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# ANCILLARY COMPUTATIONAL TOOLS FOR THE ANALYSIS OF STRUCTURAL SYSTEMS

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**Abstract.** *Ancillary software tools are developed for the analysis and response quantification of structural systems in a systematic and automated fashion. The open application programming interface (OAPI) of the structural analysis software SAP2000 is exploited for this purpose and a procedure is presented for the analysis of linear systems in conjunction with MATLAB, featuring model updating, analysis execution and extraction of the results. This is facilitated by the development of ancillary functions that complement and extend existing capabilities of the OAPI. Standalone solvers are next presented for the dynamic response evaluation of bilinear hysteretic, rigid-plastic, and rocking oscillators with arbitrary excitation force. The dynamical systems considered, are indicative of distinct nonlinear behaviour and representative of a wide spectrum of structures of engineering interest. The features at the current development stage are finally showcased on the response-surface-based reliability analysis of a truss structure and the response quantification of stochastically driven nonlinear secondary oscillators. The ancillary tools developed allow convenient simplifications in the modelling and analysis process, error tractability, and can assist learning on the subject of structural dynamics. They may find application in parametric and stochastic analysis and form the basis for the development of additional modules.*

**Keywords:** API, nonlinear oscillators, SAP2000, software development, structural analysis.

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# 1 INTRODUCTION

In engineering practice, the analysis and design of structural systems is conventionally carried out with the aid of commercial structural analysis software. These are usually treated as black boxes with models of varying degrees of complexity that have to be extensively and often iteratively run in parallel by different design teams. Although such software are reliable in the functionalities they offer, they are usually limited by the exclusion of state-of-the-art advanced analysis features that become available within the academic community. Nevertheless, they benefit from the use of general-purpose solvers which comes at the expense of prohibitively time-consuming runs as well as strenuous pre- and post-processing tasks.

In an attempt to circumvent these limitations, the vast majority of software packages offer complementary open application programming interfaces (OAPI) that permit the development of custom engineering applications. In the case of the structural analysis software SAP2000 [1], for instance, this is solely left to the discretion of the user which encounters limited support and guidelines. Despite the apparent value, only a handful of the contributions available in the literature take advantage of the OAPI [2] and at present, none of these assistant simulation-based tools is designed to be extended by the structural engineering research community.

In this paper, we present ancillary computational tools for the analysis and response quantification of structural systems in a systematic and automated fashion. In a first stage, an overview of the SAP2000 OAPI is provided, and a procedure is presented for the analysis of linear systems in conjunction with MATLAB [3], featuring model updating, analysis execution and extraction of the results. This is facilitated by the use of ancillary functions that are developed to complement and extend existing capabilities of the SAP2000 OAPI. In a second stage, standalone solvers are presented for the dynamic response evaluation of bilinear hysteretic, rigid-plastic and free-standing rocking oscillators in presence of a general-type of excitation. The dynamical systems considered, are indicative of distinct nonlinear behaviour and representative of a wide spectrum of primary as well as secondary structures of engineering interest. Finally, the features at the current development stage are showcased on two examples: *i*) the response-surface-based reliability analysis of a truss structure facilitated through the use of the SAP2000 OAPI; and *ii*) the use and interoperability of the standalone solvers developed with SAP2000 for the response quantification of nonlinear cascaded secondary oscillators due to stochastic excitation. Basis of this contribution are the preliminary investigations carried out in [4, 5].

The proposed procedure, suited for both the industrial and research communities, allows convenient simplifications in the modelling and analysis process, error tractability, extendibility and can assist learning on the subject of structural dynamics in graduate and post-graduate level. It is particularly useful for parametric, stochastic analysis and optimisation and will form the basis for the development of additional modules.

# 2 ANCILLARY TOOLS FOR STRUCTURAL ANALYSIS WITH SAP2000

## 2.1 SAP2000 Open Application Programming interface

The Open Application Programming Interface (OAPI) of SAP2000 is a programming tool that permits coupling of SAP2000 with third-party software, thus providing a path for two-way exchange of model information, for automating the processes required for the construction, analysis, and design of structural models with SAP2000 as well as retrieval of customised results (Figure 1). Numerous programming languages can be used for this purpose, including Visual Basic for Applications (VBA), Visual Basic, Visual C#, Visual Fortran, Visual C++, MATLAB and Python.

Initiation of the link process requires pre-existing installation of the SAP2000 software and the integrated development environment (IDE). The procedure then requires initialisation of the IDE-SAP2000 assembly and sequential run of a set of predefined functions, each corresponding to one task within SAP2000, closely resembling the point-and-click procedure followed during conventional use of the SAP2000 software. It is noted that the syntax of each function depends on the choice of the IDE, and that not all the functionalities of SAP2000 are supported by the OAPI. The *CSi OAPI Documentation.chm* file, however, provides a list of the supported functions along with the syntax and description of the arguments it handles, as well as basic example code for each programming language supported.

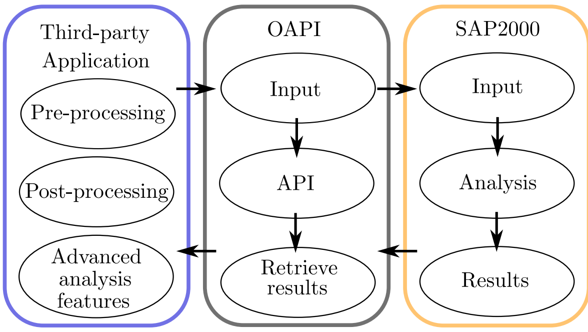


Figure 1: Schematic of the SAP2000 OAPI.

