Classification of Electromagnetic Waves Supported by Waveguiding Structures (extending along the z-axis)

TEM Waves

$\boxed{E_z = 0}$ and $\boxed{H_z = 0}$

Both $\bar{E}$ and $\bar{H}$ fields lie on the transverse plane. They do NOT have components along the waveguide axis.

TE Waves

(H modes)

$\boxed{E_z = 0}$ but $\boxed{H_z \neq 0}$

Only $\bar{E}$ field lies on the transverse plane. $\bar{H}$ field has a non-zero component along the z-axis.

TM Waves

(E modes)

$\boxed{H_z = 0}$ but $\boxed{E_z \neq 0}$

Only $\bar{H}$ field lies on the transverse plane. $\bar{E}$ field has a nonzero component along the z-axis.

Note: The principal mode in "two-conductor" transmission lines is the TEM mode. When the distance (d) between the conductors become larger than $\frac{\lambda}{4}$, TE and TM modes become also possible. Such "higher modes" are not desirable in transmission lines, so $d < \frac{\lambda}{4}$ condition is almost always satisfied.

Note: On the other hand, the waveguides are "single conductor" structures - for that reason, they can NOT carry TEM waves. Only TE and TM modes (waves) can propagate along a waveguide.

TRANSMISSION LINES

- A transmission line (TL) is a "wave guiding" structure composed of two separate conductors.

- A TL can support the propagation of TEM, TM and TE waves in general. The TEM (Transverse Electromagnetic) mode is the "Principal Mode" of a TL, supported when the distance (d) between its conductors is less than $\frac{\lambda}{4}$.

- For $d > \frac{\lambda}{4}$, TE (Transverse Electric) and TM (Transverse magnetic) modes are also possible but these "higher order" modes are not desired in TL operation.

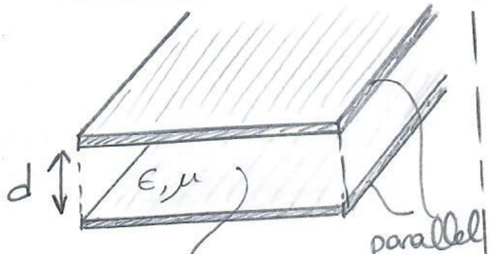
- While operating with TEM modes, $\bar{E}$, $\bar{H}$ and $\hat{n}$ (direction of propagation along the TL) vectors are mutually perpendicular to each other (similar to uniform plane wave propagation).

- Transmission Lines can be analyzed, in general, by using the Electromagnetic Field Theory to determine $\bar{E}(\bar{r},t)$ and $\bar{H}(\bar{r},t)$. Alternatively, for the TEM mode operation, transmission lines can be analyzed by using the "circuit theory approach" to determine traveling voltage $v(z,t)$ and current $i(z,t)$ waves.

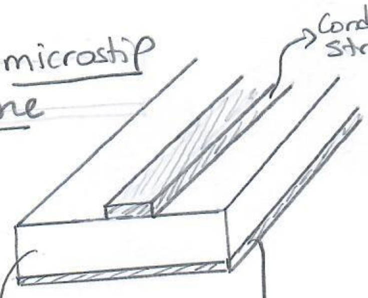
- Let "l" be the length of a given TL. For $l \ll \frac{\lambda}{4}$ conventional circuit theory can be used. Otherwise, the transmission line theory in terms of traveling voltage and current waves must be used.

Types of Transmission Lines

Parallel-plate TLs

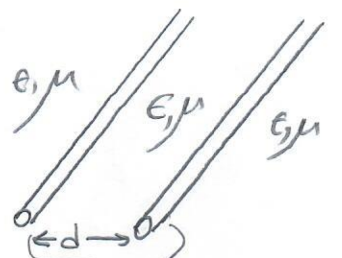


a microstrip line



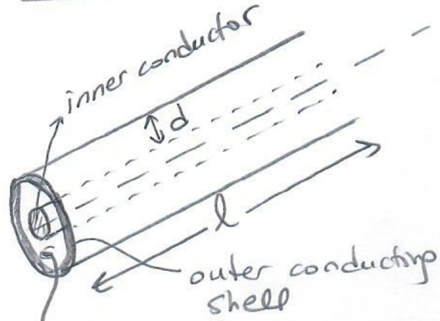
(Strip lines and microstrip lines are types of parallel plate TLs. They can be used at higher microwave frequencies)

Two-wire TLs



(used in overhead power lines, for ex.)

