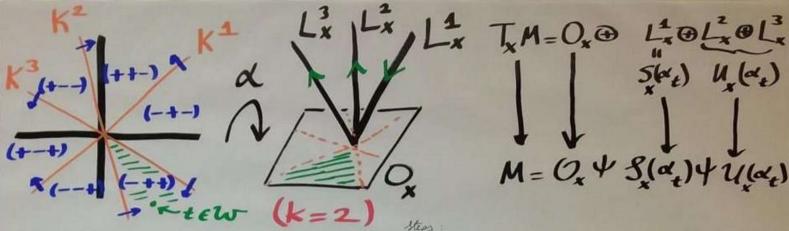
Katok - Rodriguez Stertz Arithmeticity for RK - actions by Alp Ugman, Pen State (azu4@psu.edu) SCGP 2022: Feesibility & Rigidity space (k=2) time crystal - K+1 - K Uhm (Work in Progress): A massimal rank positive entropy action of Rk (k7,2) by C'(171) diffeomorphisms is measure theoretically isomorphic to a (k+1)-dimensional space crystal corrying an affine lartan action of Zet and sticling along a k-dimensional time crystal. Partially Funded by : Kotal Berter & NSF



Precise Statement

We make a contribution to the study of global measure rigidity. The precise statement is so follows:

(4. 11) & (bby @ let pire , \$ 8, k)

4 Here \$ 1. k. The Deft of The a This) is the K-time change of the suspension of 8; the time crystal is the factor torus The and the space crystal is the elgebraic flow That "larger nears. Vie 2 2 10 the linear part of Vo is hyperbole and olymphoble

Contest & Pravious Work

is one can evaloper of consider 22 potion on and abnurrianal manifolds with ent >0, constant time change of suspensions of such actions would estimf the hypotheses of the above shedren. Haliam-K-RH actual if there were other RK- actions satisfying the hypotheses (Problem #4 in these 2018 Annals paper); ver conclusion is no

to the above theorem can be considered as a normal forming toppartion version of a theorem proved by Matmento is 2001, said theorem has the analogous statement for Co actions with sufficiently many continuement I uniformly normally hyperbolic elements to the orbit foliation in particular he measure is involved.

of the nost important hypothem in the above theorem is the positive entropy topotheris ($\forall e \in \mathbb{R}^k \mid 0 : est_p(ke) > 0$), obegang or readering it would be a major advancement in the area. (Furstending, Ruddish, Helsh, System Relin, RH, ...) In our sutting it implies that the Lagran hypoplans granded by the higher none Oseladets Theorem grandes the Kepter produce the protion!

to the main stations in to extend the machinery K-RH used in their 2026 arithmeticity paper we extend the machinery for the actions to the arithmeticity theorem in the the energy of the above theorem; their time crystal is a fait cyclic grope.

The acting group is abelian, the bank machinery for rowing from hyperboliaty (Oselebet, Penn, dedroppin, "Sound) that works for one element works for the whole action. By the portire estropy assurption we have the X picture. In particular any (rank one) dyapower subburdle is integrable; as any such alburdle is the stable burdle of some elevent of the action.

(1) By K-K the (dinerium one) disaponer foliations extent measurable mentationary linearizations, which endow them with C (444 (R), R) - observes the orbit foliation two has with C (444 (R), R) - observes the orbit foliation two has with a ratical affine structure. Assembling the Lyapunor foliations a ratical affine structure "politic C affine structures for stable and contable foliations (for any week chamber we of (p. x)). The affine structure of the orbit foliation is compatible with these affine structures

(2) by K-K-RH, pr-leby, (orbit-) stable and (orbit-) sustable holonomies are well defined, they are diagonal linear in affix charts

3 Wang the dynamical holonomies and affine structures we can define a developing map day R3. R4. R4 -> M Unspelle satisfactions of mall process of orbit-stable foliations cover M p-22, so slave pushes blogs forward to p.

(B) By construction the symmetry group you of duty is a subgroup of (R*XD) \(AH(R*) \(D) = (\cdots, \cdots) \), \(S^{2r} \) is the homeotime group of (an). There is a natural splitting \(S^{2r} = S^{2r} \cdot S^{2r} \)

3) we have

Precise Statement: 6 The acting group is abelian; the basic machinery for We make a contribution to the study of global measure rominiform hyporbolisty (Oseledet, Penn, Ledrappin, Young) that nigidity. The precise statement is as follows: works for one element works for the whole action.

Theorem: Let $r \in \mathbb{R}_{> 1}$ (regularity), $k \in \mathbb{Z}_{1/2}$ (rank), d = 2k + 1(dimension), M be a compact C^{∞} manifold of dimension d, $d : \mathbb{R}^k \to Diff^{\circ}(M, w)$ be a locally tree executive action preserving. In particular any (rank one) Lyapunov subbundle is integrable;

x(--+ (++1)) (k=2) steps

 $M = \mathcal{O}_{x} \Psi \mathcal{S}(\alpha_{t}) \Psi \mathcal{U}(\alpha_{t})$

Precise Statement:

We make a contribution to the study of global measure rigidity. The precise statement is as follows:

Theorem: Let r E R>s (regularity), k EZL712 (rank), d=2k+1 (dimersion), M be a compact (manifold of dimersion d,

d. R Diff (M, M) be a locally free ergodic action preserving a boal probability measure is on M. If $\forall t \in \mathbb{R}^k \mid 0 : ent_{\mu}(x_t) > 0$,

then IKE GL(k, R) and an affine Cartan action 8. Zik Aff (The These These or This = This/ I) such that measure theoretically

(M, X) = (bb pk & let pkrs, to 8,K) * Here to K. Rk - Diff (T & D. T k12) is the K-time change of

the wopenion of 8; the "time crustal" is the factor torus The and the space crystal is the algebraic fiber The larton" means: It & Zt 10: the linear part of 8t is hyporbola and digonalisable.

Contest & Pravious Work:

One can enalogously consider ZL action on ked dimensional manifolds with eat >0; constant time changes of suspensions of such actions would ratiofy the hypotheses of the above theorem. Halinin-K-RH asked if there were other R'- actives satisfying

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(6) The acting group is abelian; the bank machinery for rominiform hyporboliaty (Oseledet, Penn, Ledrappin, Young) that works for one element works for the whole action.

By the positive entropy assumption we have the X picture In particular any (nach one) Lyapunov subbundle is integrable; as any such subundle is the stable bundle of some cleant

1) By K-K the (dimension one) Lyapunor foliations admit measurable manstationary linearizations, which endow them with a (Aff (R), R) - structures. The orbit foliation too has a ratural affine structure. Assembling the Lyapunar foliations produces "split" C affine structures for stable and custable foliations (for any weyl chamber w of (year)). The affine structure of the orbit foliation is compatible with these

affine structures. (2) by K-K-RH, Maleby; (orbit-) stable and (orbit-) wistable holoromies are well defined; they are diagonal linear in affine charts.

3 Using the dynamical holonomies and affine structures we can define a developing map devi : R " x R x R" -> M. Vensable saturations of mall pieces of orbit-stable foliations cover H pr-se, so elevino

then IKE GL(E,R) and an affine Cartan action 8. : Zik Aff (# " of the action. (T k+1 = T k+1 or T x+1 = T k+1/1) such that meanine theoretically (H, X) = (bb Rk & bb Hk+1, to 8, K) # Here to ".K: PK - Diff a (T & D T 6+2) is the K-time change of the suspension of 8; the "time crustal" is the factor torus The and the "space crystal" is the algebraic fiber # k+1 "lartan" means: It = Ze 10: the linear part of 8t is hyperbola and digunalizable. Contest & Previous Work: & One can analogously consider 22 actions on k+2 dimensional manifolds with ent >0; constant time changes of suspensions of such askons would satisfy the hypotheses of the above theorem. Halinin-K-RH asked if there were other TR - actions satisfying the hypotheses (Problem #4 in their 2011 Annals paper); our conclusion is : no. * The above theorem can be considered as a noniniformly properbolic version of a theorem proved by Matrimoto in 2001; saral theorem has the analogous statement for Co actions with sufficiently many codimension I uniformly normally hyperbolic elements to the orbit foliation. In particular to measure is of the nost important hypothesis in the above theorem is the positive entropy topotheris ($\forall t \in \mathbb{R}^k \mid 0 : ent_{\mu}(x_t) > 0$); strapping or weakening it would be a major advancement in the area. (Furstenberg, Rudolph, Katoh, Spetzier, Kalinin, RH, ...). In our setting it implies that the Lyaquar hyperplanes or wrided by the hill

D by K-K the (dimension one) Lyapunor foliations admit measurable hanstationary linearizations, which endows them with C' (Aff(R), R) - structures. The orbit foliation too has a natural affine structure. Assembling the Lyapunor foliations produces "split" C' affine structures for stable and custable foliations (for any weyl chamber w of (p, a)). The affine structure of the orbit foliation is compatible with these affine structures.

(2) By K-K-RH, M-leby; (orbit-) stable and (orbit-) unstable holonomies are well defined; they are diagonal linear in affine charts.

(3) Wing the dynamical holonomies and affine structures we can define a developing map dux: R* x R* > M. Unstable saturations of define a developing map dux: R* x R* X R* > M. Unstable saturations of

snall pieces of orbit-stable foliations cover M μ - \Re , so always pushes lebers forward to μ .

(4) By construction the symmetry group \mathcal{G}_{x} of dw_{x}^{w} is a subsprease of $\mathbb{R}^{d} \times \mathbb{D}_{s} \leq AH(\mathbb{R}^{d})$ ($\mathcal{D}_{d} = (\overset{\circ}{\cdot}, \overset{\circ}{\cdot})$); \mathcal{G}_{x}^{w} is the homoclime group of (μx) . There is a natural splitting $\mathcal{G}_{x}^{w} = \overset{\circ}{\mathcal{G}_{x}}^{w} \overset{\circ}{\mathcal{G}_{x}}^{w}$.

(3) We have

Rd dewx, M

Rk+4/gsu - R/gsu

Rd/gsu

space
crystal

time - Rd/Rk+3 yk

time crystal

Finally Sx = Zk and ysu = Zk+1 or Zk+1 x Et].

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Rd dew. M

Rd/gro and Rk+1/gra - Rd/gro

Rd/gro

space crystal

Finally grant and grant crystal

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