

University of Utah

Spring 2024

MATH 2270-002 Midterm 1 Questions

Instructor: Alp Uzman

February 9 2024, 8:35 AM - 9:25 AM

| Surname: | |
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| First Name: | |
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Before turning the page make sure to read and sign the exam policy document, distributed separately.



1. [65 points] Consider the following system of linear equations:

$$x_1 + 3x_2 + 8x_3 - x_4 = 0$$

$$x_1 - 3x_2 - 10x_3 + 5x_4 = 0$$

$$x_1 + 4x_2 + 11x_3 - 2x_4 = 0$$

- (a) **[10 points]** How many equations are there in the system? How many unknowns (aka variables) are there in the system?
- (b) **[10 points]** Store the unknowns in a column vector x and write the system in matrix form Ax = b.

(c) **[10 points]** Write the augmented matrix that corresponds to the system.

(d) **[10 points]** Perform elementary row operations and compute an echelon form of the augmented matrix. Clearly state the elementary row operations you are applying in each step.

(e) [10 points] Find all solutions of the system.



(f) [10 points] What is the reduced echelon form of the coefficient matrix A?

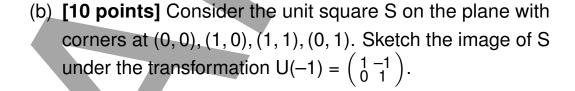
(g) [5 points] What is the rank and nullity of the coefficient matrix A?

2. **[32 points]** Consider the following two families of linear transformations of the plane:

$$G(t) = \begin{pmatrix} e^{t/2} & 0 \\ 0 & e^{-t/2} \end{pmatrix}, \quad U(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}.$$

Here $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \cdots$ is the Euler's number.

(a) **[10 points]** Consider the unit square S on the plane with corners at (0,0),(1,0),(1,1),(0,1). Sketch the image of S under the transformation $G(2) = \begin{pmatrix} e & 0 \\ 0 & e^{-1} \end{pmatrix}$.







(c) [4 points] Compute $G(t)^{-1}$.

(d) [4 points] Compute $U(s)^{-1}$.

(e) [4 points] Compute $G(t) U(s) G(t)^{-1}$.

Initials:

3. **[2 points]** Verify that the interchange $(R_i \leftrightarrow R_j)$ of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types (replacement $R_i \leftarrow aR_i + bR_j$ and scaling $R_i \leftarrow cR_i$).



4. **[1 point]** How many distinct 5×9 reduced echelon forms are there?