Coaxial TLs



(used in connections between electronic devices such as between the antenna and TV receiver etc.)

- EM fields are confined in the dielectric region
- very small EM Interference from external world.

(*) If the conductors are not perfect, "conductor losses" occur due to finite conductivity $\sigma_c < \infty$ of the metal sections. Also, if the dielectric is not perfect, "dielectric losses" occur due to non-zero conductivity $\sigma_d \neq 0$ of the dielectric section.

Leakage currents exist within lossy dielectrics (i.e. if $\sigma_d \neq 0$)

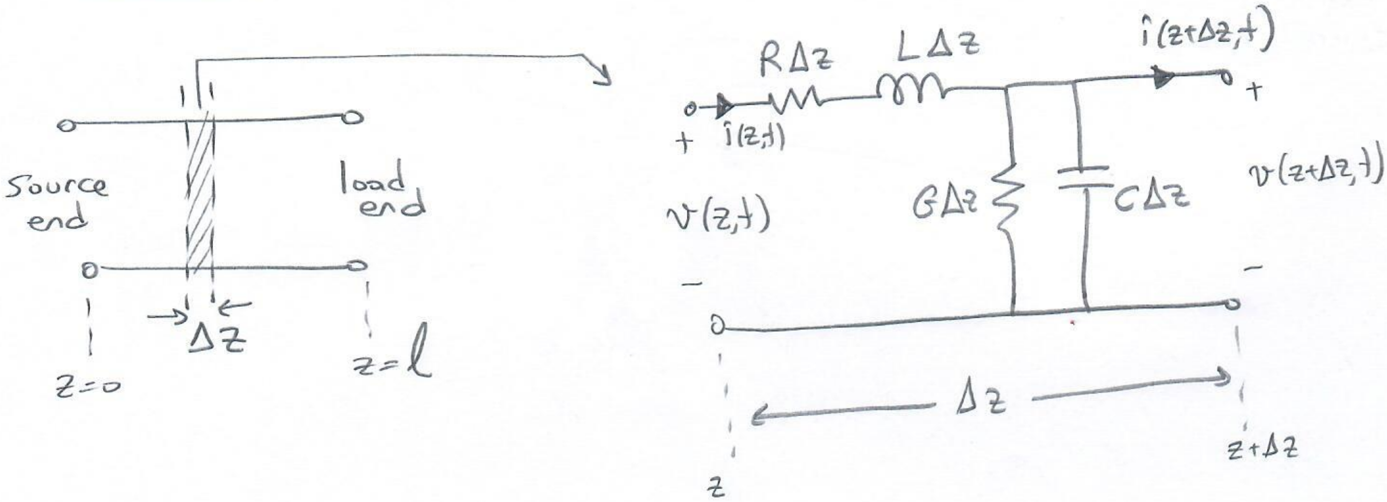
Ex: Coaxial TL

$\bar{J} = \sigma_d \bar{E}$ exists if $\sigma_d \neq 0$

$\downarrow$

cause Joule's heating loss within the dielectric region.

Use of KVL and KCL to obtain the "Telegrapher's Equation" in Transmission Lines



Distributed parameters of the transmission line $\Leftarrow$ $\left\{ C \; (F/m), \quad L \; (H/m), \quad R \; (\Omega/m), \quad G \; (\mho/m) \text{ or } (S/m) \right\}$

- C (F/m) → capacitive effects $(\bar{E})$
- L (H/m) → inductive effects $(\bar{H})$
- R (Ω/m) → conductor losses (due to $\sigma_c < \infty$)
- G $(\mho/m)$ or (S/m) → dielectric losses (due to $\sigma_d \neq 0$)

