University of Utah Spring 2026

# MATH 2270-006 PSet 4 Specification

Instructor: Alp Uzman

Subject to Change; Last Updated: 2026-02-06 12:01:42-07:00

## 1 Background

This problem set is about the concepts of bases for linear subspaces, and block matrices. It begins by examining linear subspaces, specifically focusing on identifying a basis for a subspace and investigating special subspaces associated with a matrix——its nullspace and columnspace. The notions of rank and nullity of a matrix are also related concepts, as rank is the dimension of the columnspace and nullity is the dimension of the nullspace. The last problem delves into block matrix algebra, highlighting the Schur complement, a concept with wide-ranging applications.

## 2 Generative AI and Computer Algebra Systems Regulations

This section applies only if you choose to use either a generative AI tool (e.g. chatbots) or a computer algebra system (CAS) while working on this problem set. If you do not use such tools, you may skip this section.

The use of these tools while completing this problem set is permitted, provided it is done responsibly and in a manner that supports your learning. Note however that in exams you will not be allowed to use any AI or CAS.

## 2.1 Disclosure

If you use such tools, you must disclose this fact in the designated section of the form you will complete as part of your submission. You must also share your reflections on which parts of the problem required your own mathematical judgment, insight, or decision-making, and which parts could reasonably be delegated to a computational or generative tool.

## 2.2 Guidelines for Responsible Use

If you use a generative AI tool or chatbot, you must adhere to the following guidelines:

- During the chat you may share with the chatbot parts or all of this specification document, as well as parts of the textbook or other sources.
- Directly asking the tool for complete problem solutions is prohibited.
- Appropriate uses include asking for conceptual explanations, checking intermediate steps, clarifying definitions or theorems, or exploring alternative solution approaches, provided that the final work submitted is your own.

- Regardless of the tools used, you are expected to understand, justify, and be able to reproduce all submitted work independently. The use of AI or CAS does not reduce or replace this expectation.

## 3 What to Submit

Submit your detailed solutions to each of the problems below. Though they may seem long, the additional text is meant to guide you by providing further context.

When documenting your solutions, be thorough. Your goal is not just to find the answer, but to create a clear, logical pathway to it that you or anyone else could follow in the future. It is likely that the textbook has the answers to some problems that are similar to some problems in this problem set; without further notice you may refer to these answers and reverse engineer them.

Make sure that each solution is properly enumerated and organized. Start the solution to each problem on a new page, and consider using headings or subheadings to structure your response clearly. This will not only aid in your thought process but also ensure that no part of your solution is overlooked during grading.

1. Consider the following sets of vectors:

**I**:

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ -5 \end{pmatrix} \right\}$$

**II**:

$$\left\{ \begin{pmatrix} 1 \\ -3 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} -3 \\ 9 \\ -6 \\ 12 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 5 \\ -3 \\ 7 \end{pmatrix} \right\}$$

**III**:

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ -1 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -6 \\ 8 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \\ -7 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ -8 \\ 9 \\ -5 \end{pmatrix} \right\}$$

For each of the sets of vectors, perform the following tasks:

(a) Find a basis for their span.

(b) Compute the dimension of their span.

2. Find a basis for the space of solutions of the following system of linear equations:

$$\begin{aligned} x_1 + 3x_2 + 2x_3 + 5x_4 - x_5 &= 0 \\ 2x_1 + 7x_2 + 4x_3 + 11x_4 + 2x_5 &= 0 \\ 2x_1 + 6x_2 + 5x_3 + 12x_4 - 7x_5 &= 0 \end{aligned}$$

3. Consider the following matrix:

$$A = \begin{pmatrix} 1 & -2 & 9 & 5 & 4 \\ 1 & -1 & 6 & 5 & -3 \\ -2 & 0 & -6 & 1 & -2 \\ 4 & 1 & 9 & 1 & -9 \end{pmatrix}$$

Perform the following tasks:

 https://alpuzman.github.io/
uzman@math.utah.edu

(a) Find an invertible matrix $L$ such that $A = LU$, where $U$ is an echelon form of $A$.

(b) Compute the rank and nullity of each of $U, A, U^T, A^T, L$.

(c) Find a basis for the columnspace of each of $U, A, U^T, A^T, L$.

(d) Find a basis for the nulspace of each of $U, A, U^T, A^T, L$.

4. Let $A, B, C, D$ be four matrices with the following dimensions:

   - $A$ is $p \times p$,
   - $B$ is $p \times q$,
   - $C$ is $q \times p$,
   - $D$ is $q \times q$.

   Store these four matrices in one $(p+q) \times (p+q)$ matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}.$$

   Much like how one can do block matrix addition and block matrix multiplication, one can also do elementary block row operations and consequently do block row reduction.

   (a) Suppose the matrix $A$ on the top left corner is invertible and abbreviate the matrix $D - CA^{-1}B$ by $M/A$. $M/A = D - CA^{-1}B$ is called the Schur complement of $A$ with respect to $M$. Using elementary block row operations, verify that $M$ block row reduces to the block echelon form

$$\begin{pmatrix} I & A^{-1}B \\ 0 & M/A \end{pmatrix}.$$


uzman@math.utah.edu

(b) Suppose $A$ is invertible and solve the following equation for matrices $X$ and $Y$:

$$M = \begin{pmatrix} I & 0 \\ X & I \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & M/A \end{pmatrix} \begin{pmatrix} I & Y \\ 0 & I \end{pmatrix}.$$

(c) Suppose both $M$ and $A$ are invertible. Find a formula for the block inverse of $M$. Such a formula would display $M^{-1}$ as a block matrix each of whose entries is a matrix in terms of the blocks $A, B, C$ and $D$:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix}.$$

In the case when each block is $1 \times 1$, so that $p = q = 1$ and $M$ is a $2 \times 2$ matrix with real entries, your formula ought to reduce to the formula for inverses of $2 \times 2$ matrices:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

# 4 How to Submit

- **Step 1 of 2:** Submit the form at the following URL:

  https://forms.gle/t2jhan68V7898gvJ9.

  You will receive a zero for this assignment if you skip this step, even if you submit your work on Gradescope on time.

- **Step 2 of 2:** Submit your work on Gradescope at the following URL:

https://www.gradescope.com/courses/1212064/assignments/7390793/,

see the Gradescope documentation for instructions.

## 5 When to Submit

This problem set is due on February 13, 2026 at 11:59 PM.

Late submissions, up to 24 hours after this deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires.