Supplement to Shaping a Swarm With a Shared Control Input Using Boundary Walls and Wall Friction

Paper-ID 66

***Abstract*—Includes algorithms and equations too lengthy for main paper, but potentially useful for the community.**

I. Video Attachments

A. Using walls for controlling covariance in square workspace

In this video the limits of the mean position that the swarm can achieve is observable when we have a particular number of robots. It also illustrates how covariance and correlation is changed when the swarm hits the walls and when the swarm has more or less robots.

B. Two robots positioning

Alg 1,2,3 has been implemented in mathematica to show how two robots are positioned in a square workspace. In this video the initial and ending position of the two robots has been changed to any place of the workspace, and the path that the robots should follow to get to ending position is drawn. The last position of the robots is one the most challenging one that the algorithm will give the solution for that.

C. n robots positioning

Alg 4 has been implemented in MATLAB to show how the two robots positioning is extendable to more robots. In this video each block will reach its goal while all the robots gets the same input using the algorithm.

D. Covariance Control

Alg. 5 has been implemented with box2D simulator. In this video the green ellipse is the desired covariance ellipse, the red ellipse is the current covariance ellipse of the swarm and the red dot is the mean position of the robots. Robots follow the algorithm to achieve the desired values for σ_{goalxy}, σ_x^2 and σ_y^2.

II. Calculations for modeling swarm as fluid in a simple planar workspace

Two workspaces are used, a square and a circular workspace.

A. Square Workspace

This section provides formulas for the mean, variance, covariance and correlation of a very large swarm of robots as they move inside a square workplace under the influence of gravity pointing in the direction β. The swarm is large, but the robots are small in comparison, and together cover an area of constant volume A. Under a global input such as gravity, they flow like water, moving to a side of the workplace and forming a polygonal shape. The workspace is

The range of possible angles for the global input angle β is [0,2π). In this range of angles, the swarm assumes eight different polygonal shapes. The shapes alternate between triangles and trapezoids when the area A<1/2, and alternate between squares with one corner removed and trapezoids when A>1/2.

Two representative formulas are attached, the outline of the swarm shapes in (II) and $\bar{x}(\beta, A)$ in (I).

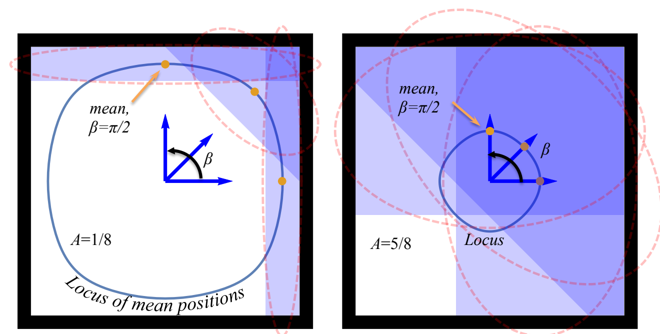

Fig. 1. A swarm in a

B. Circle Workspace

The area under a chord of a circle is the area of a sector less the area of the triangle originating at the circle center: provement of area formula: $A = S(sector) - S(triangle) = 1/2LR - 1/2C(1-h)$, thus

$$A = (1/2)\left[LR - c(R-h)\right] \tag{3}$$

where L is arc length, c is chord length, R is radius and h is height. Solving for L and C gives

$$L = 2\cos^{-1}(1-h) \tag{4}$$

$$C = 2\sqrt{h(2-h)} \tag{5}$$

Therefore the area under a chord is

$$\cos^{-1}(1-h) - (1-h)\sqrt{(2-h)h} \tag{6}$$

The variance of x and y are:

$$\sigma_x^2(h) = \frac{64(h-2)^3 h^3}{144\left(\sqrt{-(h-2)h}(h-1) + \arccos(1-h)\right)^2} + \frac{9\left(\sqrt{-(h-2)h}(h-1) + \arccos(1-h)\right)\left(\sin(4\arcsin(1-h)) + 4\arccos(1-h)\right)}{144\left(\sqrt{-(h-2)h}(h-1) + \arccos(1-h)\right)^2} \tag{7}$$

$$\sigma_y^2(h) = \frac{12\arccos(1-h) - 8\sin(2\arccos(1-h)) + \sin(4\arccos(1-h))}{48\left(\sqrt{-(h-2)h}(h-1) + \arccos(1-h)\right)} \tag{8}$$

$$
\bar{x}(\beta, A) =
\begin{array}{ll}
A \leq \frac{1}{2} : & \begin{cases}
-\frac{\tan^2(\beta)}{24A} - \frac{A}{2} + 1 & 0 \leq \beta \leq \tan^{-1}(2A) \vee 2\pi - \tan^{-1}(2A) < \beta \leq 2\pi \\
1 - \frac{1}{3}\sqrt{2}\sqrt{A\tan(\beta)} & \tan^{-1}(2A) < \beta \leq \frac{\pi}{2} - \tan^{-1}(2A) \\
\frac{\cot(\beta)}{12A} + \frac{1}{2} & \frac{\pi}{2} - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \frac{\pi}{2} \\
\frac{1}{3}\sqrt{2}\sqrt{-A\tan(\beta)} & \tan^{-1}(2A) + \frac{\pi}{2} < \beta \leq \pi - \tan^{-1}(2A) \\
\frac{\tan^2(\beta)}{24A} + \frac{A}{2} & \pi - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \pi \\
\frac{1}{3}\sqrt{2}\sqrt{A\tan(\beta)} & \tan^{-1}(2A) + \pi < \beta \leq \frac{3\pi}{2} - \tan^{-1}(2A) \\
\frac{1}{2} - \frac{\cot(\beta)}{12A} & \frac{3\pi}{2} - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \frac{3\pi}{2} \\
1 - \frac{1}{3}\sqrt{2}\sqrt{-A\tan(\beta)} & \tan^{-1}(2A) + \frac{3\pi}{2} < \beta \leq 2\pi - \tan^{-1}(2A)
\end{cases} \\
\frac{1}{2} < A < 1 : & \begin{cases}
-\frac{\tan^2(\beta)}{24A} - \frac{A}{2} + 1 & 0 \leq \beta \leq \tan^{-1}\left(\frac{1}{2}, 1 - A\right) \vee 2\pi - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) < \beta \leq 2\pi \\
\frac{2\sqrt{2}\sqrt{(1-A)\tan(\beta)}(A-1)+3}{6A} & \tan^{-1}\left(\frac{1}{2}, 1 - A\right) < \beta \leq \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) \\
\frac{6A+\cot(\beta)}{12A} & \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \frac{\pi}{2} \\
\frac{-2\sqrt{2}\sqrt{(A-1)\tan(\beta)}(A-1)+6A-3}{6A} & \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \frac{\pi}{2} < \beta \leq \pi - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) \\
\frac{\tan^2(\beta)}{24A} + \frac{A}{2} & \pi - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \pi \\
\frac{2\sqrt{2}\sqrt{(1-A)\tan(\beta)}(1-A)+6A-3}{6A} & \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \pi < \beta \leq \frac{3\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) \\
\frac{1}{2} - \frac{\cot(\beta)}{12A} & \frac{3\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1 - A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \frac{3\pi}{2} \\
\frac{2\sqrt{2}\sqrt{(A-1)\tan(\beta)}(A-1)+3}{6A} & \tan^{-1}\left(\frac{1}{2}, 1 - A\right) + \frac{3\pi}{2} < \beta \leq 2\pi - \tan^{-1}\left(\frac{1}{2}, 1 - A\right)
\end{cases} \\
A = 1 : & \frac{1}{2}
\end{array}
\tag{1}
$$

