University of Utah — Spring 2026

# MATH 2270-006 PSet 3 Specification

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Subject to Change; Last Updated: 2026-01-23 11:13:07-07:00

## 1 Background

This problem set is centered on algebra with matrices, covering matrix addition, scalar multiplication, transposes, as well as matrix multiplication and commutators. One of the key properties we'll explore is that matrix multiplication, for certain square matrices, exhibits similarities to number multiplication. For instance, some square matrices $A$ have an inverse matrix $A^{-1}$, defined as a square matrix of the same dimensions as $A$ satisfying the equations

$$AA^{-1} = I = A^{-1}A,$$

$I$ being the identity matrix, that is, the diagonal matrix with ones on the diagonal. While a crucial condition[1] for a square matrix to be invertible will be discussed later in the course, at this stage what is important is to develop familiarity with row reduction techniques to compute the inverses of given matrices, assuming they exist.

[1] For inverting numbers, aka $1 \times 1$ matrices, this condition simply states that the number cannot be zero, as "dividing by zero" is undefined.

**Commutator** A significant point of divergence between number multiplication and matrix multiplication of square matrices is the lack of commutativity in the latter. Generally,

$$AB \neq BA,$$

which means it's vital to distinguish between these two products. This difference is quantified by the **commutator (or bracket)**

$$[A, B] = AB - BA.$$

**Arithmetic with Complex Numbers** Some problems in this problem set involve complex numbers. In case you need a refresher on how to do arithmetic with complex numbers, here is all you need to know for this problem set: A complex number is by definition an expression of the form

$$a + ib,$$

where a and b are two real numbers, and i is the so-called imaginary unit. Thus in a sense one complex number is two real numbers written in a peculiar way. One can then add two complex numbers like so:

$$(a + ib) + (c + id) = (a + c) + i(b + d),$$

and multiply two complex numbers like so:

$$(a + ib)(c + id) = (ac - bd) + i(ad + bc).$$

Note that in particular by plugging in $a = 0, b = 1, c = 0, d = 1$ we have


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$$i^2 = (0 + i1)(0 + i1) = (0 \cdot 0 - 1 \cdot 1) + i(0 \cdot 1 + 1 \cdot 0) = -1 + i0 = -1,$$

so that one can think of i as a number whose square is $-1$!

One denotes the set of complex numbers by $\mathbb{C}$. The topics we have covered in this class so far extends to the case when one uses complex numbers instead of real numbers. In particular, allowing "scalars" to be complex numbers, one can consider systems of linear equations with complex coefficients, and do matrix algebra with matrices with complex entries.

# 2 Generative AI and Computer Algebra Systems Regulations

This section applies only if you choose to use either a generative AI tool (e.g. chatbots) or a computer algebra system (CAS) while working on this problem set. If you do not use such tools, you may skip this section.

The use of these tools while completing this problem set is permitted, provided it is done responsibly and in a manner that supports your learning. Note however that in exams you will not be allowed to use any AI or CAS.

## 2.1 Disclosure

If you use such tools, you must disclose this fact in the designated section of the form you will complete as part of your submission. You must also share your reflections on which parts of the problem required your own mathematical judgment, insight, or decision-making, and which parts could reasonably be delegated to a computational or generative tool.

### 2.2 Guidelines for Responsible Use

If you use a generative AI tool or chatbot, you must adhere to the following guidelines:

- During the chat you may share with the chatbot parts or all of this specification document, as well as parts of the textbook or other sources.
- Directly asking the tool for complete problem solutions is prohibited.
- Appropriate uses include asking for conceptual explanations, checking intermediate steps, clarifying definitions or theorems, or exploring alternative solution approaches, provided that the final work submitted is your own.
- Regardless of the tools used, you are expected to understand, justify, and be able to reproduce all submitted work independently. The use of AI or CAS does not reduce or replace this expectation.

## 3 What to Submit

Submit your detailed solutions to each of the problems below. Though they may seem long, the additional text is meant to guide you by providing further context.

