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University of Utah Spring 2024

MATH 2270-002
Final Exam Questions

Instructor: Alp Uzman

May 1, 2024, 8:00 AM - 10:00 AM

Surname:

First Name:

uNID:

Before turning the page make sure to read and sign the exam policy document, distributed separately.

1. **[35 points]** Consider the following system of linear equations:

$$x_1 + 3x_2 + 4x_3 + 5x_4 = 0$$
$$2x_1 + 6x_2 + 9x_3 + 5x_4 = 0$$

(a) **[10 points]** Solve the system.

(b) **[10 points]** What is the dimension of the space of solutions?

(c) **[10 points]** Find a basis for the space of solutions.

(d) **[5 points]** Find a basis for the orthogonal complement of the space of solutions.

2. **[15 points]** Consider the following two matrices:

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{pmatrix}$$

Compute the matrix $B - 2A$.

3. **[25 points]** Consider the following matrix:

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) **[10 points]** Is A diagonalizable?

(b) **[5 points]** Compute the trace and determinant of A.

(c) **[5 points]** Is A invertible? If yes, compute the determinant of A^{-1}.

(d) **[5 points]** Compute the rank and nullity of A^T.

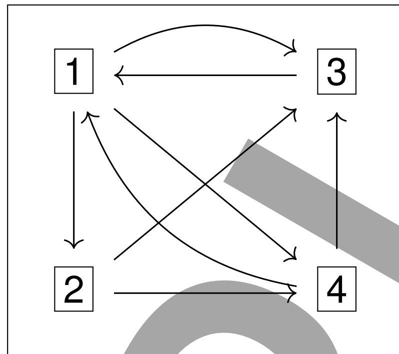
4. **[15 points]** Consider the following matrix:

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \\ 2 & -2 \end{pmatrix}.$$

(a) **[10 points]** Compute the singular values of A.

(b) **[5 points]** Find an orthonormal basis of $\mathbb{R}^2$ consisting of right singular vectors of A.

5. **[5 points]** Write the hyperlink matrix associated to the following network:



6. **[2 points]** This question is about the relation between rank one matrices and outer products of vectors.

   (a) **[1 point]** Verify that if $\mathrm{u} \in \mathbb{R}^{\mathrm{m}}$ and $\mathrm{v} \in \mathbb{R}^{\mathrm{n}}$ are two nonzero vectors, then the $\mathrm{m} \times \mathrm{n}$ matrix uv^{T} has rank one.

(b) **[1 point]** Verify that if A is a rank one $m \times n$ matrix with real entries, then there are two nonzero vectors $u \in \mathbb{R}^m$ and $v \in \mathbb{R}^n$ such that $A = uv^T$.

7. **[2 points]** Find all real numbers a, b, c such that the three functions

$$\sin(at),\ \sin(bt),\ \sin(ct)$$

are linearly independent.

8. **[1 point]** You are given the following scalar multiplication of a six dimensional vector with integer entries by 37:

$$37 \begin{pmatrix} 5492 \\ 11213 \\ 21180 \\ 7804 \\ 4120 \\ 18937 \end{pmatrix} = \begin{pmatrix} 203204 \\ 414881 \\ 783660 \\ 288748 \\ 152440 \\ 700669 \end{pmatrix}.$$

Storing each digit of each entry of the outcome vector in an entry, one obtains the following 6×6 matrix:

$$A = \begin{pmatrix} 2 & 0 & 3 & 2 & 0 & 4 \\ 4 & 1 & 4 & 8 & 8 & 1 \\ 7 & 8 & 3 & 6 & 6 & 0 \\ 2 & 8 & 8 & 7 & 4 & 8 \\ 1 & 5 & 2 & 4 & 4 & 0 \\ 7 & 0 & 0 & 6 & 6 & 9 \end{pmatrix}$$

Verify that the determinant of A is an integer multiple of 37.

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