University of Utah Spring 2024

MATH 2270-002 Midterm 3 Practice Problems

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Subject to Change; Last Updated: 2024-04-05 08:15:04-06:00

This document is designed to prepare you for the third midterm and is divided into two main sections. The first section contains is a list of problems from previous problem sets that you should focus on. The second section introduces new problems covering topics that may not have been extensively explored in prior problem sets.

As you approach these practice problems, aim for thoroughness and clarity in your solutions, much like you would for regular problem set problems. However, given the stricter time constraints of the midterm, the expected level of detail in your solutions will be adjusted accordingly. A helpful strategy for exam preparation is to first solve a problem with as much detail as possible, without concern for time. Then, revisit your solution to condense it, focusing on optimizing presentation within the constraints of the exam format.

This document is for you to practice for the midterm and there is nothing to turn in. Consequently, the effort you invest in these practice problems will not directly contribute to your course grade. Nevertheless, it's worth mentioning that some of the problems in the second section may appear in future problem sets.

1 From Previous Problem Sets

For the third midterm, prioritize the following problems from previous problem sets. This emphasis does not necessarily imply that problems similar to those not listed here will be excluded from the exam.

PSet 8 1,2,3,5,6

PSet 9 1,2

2 Extra Problems

1. Choose n to be a positive integer not larger than 6 and choose a symmetric n $\times$ n matrix A.

   (a) Compute all eigenvalues of A.

   (b) Find an orthonormal basis for each eigenspace of A.

   (c) Compute the algebraic and geometric multiplicities of each distinct eigenvalue of A.

   (d) Find an orthonormal basis of $\mathbb{R}^n$ that is also an eigenbasis of A.

   (e) Diagonalize A, if A is diagonalizable.

   (f) Orthogonally diagonalize A, if A is orthogonally diagonalizable.

   (g) Write the quadratic form $q : \mathbb{R}^n \to \mathbb{R}$ determined by A explicitly.

2. Choose n to be a positive integer not larger than 6 and choose a quadratic form $q : \mathbb{R}^n \to \mathbb{R}$.

(a) Write the symmetric matrix corresponding to q.

(b) Find an orthogonal change of coordinates matrix Q such that the quadratic form $q \circ Q$ has no mixed terms.

(c) Find the principal axes of q.

(d) Classify q as positive definite, positive semi-definite, negative semi-definite, negative definite, or indefinite.

3. Choose m and n to be positive integers not larger than 6 and choose an $m \times n$ matrix A.

(a) Compute the singular values of A.

(b) Compute the left singular vectors of A.

(c) Compute the right singular vectors of A.

(d) Compute the singular value decomposition of A.