Robotic Project

MAE C263A
Team 7

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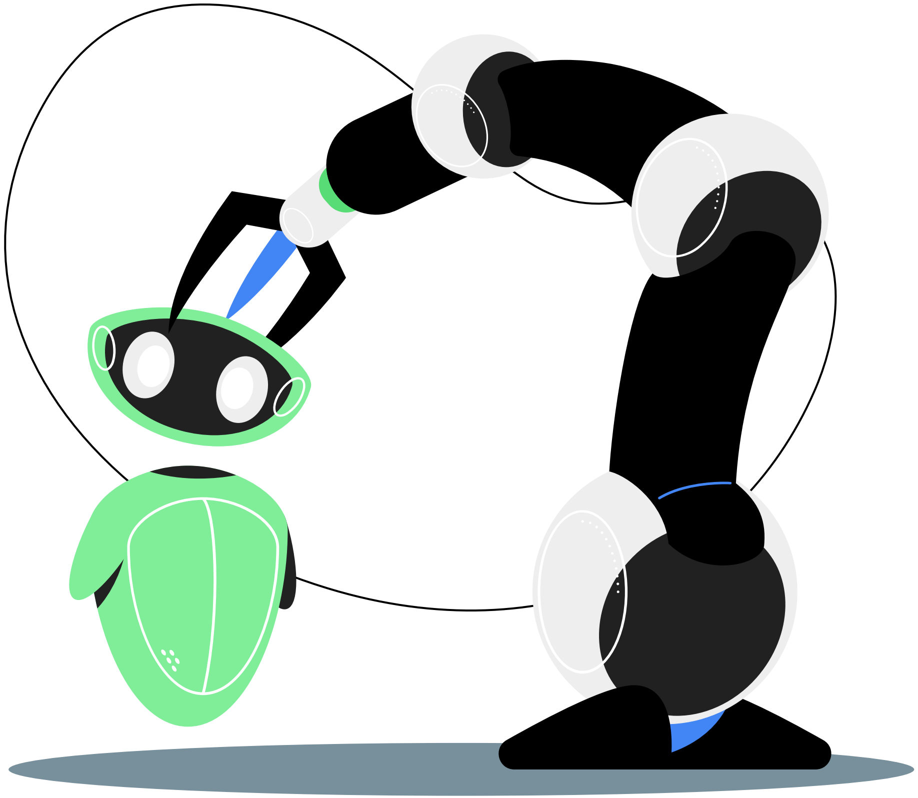


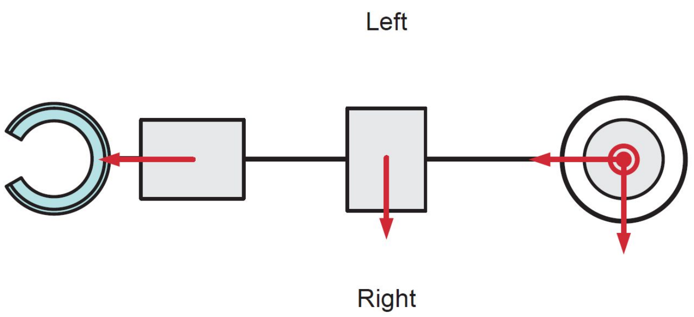
Initial Design Goals

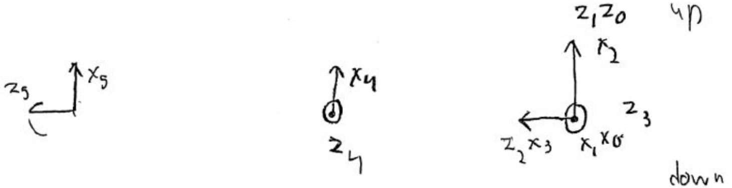
- Steps for success:
    - Move to a known home configuration of the robot arm.
    - Move to a hard-coded position
    - Pick up a cup
    - Move it to a predetermined location.
    - Deposit the payload.

- 5 DOF Robotic Arm
    - 3 DOF in spherical Joint at base
    - 1 DOF elbow
    - 1 DOF wrist
    - *1 DOF to actuate end effector

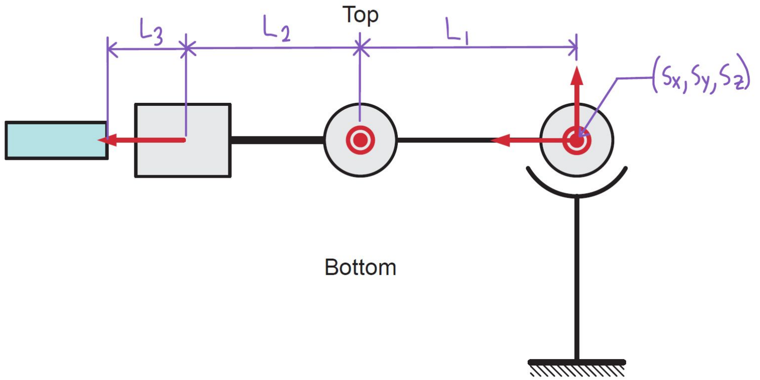
Kinematics (sketch)

Top View





Right View



Forward Kinematics DH Parameters

i-1	i	i+1	α_{i-1}	a_{i-1}	d_i	θ_i
0	1	2	0	0	0	θ_1
1	2	3	-90	0	0	θ_2
2	3	4	-90	0	0	θ_3
3	4	5	0	L1	0	θ_4
4	5	*	90	0	L2	θ_5

Forward Kinematics Transfer Functions

$$ {}^{0}_{1}T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_3 & -C_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} C_4 & -S_4 & 0 & L1 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{5}T = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & -L2 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

${}^{0}_{5}T$

```
[  C5*(C4*(S1*S3 + C1*C2*C3) + S4*(C3*S1 - C1*C2*S3)) - C1*S2*S5, - S5*(C4*(S1*S3 + C1*C2*C3) + S4*(C3*S1 - C1*C2*S3)) - C1*C5*S2, S4*(S1*S3 + C1*C2*C3) - C4*(C3*S1 - C1*C2*S3), a1*(S1*S3 + C1*C2*C3) - a2*(C4*(C3*S1 - C1*C2*S3) - S4*(S1*S3 + C1*C2*C3))]
[- C5*(C4*(C1*S3 - C2*C3*S1) + S4*(C1*C3 + C2*S1*S3)) - S1*S2*S5,   S5*(C4*(C1*S3 - C2*C3*S1) + S4*(C1*C3 + C2*S1*S3)) - C5*S1*S2, C4*(C1*C3 + C2*S1*S3) - S4*(C1*S3 - C2*C3*S1), a2*(C4*(C1*C3 + C2*S1*S3) - S4*(C1*S3 - C2*C3*S1)) - a1*(C1*S3 - C2*C3*S1)]
[                      C5*(S2*S3*S4 - C3*C4*S2) - C2*S5,                      - C2*C5 - S5*(S2*S3*S4 - C3*C4*S2),                  -S(3 + 4)*S2,                                  -S2*(a2*S(3 + 4) + a1*C3)]
[                                                     0,                                                      0,                             0,                                                          1]
```

Inverse Kinematics

Started with $Px^2 + Py^2 + Pz^2$ in order to get theta4

$$\theta_4 = A\tan 2\left[\frac{P_x^2+P_y^2+P_z^2-L_1^2-L_2^2}{2L_1L_2},\ \pm\sqrt{1-\frac{P_x^2+P_y^2+P_z^2-L_1^2-L_2^2}{2L_1L_2}}\right]$$

Theta3 was next, using:

$$P_z = -S_2\left[L_2 S_{34} + L_1 C_3\right]$$

$$r_{33} = -S_{34} S_2$$

Getting:

$$\theta_3 = \tan^{-1}\left[\frac{1-a\sin\theta_4}{a\cos\theta_4}\right] \quad a = \frac{P_z - L_2 r_{33}}{r_{33} L_1}$$

