University of Utah Spring 2024

MATH 2270-002 Midterm 2 Practice Problems

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This document is designed to prepare you for the second midterm and is divided into two main sections. The first section contains is a list of problems from previous problem sets that you should focus on. The second section introduces new problems covering topics that may not have been extensively explored in prior problem sets.

As you approach these practice problems, aim for thoroughness and clarity in your solutions, much like you would for regular problem set problems. However, given the stricter time constraints of the midterm, the expected level of detail in your solutions will be adjusted accordingly. A helpful strategy for exam preparation is to first solve a problem with as much detail as possible, without concern for time. Then, revisit your solution to condense it, focusing on optimizing presentation within the constraints of the exam format.

This document is for you to practice for the midterm and there is nothing to turn in. Consequently, the effort you invest in these practice problems will not directly contribute to your course grade. Nevertheless, it's worth mentioning that some of the problems in the second section may appear in future problem sets.

1 From Previous Problem Sets

For the second midterm, prioritize the following problems from previous problem sets. This emphasis does not necessarily imply that problems similar to those not listed here will be excluded from the exam.

PSet 5 1,2,3,4,5,6

PSet 6 1,2,5,6,7

2 Extra Problems

In what follows V is a finite dimensional vector space in one of the following forms:

- a linear subspace of a space of n-tuples of numbers:

$$V \subseteq \mathbb{R}^n, \qquad \text{or}$$

- a linear subspace of a space of matrices:

$$V \subseteq \mathcal{M}(m \times n; \mathbb{R}), \qquad \text{or}$$

- a linear subspace of the space of all single variable, real valued functions:

$$V \subseteq \mathcal{F}(\mathbb{R}; \mathbb{R}).$$

Choose such a V accordingly, and perform the following tasks:

1. Find two bases β, γ of V such that no vector in β is in γ (and vice versa). Verify that your claimed bases are indeed bases by verifying that they span V and are linearly independent.

2. Write any vector in β as a linear combination of vectors in β. Based on these linear combinations, write the column matrices of all vectors in β relative to the basis β.

3. Write any vector in β as a linear combination of vectors in γ. Based on these linear combinations, write the column matrices of all vectors in β relative to the basis γ.

4. Write any vector in γ as a linear combination of vectors in β. Based on these linear combinations, write the column matrices of all vectors in γ relative to the basis β.

5. Write any vector in γ as a linear combination of vectors in γ. Based on these linear combinations, write the column matrices of all vectors in γ relative to the basis γ.

6. Find the change-of-basis matrix from β to γ[1].

7. Find the change-of-basis matrix from γ to β.

8. Take the identity transformation id : V $\to$ V defined by id(v) = v.

   (a) Write the matrix of id relative to each of the following four options:

      i. β as input basis and β as output basis,
      ii. β as input basis and γ as output basis,
      iii. γ as input basis and β as output basis,
      iv. γ as input basis and γ as output basis.

[1] According to the universal diagram formalism this matrix is $\mathcal{M}_{\gamma\leftarrow\beta}(\,\mathrm{id}\,)$, where id : V $\to$ V is the identity transformation defined by id(v) = v.

(b) For any two of the four options above, compute the invertible matrix that transforms one to the other via left multiplication.

9. Take a linear transformation T : V $\to$ V.

   (a) Write the matrix of T relative to each of the following four options:

      i. β as input basis and β as output basis,
      ii. β as input basis and γ as output basis,
      iii. γ as input basis and β as output basis,
      iv. γ as input basis and γ as output basis.

   (b) For any two of the four options above, compute the invertible matrix that transforms one to the other via left multiplication.

10. Construct as many linear transformations from V to itself as possible.

11. Construct as many invertible linear transformations from V to itself as possible.

12. Describe all linear transformations from V to itself. Your description may refer to one or both of the bases β and γ.