## 2.2 Using the SAP2000 OAPI for the analysis of linear systems

A procedure is presented in the following for automated analysis of linear structural systems with SAP2000 and MATLAB. The procedure is decomposed in the following main steps:

**A**: Construct the structural model in SAP2000. This typically consists of model generation, definitions as well as assignments. Preparing the model through the IDE is in most cases permissible, however, it can be cumbersome and it is therefore avoided.

**B**: Initialise the MATLAB-SAP2000 link. The function ***fx_intlSAP.m*** has been prepared for this purpose which requires an input directory of the pre-existing SAP2000 model. It is worth noting that the analyst is encouraged to use the $.sdb$ model file rather than importing the $.\$2k$ one. Use of the latter was found problematic in models with general frame sections.

**C**: Update the model. A new iteration of the model is obtained by updating the model input parameters via the OAPI. For a structure whose joint loads are to be varied, for instance, the function ***fx_JntLF.m*** has been prepared for assigning loads to point objects.

To proceed further, distinction is made on whether the analysis can be carried out externally. If the finite element model remains unchanged for each model iteration, then the analysis can be performed directly in MATLAB. This can be advantageous, as it provides the analyst with control and flexibility, limiting strenuous pre- and post-processing tasks.

**D1**: Analyse the structure in MATLAB. This is facilitated by the use of the function ***fx_getMK.m*** which has been prepared to construct the mass matrix and the global stiffness matrix by exporting and ordering these from the SAP2000 model. It is noted that this functionality further complements the existing capabilities of the OAPI.

**D2**: Analyse the structure in SAP2000. The function ***fx_runSAP.m*** runs the analysis in SAP2000. The load cases to run and the analysis options used are those configured in step **A**.

**E**: Retrieve analysis results. To reduce transferring times, the analyst is encouraged (in step **A**), to define a group containing the joints where the response is to be measured. The function ***fx_getJD*** can then be used to extract the diplacement response envelope, time history or the last step, for the joints of this group. This function has been constructed by piecing together various contributions from the *CSi OAPI Documentation.chm* file. Similar functions can be constructed to retrieve velocity or acceleration results.

**F**: Unlock model and break the MATLAB-SAP2000 link. If further analyses are to be carried out, the SAP2000 model is unlocked via the existing function ***SetModelIsLocked*** and the procedure is repeated from step **C**. When no more analyses are required, the MATLAB-SAP2000 link is interrupted through the function ***fx_closeSAP.m***.

Use of the functions presented in the previous has been tested with SAP2000 v.20 [1]. They extend, facilitate and simplify the existing capabilities of the SAP2000 OAPI, offering efficiency and readability, thus encouraging future developments for other IDEs. The proposed procedure, aimed at parametric, stochastic and optimisation analysis, provides recommendations to the user and has been successfully employed in our previous investigations on the dynamic analysis of composite beam structures [5] as well as the dynamic analysis of steel frames [4]. It will be demonstrated next through two illustrative examples.

## 3 SOLVERS FOR BILINEAR, RIGID-PLASTIC AND ROCKING OSCILLATORS

In this section, solvers are presented for the dynamic response evaluation of bilinear hysteretic, rigid-plastic and free-standing rocking oscillators, due to a general-type of excitation. The dynamical systems considered herein are indicative of distinct nonlinear behaviour, representative of a wide spectrum of primary and secondary structures of engineering interest.

### 3.1 Governing equations

#### 3.1.1 Bilinear oscillator

The case of a bilinear oscillator is first considered, as depicted in Figure 2(a), whose vibration is ruled by:

$$\ddot{u}_\mathrm{s}(t) = -2\,\zeta_\mathrm{s}\,\omega_\mathrm{s}\,\dot{u}_\mathrm{s}(t) - \frac{f_\mathrm{b}\left(u_\mathrm{s}(t), \dot{u}_\mathrm{s}(t)\right)}{m_\mathrm{s}} - \ddot{\xi}(t)\,, \tag{1}$$

in which $\ddot{\xi}(t)$ is the horizontal base acceleration, where the overdot denotes differentiation with respect to time; $u_\mathrm{s}(t)$ is the unidirectional displacement, relative to the ground; $\zeta_\mathrm{s}$ and $\omega_\mathrm{s} = \sqrt{k_\mathrm{s}/m_\mathrm{s}}$ are the viscous damping ratio and circular frequency, $k_\mathrm{s}$ and $m_\mathrm{s}$ being the elastic stiffness and mass, respectively.

In the above, $f_\mathrm{b}$ represents the bilinear restoring force (Figure 2(d)), given by:

$$f_\mathrm{b}\left(u_\mathrm{s}(t), \dot{u}_\mathrm{s}(t)\right) = \psi\,\omega_\mathrm{s}^2\,m_\mathrm{s}\,u_\mathrm{s}(t) + a_\mathrm{s}\,m_\mathrm{s}\,(1-\psi_\mathrm{s})\,z_\mathrm{s}(t)\,, \tag{2}$$

where $\psi$, in the range $0 \leqslant \psi \leqslant 1$, is the post-to-pre-yield stiffness ratio; $a_\mathrm{s}$ is the specific strength of the system; and $z_\mathrm{s}(t)$ is an auxiliary state variable satisfying $|z_\mathrm{s}(t)| \leq 1$, ruled by:

$$\dot{z}_\mathrm{s}(t) = \frac{\dot{u}_\mathrm{s}(t)\,\omega_\mathrm{s}^2}{a_\mathrm{s}}\left[1 - H\left(\dot{u}_\mathrm{s}(t)\right)H\left(z_\mathrm{s}(t)-1\right) - H\left(-\dot{u}_\mathrm{s}(t)\right)H\left(-z_\mathrm{s}(t)-1\right)\right]\,, \tag{3}$$

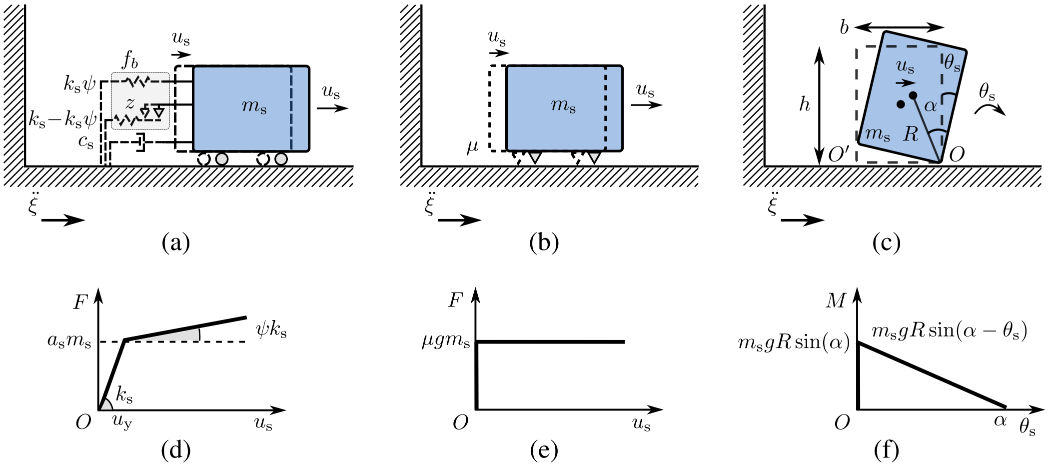


Figure 2: Bilinear (a), sliding (b) and rocking (c) oscillators and corresponding free-body diagrams (d, e, f).

where $H(\bullet)$ denotes the Heaviside unit step function, such that $H(x) = +1$ if $x \geq 0$ and $H(x) = 0$ if $x < 0$.

Setting $\psi = 0$ in Eq. (2) results to an elastic-perfectly-plastic oscillator; alternatively, setting $\psi = 1$ results to a linear one, with $f_b(t) = \omega_s^2\, m_s\, u_s(t)$.

### 3.1.2 Sliding block

The rigid-perfectly plastic oscillator is considered next (Figure 2(b)). The system exhibits infinite pre-yielding stiffness and infinite ductility, and no motion during the sticking phase (i.e. $u_s = \dot{u}_s = 0$). During the sliding motion regime (i.e. $\dot{u}_s(t) \neq 0$) its vibration is governed by:

$$\ddot{u}_s(t) = -\frac{f_s(\dot{u}_s(t))}{m_s} - \ddot{\xi}(t)\,, \tag{4}$$

where $f_s$ is the associated restoring force (Figure 2(e)) and can be regarded as the limiting case of Eq. (2), when $\psi = 0$ and $\omega_s \to +\infty$, given by:

$$f_s(\dot{u}_s(t)) = a_s\, m_s\, \mathrm{sgn}(\dot{u}_s(t))\,; \quad |\dot{u}_s(t)| > 0\,, \tag{5}$$

in which $\mathrm{sgn}(\bullet)$ is the signum function (i.e. $\mathrm{sgn}(x) = +1$ if $x > 0$, $\mathrm{sgn}(x) = -1$ if $x < 0$, and $\mathrm{sgn}(x) = 0$ if $x = 0$), and the specific strength is given by $a_s = \mu\, g$, $\mu$ being the coefficient of sliding friction assuming horizontal contact surface and $g$ the acceleration due to gravity.

The condition of initiation for sliding motion is $\left|\ddot{\xi}(t)\right| = a_s$. Following initiation, an instantaneous stop or a full stop can occur in the system once the velocity drops to zero. In the former case, the motion will reverse or it will continue in the same direction, while in the latter case the system will remain at rest until the initiation condition is meet again.

### 3.1.3 Rocking block

The case of a rectangular free-standing block exhibiting pure rocking motion, is finally considered. In this regard, the coefficient of sliding friction is assumed to be sufficiently large (i.e. $\mu \to +\infty$), and the block oscillates about its centres of rotation $O$ and $O'$, as illustrated in Figure 2(c).

The equation governing the response during the rocking regime of motion is given by:

$$\ddot{\theta}_{\mathrm{s}}(t) = -p^2 \left[ \sin\left(\alpha \operatorname{sgn}\left(\theta_{\mathrm{s}}(t)\right) - \theta_{\mathrm{s}}(t)\right) + \frac{1}{g} \cos\left(\alpha \operatorname{sgn}\left(\theta_{\mathrm{s}}(t)\right) - \theta_{\mathrm{s}}(t)\right) \ddot{\xi}(t) \right] , \qquad (6)$$

where $\theta_{\mathrm{s}}(t)$ is the response rotation of the block; $\alpha = \tan^{-1}(b/h)$ is the slenderness angle, being a function of the width $b$ and height $h$; $p = \sqrt{3\, g/(4\, R)}$ is a geometrical parameter, where $R$ is half the block's diagonal.

