CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a) i) $|z^2| = z.\bar{z}$
   3(x+yj) + 4 = 2j - (x-yj)
   3x+4+3yj=2j-x+yj
   (4x+4)+(2y-2)j = 0
   Then x = -1 , y=1. So $|z^2| = (-1+j).(-1-j) = 1+1 = 2$

   ii)

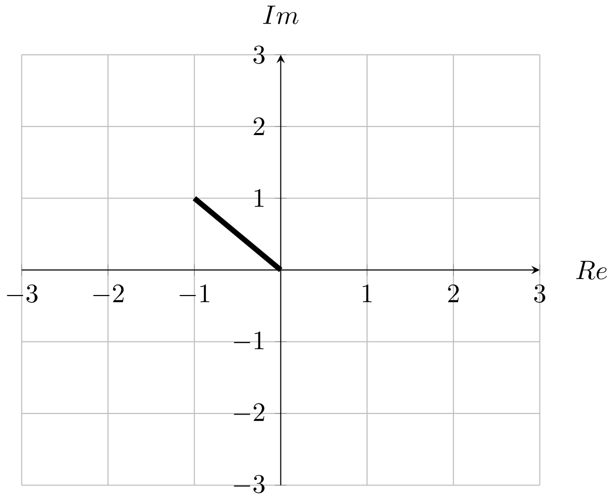

Figure 1: $z = -1 + j$

   (b)

$$z^3 = 4^3.e^{j\frac{\pi}{2}}$$
$$z = 4.e^{j(\frac{\pi}{6}+\frac{2\pi}{3}k)}\ for\ k = 0,1,2$$

   (c)

$$\frac{\sqrt{2}e^{-j\cdot\frac{\pi}{4}}.2e^{j\cdot\frac{\pi}{3}}}{\sqrt{2}e^{j\frac{\pi}{4}}} = 2.e^{-j\frac{\pi}{6}}$$
$$Then\ r = 2 \quad \theta = -\frac{\pi}{6}$$

   (d)

$$-j.(cos(\frac{\pi}{2}) + j.sin(\frac{\pi}{2}))$$
$$= -j.cos(\frac{\pi}{2}) + sin(\frac{\pi}{2}) = 1$$
$$= 1.e^{j.2\pi}$$

2. Firstly,corresponding t values in $y(t) = x(\frac{1}{2}t + 1)$ should be found.

When t=-2 in $x(t)$, $\frac{1}{2}t+1=-2 \rightarrow t=-6$,

When t=-1 in $x(t)$, $\frac{1}{2}t+1=-1 \rightarrow t=-4$,

When t= 0 in $x(t)$, $\frac{1}{2}t+1=0 \rightarrow t=-2$,

When t= 1 in $x(t)$, $\frac{1}{2}t+1=1 \rightarrow t=-0$,

When t= 2 in $x(t)$, $\frac{1}{2}t+1=2 \rightarrow t=2$.

In $x(t)$ graph , value from $-\infty$ to -2 (which is 0) is value for $y(t)=x(\frac{1}{2}t+1)$ from $-\infty$ to -6 .
In $x(t)$ graph , value from -2 to -1 is value for $y(t)=x(\frac{1}{2}t+1)$ from -6 to -4 .
In $x(t)$ graph , value from -1 to 1 (which is 1) is value for $y(t)=x(\frac{1}{2}t+1)$ from -4 to 0 .
In $x(t)$ graph , value from 1 to 2 is value for $y(t)=x(\frac{1}{2}t+1)$ from 0 to 2 .
In $x(t)$ graph , value from 2 to ∞ (which is 0) is value for $y(t)=x(\frac{1}{2}t+1)$ from 2 to ∞ .

So,the graph of $y(t)=x(\frac{1}{2}t+1)$ is ,

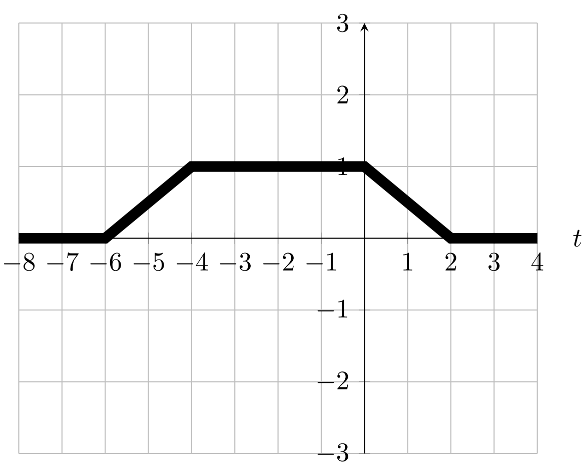

Figure 2: t vs. $x(1/2t+1)$.