TABLE I

$\bar{x}$ IN A UNIT-SQUARE WORKSPACE

III. Algorithm for generating desired y spacing between two robots using wall friction

$$
\mathrm{RobotRegion}(\beta, A) = \begin{cases}
A \leq \frac{1}{2} : \begin{cases}
\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ -A - \frac{\tan(\beta)}{2} + 1 & 1 \\ -A + \frac{\tan(\beta)}{2} + 1 & 0 \end{pmatrix} & 0 \leq \beta \leq \tan^{-1}(2A) \vee 2\pi - \tan^{-1}(2A) < \beta \leq 2\pi \\
\begin{pmatrix} 1 & 1 \\ 1 - \sqrt{2}\sqrt{A\tan(\beta)} & 1 \\ 1 & 1 - \sqrt{2}\sqrt{A\cot(\beta)} \end{pmatrix} & \tan^{-1}(2A) < \beta \leq \frac{\pi}{2} - \tan^{-1}(2A) \\
\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & -A + \frac{\cot(\beta)}{2} + 1 \\ 1 & -A - \frac{\cot(\beta)}{2} + 1 \end{pmatrix} & \frac{\pi}{2} - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \frac{\pi}{2} \\
\begin{pmatrix} 0 & 1 \\ \sqrt{2}\sqrt{-A\tan(\beta)} & 1 \\ 0 & 1 - \sqrt{2}\sqrt{-A\cot(\beta)} \end{pmatrix} & \tan^{-1}(2A) + \frac{\pi}{2} < \beta \leq \pi - \tan^{-1}(2A) \\
\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ A - \frac{\tan(\beta)}{2} & 1 \\ A + \frac{\tan(\beta)}{2} & 0 \end{pmatrix} & \pi - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \pi \\
\begin{pmatrix} 0 & 0 \\ 0 & \sqrt{2}\sqrt{A\cot(\beta)} \\ \sqrt{2}\sqrt{A\tan(\beta)} & 0 \end{pmatrix} & \tan^{-1}(2A) + \pi < \beta \leq \frac{3\pi}{2} - \tan^{-1}(2A) \\
\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & A - \frac{\cot(\beta)}{2} \\ 0 & A + \frac{\cot(\beta)}{2} \end{pmatrix} & \frac{3\pi}{2} - \tan^{-1}(2A) < \beta \leq \tan^{-1}(2A) + \frac{3\pi}{2} \\
\begin{pmatrix} 1 & 0 \\ 1 - \sqrt{2}\sqrt{-A\tan(\beta)} & 0 \\ 1 & \sqrt{2}\sqrt{-A\cot(\beta)} \end{pmatrix} & \tan^{-1}(2A) + \frac{3\pi}{2} < \beta \leq 2\pi - \tan^{-1}(2A)
\end{cases} \\
\frac{1}{2} < A < 1 : \begin{cases}
\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ (1-A) - \frac{\tan(\beta)}{2} & 1 \\ (1-A) + \frac{\tan(\beta)}{2} & 0 \end{pmatrix} & 0 \leq \beta \leq \tan^{-1}\left(\frac{1}{2}, 1-A\right) \vee 2\pi - \tan^{-1}\left(\frac{1}{2}, 1-A\right) < \beta \leq 2\pi \\
\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & \sqrt{2}\sqrt{(1-A)\cot(\beta)} \\ \sqrt{2}\sqrt{(1-A)\tan(\beta)} & 0 \end{pmatrix} & \tan^{-1}\left(\frac{1}{2}, 1-A\right) < \beta \leq \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1-A\right) \\
\begin{pmatrix} 0 & 1 \\ 1 & 1 \\ 1 & (1-A) - \frac{\cot(\beta)}{2} \\ 0 & (1-A) + \frac{\cot(\beta)}{2} \end{pmatrix} & \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1-A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \frac{\pi}{2} \\
\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 - \sqrt{2}\sqrt{-(1-A)\tan(\beta)} & 0 \\ 1 & \sqrt{2}\sqrt{-(1-A)\cot(\beta)} \end{pmatrix} & \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \frac{\pi}{2} < \beta \leq \pi - \tan^{-1}\left(\frac{1}{2}, 1-A\right) \\
\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ -(1-A) - \frac{\tan(\beta)}{2} + 1 & 1 \\ -(1-A) + \frac{\tan(\beta)}{2} + 1 & 0 \end{pmatrix} & \pi - \tan^{-1}\left(\frac{1}{2}, 1-A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \pi \\
\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 - \sqrt{2}\sqrt{(1-A)\tan(\beta)} & 1 \\ 1 & 1 - \sqrt{2}\sqrt{(1-A)\cot(\beta)} \end{pmatrix} & \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \pi < \beta \leq \frac{3\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1-A\right) \\
\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -(1-A) + \frac{\cot(\beta)}{2} + 1 \\ 1 & -(1-A) - \frac{\cot(\beta)}{2} + 1 \end{pmatrix} & \frac{3\pi}{2} - \tan^{-1}\left(\frac{1}{2}, 1-A\right) < \beta \leq \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \frac{3\pi}{2} \\
\begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \\ \sqrt{2}\sqrt{-(1-A)\tan(\beta)} & 1 \\ 0 & 1 - \sqrt{2}\sqrt{-(1-A)\cot(\beta)} \end{pmatrix} & \tan^{-1}\left(\frac{1}{2}, 1-A\right) + \frac{3\pi}{2} < \beta \leq 2\pi - \tan^{-1}\left(\frac{1}{2}, 1-A\right)
\end{cases}, \\
A = 1 : \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}
\end{cases} \tag{2}
$$

TABLE II

ROBOTREGIONS IN A UNIT-SQUARE WORKSPACE

Algorithm 1 GenerateDesired$y$-spacing(s_1, s_2, e_1, e_2, L)

Require: Knowledge of starting (s_1, s_2) and ending (e_1, e_2) positions of two robots. $(0, 0)$ is bottom corner, s_1 is rightmost robot, L is length of the walls. Current position of the robots are (r_1, r_2).

Ensure: $r_{1x} - r_{2x} \equiv s_{1x} - s_{2x}$

$\Delta s_y \leftarrow s_{1y} - s_{2y}$
$\Delta e_y \leftarrow e_{1y} - e_{2y}$
$r_1 \leftarrow s_1$, $r_2 \leftarrow s_2$
if $\Delta e_y < 0$ **then**
  $m \leftarrow (L - \max(r_{1y}, r_{2y}), 0)$ ▷ Move to top wall
else
  $m \leftarrow (-\min(r_{1y}, r_{2y}), 0)$ ▷ Move to bottom wall
end if
$m \leftarrow m + (0, -\min(r_{1x}, r_{2x}))$ ▷ Move to left
$r_1 \leftarrow r_1 + m$, $r_2 \leftarrow r_2 + m$ ▷ Apply move
if $\Delta e_y - (r_{1y} - r_{2y}) > 0$ **then**
  $m \leftarrow (\min(|\Delta e_y - \Delta s_y|, L - r_{1y}), 0)$ ▷ Move top
else
  $m \leftarrow (-\min(|\Delta e_y - \Delta s_y|, r_{1y}), 0)$ ▷ Move bottom
end if
$m \leftarrow m + (0, \epsilon)$ ▷ Move right
$r_1 \leftarrow r_1 + m$, $r_2 \leftarrow r_2 + m$ ▷ Apply move
$\Delta r_y = r_{1y} - r_{2y}$
if $\Delta r_y \equiv \Delta e_y$ **then**
  $m \leftarrow (e_{1x} - r_{1x}, e_{1y} - r_{1y})$
  $r_1 \leftarrow r_1 + m$, $r_2 \leftarrow r_2 + m$ ▷ Apply move
  return (r_1, r_2)
else
  return GenerateDesired$y$-spacing(r_1, r_2, e_1, e_2, L)
end if