

Syllabus for MATH 3210-001: Foundations of Analysis I

Instructor: Alp Uzman

Subject to Change; Last Updated: 2024-12-28 17:23:12-07:00

Contents

1 Staff and Meeting Hours	2
2 Course Description & Prerequisites	2
3 Expectations	3
4 Learning Objectives	3
5 Tentative Schedule	5
6 Grades	8
7 Books & Other Resources	9
8 Lectures	9
9 Problem Sets	10
10 Midterm Exams	11
11 Final Exam	11
12 The Math Center	12

13 Accessibility	12
14 Inclusivity and Safety	12
15 Academic Honesty	13
16 Acknowledgements	14

1 Staff and Meeting Hours

Instructor: Alp Uzman (uzman@math.utah.edu)

Lectures: Mon Tue Wed Fri 9:40 AM - 10:30 AM @ **AEB** 360

Canvas: <https://utah.instructure.com/courses/1033505>

Office Hours: By appointment

3210 Coordinator: Dragan Milicic (milicic@math.utah.edu)

2 Course Description & Prerequisites

MATH 3210 is a course focusing on single variable calculus from a rigorous point of view, serving as a transition course from calculus to analysis. As outlined in the [general catalog](#), it is worth 4 credits and has the following prerequisites:

'C' or better in (MATH 1260 OR MATH 1321 OR MATH 2210 OR MATH 2310 OR MATH 3140) AND (MATH 2200 OR MATH 2250 OR MATH 2270 OR MATH 2271)

This is a proof based class, and the emphasis will be on developing "[mathematical maturity](#)", in particular on rigorous argumentation, rather than calculation of the values of certain limits, integrals or

derivatives. We'll start with elementary logic and set theory, discuss main methods of documentation that are up to the modern standards of mathematical rigor, and then discuss differential and integral calculus of single variable, real valued functions. As adjacent topics we'll study the convergence and divergence of sequences and series of numbers and functions.

3 Expectations

As a student, you are expected to:

- attend all lectures (four times per week),
- complete 12 problem sets,
- take 2 in-class midterm exams, and
- take a final exam.

According to the [university regulations](#), you should expect a workload of approximately 8 to 12 hours per week outside of lecture hours. You will have some familiarity with many ideas and notions we will discuss throughout the semester. However the focus on rigor can be hard to get accustomed to. Missing lectures or assignments can make it challenging to catch up. Please stay engaged and reach out if you encounter any disruptions. We're here to help, but your hard work and dedication will be essential to getting back on track!

4 Learning Objectives

Among the objectives in MATH 3210 are that you learn how to:

- use logical quantifiers,
- describe statements and the operations of AND, OR, and NOT in terms of sets,
- identify injective, surjective and bijective relations and functions,
- parse mathematical proofs,
- distinguish levels of rigor in mathematical prose,
- write proofs in accordance with modern standards of mathematical rigor,
- approach numbers from a bottom-up point of view and make use of the completeness axiom and the Archimedean property,
- use the notions of supremum and infimum,
- derive the consequences of the Triangular Inequality, for numbers and functions,
- use sequences and series of numbers and functions,
- identify when a sequence or series converges and in which sense,
- use the notions of accumulation points, limit inferior and limit superior of a sequence of numbers,
- prove and use the fundamental theorems governing continuous functions, including the Maximum Principle and Intermediate Value Theorem,
- distinguish continuous functions from uniformly continuous functions,

- prove and use the fundamental theorems governing differentiable functions, including the Chain Rule, Inverse Function Theorem, Mean Value and Rolle Theorems, and L'Hôpital's Rule,
- define the notion of Riemann integrability,
- prove and use the Fundamental Theorems of Calculus,
- prove and use the fundamental theorems governing Riemann integrable functions, including the Change of Variables Formula and integration by parts,
- define elementary functions like exponential and logarithm,
- prove and use Taylor approximation.

5 Tentative Schedule

The course's subject matter is divided by weeks within the **academic calendar** as follows. For your convenience the approximate corresponding sections in the main textbook are also listed. Time permitting, additional topics¹ may be covered.

Week 1 (Week of 01/06): Functions (§1.1) Logic: "For all", "there exists". Set theory: "element of", "subset of", intersection, union, complement. Relations and functions: composition, inversion, graph of a function, injection (one-to-one), surjection (onto), bijection, reflexivity, transitivity, symmetry, antisymmetry. Proof methods: direct proof, proof by contrapositive, proof by contradiction, proof by induction.

