University of Utah Spring 2026

MATH 2270-006
Midterm 1 Questions

Instructor: Alp Uzman

February 6 2026, 9:40 AM - 10:30 AM

Surname: KEY

First Name:

uNID:

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1. **[15 points]** Write the following system of linear equations in matrix form:

$$x_2 + 4x_3 = -4$$
$$x_1 + 3x_2 + 3x_3 = -2$$
$$3x_1 + 7x_2 + 5x_3 = 6$$

$$\begin{pmatrix} 0 & 1 & 4 \\ 1 & 3 & 3 \\ 3 & 7 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}.$$

2. **[15 points]** Write the following system of linear equations in augmented matrix form:

$$x_1 - 3x_3 = 8$$
$$2x_1 + 2x_2 + 9x_3 = 7$$
$$x_2 + 5x_3 = -2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right).$$

3. **[20 points]** Find all solutions of the following system of linear equations. **Partial credit will be given for wrong numerical answers only if you also check to see if your answer solves the system.**

$$x_1 + 3x_2 + 8x_3 - x_4 = 0$$
$$x_1 - 3x_2 - 10x_3 + 5x_4 = 0$$
$$x_1 + 4x_2 + 11x_3 - 2x_4 = 0$$

$$\left(\begin{array}{cccc|c} 1 & 3 & 8 & -1 & 0 \\ 1 & -3 & -10 & 5 & 0 \\ 1 & 4 & 11 & -2 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 3 & 8 & -1 & 0 \\ 0 & -6 & -18 & 6 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 3 & 8 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 1 & 3 & -1 & 0 \end{array}\right)$$

$$\downarrow$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$\Leftarrow$

$$x_1 - x_3 + 2x_4 = 0$$
$$x_2 + 3x_3 - x_4 = 0$$
x_3, x_4 free.

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4. **[30 points]** For this problem consider the following matrix:

$$A = \begin{pmatrix} 1 & 3 & -2 & 2 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{pmatrix}.$$

(a) **[15 points]** Compute the rank (aka r-number) of A.

$$\begin{pmatrix} 1 & 3 & -2 & 2 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ 0 & -4 & -4 & 13 \end{pmatrix} \to \begin{pmatrix} 0 & 1 & 1 & -5 \\ 1 & 0 & -5 & 17 \\ 0 & 0 & 0 & \blacksquare \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \leftarrow \begin{pmatrix} 1 & 0 & -5 & 17 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3 pivot cols $\Rightarrow$ rank(A) = 3.

(b) **[15 points]** Compute AA^T.

$$AA^T = \begin{pmatrix} 1 & 3 & -2 & 2 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{pmatrix}_3 \begin{pmatrix} 1 & 1 & -2 \\ 3 & 2 & -8 \\ -2 & -3 & 2 \\ 2 & 7 & -1 \end{pmatrix}_3$$

$$= \begin{pmatrix} 1+9+4+4 & 1+6+6+14 & -2-24-4-2 \\ 1+6+6+14 & 1+4+9+49 & -2-16-6-7 \\ -2-24-4+2 & -2-16-6-7 & 4+64+4+1 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 27 & -32 \\ 27 & 63 & -31 \\ -32 & -31 & 73 \end{pmatrix}$$

$(AA^T)^T = (A^T)^T A^T$

$= A A^T$

$\Rightarrow AA^T$ sym!

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5. **[15 points]** Determine if the following matrix is invertible:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

No: cols are LD (3rd col = 4th col)

$\Rightarrow$ rank(A) < 5 $\Rightarrow$ not inv.

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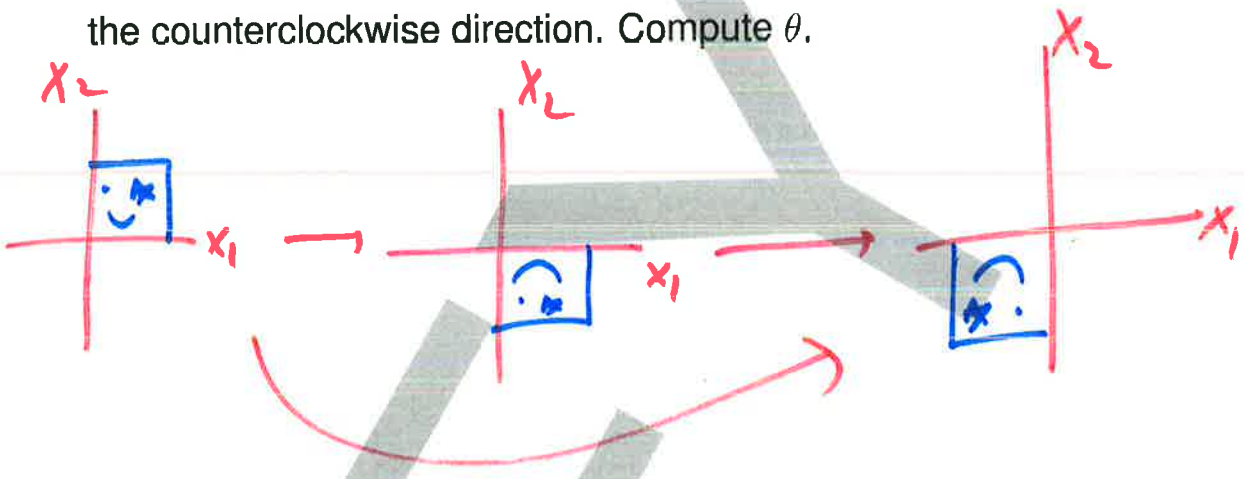
6. **[5 points]** Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects points through the horizontal x_1-axis and then reflects points through the vertical x_2-axis. You are also given that there is an angle θ such that T can be described as the linear transformation that rotates points about the origin by θ in the counterclockwise direction. Compute θ.



T is rotation by 180°.

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