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# Handout for MATH 036: Study Questions for “Bilateral Symmetry” (*Symmetry*, Chapter 1)

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- (i) How are beauty and symmetry related, according to Weyl? Observe that while the former is subjective the latter is objective. Could a deep study of symmetry lead to an objective theory of beauty (or aesthetics)? If yes, is this something to be worried about? Also keep in mind the quote from Dagobert on p. 16.
- (ii) What is the typical scheme for all theoretical knowledge? Is there a typical scheme for other kinds of knowledge (e.g. common knowledge)?
- (iii) Compare (platonic) idealism and realism. Which side is Weyl on? According to this how would you classify him, as a mathophysicist or a mathologist? Would this classification be legitimate?
- (iv) Find all double-headed eagles used as symbols throughout history. Also research their meanings. Similarly, find all many-headed animals used as symbols throughout history (such as Cerberus from Greek mythology). Compare the meanings of many-headed animals to the meanings of double-headed eagles.
- (v) On p. 13 Weyl makes a distinction between the oriental and the occidental: “For in contrast to the orient, occidental art, like life itself, is inclined to mitigate, to loosen, to modify, even to break strict symmetry.”. Do you agree with Weyl? (Eastern) martial arts, Celtic knots, and knitting are important examples to keep in mind in this regard.
- (vi) Research whether or not the houses of worship of all religions (that has a book or otherwise) have bilateral symmetry.
- (vii) What does Leibniz mean when he uses the word “indiscernible” in a technical sense?
- (viii) On p. 18 Weyl mentions one-to-one mappings (or transformations). These are precisely the one-to-one correspondences we talked about it class when discussing Halmos’ article, except perhaps one-to-one mappings preserve additional structures. To give an example, recall the one-to-one correspondence between the fingers of the two hands. If we wanted to talk about a further structure on these “sets of fingers”, we could refer to biology to recognize the uniqueness of the thumb among all other fingers. Indeed, the development of opposable thumbs is crucial to the mastery of utilizing tools. Then the one-to-one correspondence that sends the thumb of the left hand to the pinky finger of the right hand is not structure-preserving, though the “natural” one-to-one correspondence we talked about is structure-preserving. Using these definitions, find all one-to-one correspondences between fingers of hands. Find also all one-to-one mappings between fingers of hands. In particular compare the numbers of such correspondences.
- (ix) Find examples of one-to-one, two-to-one, three-to-one, four-to-one, ..., many-to-one mappings. (Hint: Consider a rubber band wrapped around itself twice. Then cast its shadow onto a flat surface by using a light source directly above it. This would be  $2 : 1$ , but from where (or what) to where (or what)?)
- (x) What is an automorphism? Compare an automorphism to a metamorphism (possibly after defining the latter term appropriately, say, a la Kafka).
- (xi) The left hemisphere of the brain controls the limbs on the right side of the body, while the right hemisphere of the brain controls the limbs on the left side of the body. Find an explanation regarding why such a crossing exists.

- (xii) On p. 25 Weyl states that “a state of equilibrium is likely to be symmetric”. What does he mean by this?
- (xiii) How does Weyl justify that “contingency is an essential feature of the world” (on p. 26)? Also see the section on emergence of the “Cellular Automata” entry on Stanford Encyclopedia of Philosophy (<https://plato.stanford.edu/entries/cellular-automata/>).
- (xiv) On p. 27 Weyl claims that “[t]he laws of nature do not determine uniquely the one world that actually exists, not even if one concedes that two worlds arising from each other by an automorphic transformation [...]”. How does this relate to our discussion of chess, and consequently, to the paradox central to dynamics?
- (xv) Recall the motto Halmos used in his article: “ontogeny recapitulates phylogeny”. Considering the last sentence of the chapter, reevaluate this motto.