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# CENG 140

## C Programming

Spring '2017-2018

Take Home Exam 1

Section 2

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### Student Information

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### Answer 1.

In the `<stdlib.h>` library, there exist a function called "rand" and both "randomIntegerMaxUnBiased" and "randomIntegerMaxNaive" use it.

Firstly, it is better to express working method of this function. This function returns a generated pseudo-random number between 0 and `RAND_MAX`. Here `RAND_MAX` value may vary between implementations but it's value is at least 32767.

Now, two implemented functions can be compared.

"randomIntegerMaxNaive" function takes an integer value which is "max" as parameter and call "rand" function. Afterwards, "rand" function returns an integer value. Then, by taking modulo of this number with respect to  $(max + 1)$ , a pseudo-random number is generated between 0 and max. If max is less than 0, the function returns 0.

Let investigate "randomIntegerMaxUnBiased" function. In this function, we define an integer "adjustedRandMax" and its value is

$$adjustedRandMax = RAND\_MAX - (RAND\_MAX \% (max + 1));$$

In this code line, "adjustedRandMax" is regulated as the closest number to the "RAND\_MAX"

which can be divided  $(max + 1)$ . The purpose of this code line is to make sure that there is no number repeating in the interval  $(0, RAND\_MAX)$  more than others when the modulo is taken. If this wasn't be done like in the "randomIntegerMaxNaive" function, there may exists a little interval at the end of  $(0, RAND\_MAX)$  which includes some numbers up to max and some of not. Thus, some of numbers may have more chance to be chosen when  $\%(max + 1)$  is applied if  $RAND\_MAX$  cannot be divided  $(max + 1)$ . Although chance of take a random integer within this residual interval is too little, it still affect the fairness of the function "randomIntegerMaxNaive".

The function "randomIntegerMaxUnBiased" returns a pseudo-random number within an interval which is all numbers have the same possibility to get taken. If "rand" function returns a value greater than adjusted max, the "rand" function runs again till the number is between 0 to "adjustedRandMax". This makes the function "randomIntegerMaxUnBiased" is more fair.

## Answer 2.

Let number of magic rooms be  $M$ ,

Number of Wumpuses be  $W$ .

Then the total number of rooms is  $M + 2$  (with start and exit room);

Total number of rooms except current room is  $M + 1$ .

### 2.a

Get eaten by a Wumpus:

Possibility to get eaten by wumpus in the 1<sup>st</sup> move is :  $\frac{W}{M+1}$ .

Possibility to get eaten by wumpus in the 2<sup>nd</sup> move is :  $\frac{M-W}{M+1} \times \frac{W}{M+1}$ . ( $M-W$  is all rooms except current room, wumpuses and exit room).

Possibility to get eaten by wumpus in the 3<sup>rd</sup> move is :  $(\frac{M-W}{M+1})^2 \times \frac{W}{M+1}$ . (In the first 2 move, the adventurer goes not to exit room and wumpuses room but in the third move, s/he goes to wumpuses room).

Then the general formula is :  $(\frac{M-W}{M+1})^{i-1} \times \frac{W}{M+1}$  (where  $i$  is number of moves).

Now, the question becomes  $\sum_{i=1}^n (\frac{M-W}{M+1})^{i-1} \times \frac{W}{M+1} = ?$

$\frac{W}{M+1}$  can be put in front of the  $\Sigma$  :

$$\frac{W}{M+1} \times \sum_{i=1}^n (\frac{M-W}{M+1})^{i-1}$$

By using the sum formula of geometric series (since the formula starts from 0, it is convenient to use another variable 'j' which is  $i-1$ ):

$$\frac{W}{M+1} \times \sum_{j=0}^{n-1} \left(\frac{M-W}{M+1}\right)^j = \frac{W}{M+1} \times \frac{1 - \left(\frac{M-W}{M+1}\right)^{j+1}}{1 - \frac{M-W}{M+1}}.$$