Apply KVL - Kirchoff's Voltage Law

$$v(z,t) = (R\Delta z)\, i(z,t) + (L\Delta z)\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t)$$

$$\underbrace{\lim_{\Delta z \to 0}\left(\frac{v(z,t)-v(z+\Delta z,t)}{\Delta z}\right)}_{-\frac{\partial v(z,t)}{\partial z}} = \lim_{\Delta z \to 0}\left[\frac{R\cancel{\Delta z}\, i(z,t) + L\cancel{\Delta z}\frac{\partial i(z,t)}{\partial t}}{\cancel{\Delta z}}\right]$$

$$\Rightarrow \boxed{-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t}} \quad (1)$$

differentiate both sides wrt z (also multiply both sides by (-1))

$$\Rightarrow \boxed{\frac{\partial^2 v}{\partial z^2} = -R\frac{\partial i}{\partial z} - L\frac{\partial^2 i}{\partial z \partial t}} \quad (1')$$

Apply KCL - Kirchoff's Current Law

$$i(z,t) = (G\Delta z)\,v(z+\Delta z,t) + (C\Delta z)\frac{\partial v(z+\Delta z,t)}{\partial t} + i(z+\Delta z,t)$$

$$\underbrace{\lim_{\Delta z\to 0}\left[\frac{i(z,t)-i(z+\Delta z,t)}{\Delta z}\right]}_{-\frac{\partial i(z,t)}{\partial z}} = \underbrace{\lim_{\Delta z\to 0}\left[\frac{G\cancel{\Delta z}\,v(z+\Delta z,t) + C\cancel{\Delta z}\,\frac{\partial v(z+\Delta z,t)}{\partial t}}{\cancel{\Delta z}}\right]}_{G\,v(z,t) + C\frac{\partial v(z,t)}{\partial t}}$$

$$\Rightarrow \boxed{-\frac{\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t}} \quad (2) \quad \Rightarrow \quad \boxed{-\frac{\partial^2 i}{\partial z\,\partial t} = G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2}} \quad (2')$$

differentiate both sides wrt time

Now, substitute $\left(-\frac{\partial i}{\partial z}\right)$ from (2) and $\left(-\frac{\partial^2 i}{\partial z\partial t}\right)$ from (2') into eqn. (1') to obtain:

$$\frac{\partial^2 v}{\partial z^2} = R\left(Gv + C\frac{\partial v}{\partial t}\right) + L\left(G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2}\right)$$

