University of Utah Spring 2026

Syllabus for MATH 3210-004: Foundations of Analysis I

Instructor: Alp Uzman

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Contents

1 Staff and Meeting Hours

Instructor: Alp Uzman (uzman@math.utah.edu)

Lectures: Mon Tue Wed Fri 8:35 AM - 9:25 AM @ LCB 121

Canvas: https://utah.instructure.com/courses/1239139

Office Hours: By appointment

3210 Coordinator: Dragan Miličić (milicic@math.utah.edu)

2 Course Description & Prerequisites

MATH 3210 is a course focusing on single variable calculus from a rigorous point of view, serving as a transition course from calculus to analysis. As outlined in the general catalog, it is worth 4 credits and has the following prerequisites:

> 'C' or better in (MATH 1260 OR MATH 1321 OR MATH 2210 OR MATH 2310 OR MATH 3140) AND (MATH 2200 OR MATH 2250 OR MATH 2270 OR MATH 2271)

This is a proof based class, and the emphasis will be on developing "mathematical maturity", in particular on rigorous argumentation, rather than calculation of the values of certain limits, integrals or derivatives. We'll start with elementary logic and set theory, discuss main methods of documentation that are up to the modern standards of mathematical rigor, and then discuss integral and differential calculus of single variable, real valued functions. As adjacent topics we'll study the convergence and divergence of sequences and series of numbers and functions.

3 Expectations

As a student, you are expected to:

- attend all lectures (four times per week),
- complete 11 problem sets,
- take 2 in-class midterm exams, and
- take a final exam.

According to the university regulations, you should expect a workload of approximately 8 to 12 hours per week outside of lecture hours. You will have some familiarity with many ideas and notions we will discuss throughout the semester. However the focus on rigor can be hard to get accustomed to. Missing lectures or assignments can make it challenging to catch up. Please stay engaged and reach out if you encounter any disruptions. We're here to help, but your hard work and dedication will be essential to getting back on track!

4 Learning Objectives

Among the objectives in MATH 3210 are that you learn how to:

- use logical quantifiers,
- describe statements and the operations of AND, OR, and NOT in terms of sets,
- identify injective, surjective and bijective relations and functions,
- parse mathematical proofs,
- distinguish levels of rigor in mathematical prose,
- write proofs in accordance with modern standards of mathematical rigor,
- approach numbers from a bottom-up point of view and make use of the completeness axiom and the Archimedean property,
- use the notions of supremum and infimum,
- derive the consequences of the Triangular Inequality, for numbers and functions,
- use sequences and series of numbers and functions,
- identify when a sequence or series converges and in which sense,
- use the notions of accumulation points, limit inferior and limit superior of a sequence of numbers,
- prove and use the fundamental theorems governing continuous functions, including the Maximum Principle and Intermediate Value Theorem,
- define and distinguish various notions of integrability,

- prove and use the fundamental theorems governing integrable functions, including the Change of Variables Formula and integration by parts,
- prove and use the fundamental theorems governing differentiable functions, including the Chain Rule, Inverse Function Theorem, and Mean Value Theorem,
- prove and use the Fundamental Theorems of Calculus,
- prove and use Taylor approximation,
- prove and use the fundamental theorems governing infinite sums, including comparison, ratio and root tests
- prove and use the fundamental theorems governing real analytic functions, including Weierstrass M-test and Cauchy-Hadamard theorem.

5 Grades

The final letter grades will be determined according to the following weights and cutoffs. The cutoffs below are not definitive; based on the overall performance it may be adjusted at the end of the semester. All such adjustments will affect all students in the section equally.

Assignment	Weight
Attendance	5%
PSets	30%
Midterm 1	20%
Midterm 2	20%
Final	25%
Total	100%

Grade	Percent
A	90%
A-	85%
B+	80%
B	75%
B-	70%
C+	65%
C	60%
C-	55%
D+	50%
D	45%
D-	40%
E	else

6 Books & Other Resources

The main textbook for MATH 3210 is the following:

> Taylor, Joseph L.. *Foundations of Analysis*. AMS. 2012.
> ISBN-13: 9780821889848.

Additionally we'll use the first chapter of Fonda's *The Kurzweil-Henstock Integral for Undergraduates* for integral calculus and the first chapter of Krantz and Parks' *A Primer of Real Analytic Functions* for real analytic function theory. PDF files of both of these chapters are available on Canvas.

Textual Supplements As supplementary texts the following are recommended:

- Lecture notes by Terence Tao available at

https://www.math.ucla.edu/
%7Etao/resource/general/131ah.1.03w/

- Lecture notes by Ali Ülger available at

https://web.archive.org/web/20120703144624/
http://home.ku.edu.tr/~aulger/RealAnalysis(AU).pdf

Further resources will be uploaded to the Canvas webpage if needed.

Previous Materials From the last time I taught this class (Spring 2025), you may find the following helpful:

- Annotated recordings of all lectures:

https://www.youtube.com/playlist?list
=PL40ydqvvyXfPGbwcE1wT2_dHVyGykxsLo

- All textual materials (including selected solutions to problem set problems and exams):

https://github.com/AlpUzman/
MATH_3210_001_SPRING_2025

7 Tentative Schedule

The course's subject matter is divided by weeks within the academic calendar as follows. For your convenience the approximate corresponding sections in the main textbook as well as supplementary materials are also listed. Time permitting, additional topics[1] may be covered.

[1] Among candidate additional topics are: derived set of a subset of reals, Period Three Implies Chaos, Sharkovsky Theorem, moduli of continuity, Lipschitz and Hölder continuity, Banach Con-

Week 1 (Week of 01/05): Functions (§1.1) Logic: "For all", "there exists". Set theory: "element of", "subset of", intersection, union, complement. Relations and functions: composition, inversion, graph of a function, injection (one-to-one), surjection (onto), bijection, reflexivity, transitivity, symmetry, antisymmetry. Proof methods: direct proof, proof by contrapositive, proof by contradiction, proof by induction.

Week 2 (Week of 01/12): Reals (§1.2-5) Natural numbers. Integers. Rational numbers. Real numbers. Completeness axiom. Archimedean Property. Absolute value. Triangular inequality. Extended reals. Supremum, infimum, maximum, minimum of a subset of reals. Supremum, infimum, maximum, minimum of a real valued function.

Week 3 (Week of 01/19): Sequences (§2.1-6) Sequences. Subsequences. Metric spaces. Convergent and divergent sequences. Accumulation points of a sequence. Monotone sequences. Cauchy sequences. Bolzano-Weierstrass Theorem. Limit inferior and limit superior of a sequence. Directed sets. Nets (aka Moore-Smith sequences). Subnets.

Week 4 (Week of 01/26): Continuity 1 (§3.1-4) Continuous functions. Maximum principle. Intermediate Value Theorem. Monotone functions. Homeomorphisms. Uniformly continuous functions. Sequences of functions. Pointwise convergence of a sequence of functions.

traction Principle, Lipschitz Inverse Function Theorem, equicontinuity, Arzelà-Ascoli Theorem, Faà di Bruno Formula, Schwarzian derivative, depth-1 feedforward neural networks, weak derivatives, Stieltjes integral, Riemann zeta function, Stirling formula. Some of these topics may show up as part of a problem set.

Week 5 (Week of 02/02): Continuity 2 (§3.4,4.1); Midterm 1 Uniform convergence of a sequence of functions. Uniform metric. Uniformly Cauchy sequences of functions. Limits of functions.

Week 6 (Week of 02/09): Integral Calculus 1 (§5.1-2, Fonda) Partitions. Refinements. Upper and Lower Darboux sums. Sampling Functions Riemann sums. Riemann integrable functions. Unsigned definite Riemann integral. Signed definite Riemann integral. Gauges. delta-fineness. Cousin Lemma. Henstock-Kurzweil integrable functions. Unsigned definite Henstock-Kurzweil integral. Signed definite Henstock-Kurzweil integral.

Week 7 (Week of 02/16): Integral Calculus 2 (Fonda) Saks-Henstock Theorem. Lebesgue integrable functions. Monotone Convergence Theorem. Dominated Convergence Theorem.

Week 8 (Week of 02/23): Integral Calculus 3 (§5.4, Fonda) Improper integrals. Hake Theorem. Integral test for series.

Week 9 (Week of 03/02): Differential Calculus (§4.2-4) Derivative of a function at a point. Differentiable functions. Chain Rule. Diffeomorphisms. Inverse Function Theorem. Rolle Lemma. Mean Value Theorem.

Week 10 (Week of 03/16): Fundamental Theorems of Calculus (§5.3, Fonda); Midterm 2 Indefinite integral (aka antiderivative aka primitive). First Fundamental Theorem of Calculus. Second Fundamental Theorem of Calculus. Change of Variables Formula.

Week 11 (Week of 03/23): Taylor Approximation (§5.3, 6.5, Fonda) Integration by Parts. Taylor Approximation. Remainder formulas for Taylor Approximation.

Week 12 (Week of 03/30): Series (§6.1-3) Formal series. Convergent, conditionally convergent and absolutely convergent series. Convergence tests for series. Arithmetic with series.

Week 13 (Week of 04/06): Real Analytic Functions 1 (§6.4-5, Krantz-Parks) Functional series. Pointwise convergence of a functional series. Uniform convergence of a functional series. Weierstrass M-Test.

Week 14 (Week of 04/13): Real Analytic Functions 2 (§6.4-5, Krantz-Parks) Power series. Radius and interval of convergence for a power series. Calculus with power series. Real analytic functions.

Week 15 (Week of 04/20): Review

8 Lectures

Attendance at all lectures is expected. Attendance at random lectures will be taken and will have an effect on the final grades. The number of lectures when attendance will be taken will also be random. A certain number of recorded attendance days may be dropped at the end of the semester.

Lecture Recordings Video recordings of lectures, excluding those reserved for reviews and midterms, will be uploaded weekly to the following YouTube playlist:

https://www.youtube.com/playlist?list=PL40ydqvvyXfPnWecw8AYQTSXShgxUge_W.

This is an experimental feature of the course. Recordings may become unavailable at any time and without notice; do not rely solely on them.

9 Problem Sets

Eleven problem sets will be assigned throughout the semester, typically on a weekly basis, excluding weeks with midterms. Each problem set is due by 11:59 PM on the Friday following its release. Late submissions, up to 24 hours after the deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires. A certain number of problem sets with lowest scores may be dropped at the end of the semester.

Grading Criteria The problem sets will be graded partly on completion and partly on accuracy, validity, and presentation/documentation.

Submission Details For each problem set, you are required to submit a form through Google Forms and digital copies of your work through Gradescope. Detailed submission instructions will be provided in each problem set specification.

Conflicting Instructions and Deadlines The submission deadlines and methods detailed in the problem set specifications (i.e., the PDF files embedded in the associated assignment webpages on Canvas), Canvas, and Gradescope may differ. Always adhere to the deadline and instructions in the specification document, as these will be the accurate guidelines.

10 Midterm Exams

There will be two in-class midterm exams scheduled as follows:

- **Midterm 1:** February 6 (Friday of Week 5) – covering topics from Week 1 through Week 5.
- **Midterm 2:** March 20 (Friday of Week 10) – covering topics from Week 6 through Week 10.

During the midterm exams, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exams will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exams were graded.

11 Final Exam

The final exam is scheduled in accordance with the university-wide final exam schedule. Details are as follows:

- **Date and Time:** April 24, from 8:00 AM to 10:00 AM.
- **Content:** The final exam will be cumulative, covering all topics from the course.

During the final exam, the use of external resources (including colleagues, books, and electronics) will not be permitted. Your exam will be digitized and uploaded to Gradescope by the staff. You will have access to view how your exam was graded.

12 The Math Center

In addition to the office hours held by the course staff, the Math Center is an excellent resource for assistance with your studies. Located in the basement connecting JWB and LCB, the Math Center offers free drop-in as well as online support.

13 Accessibility

The University of Utah seeks to provide equal access to its programs, services, and activities for people with disabilities. If you will need accommodations in this class, reasonable prior notice needs to be given to the Center for Disability & Access, SSB 350, (801) 581-5020. CDA will work with you and the staff to make arrangements for accommodations. All written information in this course can be made available in an alternative format with prior notification to the Center for Disability & Access.

14 Inclusivity and Safety

Title IX makes it clear that violence and harassment based on sex and gender (which includes sexual orientation and gender identity/expression) is a civil rights offense subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories such as race, national origin, color, religion, age, status as a person with a disability, veteran's status or genetic information.

If you or someone you know has been harassed or assaulted, you are encouraged to report it to:

- **Title IX Coordinator** in the Office of Equal Opportunity and Affirmative Action, LAW OEO Suite, (801) 581-8365, or
- **Office of the Dean of Students**, UNION 270, (801) 581-7066.

For support and confidential consultation, contact the Center for Campus Wellness, SSB 350, (801) 581-7776. To report to the police, contact the Campus Security, (801) 585-2677(COPS). For further safety resources, see SafeU.

15 Academic Honesty

You are expected to adhere to University of Utah policies regarding academic honesty. This means that ultimately all work you submit must be your own, created without unauthorized assistance, and all external resources, including generative tools and computer algebra systems, must be properly cited and documented within your submissions. Any student who engages in academic dishonesty or who violates the professional and ethical standards may be subject to academic sanctions as per the University of Utah's Student Code.

Generative AI and Computer Algebra Systems In this course, you may use generative tools like ChatGPT[2] as well as computer algebra systems such as MATLAB under specific guidelines. These tools are permitted to guide your understanding of problem sets and topics. When using them, you must submit logs of your interactions as part of your problem set submissions. These tools can also be used to verify calculations, but remember, computations are generally expected to be done manually unless explicitly stated otherwise. You

[2] As a U of U student, you have access to advanced models through ChatGPT Enterprise EDU Access.

are responsible for ensuring the validity, accuracy, and relevance of your submissions, whether or not you use these tools. While encouraged, the use of such technology is not required for success in this class. The policy governing the use of such technology is specific to this course and section; other courses and sections may have different policies.

Regret Clause If you commit an unreasonable act but bring it to the staff's attention within 48 hours of the relevant submission, sanctions may be limited to that submission only, rather than leading to further disciplinary action. This clause will not be applied in the case of repeated violations.

16 Acknowledgements

The staff used ChatGPT in retouching this syllabus for clarity and readability.