Since  $\frac{M-W}{M+1}$  is between  $(0, 1)$ , this series is converges when  $(n \rightarrow \infty)$  and it is equal to

$$\frac{W}{M+1} \times \frac{1}{1 - \frac{M-W}{M+1}}.$$

## 2.b

Exit the dungeon without the gold :

Possibility to exit the dungeon without the gold in the 1<sup>st</sup> move is :  $\frac{1}{M+1}$ . (directly go to the exit room, (m+1) is all rooms except current which is start room)

Possibility to exit the dungeon without the gold in the 2<sup>nd</sup> move is :  $\frac{M-W-1}{M+1} \times \frac{1}{M+1}$ . (M-W-1 is all rooms except current room, wumpuses, gold room and exit room).

Possibility to exit the dungeon without the gold in the 3<sup>rd</sup> move is :  $\left(\frac{M-W-1}{M+1}\right)^2 \times \frac{1}{M+1}$ . (In the first 2 move, the adventurer goes not to exit room, wumpuses room and gold room and in the third move, s/he goes to exit room).

Then the general formula is :  $\left(\frac{M-W-1}{M+1}\right)^{i-1} \times \frac{1}{M+1}$  (where i is number of moves).

Now, the question becomes  $\sum_{i=1}^n \left(\frac{M-W-1}{M+1}\right)^{i-1} \times \frac{1}{M+1} = ?$

$\frac{1}{M+1}$  can be put in front of the  $\Sigma$  :

$$\frac{1}{M+1} \times \sum_{i=1}^n \left(\frac{M-W-1}{M+1}\right)^{i-1}$$

By using the sum formula of geometric series (since the formula starts from 0, it is convenient to use another variable 'j' which is i-1.):

$$\frac{1}{M+1} \times \sum_{j=0}^{n-1} \left(\frac{M-W-1}{M+1}\right)^j = \frac{1}{M+1} \times \frac{1 - \left(\frac{M-W-1}{M+1}\right)^{j+1}}{1 - \frac{M-W-1}{M+1}}.$$

Since  $\frac{M-W-1}{M+1}$  is between  $(0, 1)$ , this series is converges when  $(n \rightarrow \infty)$  and it is equal to

$$\frac{1}{M+1} \times \frac{1}{1 - \frac{M-W-1}{M+1}}.$$

## 2.c

Exit the dungeon with the gold :

Possibility to exit the dungeon with the gold in the 1<sup>st</sup> move is : 0 (Impossible)

Possibility to exit the dungeon with the gold in the  $2^{nd}$  move is :  $\frac{1}{M+1} \times \frac{1}{M+1}$ . (First adventurer goes to gold room and directly goes to the exit room).

Possibility to exit the dungeon with the gold in the  $3^{rd}$  move is :  $\binom{2}{1} (\frac{1}{M+1})^2 \times \frac{M-W}{M+1}$ .  
(In the first 2 move, the adventurer goes not to exit room, wumpuses room but goes gold room and in the third move, s/he goes to exit room)  
( $\binom{2}{1}$  is coming from adventurer goes to gold room either in the first move or second move).

Then the general formula is :  $\binom{i-1}{1} \times (\frac{M-W}{M+1})^{i-1} \times (\frac{1}{M+1})^2$  (where i is number of moves).

Now, the question becomes  $\sum_{i=1}^n \binom{i-1}{1} \times (\frac{M-W}{M+1})^{i-1} \times (\frac{1}{M+1})^2 = ?$

$(\frac{1}{M+1})^2$  can be put in front of the  $\Sigma$  :

$$(\frac{1}{M+1})^2 \times \sum_{i=1}^n \binom{i-1}{1} \times (\frac{M-W}{M+1})^{i-1}$$

Since  $\binom{i-1}{1} = i - 1$ ;

$$(\frac{1}{M+1})^2 \times \sum_{i=1}^n (i - 1) \times (\frac{M-W}{M+1})^{i-1}.$$

### Answer 3.

IN THE MEMORY :):):)