¹Among candidate additional topics are: derived set of a subset of reals, Period Three Implies Chaos, Sharkovsky Theorem, moduli of continuity, Lipschitz and Hölder continuity, Banach Contraction Principle, Lipschitz Inverse Function Theorem, equicontinuity, Arzelà-Ascoli Theorem, Faà di Bruno Formula, Schwarzian derivative, depth-1 feedforward neural networks, weak derivatives, Stieltjes integral, Riemann zeta function, Stirling formula. Some of these topics may show up as part of a problem set.

Week 2 (Week of 01/13): Reals (§1.2-5) Natural numbers. Integers. Rational numbers. Real numbers. Completeness axiom. Archimedean Property. Absolute value. Triangular inequality. Extended reals. Supremum, infimum, maximum, minimum of a subset of reals. Supremum, infimum, maximum, minimum of a real valued function.

Week 3 (Week of 01/20): Sequences (§2.1-6) Sequences. Subsequences. Metric spaces. Convergent and divergent sequences. Accumulation points of a sequence. Monotone sequences. Cauchy sequences. Bolzano-Weierstrass Theorem. Limit inferior and limit superior of a sequence.

Week 4 (Week of 01/27): Continuity 1 (§3.1-4) Continuous functions. Maximum principle. Intermediate Value Theorem. Monotone functions. Homeomorphisms. Uniformly continuous functions. Sequences of functions. Pointwise convergence of a sequence of functions.

Week 5 (Week of 02/03): Continuity 2 (§3.4,4.1); Review & Midterm 1 Uniform convergence of a sequence of functions. Uniform metric. Uniformly Cauchy sequences of functions. Limits of functions.

Week 6 (Week of 02/10): Differential Calculus 1 (§4.2) Derivative of a function at a point. Differentiable functions. Chain Rule. Diffeomorphisms. Inverse Function Theorem.

Week 7 (Week of 02/17): Differential Calculus 2 (§4.3-4) Mean Value Theorem. Rolle Theorem. L'Hôpital's Rule.

Week 8 (Week of 02/24): Integral Calculus 1 (§5.1-2) Partitions. Refinements. Upper and Lower Riemann sums. Riemann integrable functions. Mean Value Theorem in terms of Integrals.

Week 9 (Week of 03/03): Integral Calculus 2 (§5.3) Indefinite Riemann integral (aka antiderivative). Signed definite Riemann integral. Unsigned definite Riemann integral. First Fundamental Theorem of Calculus. Second Fundamental Theorem of Calculus.

Week 10 (Week of 03/17): Integral Calculus 3 (§5.3-4) Change of Variables Formula. Integration by Parts. Elementary functions.

Week 11 (Week of 03/24): Integral Calculus 4 (§5.4); Review & Midterm 2 Improper Riemann integral. Cauchy Principal Value.

Week 12 (Week of 03/31): Series 1 (§6.1-3) Formal series. Convergent, conditionally convergent and absolutely convergent series. Convergence tests for series. Riemann Rearrangement Theorem. Arithmetic with series.

Week 13 (Week of 04/07): Series 2 (§6.4) Functional series. Point-wise convergence of a functional series. Uniform convergence of a functional series. Weierstrass M-Test.

Week 14 (Week of 04/14): Series 3 (§6.5) Power series. Real analytic functions. Radius and interval of convergence for a power series. Calculus with power series. Taylor Approximation. Remainder formulas for Taylor Approximation.

Week 15 (Week of 04/21): Review

6 Grades

The final letter grades will be determined according to the following weights and cutoffs. The cutoffs below are not definitive; based on the overall performance it may be adjusted at the end of the semester. All such adjustments will affect all students in the section equally.

Assignment	Weight
Attendance	5%
PSets	30%
Midterm 1	20%
Midterm 2	20%
Final	25%
Total	100%

Grade	Percent
A	90%
A-	85%
B+	80%
B	75%
B-	70%
C+	65%
C	60%
C-	55%
D+	50%
D	45%
D-	40%
E	else

7 Books & Other Resources

The **main textbook** for MATH 3210 is the following:

Taylor, Joseph L.. *Foundations of Analysis*. AMS. 2012.
ISBN-13: 9780821889848.

As supplements lecture notes by **Terence Tao** available at

[https://www.math.ucla.edu/
%7Etao/resource/general/131ah.1.03w/](https://www.math.ucla.edu/%7Etao/resource/general/131ah.1.03w/)

and lecture notes by **Ali Ülger** available at

[https://web.archive.org/web/20120703144624/
http://home.ku.edu.tr/~aulger/RealAnalysis\(AU\).pdf](https://web.archive.org/web/20120703144624/http://home.ku.edu.tr/~aulger/RealAnalysis(AU).pdf)

are recommended. Further resources will be uploaded to the Canvas webpage if needed.