Organize this partial differential equation (pde) as

$$\boxed{\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}}$$

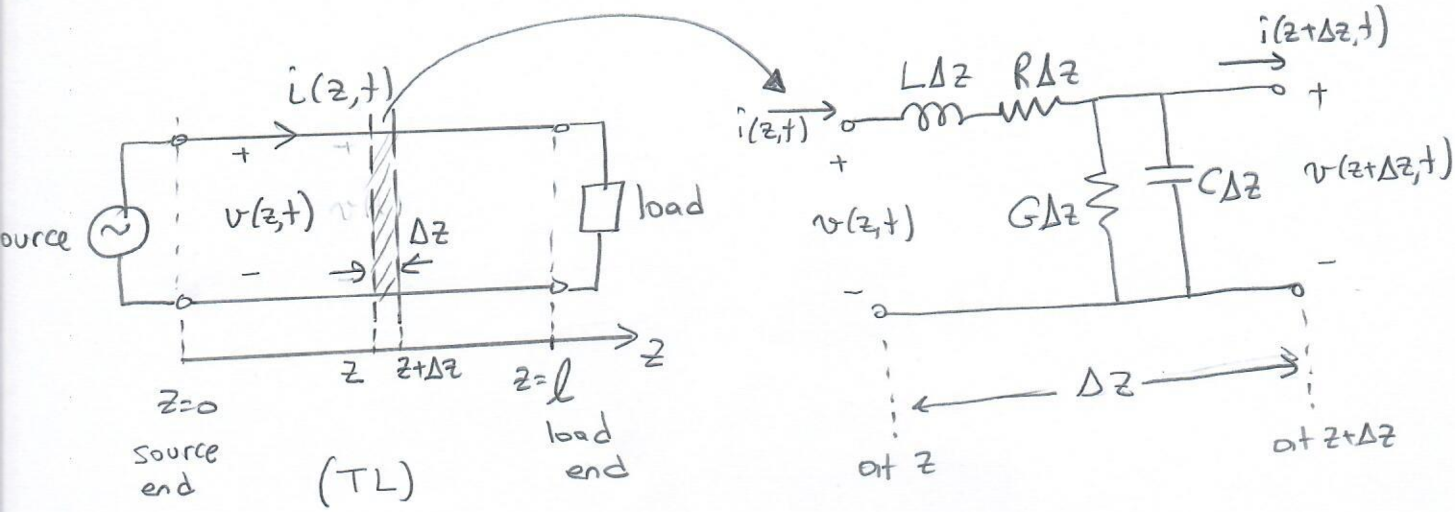
Telegrapher's Eqn. for $v(z,t)$

($i(z,t)$ satisfies the same pde.)

For lossless TLs, $R=G=0$

$$\Rightarrow \boxed{\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2}}$$

(wave equation for $v(z,t)$) $\Rightarrow$ Solution is a traveling wave in the form $f(z\mp vt)$ with $v = \frac{1}{\sqrt{LC}}$ velocity.



An infinitesimal section of a T.L. can be represented by a "Distributed Circuit Model" using the "per-unit length" parameters $R\,(\Omega/m)$, $L\,(H/m)$, $C\,(F/m)$, $G\,(\mho/m)$ uniformly and continuously.

Applying the KVL and KCL to the circuit above, it has been shown that we obtain:

$$\boxed{\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}}$$

and

$$\boxed{\frac{\partial^2 i}{\partial z^2} = RGi + (RC+LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}}$$

Telegrapher's Equations for $v(z,t)$ and $i(z,t)$

For monochromatic excitation of TLs, phasor domain analysis can be used where $\boxed{v(z,t) \longleftrightarrow V(z)}$ and $\boxed{i(z,t) \longleftrightarrow I(z)}$.

Therefore, in "AC Steady-State Analysis", letting $\boxed{\frac{\partial^n}{\partial t^n} \longleftrightarrow (j\omega)^n}$, we can get:

$$\frac{d^2 V(z)}{dz^2} = RGV(z) + (RC+LG)j\omega V(z) + LC(j\omega)^2 V(z)$$

$$\frac{d^2V(z)}{dz^2} - \Big[\underbrace{RG + j\omega RC}_{R(G+j\omega C)} + \underbrace{j\omega LG + (j\omega L)(j\omega C)}_{j\omega L(G+j\omega C)}\Big]V(z) = 0$$

$$\boxed{\frac{d^2V(z)}{dz^2} - \big[(R+j\omega L)(G+j\omega C)\big]V(z) = 0}$$

let $\boxed{\gamma^2 = (R+j\omega L)(G+j\omega C)}$ (γ: Propagation constant)

$$\Rightarrow \boxed{\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0}$$

needs to be solved for the traveling voltage phasor $V(z)$

Similarly, $\boxed{\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0}$ for the traveling current phasor $I(z)$.

Then $v(z,t) = Re\{V(z)e^{j\omega t}\}$ can be obtained.

and $i(z,t) = Re\{I(z)e^{j\omega t}\}$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0 \Longrightarrow \boxed{V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}}$$

is the general solution for $V(z)$ where V^+ and V^- are some arbitrary constants.

with $\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta}$

Propagation constant γ is complex in general.

α: Attenuation constant

β: Phase constant

$$V(z) = \underbrace{V^{+} e^{-\alpha z} e^{-j\beta z}}_{\substack{\text{forward traveling} \\ \text{wave (in } \hat{a}_z \text{ dir.)} \\ \text{(from source to load)}}} + \underbrace{V^{-} e^{\alpha z} e^{j\beta z}}_{\substack{\text{backward traveling} \\ \text{wave (in } -\hat{a}_z \text{ dir.)} \\ \text{(from load to source)}}}$$

$$\boxed{v = \frac{\omega}{Im(\gamma)} = \frac{\omega}{\beta}}$$

Velocity of propagation over TL (phase velocity)

$$\boxed{\lambda = \frac{2\pi}{Im(\gamma)} = \frac{2\pi}{\beta}}$$

wavelength of propagation

Compare propagation constants of a TL and a u.p.w

$$\gamma_{TL} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma_{u.p.w} = \sqrt{?\, j\omega\mu\, (\sigma + j\omega\epsilon)}$$

No term exists in γ_{upw} corresponding to R of γ_{TL}! Why? (no conductor losses in u.p.w. propagation in an unbounded lossy space)

Now consider eqn. ① of KVL application:

$$-\frac{\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t} \longleftrightarrow -\frac{dV(z)}{dz} = RI(z) + j\omega L I(z)$$
$$= (R + j\omega L) I(z)$$

$$\Rightarrow I(z) = -\frac{1}{R + j\omega L} \frac{dV(z)}{dz}$$
$$= -\frac{1}{R + j\omega L} \frac{d}{dz}\left[V^{+} e^{-\gamma z} + V^{-} e^{\gamma z}\right]$$

$$\Rightarrow \quad I(z) = -\frac{1}{R+j\omega L}\left[-\gamma' V^+ e^{-\gamma' z} + \gamma' V^- e^{\gamma' z}\right]$$

$$I(z) = \frac{\gamma'}{R+j\omega L}\left[V^+ e^{-\gamma' z} - V^- e^{\gamma' z}\right]$$

where $\dfrac{\gamma'}{R+j\omega L} = \dfrac{\sqrt{(R+j\omega L)(G+j\omega C)}}{R+j\omega L} = \sqrt{\dfrac{G+j\omega C}{R+j\omega L}} \triangleq Y_o$

$$\Rightarrow \quad \boxed{Z_o \triangleq \frac{1}{Y_o} \triangleq \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

Characteristic Impedance of a TL (Ω)

(Compare with $\eta_{upw} = \sqrt{\dfrac{j\omega\mu}{\sigma+j\omega\epsilon}}$: intrinsic impedance (Ω) for u.p.w propagation in a lossy medium)

$$\Rightarrow \quad \boxed{I(z) = \frac{1}{Z_o}\left(V^+ e^{-\gamma z} - V^- e^{\gamma z}\right)}$$

Or define $\dfrac{V^+}{Z_o} = I^+$ and $\dfrac{-V^-}{Z_o} = I^-$

to write

$$\boxed{I(z) = I^+ e^{-\gamma z} + I^- e^{\gamma z}} \qquad \boxed{V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}}$$

where $\boxed{\dfrac{V^+}{I^+} = Z_o = -\dfrac{V^-}{I^-}}$

note the (-) sign!

Note that $\dfrac{V(z)}{I(z)} \neq Z_o$ in general

Lossless Transmission Lines $\Longleftrightarrow$ $\boxed{R = 0 \text{ and } G = 0}$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}\Big|_{R=G=0} = \sqrt{(j\omega L)(j\omega C)} = \boxed{j\omega\sqrt{LC} = \gamma_{\substack{lossless\\TL}}}$$

$$\gamma = \alpha + j\beta \quad (\text{in general}) \Longrightarrow \boxed{\begin{array}{c}\alpha = 0\\ \beta = \omega\sqrt{LC}\end{array}}$$ in lossless Transmission Lines!

(Compare to $\beta_{upw} = \omega\sqrt{\mu\epsilon}$ in a lossless medium)

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}\Bigg|_{R=G=0} = \sqrt{\frac{\cancel{j\omega} L}{\cancel{j\omega} C}} = \sqrt{\frac{L}{C}} \Longrightarrow \boxed{Z_{o\,\substack{lossless\\TL}} = \sqrt{\frac{L}{C}}}$$

(Compare to $\eta_{upw} = \sqrt{\frac{\mu}{\epsilon}}$ in a lossless medium)

$$v = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}} = \frac{1}{\sqrt{LC}} \Longrightarrow \boxed{v_{\substack{lossless\\TL}} = \frac{1}{\sqrt{LC}}}$$

(Compare to $v_{u.p.w} = \frac{1}{\sqrt{\mu\epsilon}}$ in a lossless medium)

Also, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{\cancel{2\pi}}{\cancel{2\pi} f\sqrt{LC}} = \frac{v}{f} \Longrightarrow \boxed{\lambda = \frac{v}{f}}$

Low-Loss Transmission Lines $\Longleftrightarrow$ $\begin{cases} R \ll \omega L \\ G \ll \omega C \end{cases}$

(most TLs behave like this at UHF and above)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{\underbrace{\cancel{RG}}_{\substack{\text{negligible}\\ \text{wrt } \omega^2 LC}} - \omega^2 LC + j\omega(RC + GL)}$$

$$\cong \sqrt{(j\omega)^2 LC + j\omega(RC + GL)}$$ (take it to the parenthesis of $(j\omega)^2 LC = -\omega^2 LC$)

$$\gamma \cong j\omega\sqrt{LC}\sqrt{1 - \frac{j\omega(RC+GL)}{\omega^2 LC}}$$

$$= j\omega\sqrt{LC}\sqrt{1 - \underbrace{j\frac{1}{\omega}\left(\frac{R}{L}+\frac{G}{C}\right)}_{\text{very small with respect to 1}}}$$

(as $R \ll \omega L \Rightarrow \frac{R}{\omega L} \ll 1$; Similarly, $\frac{G}{\omega C} \ll 1$)

Use Binomial Theorem to approximate the square-root term as

$$\sqrt{1-a} \approx 1 - \frac{a}{2} \quad \text{where } \begin{cases} a = \frac{j}{\omega}\left(\frac{R}{L}+\frac{G}{C}\right) \\ \text{and } |a| \ll 1 \end{cases}$$

$$\Rightarrow \gamma \cong j\omega\sqrt{LC}\left[1 - \frac{j}{2\omega}\left(\frac{R}{L}+\frac{G}{C}\right)\right]$$

(write in the $\gamma = \alpha + j\beta$ form)

$$\Rightarrow \gamma \cong \frac{1}{2\cancel{\omega}}\cancel{\omega}\sqrt{LC}\left(\frac{R}{L}+\frac{G}{C}\right) + j\omega\sqrt{LC}$$

$$\gamma \cong \underbrace{\frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)}_{\alpha} + j\underbrace{\omega\sqrt{LC}}_{\beta} = \alpha + j\beta$$

$$\Rightarrow \boxed{\begin{aligned} \alpha &\cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \\ \beta &\cong \omega\sqrt{LC} \end{aligned}}$$

for low-loss transmission lines. (where $Z_0 = \sqrt{\frac{L}{C}}$)

see page (TL-11)

Note that $\beta_{\substack{\text{lossless}\\ \text{TL}}} \cong \beta_{\substack{\text{low-loss}\\ \text{TL}}} = \omega\sqrt{LC}$

$\Rightarrow$ expressions for $v = \frac{\omega}{\beta}$ and $\lambda = \frac{2\pi}{\beta}$ are also approximately the same in lossless and low-loss transmission line cases!

Also, $$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L\left(1+\frac{R}{j\omega L}\right)}{j\omega C\left(1+\frac{G}{j\omega C}\right)}}$$

$$= \sqrt{\frac{L}{C}}\Big(1+\underbrace{\frac{R}{j\omega L}}_{\text{very small wrt unity}}\Big)^{1/2}\Big(1+\underbrace{\frac{G}{j\omega C}}_{\text{very small wrt unity}}\Big)^{-1/2}$$

(applying Binomial Thm - again)

$$Z_0 \approx \sqrt{\frac{L}{C}}\left(1+\frac{R}{2j\omega L}\right)\left(1-\frac{G}{2j\omega C}\right)$$

$$\approx \sqrt{\frac{L}{C}}\Big[1+\underbrace{\frac{1}{2j\omega}\left(\frac{R}{L}-\frac{G}{C}\right)}_{\text{a very small reactive part!}} + \underbrace{\cancel{\frac{RG}{4\omega^2 LC}}}_{\text{negligible (very small)}}\Big]$$

$$\boxed{Z_0 \approx \sqrt{\frac{L}{C}} \underbrace{- j\frac{1}{2}\sqrt{\frac{L}{C}}\left(\frac{R}{\omega L}-\frac{G}{\omega C}\right)}_{\text{a very small reactive part which is usually neglected}}}$$

$$\boxed{Z_0 \approx \sqrt{\frac{L}{C}}}$$ as in the lossless TL case.