3. (a) This is the graph for x[-n] ,

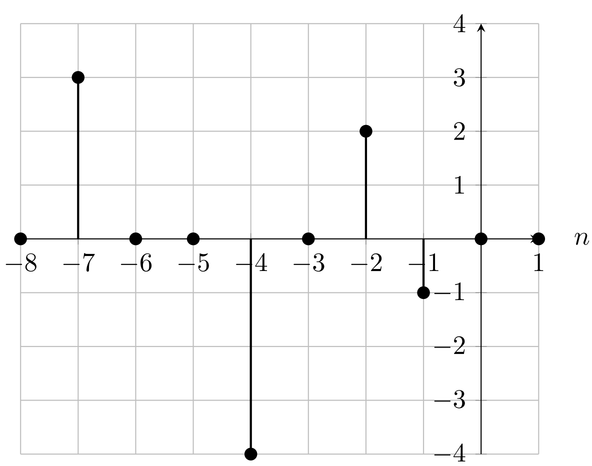

Figure 3: n vs. $x[-n]$.

And , below is a graph for $x[2n+1]$,

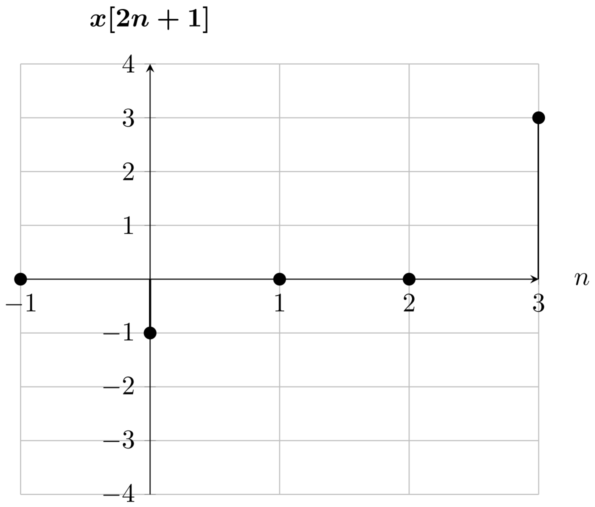

Figure 4: n vs. $x[2n + 1]$.

So , the graph of $x[-n] + x[2n + 1]$ is ,

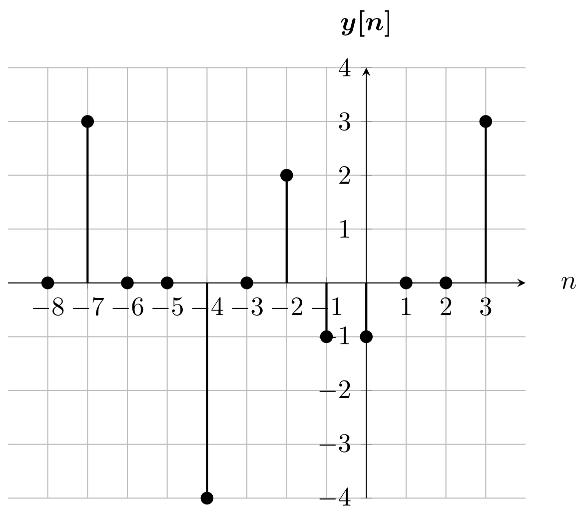

Figure 5: n vs. $x[-n] + x[2n + 1]$.

(b) Unit impulse function $\delta(n - n_0)$ is :
1 at $n = n_0$,
0 for all values for n except n_0.

Thus,
$x[n] + x[2n + 1] = 3\delta[n - 3] - \delta[n] - \delta[n + 1] + 2\delta[n + 2] - 4\delta[n + 4] + 3\delta[n + 7]$

4. (a) Lets say parts with cosine and sine has the fundamental periods N_1 and N_2 respectively.
For N_1:

$$\begin{aligned} 3cos[\frac{13\pi n}{10}] &= 3cos[\frac{13\pi(n + N)}{10}] \\ \frac{13\pi n}{10} + 2\pi k &= \frac{13\pi n}{10} + \frac{13\pi N}{10} \\ N &= \frac{20k}{13} \quad k = 13, 26, ... \\ Then\ N_1 &= 20 \end{aligned}$$

For N_2:

$$
\begin{aligned}
5sin[\frac{7\pi n}{3} &- \frac{2\pi}{3}] = 5sin[\frac{7\pi(n+N)}{3} - \frac{2\pi}{3}] \\
\frac{7\pi n}{3} - \frac{2\pi}{3} + 2\pi k &= \frac{7\pi n}{3} + \frac{7\pi N}{3} - \frac{2\pi}{3} \\
N &= \frac{6k}{7} \quad k = 7, 14, ... \\
Then\ N_2 &= 6
\end{aligned}
$$

Therefore, fundamental period of the given signal is $N_0 = 60$.

(b)

$$
\begin{aligned}
&5sin[3n - \frac{\pi}{4}] = 5sin[3(n+N) - \frac{\pi}{4}] \\
&3n - \frac{\pi}{4} + 2\pi k = 3n + 3N - \frac{\pi}{4} \\
&N = \frac{2\pi k}{3} \\
&Since\ period\ needs\ to\ be\ an\ integer\ in\ discrete\ time\ periodic\ signals,\ this\ signal\ is\ not\ periodic. \\
&We\ cannot\ get\ an\ integer\ value\ for\ N\ because\ of\ the\ \pi\ in\ the\ numerator.
\end{aligned}
$$

(c) Fundamental period of a continuous signal is given by $T_0 = \frac{2\pi}{|\omega_0|}$ equation.
In $k \times cos(\omega_0 t + \theta) = 2cos(3\pi t - \frac{2\pi}{5})$,
(3π) is the ω_0 and $(-\frac{2\pi}{5})$ is θ .
Thus , fundamental period T_0 is $\frac{2\pi}{|\omega_0|} = \frac{2\pi}{|3\pi|}$

$$\boxed{T_0 = \frac{2}{3}}$$

(d) This signal is periodic with T if it satisfies this equation :
$-je^{j5t} = -je^{j5(t+T)}$.
Since $-je^{j5(t+T)} = -je^{j5t} \times e^{j5T}$,
$-je^{j5t} = -je^{j5(t+T)} = -je^{j5t} \times e^{j5T} \rightarrow 1 = e^{j5T}$.
Because $e^{j2\pi k} = 1$,$= e^{j5T} = e^{j2\pi k} \rightarrow 5T = 2\pi k \rightarrow T = \frac{2\pi k}{5}, \forall k$.
Equation for T is found , $T = \frac{2\pi k}{5}$.However fundamental period T_0 is the smallest positive value of T which holds the equation , that is value of k is 1 and

$$\boxed{T_0 = \frac{2\pi}{5}}$$

5. The signal is not symmetric with respect to the y-axis, so it's not even. It is also not symmetric with respect to the origin which means it's not odd as well.

