this clear. Again, C is usually written as a matrix, and not a set.

In equation (9), should u have the subscript x?

After equation (10), you say “the controllable states of the swarm are the average position and average velocity”. It would be more appropriate to say “the average position and average velocity are controllable”, since you are able to control any two states, and you are just choosing to control the mean position and velocity. Also, remove the end-of-proof symbol, since a proof was never begun.

In equation (11), should \bar{x} have the subscript p to be consistent with equations (8) and (9)?

In the equation between (12) and (13), does the left hand side mean [(d/dt) sigma]^2 or (d/dt)[sigma^2]? This also applies to equations (15) and (17).

In equation (13), what is r?

After equation (13), it is not clear where the value r+sqrt{3 sigma^2_{goal}} comes from.

When giving the value of sigma^2_{optimal}, what values of n and r are used to compute 0.28?

In the paragraph beginning before equation (14), what origin are you referring to when you say “we will prove that the origin is asymptotically stable”?

A picture describing the control law in equation (16) might be useful.

In equation (17), should the term W be multiplied by epsilon as in the equation between equations (12) and (13)? Also, sigma^2_{min} is used in this equation, but is not defined until the next Section.

In the line after equation (17), you state that dot{V} is negative definite. However, when the control is “move to wall”, the variance does not begin to decrease until the robots reach the wall. In fact, the variance continues to increase due to the noise until the robots reach the wall. The heuristic of moving toward the wall to decrease the variance is clear, and I’m not sure that you need the Lyapunov arguments, especially when the claim that dot{V} is negative definite is not always true.

In Algorithm 1, you mention that there is a hysteresis component. Where exactly is the hysteresis term in the algorithm?

In Algorithm 1, line 9, the two y’s should be x’s.

In Figure 4, it would help to use the labels “K_p” and “K_d” instead of “p” and “d”.

In Figure 5, it would help to use “W” instead of “w” in the legend so that it is more consistent with equation (12).

After equation (20), what do you mean by “the variance integrates over time”?

Section V:

I don’t understand your counterexample for open loop gathering with a ‘T’ shaped obstacle. What do you mean by “bottom of a ‘T’”?