Inverse Kinematics

Theta2:

$$\sin\theta_2 = \frac{-r_{33}}{s(\theta_3+\theta_4)} \qquad \theta_2 = A\tan 2\left[\frac{-r_{33}}{s(\theta_3+\theta_4)},\ \pm\sqrt{1+\frac{r_{33}}{s(\theta_3+\theta_4)}}\right.$$

Theta5 started with this:

$$r_{31} = c_5[s_2 s_3 s_4 - c_3 c_4 s_2] - c_2 s_5$$

$$r_{32} = -s_5[s_2 s_3 s_4 - c_3 c_4 s_2] - c_2 c_5$$

In order to get:

$$\boxed{\theta_5 = A\tan 2\left(\frac{-r_{31}c_2 - r_{32}b}{b^2+c_2^2},\ \pm\sqrt{1+\frac{r_{31}c_2+r_{32}b}{b^2+c_2^2}}\right)}$$

$$b = s_2 s_3 s_4 - s_3 c_4 s_2$$

Inverse Kinematics

Finally, going back to find Theta1:

$$P_x C_1 + P_y S_1 = C_2 \left(L_2 S(3+4) + L_1 \cos_3 \right)$$

$$-P_x S_1 + P_y C_1 = L_2 C(3+4) - L_1 S_3$$

$$\sin\theta_1 = \frac{P_y C_2 [L_2 S(3+4) + L_1 C_3] - P_x [L_2 C(3+4) - L_1 S_3]}{P_x^2 + P_y^2}$$

$$\theta_1 = \mathrm{Atan2}\left(\sin\theta_1 , \pm\sqrt{1-\sin\theta_1} \right)$$

Matlab Simulation

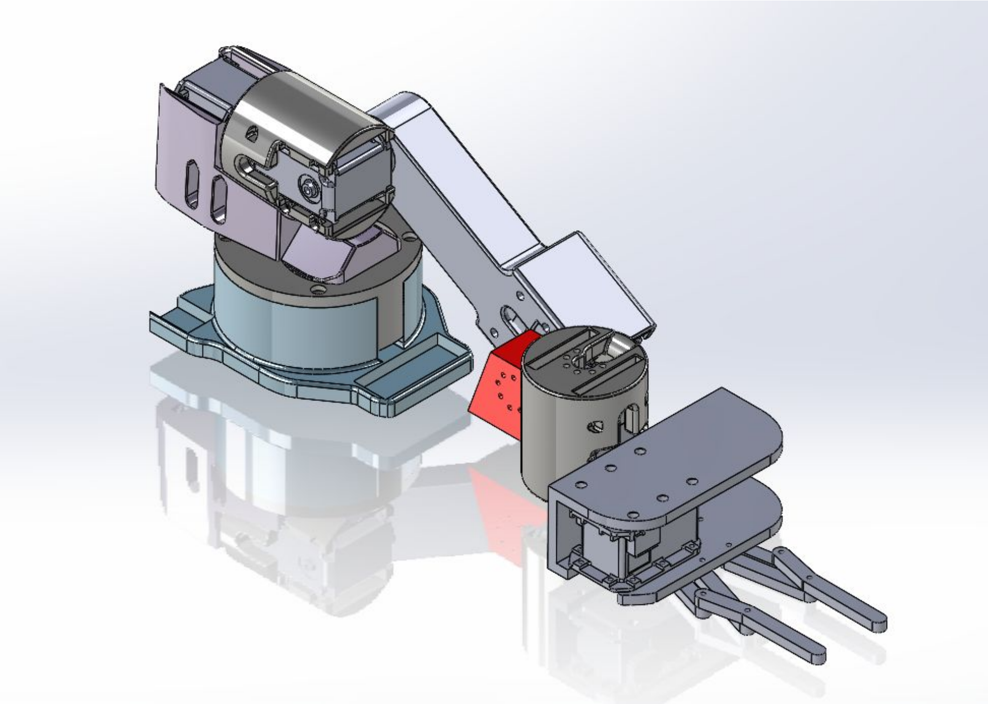
```
L1 = c(1);
L2 = c(2);
L3 = c(3);
t1 = joint(1);
t2 = joint(2);
t3 = joint(3);
t4 = joint(4);
t5 = joint(5);
% DH parameters
DH = [0 0 0 t1;...
   -pi/2 0 0 t2;...
   -pi/2 0 0 t3;...
   0 L1 0 t4;...
   pi/2 0 L2 t5];
```

Matlab Simulation

```
L1 = 0;
L2 = 0;
L3 = 0;
c = [L1 L2 L3];
% Joint Space
for i = 1:N
   p = [x(i) y(i) z(i)]';
   T0e = [R p;0 0 0 1];
   [theta1(i),theta2(i),theta3(i),theta4(i),theta5(i)] = IK(T0e,c);
end
t1 = unwrap(theta1);
t2 = unwrap(theta2);
t3 = unwrap(theta3);
t4 = unwrap(theta4);
t5 = unwrap(theta5);
joint = [t1;t2;t3;t4;t5];
```

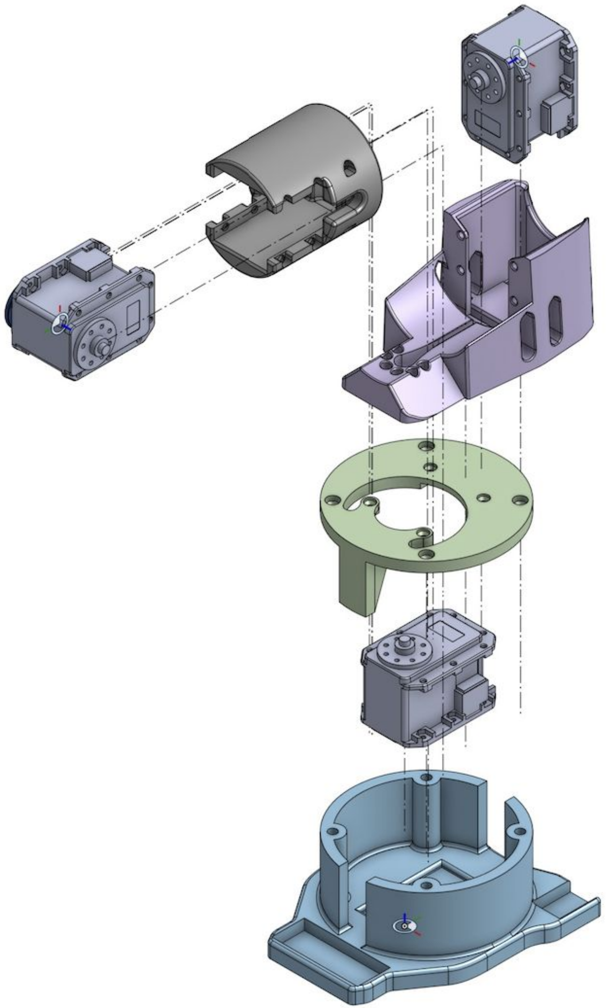
Final Assembly

- Spherical joint keeps CoM near base
- Printed out of Nylon
- Individual testing determined nylon and servos can support weight easily