$$Ev\{x[n]\} = \frac{x[n] + x[-n]}{2}$$

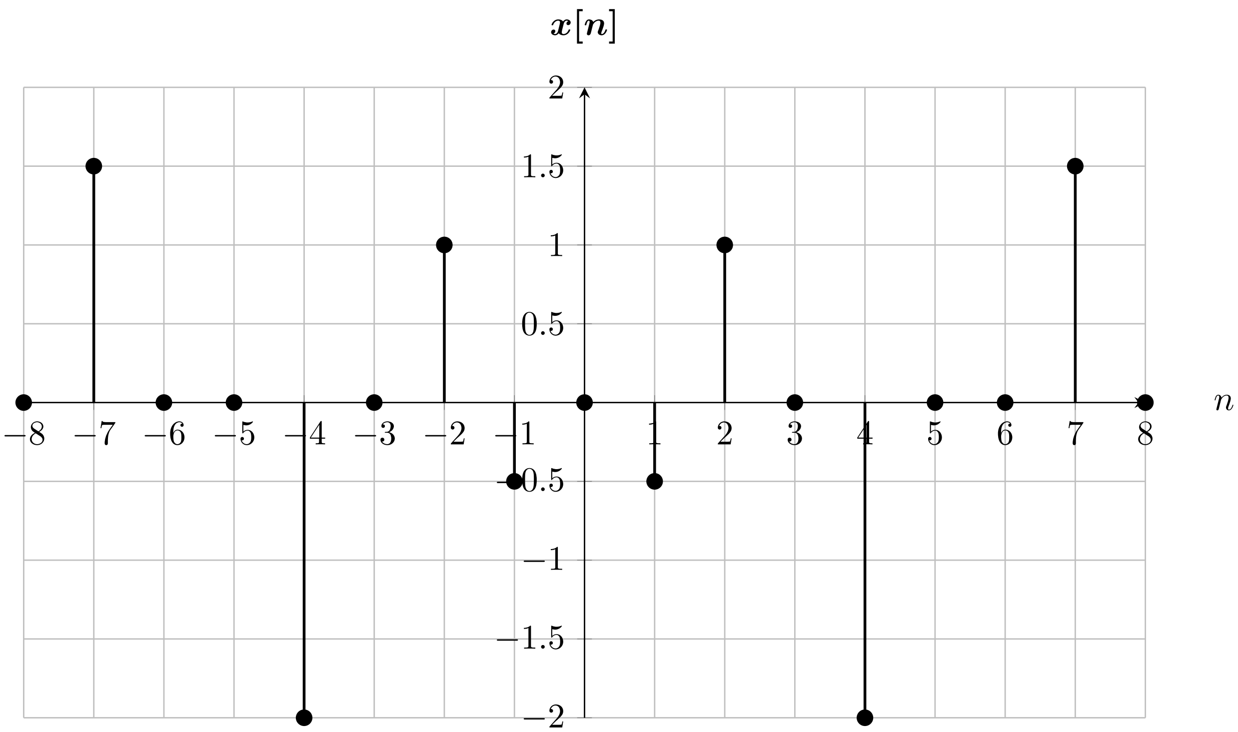

Figure 6: Even decomposition of the signal

$$Odd\{x[n]\} = \frac{x[n] - x[-n]}{2}$$

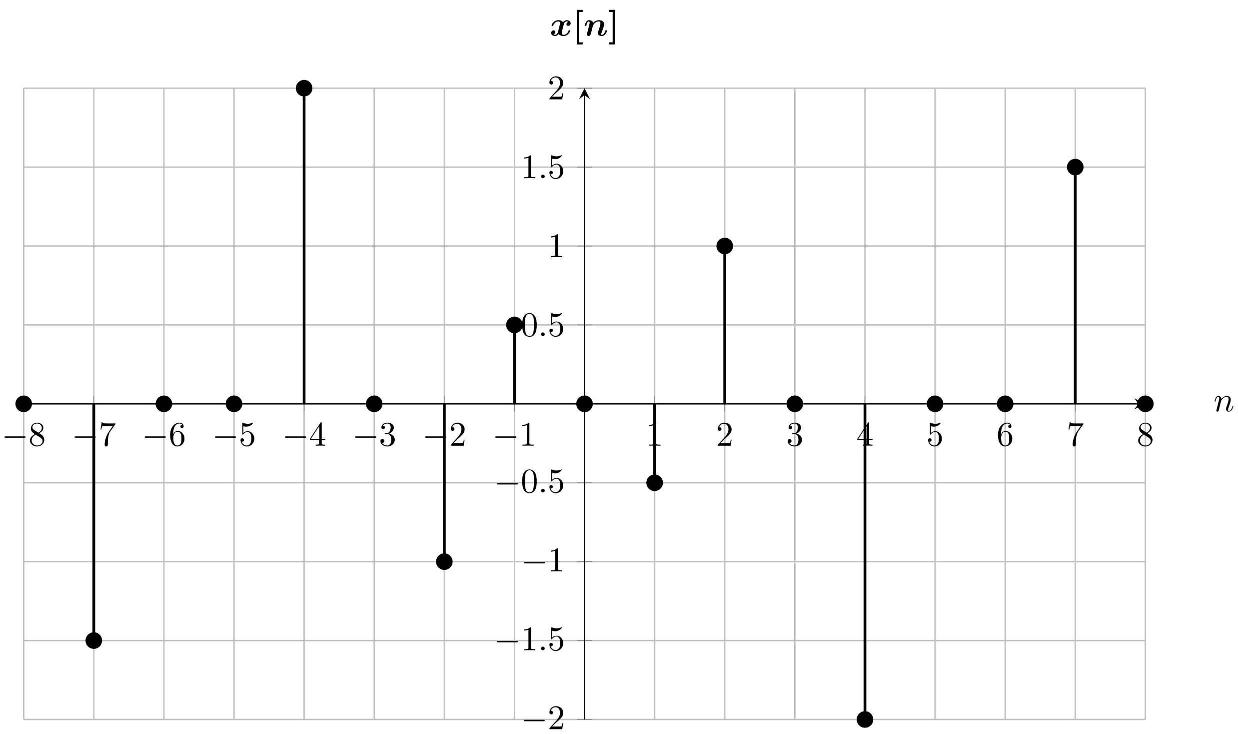

Figure 7: Odd decomposition of the signal

6. (a) $y(t) = x(2t - 3)$

1)Memory property is investigated.

If system's output at t_0 depends on only input at t_0 , this system is memoryless.Otherwise, it is said to have memory .

In this system , output at $t_0 = 3$ depends on only input at $t_0 = 3$.However , for other all options , this system's output at t_0 does not depends on input at t_0.
Thus,this system has memory.

2)Stability property is investigated.

A system is stable when bounded inputs lead to bounded outputs.Bounded means that for any t value in range $(-\infty, \infty)$, $x(t)$ does not have $-\infty$ or ∞,i.e. it's value is finite.

This system takes $x(t)$ as an input and give output as $x(2t-3)$.

Let $x(t)$ is bounded(unbounded inputs don't affect stability , bounded inputs are required).
This means that for all t values in $(-\infty, \infty)$, x(t) is finite.That's also means that for all $x(2t-3)$ values are finite because also another t_0 value maps that point , $(t_0, x(t_0))$ and we know that it is finite.

Thus , this system is stable.

3)Causality property is investigated.

If system's output at t_0 depends on inputs $(-\infty, t_0]$, this system is causal.If system's output depends on future input at least once , this system is non-causal.
Inputs in range $(-\infty, 3]$ does not affect the causality.However,system's output depends on future inputs when input is greater than 3.
Thus,this system is non-causal.

4)Linearity property is investigated.

A system is linear when superposition principle holds.

Let $y_1(t) = x_1(2t-3)$ and $y_2(t) = x_2(2t-3)$.