As shown in Figure 2(f), the block initially possess infinite rotational stiffness until the applied moment about one of the pivot points $O$ and $O'$ reaches the value $|M_{\mathrm{s}}| = m_{\mathrm{s}}\, g\, R \sin(\alpha)$, and a softening branch initiates, reaching $M_{\mathrm{s}} = 0$ at the tipping condition $|\theta_{\mathrm{s}}| = \alpha$.

The initiation condition for Eq. (6) is $\left|\ddot{\xi}(t)\right| = g\, \tan(\alpha)$. Following initiation, a change in the sign of the rotation $\theta_{\mathrm{s}}(t)$ will correspond to an impact and (assuming that the block does not bounce back) the pivot point will switch from $O$ to $O'$ (or vice versa). It is assumed that the post-impact rotational velocity, $\dot{\theta}_{\mathrm{s}}^{+}$, is a fraction of the velocity prior to impact, $\dot{\theta}_{\mathrm{s}}^{-}$, that is: $\dot{\theta}_{\mathrm{s}}^{+} = \varepsilon\, \dot{\theta}_{\mathrm{s}}^{-}$, where $\varepsilon$ is the coefficient of restitution, with $0 < \varepsilon \leq \varepsilon_{\max} < 1$, $\varepsilon_{\max} = 1 - 3\sin^2(\alpha)/2$ being the maximum value allowed.

### 3.2 Solvers for nonlinear dynamic response quantification

The dynamical systems considered in the previous are of piecewise linear form and therefore a highly efficient numerical procedure can be employed for quantifying the response. Specifically, each regime of motion is separately considered, and the solution is developed by interpolating the excitation over each time interval. Accordingly, recurrence formulae are derived for the response state vector of each system from exact solution of the associated equation of motion. To this end, the only requirement is that the time step is sufficiently small to closely approximate the excitation. The overall time history is finally constructed by piecing together the individual segments. Details on the methodology and the resulting expressions are provided in [6].

The solutions presented in [6] have been implemented in C++ along with an iterative procedure based on the bisection method [7] to identify state events (i.e. transition points of piecewise solutions such as the initiation and change in the regime of motion) and break down the solution in parts which have been later pieced together.

In order to confirm the validity of the solvers the solution has been compared to a MATLAB [3] implementation that has been prototyped using build-in Ordinary Differential Equation solvers. Specifically, ODE45 has been used, which is based on an explicit fourth- and fifth-order Runge-Kutta formulation. The implementation has been performed with consistent initial conditions and by setting MATLAB's odeset parameter values AbsTol = RelTol = $10^{-8}$ and Refine $= 4$, which refer to relative and absolute solution tolerances and interpolation output, respectively. The option 'Events' has been invoked to approximately identify state events.

The resulting standalone executables ***B1.exe***, ***S1.exe*** and ***R1.exe*** corresponding to the bilinear, rigid-plastic and rocking oscillators, respectively, accept their input arguments and export their results in standard fixed-length txt files which can be further processed by third-party software. Details on accessing the solvers along with their user instructions are provided in § 5.

They facilitate accurate and efficient dynamic analysis of the systems under consideration in presence of a general-type of base excitation and may find application in the stochastic analysis and optimisation of structural systems.

Table 1: Input random variable definition.

| Variable | Distribution | Mean | CoV |
|---|---|---|---|
| $E_1, E_2$ [N/m$^2$] | Lognormal | $2.1 \times 10^{11}$ | 0.10 |
| $A_1$ [m$^2$] | Lognormal | $2.0 \times 10^{-3}$ | 0.10 |
| $A_2$ [m$^2$] | Lognormal | $1.0 \times 10^{-3}$ | 0.10 |
| $f_1 - f_6$ [N] | Gumbel | $5.0 \times 10^{4}$ | 0.15 |

## 4 ILLUSTRATIVE EXAMPLES

Two examples are presented in the following for the analysis of structural systems with the ancillary tools developed. In the first example, the linear static analysis of a truss structure is facilitated through the use of the SAP2000 OAPI, with the purpose of quantifying the response statistics due to system uncertainties. The second example demonstrates the use and interoperability of the solvers presented in the previous with SAP2000, for the response quantification of nonlinear cascaded secondary oscillators in presence of stochastic excitation.

### 4.1 Example 1: Reliability analysis of a truss structure

#### 4.1.1 Problem statement

The case of a truss structure is considered [8, 9], comprising of 11 horizontal and 12 diagonal members, as depicted in Figure 3. A finite element (FE) model is constructed in SAP2000 using 23 bar elements. The model is characterised by the input random variable vector $\mathbf{X} = \{E_1, E_2, A_1, A_2, f_1, \ldots, f_6\}$, consisting of a set of $n = 10$ independent random variables, where $E_1, E_2$ denote the Young's moduli and $A_1, A_2$ the cross sectional areas, of the horizontal and diagonal members, respectively, and $f_1 - f_6$ are applied loads, whose probabilistic definition is summarised in Table 1.

Requirement for this problem is the quantification of the response statistics of the midspan deflection $u$ and the probability of failure $P_f = \text{Prob}\,[g(\mathbf{X}) < 0]$, $g(\mathbf{X})$ being the limit state function, defined as:

$$g(\mathbf{X}) = u_{\max} - |u(\mathbf{X})| \leq 0\,, \tag{7}$$

where $u_{\max} = 0.11$ is the maximum permissible value of $u$.

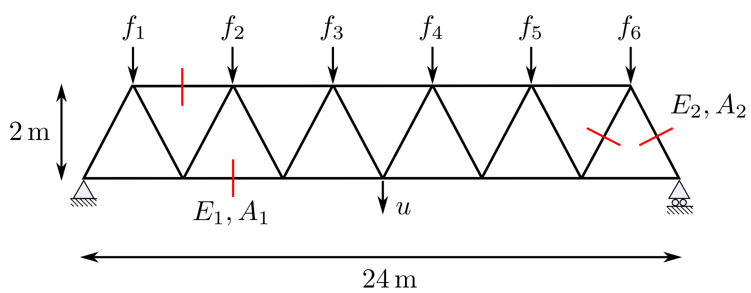


Figure 3: Truss structure [8].

### 4.1.2 Response surface-based metamodel

Owing to the presence of system uncertainties, the stiffness matrix is a random variable and therefore analysis is required within SAP2000. As $g(\mathbf{X})$ is algorithmically known, a response surface (RS) metamodel is constructed with the purpose of limiting the number of FE runs. Provided the fitted surface is an adequate approximation of the true response function, the analysis will be approximately equivalent to the one of the actual system.

In doing this, $g(\mathbf{X})$ is approximated with a quadratic polynomial of the form:

$$\tilde{g}(\mathbf{X}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \sum_{i=1}^{n} \alpha_{i,i} x_i^2 + \sum_{i=1}^{n-1} \sum_{j>i}^{n} \alpha_{i,j} x_i x_j \,, \tag{8}$$

where $\alpha_i$, $\alpha_{i,i}$ and $\alpha_{i,j}$ are a set of $s = 1 + (n^2 + 3\,n)/2 = 66$ coefficients to be identified, conveniently collected in the vector:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_0 & \alpha_1 & \cdots & \alpha_n & \alpha_{1,1} & \cdots & \alpha_{n,n} & \alpha_{1,2} & \cdots & \alpha_{n-1,n} \end{bmatrix}^\top . \tag{9}$$

The values of the coefficients are determined via a set of sample points from the true limit state function, $g(\mathbf{X})$. In doing this, a circumscribed central composite design is adopted, widely used for fitting a second-order RS. This requires $r$ FE runs with SAP2000 for evaluation of $g(\mathbf{X})$ symmetrically around the mean values of the random variables i.e. $x_i^{\pm} = \mu_i \pm h\,\sigma_i$, where $h = 0.95$ controls the size of the sampling domain. It is worth noting here that, if the shape of the true limit state function is not linear or quadratic the choice of $h$ can significantly affect the accuracy of the approximation.

On minimising the error,

$$\varepsilon(\boldsymbol{\alpha}) = \sum_{k=1}^{r} \left( g(\mathbf{x}_k) - \tilde{g}(\mathbf{x}_k) \right)^2 \,, \tag{10}$$

with respect to $\boldsymbol{\alpha}$, the solution is given by,

$$\boldsymbol{\alpha} = \left( \mathbf{V}^\top \cdot \mathbf{V} \right)^{-1} \cdot \mathbf{V}^\top \cdot \mathbf{y} \,, \tag{11}$$

where $\mathbf{y}$ is a vector whose generic component is $g(\mathbf{x}_k)$, for the set of fitting points $\mathbf{x}_k, k = 1, \ldots, r$, and $\mathbf{V}$ is a matrix of size $(r \times s)$ collecting the associated polynomial terms:

$$\mathbf{V} = \begin{bmatrix} 1 & x_{1,1} & \cdots & x_{n,1} & x_{1,1}^2 & \cdots & x_{n,1}^2 & x_{1,1} x_{2,1} & \cdots & x_{n-1,1} x_{n,1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1,r} & \cdots & x_{n,r} & x_{1,r}^2 & \cdots & x_{n,r}^2 & x_{1,r} x_{2,r} & \cdots & x_{n-1,r} x_{n,r} \end{bmatrix} . \tag{12}$$

Analysis is finally performed using the fitted surface $\tilde{g} = \tilde{\mathbf{V}} \cdot \boldsymbol{\alpha}$, where $\tilde{\mathbf{V}}$, of size $(N \times s)$, retains a similar form as in Eq. (12) and is populated using $N$ realisations of the vector of random variables.

### 4.1.3 Response quantification

The response probability density function (PDF) of $u$ and the probability of failure $P_f$ have been quantified using the RS model ($r = 178$ FE runs have been used with SAP2000) and have been compared to the reference FE solution. The results for both the RS and reference solution

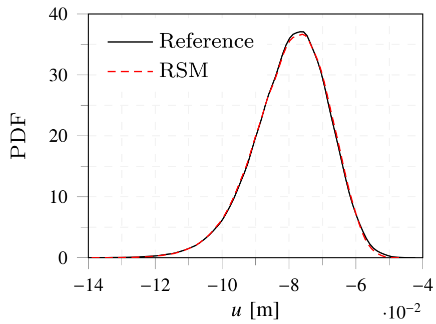


Figure 4: Response probability density function.

were obtained via crude Monte Carlo simulation with $N = 1 \times 10^6$ realisations. Analysis with SAP2000 was facilitated by use of the OAPI.

