DO NOT
WRITE
HERE

University of Utah Spring 2024

MATH 2270-002
Midterm 1 Questions

Instructor: Alp Uzman

February 9 2024, 8:35 AM - 9:25 AM

Surname:

First Name:

uNID:

Before turning the page make sure to read and sign the exam policy document, distributed separately.

1. **[65 points]** Consider the following system of linear equations:

$$\begin{aligned} x_1 + 3x_2 + 8x_3 - x_4 &= 0 \\ x_1 - 3x_2 - 10x_3 + 5x_4 &= 0 \\ x_1 + 4x_2 + 11x_3 - 2x_4 &= 0 \end{aligned}$$

(a) **[10 points]** How many equations are there in the system? How many unknowns (aka variables) are there in the system?

(b) **[10 points]** Store the unknowns in a column vector x and write the system in matrix form Ax = b.

(c) **[10 points]** Write the augmented matrix that corresponds to the system.

(d) **[10 points]** Perform elementary row operations and compute an echelon form of the augmented matrix. Clearly state the elementary row operations you are applying in each step.

(e) **[10 points]** Find all solutions of the system.

(f) **[10 points]** What is the reduced echelon form of the coefficient matrix A?

(g) **[5 points]** What is the rank and nullity of the coefficient matrix A?

2. **[32 points]** Consider the following two families of linear transformations of the plane:

$$G(t) = \begin{pmatrix} e^{t/2} & 0 \\ 0 & e^{-t/2} \end{pmatrix}, \quad U(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}.$$

Here $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \cdots$ is the Euler's number.

(a) **[10 points]** Consider the unit square S on the plane with corners at $(0,0), (1,0), (1,1), (0,1)$. Sketch the image of S under the transformation $G(2) = \begin{pmatrix} e & 0 \\ 0 & e^{-1} \end{pmatrix}$.

(b) **[10 points]** Consider the unit square S on the plane with corners at $(0,0), (1,0), (1,1), (0,1)$. Sketch the image of S under the transformation $U(-1) = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

(c) **[4 points]** Compute $G(t)^{-1}$.

(d) **[4 points]** Compute $U(s)^{-1}$.

(e) **[4 points]** Compute $G(t)\,U(s)\,G(t)^{-1}$.

3. **[2 points]** Verify that the interchange ($R_i \leftrightarrow R_j$) of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types (replacement $R_i \leftarrow aR_i + bR_j$ and scaling $R_i \leftarrow cR_i$).

4. **[1 point]** How many distinct 5×9 reduced echelon forms are there?