Then $\alpha_1 y_1(t) + \alpha_2 y_2(t) = \alpha_1 x_1(2t-3) + \alpha_2 x_2(2t-3)$.

When $(\alpha_1 x_1(t) + \alpha_2 x_2(t))$ input is given to the system , if output is exactly same with $\alpha_1 x_1(2t-3) + \alpha_2 x_2(2t-3)$, this system is said to be linear.

This system takes $x(t)$ as an output and produces $x(2t-3)$,namely $x(t) \rightarrow h \rightarrow x(2t-3)$.
Since this system functionality is changing the "t" in the input to the "2t-3" and it does not add a coefficient to output function , $\alpha_1 x_1(t)$ is mapped to $\alpha_1 x_1(2t-3)$ and $\alpha_2 x_2(t)$ is mapped to $\alpha_2 x_2(2t-3)$.Then $\alpha_1 x_1(t) + \alpha_2 x_2(t)$ input is mapped to $\alpha_1 x_1(2t-3) + \alpha_2 x_2(2t-3)$.

Thus, this system is linear.

5)Invertibility property is investigated.

If an invertible system produces the output y(t) for the input x(t), then its inverse produces the output x(t) for the input y(t).

If a system exists that takes $2t-3$ as input and gives output as t,namely the inverse function of it,this system is said to be invertible system.

Since $w(t) = \frac{t+3}{2}$ function makes this inverse transition and defined everywhere , h^{-1}system exists.

$h(x(t)) = y(t) = x(2t-3) \rightarrow h^{-1}(x(2t-3)) = y^{-1}(t) = x(t)$

Thus,this system is invertible.

6)Time-invariance property is investigated.

Let system H maps to $x(t)$ to $y(t)$.
When a delay is applied to input $x(t + Delay)$, if this delay directly equates the output i.e. $y(t + Delay)$, this system is said to be time invariant.Otherwise it is time variant.

In this system , delayed input $x_d(t-t_0)$ is transformed to $y(t-t_0) = x(2(t-t_0)-3) = x(2t-2t_0-3)$.

$y(t) = x(2t-3)$ is output for $x(t)$.Now, delay operation is applied to output.

$y(t-t_0) = x(2t-t_0-3)$.