Figure 4 shows that the RS closely approximates the response PDF of $u$. Specifically, the calculated values of the first three statistical moments are in excellent agreement with the reference values reported in [9]. Further, the calculated $P_f = 0.00814$ is in good agreement with the reference FE solution $P_f^{\mathrm{ref}} = 0.00898$.

### 4.2 Example 2: Stochastic response quantification of nonlinear cascaded secondary oscillators to white-noise and filtered-white noise process

#### 4.2.1 Problem statement

The case of a multi-degree-of-freedom primary system [6] is next considered, comprising of a planar 5-storey single-bay moment-resisting frame, as depicted in Figure 5. A linear model of the structure is constructed using SAP2000. Floors are assumed rigid in their own plane, and the self-weight and super-dead load are the two sources of mass, lumped at the floor level. The total number of DoFs is $n_{\mathrm{p}} = 75$ (i.e. 15 DoFs per storey, 2 finite elements are used for each frame element) and the fundamental circular frequency in the direction of interest $x$ is $\omega_{\mathrm{p}} = 14.76\,\mathrm{rad/s}$, corresponding to a participation of $84\%$ of the modal mass.

The structure is subjected to the unidirectional action of a horizontal ground acceleration, successively modelled as white noise and filtered white noise processes, commonly used in earthquake engineering applications. The latter is characterised by the well-known Kanai-

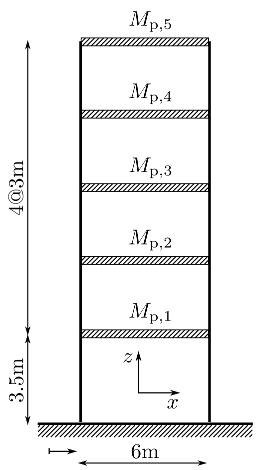


Figure 5: Primary structure [6].

Tajimi [10, 11] spectral density function:

$$S_{\ddot{\xi}}(\omega) = S_0 \cdot \frac{1 + 4\,\zeta_g^2\,(\omega/\omega_g)^2}{\left(1 - (\omega/\omega_g)^2\right)^2 + 4\,\zeta_g^2\,(\omega/\omega_g)^2}\,; \quad -\infty < \omega < \infty\,, \tag{13}$$

where $S_0$ represents a constant power spectral density level due to white noise, and the parameters $\omega_g$ and $\zeta_g$ denote the characteristic frequency and damping ratio of the soil layer, respectively. The filter represented by the second term in Eq. (13), thus attenuates the frequency content for $\omega > \omega_g$ as $\omega \to \infty$, and amplifies the frequencies in the vicinity of $\omega = \omega_g$. Neglecting the presence of the filter results to white noise modelling of the excitation, which implies infinite power of the resulting process, an unphysical yet mathematically convenient idealisation.

A secondary system is attached to the primary structure, at roof level, and is successively modelled as (i) linear, (ii) bilinear and (iii) sliding SDoF oscillator in cascade (see: § 3.1). Neglecting the feedback action of the secondary oscillator onto the primary one is in fact admissible provided the oscillator is sufficiently light and it does not vibrate close to or is in tune with the primary structure [6].

Requirement for this problem is the quantification of the full probabilistic structure of the response of each secondary system, due to white noise and filtered white noise modelling of the earthquake excitation.

#### 4.2.2 Response quantification

Due to the linearity and deterministic form of the primary system considered, the analysis is carried out externally after extracting the mass and stiffness matrices from the SAP2000 model (see: § 2), thus avoiding strenuous pre- and post-processing tasks required by the Monte Carlo simulations. Further, owing to the cascade approximation, the analysis is independently carried out for the primary structure by means of classical modal analysis, retaining only a single mode of vibration. Such an approximation is permissible due to the high participation of modal mass.

A suite of 5000 synthetic ground motions are first generated through the summation of cosines with amplitudes and frequencies characterised by the power spectrum under consideration, and uniformly distributed over the interval $[0, 2\pi]$ [12]. In doing this, the frequency

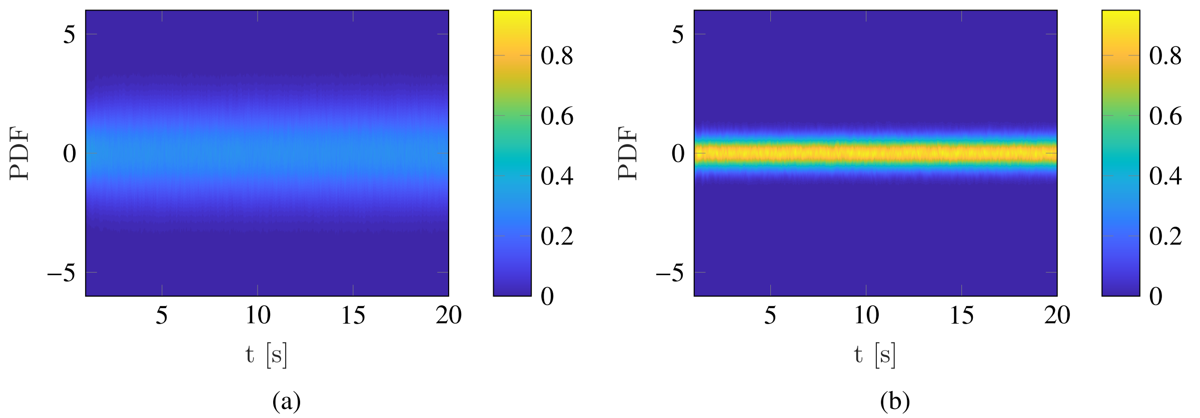


Figure 6: Response PDF of the absolute acceleration of the primary structure ($\omega_{\rm p} = 14.76\,\mathrm{rad/s}$, $\zeta_{\rm p} = 0.05$) due to: (a) white noise ($S_0 = 0.00214\,\mathrm{m^2/s^3}$); and (b) filtered white noise ($S_0 = 0.00214\,\mathrm{m^2/s^3}$, $\omega_g = 5\,\mathrm{rad/s}$, $\zeta_g = 0.2$) base excitation evaluated through 5000 Monte Carlo realisations.

interval $[0, \tilde{\omega}]$ is considered, where $\tilde{\omega} = 100$ is an upper cut-off frequency, beyond which the spectral density is regarded negligible. Furthermore, a soft soil is assumed with parameters $S_0 = 0.00214\,\mathrm{m}^2/\mathrm{s}^3$, $\omega_g = 5\,\mathrm{rad/s}$ and $\zeta_g = 0.2$ [13]. The resulting generated ground motions of variance $\sigma^2_{\ddot{u}_\xi} = 0.428$ and $\sigma^2_{\ddot{u}_\xi} = 0.097$, corresponding to white noise and filtered white noise, respectively, closely approximating the variance of the power spectrum.

The absolute acceleration response time history of the primary structure is then numerically determined at roof level. Figure 6 shows the evolution of the resulting stationary response PDF for $t \geq 1$, quantified through the kernel nonparametric probability density esimate, for the two forms of excitation under consideration.

