University of Utah Spring 2026

MATH 2270-006 PSet 1 Specification

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Subject to Change; Last Updated: 2026-01-08 20:12:02-07:00

1 Background

This problem set focuses on solving systems of linear equations, elementary row operations and echelon forms. While throughout this course we'll learn how to engage with linearities at conceptually higher and higher levels, often once we unpack the definitions the matter at hand will reduce to solving a certain system of linear equations.

2 Generative AI and Computer Algebra Systems Regulations

This section applies only if you choose to use either a generative AI tool (e.g. chatbots) or a computer algebra system (CAS) while working on this problem set. If you do not use such tools, you may skip this section.

The use of these tools while completing this problem set is permitted, provided it is done responsibly and in a manner that supports your learning. Note however that in exams you will not be allowed to use any AI or CAS.

2.1 Disclosure

If you use such tools, you must disclose this fact in the designated section of the form you will complete as part of your submission. You must also share your reflections on which parts of the problem required your own mathematical judgment, insight, or decision-making, and which parts could reasonably be delegated to a computational or generative tool.

2.2 Guidelines for Responsible Use

If you use a generative AI tool or chatbot, you must adhere to the following guidelines:

- During the chat you may share with the chatbot parts or all of this specification document, as well as parts of the textbook or other sources.
- Directly asking the tool for complete problem solutions is prohibited.
- Appropriate uses include asking for conceptual explanations, checking intermediate steps, clarifying definitions or theorems, or exploring alternative solution approaches, provided that the final work submitted is your own.
- Regardless of the tools used, you are expected to understand, justify, and be able to reproduce all submitted work independently. The use of AI or CAS does not reduce or replace this expectation.

3 What to Submit

Submit your detailed solutions to each of the problems below. Though they may seem long, the additional text is meant to guide you by providing further context.

When documenting your solutions, be thorough. Your goal is not just to find the answer, but to create a clear, logical pathway to it that you or anyone else could follow in the future. It is likely that the textbook has the answers to some problems that are similar to some problems in this problem set; without further notice you may refer to these answers and reverse engineer them.

For instance problem 2 below is based on, **but not exactly the same as**, Exercise 38 of §1.1 in the book. The book has the answer to Exercise 37 of the same section, which is a variant of Exercise 38. Going forward, and possibly for the rest of this problem set, such correspondences will not be mentioned; it'll be up to you to locate them (if need be)!

Make sure that each solution is properly enumerated and organized. Start the solution to each problem on a new page, and consider using headings or subheadings to structure your response clearly. This will not only aid in your thought process but also ensure that no part of your solution is overlooked during grading.

1. Consider the following three systems of linear equations:

   I:

$$\begin{aligned} x_2 + 4x_3 &= -4 \\ x_1 + 3x_2 + 3x_3 &= -2 \\ 3x_1 + 7x_2 + 5x_3 &= 6 \end{aligned}$$

II:

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

III:

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

For each of these systems, perform the following tasks:

(a) Write the system in matrix form.

(b) Write the augmented matrix of the system.

(c) Solve the system.

(d) Verify that your solution is correct by plugging the values back into the original system.

2. Suppose a, b, c, and d are constants such that the system below is consistent for all possible values of f and g. What can you say about the numbers a, b, c, and d?

$$\begin{aligned} ax_1 + bx_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$$

3. Let

$$A = \begin{pmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{pmatrix}, \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

Describe the set of all b for which the equation $Ax = b$ does not have a solution.

4. List all possible echelon forms of a 4×5 matrix. Use the symbol ■ for an anonymous nonzero number and the symbol ★ for an anonymous number that is possibly zero.

5. Consider the following three matrices:

I:
$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

II:
$$A = \begin{pmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{pmatrix}$$

III:
$$A = \begin{pmatrix} 1 & 3 & -2 & 2 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{pmatrix}$$

In this problem, your job is to use WolframAlpha[1] to analyze the given matrices. For each of these matrices A, record the numerical values of the following **(We'll learn later what these words actually mean.)**:

- rank,

[1] The free version of WolframAlpha will suffice.

- nullity,
- determinant,
- trace,
- eigenvalues,
- singular values.

4 How to Submit

- **Step 1 of 2:** Submit the form at the following URL:

  https://forms.gle/ah2TVdmJCU5vksC86.

  You will receive a zero for this assignment if you skip this step, even if you submit your work on Gradescope on time.

- **Step 2 of 2:** Submit your work on Gradescope at the following URL:

  https://www.gradescope.com/courses/1212064/assignments/7390792/,

  see the Gradescope documentation for instructions.

5 When to Submit

This problem set is due on January 16, 2026 at 11:59 PM.

As per the course's syllabus, late submissions, up to 24 hours after this deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires.