
University of Utah

Spring 2024

MATH 2270-002

Midterm 2 Questions

Instructor: Alp Uzman

March 15 2024, 8:35 AM - 9:25 AM

Surname:

First Name:

uNID:

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1. **[70 points]** Consider the permutation P defined by

$$P \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} x_4 \\ x_1 \\ x_3 \\ x_2 \\ x_5 \end{pmatrix}.$$

- (a) **[15 points]** Write the permutation P in matrix form.
- (b) **[10 points]** Compute P^T .
- (c) **[15 points]** Compute P^{-1} if P is invertible. If P is not invertible, state that P^{-1} does not exist.

(d) **[15 points]** Compute the determinant of P .

(e) **[15 points]** Compute the determinant of P^T .

2. **[15 points]** Let S be the parallelogram determined by the vectors

$$b_1 = \begin{pmatrix} -3 \\ 5 \end{pmatrix}, \quad b_2 = \begin{pmatrix} -3 \\ 8 \end{pmatrix}$$

and let $A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation given by

$$A = \begin{pmatrix} 3 & -4 \\ -4 & 6 \end{pmatrix}.$$

What is the area of the image of S under the linear transformation A ?

3. **[12 points]** Consider the following two bases of \mathbb{R}^2 :

$$\beta = ((7, 5), (-3, -1)), \quad \gamma = ((1, -5), (-2, 2)).$$

(a) **[6 points]** Compute the change-of-basis matrix from β to γ .

- (b) **[6 points]** Compute the change-of-basis matrix from γ to β .

4. **[3 point]** Let \mathcal{M} be the vector space of all 2×2 matrices with real entries. What is the dimension of the space of all linear transformations from \mathcal{M} to \mathcal{M} that commute with the transpose operation?

SAMPLE