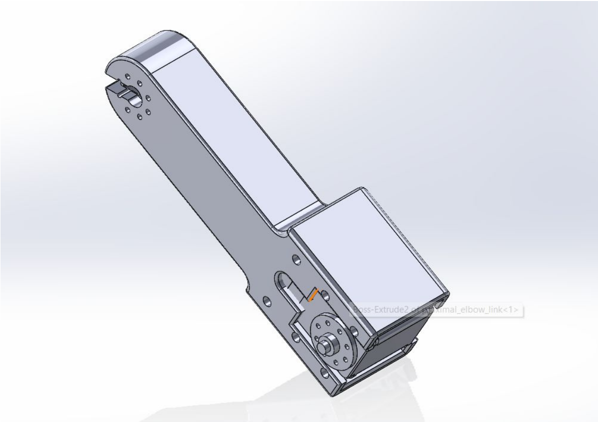
Spherical Joint - Exploded View

- Yaw, Roll, Pitch
- Minimize distance between links

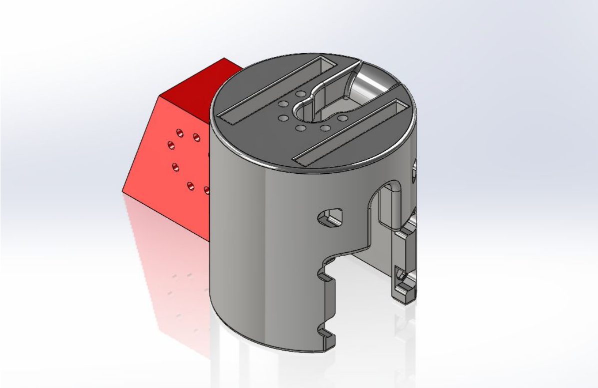


Elbow Joints

- Proximal:
  - Length dependent on rest position
  - Parallel joint axes
- Distal:
  - What you see is just a placeholder, we had an oopsies
  - Short link to minimize moment, tradeoff with reachable WS
  - Joint axis gives wrist rotation



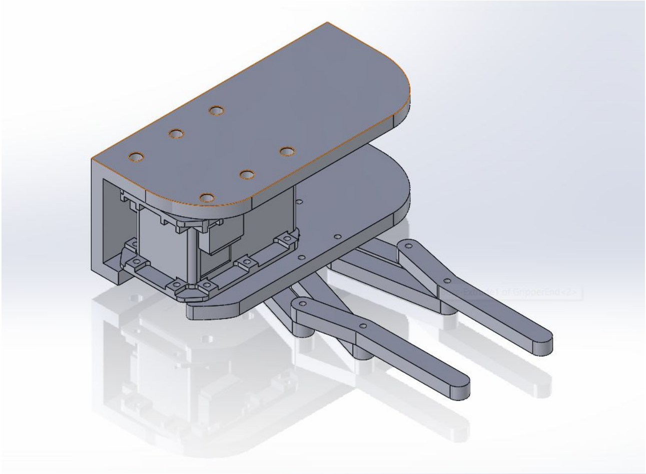
Proximal Elbow Link



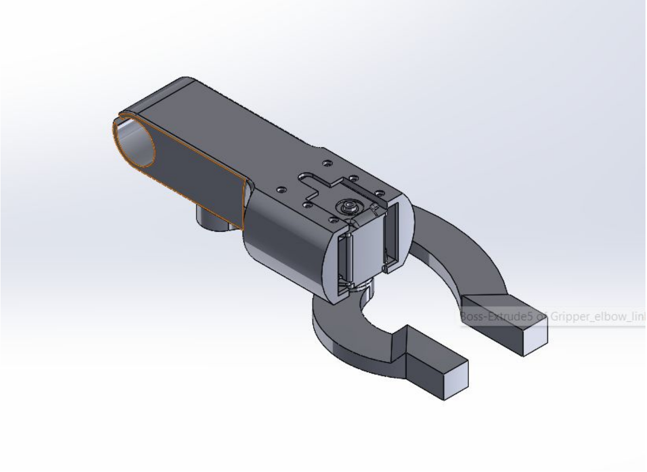
“Distal Elbow Link”

End Effector

- 4 Bar linkage with parallel sides
  - Grippers remain parallel
- Notably clunky with servo



- Considering alternatives:
- Clamp with only one moving arm
  - Gripper rotates around motor



The Next Steps

- Optimize CAD
- Remaining 3D Printing/Prototyping (proximal elbow - EE)
- Final Assembly
- Inverse Kinematics Matlab simulation
- Report Writing