8 Lectures

Attendance at all lectures is expected. Attendance at random lectures will be taken and will have an effect on the final grades. The number of lectures when attendance will be taken will also be random. A certain number of recorded attendance days may be dropped at the end of the semester.

On weeks when a midterm is scheduled (i.e. Weeks 5 and 11), first two classes will focus on topics as usual; whereas the class before the midterm is reserved for review. Likewise, the two classes on the last week (i.e. Week 15) will be reserved for review.

Lecture Recordings Video recordings of lectures, excluding those reserved for reviews and midterms, will be uploaded weekly to the following YouTube playlist:

https://www.youtube.com/playlist?list=PL40ydqvvvXfPGbwcE1wT2_dHVyGykxsLo.

**This is an experimental feature of the course.
Recordings may become unavailable at any time
and without notice; do not rely solely on them.**

9 Problem Sets

Twelve problem sets will be assigned throughout the semester, typically on a weekly basis, excluding weeks with midterms. Each problem set is due by 11:59 PM on the Friday following its release. Late submissions, up to 24 hours after the deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires. A certain number of problem sets with lowest scores may be dropped at the end of the semester.

Grading Criteria The problem sets will be graded partly on completion and partly on accuracy, validity, and presentation/documentation.

Submission Details For each problem set, you are required to submit a form through Google Forms and digital copies of your work through Gradescope. Detailed submission instructions will be provided in each problem set specification.

Conflicting Instructions and Deadlines The submission deadlines and methods detailed in the problem set specifications (i.e., the PDF files embedded in the associated assignment webpages on Canvas), Canvas, and Gradescope may differ. Always adhere to the deadline and instructions in the specification document, as these will be the accurate guidelines.

10 Midterm Exams

There will be two in-class midterm exams scheduled as follows:

- **Midterm 1:** February 7 (Friday of Week 5) – covering topics from Week 1 through Week 5.
- **Midterm 2:** March 28 (Friday of Week 11) – covering topics from Week 6 through Week 11.

During the midterm exams, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exams will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exams were graded.

11 Final Exam

The final exam is scheduled in accordance with the university-wide [final exam schedule](#). Details are as follows:

- **Date and Time:** April 25, from 8:00 AM to 10:00 AM.
- **Content:** The final exam will be cumulative, covering all topics from the course.

During the final exam, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exam will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exam was graded.

12 The Math Center

In addition to the office hours held by the course staff, the **Math Center** is an excellent resource for assistance with your studies. Located in the basement connecting **JWB** and **LCB**, the Math Center offers free drop-in as well as online support.

13 Accessibility

The University of Utah seeks to provide equal access to its programs, services, and activities for people with disabilities. If you will need accommodations in this class, reasonable prior notice needs to be given to the **Center for Disability & Access**, **UNION 162**, **(801) 581-5020**. CDA will work with you and the staff to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability & Access.

14 Inclusivity and Safety

Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses

against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information.

If you or someone you know has been harassed or assaulted, you are encouraged to report it to:

- **Title IX Coordinator** in the **Office of Equal Opportunity and Affirmative Action**, **PARK 135**, **(801) 581-8365**, or
- **Office of the Dean of Students**, **UNION 270**, **(801) 581-7066**.

For support and confidential consultation, contact the **Center for Student Wellness**, **SSB 328**, **(801) 581-7776**. To report to the police, contact the Department of Public Safety, **(801) 585-2677(COPS)**. For further safety resources, see **SafeU**.

15 Academic Honesty

You are expected to adhere to University of Utah policies regarding academic honesty. This means that ultimately all work you submit must be your own, created without unauthorized assistance, and all external resources, including generative tools and computer algebra systems, must be properly cited and documented within your submissions. Any student who engages in academic dishonesty or who violates the professional and ethical standards may be subject to academic sanctions as per the University of Utah's **Student Code**.

Generative AI and Computer Algebra Systems In this course, you may use generative tools like **ChatGPT** as well as computer algebra systems such as **MATLAB** under specific guidelines. These tools are permitted to guide your understanding of problem sets and

topics. When using them, you must submit logs of your interactions as part of your problem set submissions. These tools can also be used to verify calculations, but remember, computations are generally expected to be done manually unless explicitly stated otherwise. You are responsible for ensuring the validity, accuracy, and relevance of your submissions, whether or not you use these tools. While encouraged, the use of such technology is not required for success in this class. The policy governing the use of such technology is specific to MATH 3210-001; other courses and sections may have different policies.

Regret Clause If you commit an unreasonable act but bring it to the staff's attention within 48 hours of the relevant submission, sanctions may be limited to that submission only, rather than leading to further disciplinary action. This clause will not be applied in the case of repeated violations.

16 Acknowledgements

The staff used ChatGPT in retouching this syllabus for clarity and readability.