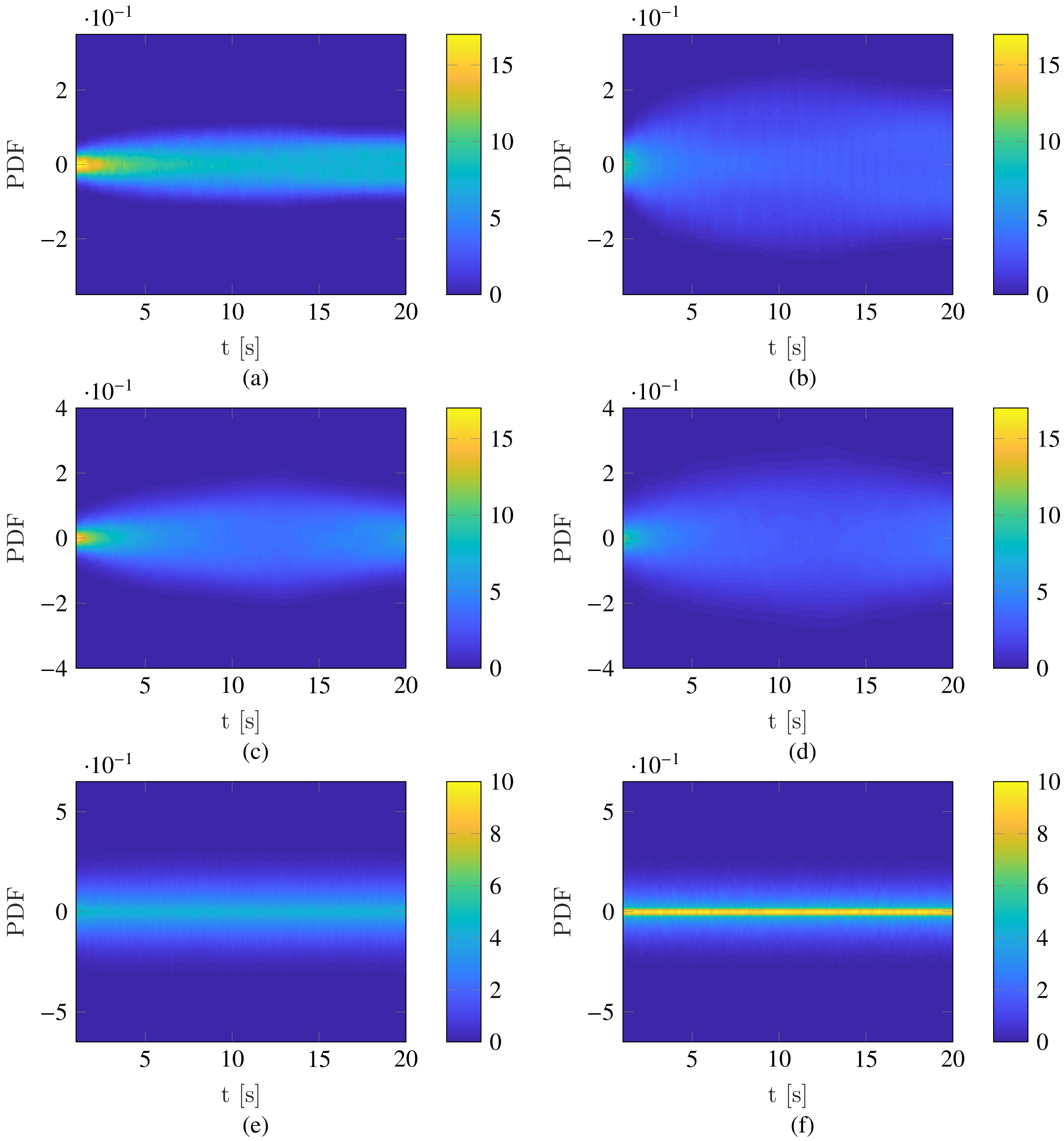


Figure 7: Response PDF of the (a, b) linear ($\omega_s = 4.43\,\mathrm{rad/s}$, $\zeta_s = 0.02$, $\psi = 1$), (c, d) bilinear ($\omega_s = 4.43\,\mathrm{rad/s}$, $\zeta_s = 0.02$, $a_s = 0.3$, $\psi = 0.01$) and (e, f) sliding ($\mu = 0.02$) secondary oscillators, due to white noise (left) and filtered white noise (right) base excitation ($S_0 = 0.00214\,\mathrm{m}^2/\mathrm{s}^3$, $\omega_g = 5\,\mathrm{rad/s}$, $\zeta_g = 0.2$) evaluated through 5000 Monte Carlo realisations.

The analysis of the three secondary systems under consideration is carried out next. We circumvent the prohibitively time-consuming runs typically encountered in conventional analysis of such strongly-nonlinear systems by exploiting the solvers presented in § 3. In doing this, each response time history of the primary structure is successively used as input to the associated solver which is treated as black box model. For the linear and bilinear system, the parameters considered are $\psi = 1$ and $a_{\mathrm{s}} = 0.3$, $\psi = 0.01$, respectively, as well as $\omega_{\mathrm{s}} = 0.3$, $\omega_{\mathrm{p}} = 4.43\,\mathrm{rad/s}$ and $\zeta_{\mathrm{s}} = 0.02$. For the sliding system a friction coefficient of $\mu = 0.02$ is assumed.

Figures 7(a) - 7(d) plot the evolution of the displacement response PDF for the linear and bilinear secondary oscillators for $t \geq 1$. The velocity response PDF of the sliding oscillator is plotted in Figures 7(e) - 7(f). The results are quantified through the 5000 Monte Carlo realisations, and are presented for the two forms of excitation considered.

## 5 CONCLUSIONS

Assistant computational tools have been developed for the analysis and response quantification of structural systems in a systematic and automated fashion.

The open application programming interface (OAPI) of the commercial structural analysis software SAP2000 has been exploited and a procedure has been presented for the analysis of linear systems in conjunction with MATLAB, featuring model updating, analysis execution and extraction of the results. In-house ancillary functions have been developed for this purpose, that complement and extend existing capabilities of the OAPI.

Standalone solvers have been presented for the dynamic response analysis of bilinear hysteretic, rigid-plastic, and rocking oscillators subjected to an arbitrary excitation force. Such dynamical systems, are indicative of distinct nonlinear behaviour and representative of a wide spectrum of primary as well as secondary structures of engineering interest. The solvers developed herein can be used in a black-box type approach with third-party software.

The features at the current development stage have been showcased on the response-surface-based reliability analysis of a truss structure, facilitated through the use of the SAP2000 OAPI, and the response quantification of stochastically driven nonlinear secondary oscillators. The second application further demonstrates interoperability of the solvers with SAP2000.

The tools developed are available, along with user instructions, at this link. They allow convenient simplifications in the modelling and analysis process, error tractability, and can assist learning on the subject of structural dynamics. They may find application in parametric and stochastic analysis and form the basis for the development of additional modules.

### References

[1] Computers and Structures. SAP2000. *Release 20.0.0*, 2013.

[2] A. Sextos and G. Balafas. Using the new SAP2000 open application programming interface to develop an interactive front-end for the modal pushover analysis of bridges. *International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 2011.

[3] The MathWorks Inc. MATLAB. *Release 8.2*, 2013.

[4] S. Kasinos, A. Palmeri, S. Maheshwari and M. Lombardo. Dynamic analysis of steel frames with uncertain semi-rigid connections. *International Conference on Structural Safety & Reliability*, 2017.

[5] S. Kasinos, A. Palmeri and M. Lombardo. Using the vibration envelope as a damage-sensitive feature in composite beam structures. *Structures*, **1**, 67–75, 2015.

[6] S. Kasinos. *Seismic response analysis of linear and nonlinear secondary structures*. PhD thesis, 2018.

[7] K. Atkinson. *An introduction to numerical analysis*. John Wiley & Sons, 2008.

[8] S. Lee and B. Kwak. Response surface augmented moment method for efficient reliability analysis. *Structural Safety*, **28**, 261–272, 2006.

[9] G. Blatman, B. Sudret and M. Berveiller. Quasi-random numbers in stochastic finite element analysis. *18 ème Congrès Français de Mécanique*, 2007.

[10] K. Kanai. An empirical formula for the spectrum of strong earthquake motions. *Bulletin of the Earthquake Research Institute, University of Tokyo, Japan*, **39**, 1961.

[11] H. Tajimi. A statistical method of determining the maximum response of a building structure during an earthquake. *Proceedings of the 2nd World Conference in Earthquake Engineering*, 1960.

[12] M. Shinozuka and G. Deodatis. Simulation of stochastic processes by spectral representation. *Applied Mechanics Reviews*, **44**, 191–204, 1991.

[13] K. Haciefendioglu and F. Birinci. Stochastic dynamic response of masonry minarets subjected to random blast and earthquake-induced ground motions. *Structural Design of Tall and Special Buildings*, **20**, 669–678, 2011.