University of Utah Spring 2024

Syllabus for MATH 2270-002: Linear Algebra

Instructor: Alp Uzman

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Contents

1 Staff and Meeting Hours

Instructor: Alp Uzman (uzman@math.utah.edu)

Lectures: Mon Tue Wed Fri 8:35 AM - 9:25 AM @ JWB 335

Office Hours: By appointment

2270 Coordinator: Peter Alfeld (pa@math.utah.edu)

2 Course Description & Prerequisites

MATH 2270 is a foundational course in linear algebra, essential for various STEM fields. As outlined in the general catalog, it is worth 4 credits and has the following prerequisites:

> C or better in ((MATH 2210 OR MATH 1260 OR MATH 1320 OR MATH 1321 OR MATH 2310) OR B or better in MATH 1220 OR AP Calculus BC score of 5.

It is not an exaggeration to say that linear algebra is the backbone of modern science. On the one hand, many systems are modelled via linear relations, and on the other hand, even when a model is inherently "nonlinear", one can try to "linearize" it to extract information about it[1]. In fact, one could say that a natural phenomenon admits

[1] An example of this is the process of computing a function's derivative, which is a form of linear approximation.

mathematical inquiry insofar as it admits such a linearization, albeit possibly in a very farfetched way.

In this course we'll start studying linear structures from a concrete and computational point of view via matrices and eventually also develop a more abstract understanding of linearities. We will discuss and study the proofs of many theorems such as the Spectral Theorem and Singular Value Decomposition, although our main concern ultimately will be the demonstration and application of such theorems. Through this course, students will gain not only theoretical knowledge but also practical skills relevant to real-world problems in science and technology.

3 Expectations

As a student, you are expected to:

- attend all lectures (four times per week),
- complete 11 problem sets,
- take 3 in-class midterm exams, and
- take a final exam.

According to the university regulations, you should expect a workload of approximately 8 to 12 hours per week outside of lecture hours. MATH 2270 is a fast-paced course covering a wide range of topics that are interconnected in intricate ways. Missing lectures or assignments can make it challenging to catch up. Please stay engaged and reach out if you encounter any disruptions. We're here to help, but your hard work and dedication will be essential to getting back on track!

4 Learning Objectives

This course aims to equip you with a robust set of mathematical skills, tools, and understanding that will be invaluable in more advanced classes and in various scientific and engineering fields. Among the objectives in MATH 2270 are that you learn how to:

- solve systems of linear equations,
- interpret matrices as linear transformations,
- perform algebra with matrices, including operatiohns like addition, scaling, multiplication and inversion.
- determine and manipulate linear coordinates,
- compute and interpret the determinant of square matrices,
- compute and interpret the spectral data of a square matrix,
- determine whether a given square matrix is diagonalizable and execute its diagonalization when possible,
- compute and interpret the inner product of two vectors, appreciating its applications in concepts like orthogonality,
- learn and apply the spectral and singular value decompositions, understanding their significance in various transformations,
- explore a variety of applications ranging from computer graphics to signal processing and PageRank.

5 Tentative Schedule

The course's subject matter is divided by weeks within the academic calendar as follows. For your convenience the approximate corresponding sections in the main textbook are also listed:

Week 1 (Week of 01/08): Systems of Linear Equations (§1.1-6) Linear Systems. Elementary Row Operations. Existence and Uniqueness of Solutions. Row Reduction. Echelon Forms. Linear Combinations. Linear Span. Matrix Equations.

Week 2 (Week of 01/15): Linear Transformations (§1.7-9) Linear Independence. Linear Transformations. Linear Geometry.

Week 3 (Week of 01/22): Algebra with Matrices (§2.1-5) Matrix Addition and Scaling. Matrix Multiplication. Matrix Powers. Transpose of a Matrix. (Multiplicative) Inverse of a Matrix. Elementary Matrices. Block Matrices. Matrix Factorizations.

Week 4 (Week of 01/29): Linear Subspaces (§2.8-9) Linear Subspaces of $\mathbb{R}^n$. Column Space of a Matrix. Null Space of a Matrix. Bases for Linear Subspaces. Linear Coordinates. Linear Dimension. Rank of a Matrix.

Week 5 (Week of 02/05): Computer Graphics (§2.7); Review & Midterm 1 Homogeneous Coordinates. Perspective Projections.

Week 6 (Week of 02/12): Determinants (§3.1-3) Determinant of a 2×2 Matrix. Determinant of a 3×3 Matrix. Determinant of an

Arbitrary Square Matrix. Determinant as Volume Distortion. Properties of Determinant.

Week 7 (Week of 02/19): Vector Spaces (§4.1-3) Axiomatic Definition of a Vector Space. Linear Subspaces. Linear Spans. Column Space of a Matrix. Null Space of a Matrix. Row Space of a Matrix. Kernel of a Linear Transformation. Image of a Linear Transformation. Linear Independence. Bases.

Week 8 (Week of 02/26): Linear Coordinates (§4.4-6) Bases as Linear Coordinates. Linear Charts. Linear Isomorphism. Linear Dimension. Change of Basis.

Week 9 (Week of 03/11): Signal Processing (§4.7-8); Review & Midterm 2 Discrete-Time Signals. Periodic Signals. Linear Time-Invariant Transformations. Linear Difference Equations.

Week 10 (Week of 03/18): Spectral Theory (§5.1-5) Eigenvalues and Eigenvectors of a Square Matrix. Characteristic Equation. Algebraic Multiplicity. Similarity. Diagonalization. Gershgorin Theorems.

Week 11 (Week of 03/25): Orthogonality (§6.1-4,6.7) Inner Product of Two Vectors in $\mathbb{R}^n$ Norm, Length, Distance. Orthogonality. Orthogonal Projection. Gram-Schmidt Process. QR Factorization. Axiomatic Definition of an Inner Product Space.

Week 12 (Week of 04/01): More Spectral Theory (§7.1-4) Symmetric Matrices. Spectral Theorem. Spectral Decomposition. Quadratic Forms. Principal Axes Theorem. Constrained Optimization of Quadratic

Forms. Singular Values of a Square Matrix. Condition Number. Moore-Penrose Inverse of a Square Matrix. Singular Value Decomposition.

Week 13 (Week of 04/08): Linear Dynamics (§5.6); Review & Midterm 3 Discrete-Time Dynamical Systems. Prey-Predator Systems. Attractors. Repellors. Saddles.

Week 14 (Week of 04/15): PageRank (§5.8-9,10.1-2) Power Method. Markov Chains. Probability Vectors. Column Stochastic Matrices. Random Surfer Model. PageRank.

Week 15 (Week of 04/22): Review

6 Grades

The final letter grades will be determined according to the following weights and cutoffs. The cutoffs below are not definitive; based on the overall performance it may be adjusted at the end of the semester. All such adjustments will affect all students in the section equally.

Assignment	Weight
PSets	35%
Midterm 1	15%
Midterm 2	15%
Midterm 3	15%
Final	20%
Total	100%

Grade	Percent
A	90%
A-	85%
B+	80%
B	75%
B-	70%
C+	65%
C	60%
C-	55%
D+	50%
D	45%
D-	40%
E	else

7 Books & Other Resources

The main textbook for MATH 2270 is the following:

> Lay, D. C., McDonald, J. J., Lay, S. R. *Linear Algebra and Its Applications, 6e*. Pearson Education (US). 2020. ISBN-13: 9780136880929.

By way of the Inclusive Access Program, an e-book will be available on Canvas. You will receive an email to your email account a week prior to class start date with the options regarding the textbook fee. Here's how it works:

- If you take no action, you will be automatically opted in and charged the textbook fee.
- If you do not wish to be charged the textbook fee, follow the instructions in the email to opt out.

As a secondary source, the linear algebra notes by Paul Dawkins of Lamar University is available on Canvas under the Pages tab.

In terms of videos, the linear algebra playlist on the Youtube channel 3Blue1Brown is recommended.

Further resources will be uploaded to the Canvas webpage if needed.

8 Lectures

Attendance at all lectures is expected, although it will not be formally kept track of.

On weeks when a midterm is scheduled (i.e. Weeks 5, 9, and 13), first two classes will focus on topics that are more "applied"; whereas the class before the midterm is reserved for review. Likewise, the two classes on the last week (i.e. Week 15) will be reserved for review.

Lecture Recordings Video recordings of lectures, excluding those reserved for reviews and midterms, will be uploaded weekly to the following YouTube playlist:

https://www.youtube.com/playlist?list=
PL40ydqvvyXfOBtKE5ZbyvjEOFfOoreeUF.