Since $x(2t-t_0-3)$ is not equal to $x(2t-2t_0-3)$, this system is time variant.

(b) ***memory*** **:**
$y(t_0) = t_0.x(t_0)$ for $t_0 \in R$
Since the output of the system depends on only the present values of the input, the system is memoryless.
stability **:**

$$Let\ x(t) = 2 \rightarrow system \rightarrow y(t) = 2t$$

When we choose $x(t) = 2$ which is bounded and the value of t goes to infinity, $y(t) = 2t$ goes to infinity as well. Therefore, $y(t)$ is unbounded and the system is not stable.
causality **:**
$y(t_0) = t_0.x(t_0)$ for $t_0 \in R$
The system is independent of future values of input as it depends on only present values of input, so the system is causal.
linearity **:**
$a_1.x_1(t) \rightarrow system \rightarrow a_1.y_1(t) = a_1.t.x_1(t)$
$a_2.x_2(t) \rightarrow system \rightarrow a_2.y_2(t) = a_2.t.x_2(t)$

$x_3(t) = a_1.x_1(t) + a_2.x_2(t) \rightarrow system \rightarrow t(a_1.x_1(t) + a_2.x_2(t)) = t.a_1.x_1(t) + t.a_2.x_2(t)$ and since this equals the summation of the above $a_1.t.x_1(t)$ and $a_2.t.x_2(t)$, the superposition principle holds. As a result, the system is linear.
invertibility **:**
This system is not invertible because it is not one-to-one. When we choose $x_1(t) = \delta(t)$ and $x_2(t) = 2\delta(t)$, outputs in both cases tend to infinity at t=0, and 0 otherwise.
time − invariance **:**
delay the input: $x(t-t_0) \rightarrow system \rightarrow y(t) = t.x(t-t_0)$
delay the output: $x(t) \rightarrow system \rightarrow y(t) \rightarrow y(t-t_0) = (t-t_0).x(t-t_0)$
Since $t.x(t-t_0)$ and $(t-t_0).x(t-t_0)$ are not equal, the system is time-variant.

(c) $y[n] = x[2n-3]$

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$y[n-n_0] = x[2n-n_0-3]$.

Since $x[2n-n_0-3]$ is not equal to $x[2n-2n_0-3]$, this system is time variant.

(d) ***memory*** **:**

$$y[n] = \sum_{k=1}^{\infty} x[n-k] = x[n-1] + x[n-2] + ...$$

Since the output depends on the past values of input, the system has memory.
stability **:**

$$Let\ x[n] = 2 \to system \to y[n] = \sum_{k=1}^{\infty} x[n-k] = \sum_{k=1}^{\infty} 2 = \infty$$

When we give the system $x[n] = 2$ which is a bounded input, output is unbounded. So, the system is not stable.
causality **:**

Since k is a positive integer ranging from 1 to infinity, output depends on only the past values of input $x[n-1]+x[n-2]+x[n-3]+...$ Therefore, the system is causal.
$\boldsymbol{linearity}$ **:**

$$a_1x_1[n] \rightarrow system \rightarrow a_1y_1[n] = a_1.\sum_{k=1}^{\infty} x_1[n-k]$$

$$a_2x_2[n] \rightarrow system \rightarrow a_2y_2[n] = a_2.\sum_{k=1}^{\infty} x_2[n-k]$$

$$x_3[n] = a_1x_1[n] + a_2x_2[n] \rightarrow system \rightarrow \sum_{k=1}^{\infty} a_1x_1[n-k] + a_2x_2[n-k] = a_1.\sum_{k=1}^{\infty} x_1[n-k] + a_2.\sum_{k=1}^{\infty} x_2[n-k]$$

Since the superposition principle holds, the system is linear.
$\boldsymbol{invertibility}$ **:**
$x[n] \rightarrow system \rightarrow y[n] \rightarrow inverse \rightarrow x[n]$
The system is invertible with inverse $w[n] = y[n+1] - y[n]$
$\boldsymbol{time - invariance}$ **:**
delay the input:

$$x[n-n_0] \rightarrow system \rightarrow y[n] = \sum_{k=1}^{\infty} x[n-n_0-k]$$

delay the output:

$$x[n] \rightarrow system \rightarrow y[n] \rightarrow y[n-n_0] = \sum_{k=1}^{\infty} x[n-n_0-k]$$

Since these two are equal, the system is time-invariant.