Exercise Problem : Distortionless Transmission Line

Show that for the condition $\boxed{\frac{R}{L} = \frac{G}{C}}$, we'll obtain:

$$\gamma = \sqrt{\frac{C}{L}}(R+j\omega L) = \alpha + j\beta$$

$\longrightarrow \boxed{\alpha = R\sqrt{\frac{C}{L}}}$ independent of frequency

$\longrightarrow \boxed{\beta = \omega\sqrt{LC}}$ linear function of frequency

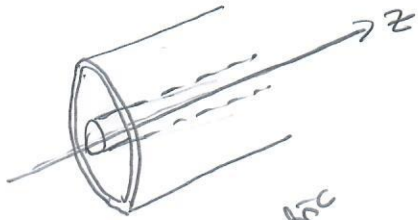
$$\Rightarrow \boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} \text{ (independent of frequency)}}$$

Also, $\boxed{Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$ as in the lossless case.

∴ All frequency components of an input wave travel at the same speed (v) experiencing the same attenuation (α) $\Rightarrow$ NO SIGNAL DISTORTION!

(*) Study section 8-3 in Cheng for TL parameters!

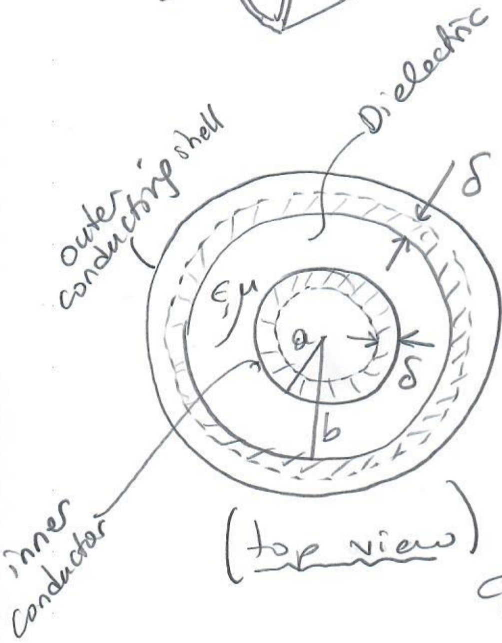
Example: Distributed Parameters of a Coaxial TL



Inner conductor radius $= a$

Outer conducting shell radius $= b$

Filled by a dielectric (ϵ, μ)



$$\boxed{L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \quad (H/m)}$$

$$\boxed{C = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \quad (F/m)}$$

Characteristic Impedance $\leftarrow Z_0 = \sqrt{\frac{L}{C}} = \cdots = \sqrt{\frac{\mu}{\epsilon}} \; \frac{\ln(b/a)}{2\pi} \quad (\Omega)$

Phase velocity $\leftarrow v = \frac{1}{\sqrt{LC}} = \cdots = \frac{1}{\sqrt{\mu\epsilon}}$: velocity of a plane wave in an unbounded medium with parameters ϵ, μ.

$R = R_{in} + R_{out}$

$$\boxed{R = \frac{1}{\sigma_c (2\pi a \delta)} + \frac{1}{\sigma_c (2\pi b \delta)} \quad (\Omega/m)}$$

AC resistance of the conductors (per unit length, for $l = 1m$.)

where $\delta = \frac{1}{\sqrt{\pi f \mu_c \sigma_c}}$ assuming a well developed skin effect.

skin depth

σ_c : conductance of the conductors, $\sigma_c < \infty$ but it is very large.

and $$\boxed{G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad (\mho/m)}$$

where $\sigma_d \neq 0$ but small (conductivity of the dielectric).