University of Utah Fall 2024

MATH 4800-001 Presentation Schedule

Instructor: Alp Uzman

Subject to Change; Last Updated: 2024-12-05 15:56:28-07:00

1 Introduction

The two lectures in week 15 (week of 12/02) are reserved for oral in-class presentations. Each presentation will be about 10 minutes long, followed by approximately 6 minutes for questions and discussion, as well as logistics for the next talk, if applicable. The plan is to have 5 presentations on the first day and 4 on the second day, for a total of 9 presentations.

In case there are delays due to some technical issues, the fifth presentation for the first day might end up getting rescheduled to be the first presentation for the second day, with the remaining presentations shifted accordingly.

2 Day 1: 12/03, Tuesday

Talk 1 by [redacted] (12:25 PM - 12:41 PM)

Title Dimension of a Solenoid's Attractor

Abstract To provide motivation and logical stability to a proof of the dimension of a Smale-Williams solenoid, we construct the iterative function from the ground up, project it into two dimensions, identify a coding system, and analyze properties of curves in the attractor. By confirming that our solenoid meets the necessary contraction, expansion, and directional requirements, we can use Károlyn Simeon's propositions from "Hausdorff dimension of non-invertible maps" to prove its dimension to be $\dim_{\mathrm{H}}(\Lambda) = 1 + \dfrac{\log 2}{\log(1/\max(\lambda, \mu))}$. We reign in the focus of this proof to use minimal constructions from areas of mathematics outside of basic dynamics.

Talk 2 by [redacted] (12:41 PM - 12:57 PM)

Title Fractal Transformations for Image Encoding and Spatial Modeling

Abstract Spatial analysis is an important tool used in a variety of research fields. Spatial variables are often stored using raster images. Raster layers can be very large, and sometimes we may be interested in many raster's over a temporal scale. Barnsley et. al. have proposed a method for image encoding using fractal transformations. I study this method and begin to examine if it offers a way to compress multiple raster's into one while maintaining spatial structure in predictable ways.

Talk 3 by [redacted] (12:57 PM - 1:13 PM)

Title The Hausdorff dimension of the distribution of digits of base-m expansions in $[0, 1)$.

Abstract I will define and show examples of the mathematical tools necessary for proving this classic result by Eggleston. I will also outline the steps needed to prove this theorem by Eggleston.

Talk 4 by [redacted] (1:13 PM - 1:29 PM)

Title An Introduction to Continued Fractions and the Gauss Transform

Abstract In this paper we take a brief look at continued fraction. We will discuss the process for generating a continued fraction for any given number $x \in \mathbb{R}$. Then we will define the Gauss bracket and convergents of a continued fraction. We will then proceed to the idea of coding for a continued fraction and introduce the Gauss Transform. Our final result will be to show that the Gauss Transform is Ergodic with respect to the Gauss Measure.

Talk 5 by [redacted] (1:29 PM - 1:45 PM)

Title A Comparison of Dimensions

Abstract In this presentation I will cover briefly the relationships between common definitions of dimensions in the context of fractals and topology, and then focus specifically on the proof of the Pontrjagin-Schrinelmann Theorem.

3 Day 2: 12/05, Thursday

Talk 6 by [redacted] (12:25 PM - 12:41 PM)

Title Fractal Properties of Brownian motion

Abstract Described by botanist Robert Brown in 1827, Brownian motion is the highly irregular observed movement of particles suspended in some medium. After Einstein's groundbreaking paper on Brownian motion in 1905 where he showed that these Brownian objects moved according to the diffusion equation $\frac{\partial \rho}{\partial t} = D\frac{\partial^2 \rho}{\partial x^2}$ the subject grew rapidly and was the basis for much research in the 20th century. For example, the existence of Brownian motion was strong experimental evidence for atomic theory, it allowed researchers to calculate the size of atoms leading to the numerical value of Avogadro's number, and laid the groundwork for statistical mechanics. Furthermore, these Brownian trails and graphs exhibit self-similar/fractal properties which leads us to some important questions; what is the Hausdorff/Minkowski dimension of a Brownian trail/graph? Does the amount of intersections that a trail will have with itself change in $\mathbb{R}^2$ vs $\mathbb{R}^3$? Are there any generalizations for this motion so we can apply it to things other than atoms and molecules?

Talk 7 by [redacted] (12:41 PM - 12:57 PM)

Title A Glimpse of Computability Theory in Fractal Geometry

Abstract In the field of theoretical computer science, a considerable amount of work is done to dichotomize problems based on their algorithmic solvability. In this talk, we consider the problem of determining whether the attractor of a given iterated function system acting on the

unit square intersects the line segment between the $(0,0)$ and $(1,1)$. After a brief introduction to computability theory, we prove that there does not exist an algorithm which solves this problem (for any given iterated function system) (in finite time).

Talk 8 by [redacted] (12:57 PM - 1:13 PM)

Title Predictions of the Future from Measurements of the Past

Abstract Most natural processes are governed by chaotic systems, yet their mathematical descriptions are often unknown. This paper demonstrates how the phase space of such systems can be reconstructed by observing only a single variable over time. Additionally, it shows how the fractal dimension of a chaotic system can be accurately estimated using incomplete information. By leveraging average mutual information (AMI) and the false nearest neighbors (FNN) algorithm, we apply Takens' Embedding Theorem to reconstruct phase spaces and analyze the impact of embedding dimensions on the correlation dimension. The results highlight the practical application and effectiveness of these techniques in understanding chaotic dynamics.

Talk 9 by [redacted] (1:13 PM - 1:29 PM)

Title Wildfire Dynamics and Fractal Dimension

Abstract Wildfires have been shown to spread in a fractal manner. This can be used to stop fires from spreading by predicting how they will move through their environment. We can analyze wildfire dynamics using a model similar to epidemic spread in a population to determine when a fire will grow and how quickly it will die out. Satellite

images of burned areas of wildfires can be analyzed to determine their fractal dimension. The fractal dimension of a fire's accessible perimeter correlates to the intensity of the fire front and how much it will spread. The density of vegetation in the area also influences the fractal dimension and fire intensity. Understanding wildfire propagation dynamics is useful for developing forest management policies and stopping the spread of burning fires. There are external factors not captured in these simple models such as climate, vegetation, and landscape, but the models still help develop an understanding of wildfire behavior.

Closing Remarks by Alp Uzman (1:29 PM - 1:45 PM)