This is an experimental feature of the course. Recordings may become unavailable at any time and without notice; do not rely solely on them.

Extra Credit for Error Reporting Typos and mistakes are inevitable in the lecture recordings. Reporting an error, once validated, earns you 0.1 extra points towards your final grade. The specifics of this process will be outlined in a separate document on Canvas.

9 Problem Sets

Eleven problem sets will be assigned throughout the semester, typically on a weekly basis, excluding weeks with midterms. Each problem set is due by 11:59 PM on the Friday following its release. Late submissions, up to 24 hours after the deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires. A certain number of problem sets with lowest scores may be dropped at the end of the semester.

Grading Criteria The problem sets will be graded partly on completion and partly on accuracy, validity, and presentation/documentation.

Submission Details For each problem set, you are required to submit a form through Google Forms and digital copies of your work through Gradescope. Detailed submission instructions will be provided in each problem set specification.

Conflicting Instructions and Deadlines The submission deadlines and methods detailed in the problem set specifications (i.e., the PDF files embedded in the associated assignment webpages on Canvas), Canvas, and Gradescope may differ. Always adhere to the deadline and instructions in the specification document, as these will be the accurate guidelines.

10 Midterm Exams

There will be three in-class midterm exams scheduled as follows:

- **Midterm 1:** February 9 (Friday of Week 5) – covering topics from Week 1 through Week 5.
- **Midterm 2:** March 15 (Friday of Week 9) – covering topics from Week 6 through Week 9.
- **Midterm 3:** April 12 (Friday of Week 13) – covering topics from Week 10 through Week 13.

During the midterm exams, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exams will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exams were graded.

11 Final Exam

The final exam is scheduled in accordance with the university-wide final exam schedule. Details are as follows:

- **Date and Time:** May 1, from 8:00 AM to 10:00 AM.
- **Content:** The final exam will be cumulative, covering all topics from the course.

During the final exam, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exam will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exam was graded.

12 The Math Center

In addition to the office hours held by the course staff, the Math Center is an excellent resource for assistance with your studies. Located in

the basement connecting JWB and LCB, the Math Center offers free drop-in as well as online support.

13 Accessibility

The University of Utah seeks to provide equal access to its programs, services, and activities for people with disabilities. If you will need accommodations in this class, reasonable prior notice needs to be given to the Center for Disability & Access, UNION 162, (801) 581-5020. CDA will work with you and the staff to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability & Access.

14 Inclusivity and Safety

Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information.

If you or someone you know has been harassed or assaulted, you are encouraged to report it to:

- **Title IX Coordinator** in the Office of Equal Opportunity and Affirmative Action, PARK 135, (801) 581-8365, or
- **Office of the Dean of Students**, UNION 270, (801) 581-7066.

For support and confidential consultation, contact the Center for Student Wellness, SSB 328, (801) 581-7776. To report to the police, contact the Department of Public Safety, (801) 585-2677(COPS). For further safety resources, see SafeU.

15 Academic Honesty

You are expected to adhere to University of Utah policies regarding academic honesty. This means that ultimately all work you submit must be your own, created without unauthorized assistance, and all external resources, including generative tools and computer algebra systems, must be properly cited and documented within your submissions. Any student who engages in academic dishonesty or who violates the professional and ethical standards may be subject to academic sanctions as per the University of Utah's Student Code.

Generative AI and Computer Algebra Systems In this course, you may use generative tools like ChatGPT as well as computer algebra systems such as MATLAB under specific guidelines. These tools are permitted to guide your understanding of problem sets and topics. When using them, you must submit logs of your interactions as part of your problem set submissions. These tools can also be used to verify calculations, but remember, computations are generally expected to be done manually unless explicitly stated otherwise. You are responsible for ensuring the validity, accuracy, and relevance of your submissions, whether or not you use these tools. While encouraged, the use of such technology is not required for success in this class. The policy governing the use of such technology is specific to MATH 2270-002; other courses and sections may have different policies.

Regret Clause If you commit an unreasonable act but bring it to the staff's attention within 48 hours of the relevant submission, sanctions may be limited to that submission only, rather than leading to further disciplinary action. This clause will not be applied in the case of repeated violations.

16 Acknowledgements

The staff used ChatGPT in retouching this syllabus for clarity and readability.