University of Utah | Spring 2026

MATH 3210-004 PSet 2 Specification

Instructor: Alp Uzman

Subject to Change; Last Updated: 2026-01-16 11:52:03-07:00

1 Background

This problem set explores foundational concepts in mathematical analysis, focusing on the properties of numbers, functions, and sets. The following four (provable) axioms regarding numbers play a central role for this and later problem sets:

Well-Ordering Principle: Any nonempty subset of positive integers has a least element.

Completeness Axiom: Any nonempty subset of real numbers that is bounded from above has a least upper bound.

Archimedean Property: Any real number is dominated by some integer.

Density of Rationals: Any positive real number dominates some positive rational number.

Really, the Well-Ordering Principle is equivalent to the Principle of Mathematical Induction; a proof of the Completeness Axiom ultimately involves some explicit construction of $\mathbb{R}$ from $\mathbb{Q}$, and the final two axioms are consequences of the Completeness Axiom. The notions of supremum and infimum are quintessential manifestations of these properties; consequently a significant part of this problem set focuses on these notions.

2 Generative AI and Computer Algebra Systems Regulations

This section applies only if you choose to use either a generative AI tool (e.g. chatbots) or a computer algebra system (CAS) while working on this problem set. If you do not use such tools, you may skip this section.

The use of these tools while completing this problem set is permitted, provided it is done responsibly and in a manner that supports your learning. Note however that in exams you will not be allowed to use any AI or CAS.

2.1 Disclosure

If you use such tools, you must disclose this fact in the designated section of the form you will complete as part of your submission. You must also share your reflections on which parts of the problem required your own mathematical judgment, insight, or decision-making, and which parts could reasonably be delegated to a computational or generative tool.

2.2 Guidelines for Responsible Use

If you use a generative AI tool or chatbot, you must adhere to the following guidelines:

- During the chat you may share with the chatbot parts or all of this specification document, as well as parts of the textbook or other sources.
- Directly asking the tool for complete problem solutions is prohibited.
- Appropriate uses include asking for conceptual explanations, checking intermediate steps, clarifying definitions or theorems, or exploring alternative solution approaches, provided that the final work submitted is your own.
- Regardless of the tools used, you are expected to understand, justify, and be able to reproduce all submitted work independently. The use of AI or CAS does not reduce or replace this expectation.

3 What to Submit

Submit your detailed solutions to each of the problems below. While the prompts may seem lengthy, the additional text is intended to guide you by providing context and helpful details.

When tackling statements in MATH 3210, follow these four steps (in no particular order):

1. **Assess the truth of the statement:** Determine, either exactly or probabilistically, whether the statement is true.

2. **Build a case:** Come up with examples or begin drafting a proof to identify potential issues or areas where the argument may break down.

3. **Construct a proof:** Provide a formal argument to support your conclusion.

4. **Perturb the statement:** Experiment with logical variations of the statement. Can you prove a stronger/weaker result, or find alternative proofs?

In this problem set, and in later problem sets as well as exams, your work must address the third step (aka constructing a proof). While work regarding the remaining three items likely will help you write a formal proof, understand the material as well as identify connections between different concepts, you do not need to turn in generalizations or multiple proofs of the same statement, nor do you need to provide examples or counterexamples (unless you are specifically asked to). However, demonstrating these steps can contribute to partial credit when appropriate.

Statements in problems will typically be presented in neutral language. For example, a problem may simply state "P" (for P a well-defined statement) rather than "Show that P" or "Prove that P is false". Your task is to determine the truth of the statement and provide a formal proof or refutation.

If a statement is false, it is highly recommended to propose a corrected version of the statement and prove this corrected version. This process, part of "perturbing the statement", will enhance your mathematical error detection and correction skills.

Unless explicitly stated otherwise, all problems require proofs. If you provide an example, you must also prove that the example satisfies the necessary conditions.

Some problems may be (variants of) theorems or exercises from the textbook. Going forward, and possibly for the rest of this problem set, such correspondences will not be mentioned; it'll be up to you to locate them (if need be)!

Make sure that each solution is properly enumerated and organized. Start the solution to each problem on a new page, and consider using headings or subheadings to structure your work clearly. This will not only aid in your thought process but also ensure that no part of your solution is overlooked during grading.

1. For any positive integer n:

$$\sum_{k=1}^{n} k^3 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2.$$

2. Let $A \subseteq \mathbb{R}$ be nonempty and bounded. Then

   (a) The set of all upper bounds of A is $[\sup(A), \infty[$.

   (b) The set of all lower bounds of A is $]-\infty, \inf(A)]$.

   (c) The smallest closed interval containing A is $[\inf(A), \sup(A)]$.

   (d) The largest open interval contained in A is $]\inf(A), \sup(A)[$.

3. Let $A, B \subseteq \mathbb{R}$. Then

   (a) $\sup(A \cup B) = \max\{\sup(A)\, ,\, \sup(B)\}$

   (b) $\inf(A \cup B) = \min\{\inf(A)\, ,\, \inf(B)\}$.

4. Consider the following functions:

   I: $f : [-1, 1] \to \mathbb{R}, x \mapsto x^2$.

   II: $f :]1, 2[\to \mathbb{R}, x \mapsto \dfrac{x}{x-1}$.

 https://alpuzman.github.io/
uzman@math.utah.edu

III: $f : [0, 1[\to \mathbb{R}, x \mapsto 2x - x^2$.

IV: $f : \mathbb{R} \to \mathbb{R}, x \mapsto 2x - x^2$.

For each function f above, perform the following tasks.

(a) Compute the supremum and infimum of f.

(b) Compute the maximum and minimum of f.

5. Let X be a set, $f, g \in F(X; \mathbb{R})$ and $c \in \mathbb{R}$. Then

(a) $\sup(cf) = c \sup(f)$.

(b) $\sup(f + g) \leq \sup(f) + \sup(g)$.

(c) $\inf(f + g) \leq \inf(f) + \sup(g)$.

(d) $\sup(fg) = \sup(f) \sup(g)$.

(e) $\sup(\min\{f, g\}) = \min\{\sup(f), \sup(g)\}$, where $\min\{f, g\} : X \to \mathbb{R}$, $x \mapsto \min\{f(x), g(x)\}$.

(f) $\sup(\max\{f, g\}) = \max\{\sup(f), \sup(g)\}$, where $\max\{f, g\} : X \to \mathbb{R}$, $x \mapsto \max\{f(x), g(x)\}$.

(g) $\sup\{f(x_1) - f(x_2) | x_1, x_2 \in X\} = \sup(f) - \inf(f)$.

(h) $\inf\{f(x_1) - f(x_2) | x_1, x_2 \in X\} = \inf(f) - \sup(f)$.

4 How to Submit

- **Step 1 of 2:** Submit the form at the following URL:

  https://forms.gle/F4JxgeeCtVSAfoeE7.

  You will receive a zero for this assignment if you skip this step, even if you submit your work on Gradescope on time.

- **Step 2 of 2:** Submit your work on Gradescope at the following URL:

  https://www.gradescope.com/courses/1212058/assignments/7390763,

  see the Gradescope documentation for instructions.

5 When to Submit

This problem set is due on January 23, 2026 at 11:59 PM.

As per the course's syllabus, late submissions, up to 24 hours after this deadline, will be accepted with a 10% penalty. Submissions more than 24 hours late will not be accepted unless you contact the course staff with a valid excuse before the 24-hour extension expires.