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MATH 2270-002 Midterm 1 Practice Problems

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This document is designed to prepare you for the first midterm and is divided into two main sections. The first section contains is a list of problems from previous problem sets that you should focus on. The second section introduces new problems covering topics that may not have been extensively explored in prior problem sets.

As you approach these practice problems, aim for thoroughness and clarity in your solutions, much like you would for regular problem set problems. However, given the stricter time constraints of the midterm, the expected level of detail in your solutions will be adjusted accordingly. A helpful strategy for exam preparation is to first solve a problem with as much detail as possible, without concern for time. Then, revisit your solution to condense it, focusing on optimizing presentation within the constraints of the exam format.

This document is for you to practice for the midterm and there is nothing to turn in. Consequently, the effort you invest in these practice problems will not directly contribute to your course grade. Nevertheless, it's worth mentioning that some of the problems in the second section may appear in future problem sets.

1 From Previous Problem Sets

For the first midterm, prioritize the following problems from previous problem sets. This emphasis does not necessarily imply that problems similar to those not listed here will be excluded from the exam.

PSet 1 1,2,4,5,8

PSet 2 1,4,5,6,10

PSet 3 2,4,6,7

2 Extra Problems

Write down a system of linear equations with at most five equations and at most five unknowns. For the system you wrote, perform the tasks below. While you are solving the problems keep in mind the following considerations:

- Many of the problems are interconnected; for example, some may essentially be restatements of others in different terms. While you won't encounter every single problem below in the exam, you may encounter questions that, at their core, seek to understand the same concepts but are phrased differently.
- The solution to certain problems may directly inform the answers to others. For instance, once you have solved a linear system, determining the number of solutions becomes straightforward. While solving the below problems, look for such relationships. When a problem requires less information, challenge yourself to answer it without relying on the solutions to more complex

questions (e.g., try to ascertain the number of solutions a system has without fully solving it).

- Sometimes, the connections you notice might just be because of the specific system you're working with. Don't take your first observation as the final answer—try out different systems to see if what you found out still holds true.

1. How many equations are there in the system?
2. How many unknowns are there in the system?
3. Store the unknowns in a column vector x and write the system in the form $Ax = b$ using matrix notation.
4. How many rows does A have?
5. How many columns does A have?
6. Interpret A as a linear transformation $A : \mathbb{R}^n \to \mathbb{R}^m$. What are the numerical values for n and m?
7. Write the system as an augmented matrix $\begin{pmatrix} A & b \end{pmatrix}$.
8. Apply elementary row operations to obtain an echelon form of the augmented matrix $\begin{pmatrix} A & b \end{pmatrix}$.
9. Find all solutions to the system.
10. Plug in the solutions you found back into the original system and verify that they are correct.
11. Does the system have no solutions, a unique solution, or infinitely many solutions?
12. Is the space of solutions of the system a linear subspace of the input space?

13. Is the space of solutions of the system an affine but not linear subspace of the input space?

14. What is the dimension of the space of solutions?

15. Apply elementary row operations to obtain the reduced echelon form of the augmented matrix $\begin{pmatrix} A & b \end{pmatrix}$.

16. Apply elementary row operations to obtain an echelon form of the coefficient matrix A.

17. Apply elementary row operations to obtain the reduced echelon form of the augmented matrix $\begin{pmatrix} A & b \end{pmatrix}$.

18. Consider one of the elementary row operations you used to obtain the reduced echelon form of A. Write that elementary row operation as an elementary matrix E and interpret it as a linear transformation $E : \mathbb{R}^n \to \mathbb{R}^m$.

    (a) What are the numerical values for n and m?

    (b) Is E invertible? If so, compute the inverse E^{-1}.

    (c) Sketch how the transformation E transforms its input space.

    (d) Is there an nonzero integer k such that $E^k = I$?

19. Find an invertible matrix L such that for U the reduced echelon form of A, A = LU.

20. Find all invertible matrices L such that for U the reduced echelon form of A, A = LU.

21. Find all matrices L such that for U the reduced echelon form of A, A = LU.

22. Which columns of A are its pivot columns?

23. How many pivot columns does A have?
24. Are the columns of A linearly independent?
25. Can any column of A be written as a linear combination of the other columns of A?
26. Are the rows of A linearly independent?
27. Can any row of A be written as a linear combination of the other rows of A?
28. What is the rank of A?
29. What is the nullity of A?
30. Find a basis for the columnspace of A.
31. Find a basis for the nullspace of A.
32. What is the maximum number of linearly independent vectors in the columnspace of A?
33. What is the maximum number of linearly independent vectors in the nullspace of A?
34. What is the minimum number of vectors that span the columnspace of A?
35. What is the minimum number of vectors that span the nullspace of A?
36. Which columns of the reduced echelon form U of A are its pivot columns?
37. How many pivot columns does the reduced echelon form of A have?

38. What is the rank of the reduced echelon form of A?
39. What is the nullity of the reduced echelon form of A?
40. Find a basis for the columnspace of the reduced echelon form of A.
41. Find a basis for the nullspace of the reduced echelon form of A.
42. Find a linear transformation that maps the columnspace of the reduced echelon form of A onto the columnspace of A.
43. Find all linear transformations that map the columnspace of the reduced echelon form of A onto the columnspace of A.
44. Find a linear transformation that maps the columnspace of A onto the columnspace of the reduced echelon form of A.
45. Find all linear transformations that maps the columnspace of A onto the columnspace of the reduced echelon form of A.
46. Which columns of A^T are its pivot columns?
47. How many pivot columns does A^T have?
48. What is the rank of A^T?
49. What is the nullity of A^T?
50. Find a basis for the columnspace of A^T.
51. Find a basis for the nullspace of A^T.
52. Is $\left[A, A^T\right]$ syntactic? If so, compute it.