When documenting your solutions, be thorough. Your goal is not just to find the answer, but to create a clear, logical pathway to it that you or anyone else could follow in the future. It is likely that the textbook has the answers to some problems that are similar to some problems in this problem set; without further notice you may refer to these answers and reverse engineer them.

Make sure that each solution is properly enumerated and organized. Start the solution to each problem on a new page, and consider using headings or subheadings to structure your response clearly. This will not only aid in your thought process but also ensure that no part of your solution is overlooked during grading.

1. Consider the following five matrices:

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{pmatrix},$$
$$C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 3 & 5 \\ -1 & 4 \end{pmatrix}, \qquad E = \begin{pmatrix} -5 \\ 3 \end{pmatrix}.$$

Some of the following expressions involving matrix addition, scalar multiplication, matrix multiplication and matrix transpose are syntactic (aka they are "grammatical"), whereas some are not. For each of the following expressions, compute the expression by writing it in one matrix if the expression is syntactic, or state that it is not syntactic otherwise.

(a) $B - 2A$

(b) $\left(\left(\left(A^T\right)^T\right)^T\right)^T$

(c) $B^T + A$

(d) $AC$

(e) $A^T C$

(f) $\left[A, B^T\right]$

(g) $\left[[C, D], AB^T\right]$

(h) $CDC^T$

(i) $\frac{C+C^T}{2}$

(j) $3C - E$

(k) $CB$

(l) $EB$

(m) $E^T B$

(n) $E^T E$

(o) $E^2$

(p) $EE^T$

2. Let $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ be an anonymous $2 \times 2$ matrix. Verify the Cayley-Hamilton identity by computing both sides and comparing the resulting matrices entry-by-entry:

$$A^2 = (a + d)A - (ad - bc)I$$

where $I = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right)$ is the $2 \times 2$ identity matrix. Note that the Cayley-Hamilton identity displays the square $A^2 = AA$ of any $2 \times 2$ matrix $A$ as a certain linear combination of $A$ and $I$.

3. Consider the following $2 \times 2$ matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Here i is the imaginary unit.

(a) Fill in the multiplication table by computing the products of matrices. Ensure the matrix from the row is always the first factor. Here $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Try to express the end results in terms of the matrices already given.

| mult. | I | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ |
|---|---|---|---|---|
| I | | | | |
| $\sigma_1$ | | | | |
| $\sigma_2$ | | | | |
| $\sigma_3$ | | | | |


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(b) Fill in the commutator table by computing the commutators of matrices. Ensure the matrix from the row is always the first entry in the commutator. Here $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Try to express the end results in terms of the matrices already given.

| comm. | I | $\sigma_1$ | $\sigma_2$ | $\sigma_3$ |
|---|---|---|---|---|
| I | | | | |
| $\sigma_1$ | | | | |
| $\sigma_2$ | | | | |
| $\sigma_3$ | | | | |


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4. Consider the following matrix:

$$A = \begin{pmatrix} 8 & 3 \\ 5 & 2 \end{pmatrix}$$

Perform the following tasks:

(a) Compute the (multiplicative) inverse $A^{-1}$ of A.

(b) Verify that $AA^{-1} = I = A^{-1}A$ by multiplying the matrices and comparing the corresponding entries.

(c) Verify that $A^{-1}(2, -1)$ is the unique solution to the system

$$\begin{aligned} 8x_1 + 3x_2 &= 2 \\ 5x_1 + 2x_2 &= -1 \end{aligned}$$

5. Compute the inverses of each of the following matrices:

(a)

$$\begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

# 4 How to Submit

- **Step 1 of 2:** Submit the form at the following URL:

https://forms.gle/xzPyyugmrojCT49s8.

You will receive a zero for this assignment if you skip this step, even if you submit your work on Gradescope on time.

- **Step 2 of 2:** Submit your work on Gradescope at the following URL:

  https://www.gradescope.com/courses/1212064/assignments/7390801/,

  see the Gradescope documentation for instructions.

## 5 When to Submit

This problem set is due on January 30, 2026 at 11:59 PM.

Late submissions, up to 24 hours after this deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires.