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University of Utah Spring 2024

MATH 2270-002
Midterm 3 Questions

Instructor: Alp Uzman

April 12 2024, 8:35 AM - 9:25 AM

Surname:

First Name:

uNID:

Before turning the page make sure to read and sign the exam policy document, distributed separately.

1. **[90 points]** Consider the following matrix:

$$A = \begin{pmatrix} 7 & 2 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{pmatrix}.$$

(a) **[20 points]** Write the characteristic equation of A.

(b) **[20 points]** Compute all eigenvalues of A.

(c) **[15 points]** For each distinct eigenvalue of A find a basis of the associated eigenspace.

(d) **[10 points]** Compute the algebraic and geometric multiplicities of each distinct eigenvalue of A.

(e) **[10 points]** Is A diagonalizable? If it is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

(f) **[5 points]** Is A orthogonally diagonalizable? If it is, find an orthogonal matrix Q and a diagonal matrix E such that $A = QEQ^T$.

(g) **[5 points]** Compute the orthogonal projection of the vector

$$b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

to the columnspace of A.

(h) **[5 points]** Compute the singular value decomposition of A.

2. **[7 points]** Compute the singular values of the following matrix:

$$A = \begin{pmatrix} 3 & 5 \\ 4 & 0 \end{pmatrix}.$$

3. **[3 points]** Let A be an $m \times n$ matrix and b be a vector in $\mathbb{R}^m$. Assume that the rank of A is n. Verify that the orthogonal projection of b onto the columnspace of A is given by

$$A(A^TA)^{-1}A^Tb.$$

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