University of Utah Spring 2026

MATH 2270-006 PSet 2 Specification

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Subject to Change; Last Updated: 2026-01-22 10:08:12-07:00

1 Background

This problem set delves into three central concepts in linear algebra: linear spans, linear independence and the interpretation of matrices as linear transformations.

We start by exploring complementary concepts of linear span and linear independence. While linear span concerns "accessibility" — the places that can be reached using a given set of vectors — linear independence focuses on "non-redundancy". It examines whether removing a vector from a set of vectors limits the reach or span that could otherwise be achieved with the full set.

The problems on matrices shifts focus from their role in linear equations to their geometric significance as linear transformations. This perspective reveals matrices as dynamic mathematical objects influencing geometric space.

2 Generative AI and Computer Algebra Systems Regulations

This section applies only if you choose to use either a generative AI tool (e.g. chatbots) or a computer algebra system (CAS) while working on this problem set. If you do not use such tools, you may skip this section.

The use of these tools while completing this problem set is permitted, provided it is done responsibly and in a manner that supports your learning. Note however that in exams you will not be allowed to use any AI or CAS.

2.1 Disclosure

If you use such tools, you must disclose this fact in the designated section of the form you will complete as part of your submission. You must also share your reflections on which parts of the problem required your own mathematical judgment, insight, or decision-making, and which parts could reasonably be delegated to a computational or generative tool.

2.2 Guidelines for Responsible Use

If you use a generative AI tool or chatbot, you must adhere to the following guidelines:

- During the chat you may share with the chatbot parts or all of this specification document, as well as parts of the textbook or other sources.

- Directly asking the tool for complete problem solutions is prohibited.
- Appropriate uses include asking for conceptual explanations, checking intermediate steps, clarifying definitions or theorems, or exploring alternative solution approaches, provided that the final work submitted is your own.
- Regardless of the tools used, you are expected to understand, justify, and be able to reproduce all submitted work independently. The use of AI or CAS does not reduce or replace this expectation.

3 What to Submit

Submit your detailed solutions to each of the problems below. Though they may seem long, the additional text is meant to guide you by providing further context.

When documenting your solutions, be thorough. Your goal is not just to find the answer, but to create a clear, logical pathway to it that you or anyone else could follow in the future. It is likely that the textbook has the answers to some problems that are similar to some problems in this problem set; without further notice you may refer to these answers and reverse engineer them.

Make sure that each solution is properly enumerated and organized. Start the solution to each problem on a new page, and consider using headings or subheadings to structure your response clearly. This will not only aid in your thought process but also ensure that no part of your solution is overlooked during grading.

1. Consider the following two vectors in $\mathbb{R}^2$:

$$u = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \qquad v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Display each of the following vectors using arrows on the xy-plane:

(a) u

(b) v

(c) $3u - 2v$

(d) $u + v - 2u - 4v + 3u$

2. Let $A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 1 \\ -4 \end{pmatrix}$. Denote by W the span of the columns of A.

(a) Is b in W? How many vectors are in W?

(b) Is the first column of A is in W?

(c) Is $W = \mathbb{R}^3$?

3. Determine if the given sets of vectors are linearly independent.

(a)

$$\left\{ \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ 6 \end{pmatrix} \right\}$$

(b)

$$\left\{ \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix} \right\}$$

https://alpuzman.github.io/
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(c)

$$\left\{ \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} \right\}$$

(d)

$$\left\{ \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

(e)

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$$

(f)

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ -8 \end{pmatrix}, \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} \right\}$$

(g)

$$\left\{ \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ -8 \end{pmatrix}, \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 41 \\ -37 \\ 1/463 \end{pmatrix} \right\}$$

4. Consider the following three 2×2 matrices:

I:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 1/3 \end{pmatrix}$$

II:

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

III:

$$A = \begin{pmatrix} 0 & -2 \\ 1 & 0 \end{pmatrix}$$

Interpret each of these matrices as a linear transformation of the plane, and for each of these matrices, perform the following tasks:

(a) Use a coordinate system to plot

$$u = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \quad v = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$

and their images under the linear transformation.

(b) Describe geometrically what the linear transformation does to an anonymous vector in $\mathbb{R}^2$.

5. In each part of this problem you are given a certain description as to how a linear transformation operates. Your job is to write down the corresponding matrix.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^4$, $T(e_1) = (2, 1, 2, 1)$ and $T(e_2) = (-5, 2, 0, 0)$, where $e_1 = (1, 0)$ and $e_2 = (0, 1)$.

(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points (about the origin) through $3\pi/2$ radians (in the counterclockwise direction).

(c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a vertical shear transformation that maps e_1 into $e_1 - 2e_2$ but leaves the vector e_2 unchanged.

(d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ first rotates points through $-3\pi/4$ radians and then reflects points through the horizontal x_1-axis.

(e) $T : \mathbb{R}^2 \to \mathbb{R}^2$ first performs a horizontal shear that transforms e_2 into $e_2 - 3e_1$ (leaving e_1 unchanged) and then reflects points through the line $x_2 = -x_1$ (the line spanned by $e_1 - e_2$).

4 How to Submit

- **Step 1 of 2:** Submit the form at the following URL:

  https://forms.gle/vPKsP5FJtW4pe45u7.

  You will receive a zero for this assignment if you skip this step, even if you submit your work on Gradescope on time.

- **Step 2 of 2:** Submit your work on Gradescope at the following URL:

  https://www.gradescope.com/courses/1212064/assignments/7390800,

  see the Gradescope documentation for instructions.

5 When to Submit

This problem set is due on January 23, 2026 at 11:59 PM.

Late submissions, up to 24 hours after this deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires.