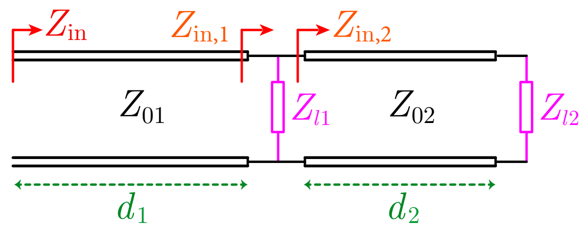
2020-2021, EE303 – Recitation 11

Question 1
Consider a combination of two loads and two lossless transmission lines.



The transmission lines are described as follows:

- $Z_{01} = 100\ \Omega$, $\quad d_1 = \lambda/4$, $\qquad$ • $Z_{02} = 200\ \Omega$, $\quad d_2 = 5\lambda/8$ $\qquad$ (1)

Given the load impedances $Z_{l1} = (100 - j200)\ \Omega$ and $Z_{l2} = (100 + j100)\ \Omega$, do the following.

- In the *second* transmission line, find the positions (measured from the load) of the first maximum and the first minimum of the voltage pattern.
- Find VSWR in both transmission lines.
- Find the input impedance of the overall combination, i.e., Z_{in}.

* Extra question: Is it possible to make VSWR in the first line *unity* by selecting a suitable Z_{l1} without changing any other item?

Solution
The reflection coefficient at the second load:

$$\Gamma_{l2} = \frac{Z_{l2} - Z_{02}}{Z_{l2} + Z_{02}} = \frac{100 + j100 - 200}{100 + j100 + 200} = \frac{-1 + j}{3 + j} = -\frac{1}{5} + j\frac{2}{5} \tag{2}$$

$$|\Gamma_{l2}| = \sqrt{\left(-\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{1}{\sqrt{5}} \tag{3}$$

$$\varphi_{l2} = \pi - \tan^{-1}[(2/5)/(1/5)] = \pi/2 + \tan^{-1}[(1/5)/(2/5)] \tag{4}$$

$$\Gamma_{l2} = |\Gamma_{l2}| \exp(j\varphi_{l2}) = \frac{1}{\sqrt{5}} \exp[j\pi/2 + j\tan^{-1}(1/2)] \tag{5}$$

First maximum/minimum in the second line (in this case, the first maximum occurs earlier than the first minimum):

$$z'_{\text{max,first}} = \frac{\lambda\varphi_{l2}}{4\pi} = \frac{\lambda(\pi/2)}{4\pi} + \frac{\lambda\tan^{-1}(1/2)}{4\pi} = \frac{\lambda}{8} + \frac{\lambda}{4\pi}\tan^{-1}(1/2) \tag{6}$$

$$z'_{\text{min,first}} = z'_{\text{max,first}} \pm \frac{\lambda}{4} = \frac{3\lambda}{8} + \frac{\lambda}{4\pi}\tan^{-1}(1/2) \tag{7}$$

VSWR for the second transmission line:

$$\text{VSWR}_2 = \frac{1+|\Gamma_{l2}|}{1-|\Gamma_{l2}|} = \frac{1+1/\sqrt{5}}{1-1/\sqrt{5}} = \frac{\sqrt{5}+1}{\sqrt{5}-1} \approx 2.62 \tag{8}$$

$$Z_{in,2} = Z_{02}\frac{Z_{l2}+jZ_{02}\tan(\beta_2 d_2)}{Z_{02}+jZ_{l2}\tan(\beta_2 d_2)} = 200\frac{100+j100+j200\tan(5\pi/4)}{200+j(100+j100)\tan(5\pi/4)} \tag{9}$$

$$= 200\frac{100+j100+j200}{200+j100-100} = 200\frac{1+j3}{1+j} = (400+j200)\ \Omega \tag{10}$$

$$Z_{in,1} = Z_{l1} \parallel Z_{in,2} = (100-j200) \parallel (400+j200) = (160-j120)\ \Omega \tag{11}$$

Reflection coefficient at the effective load of the first transmission line:

$$\Gamma_{l1} = \frac{Z_{in,1}-Z_{01}}{Z_{in,1}+Z_{01}} = \frac{160-j120-100}{160-j120+100} = \frac{3-j6}{13-j6} \approx 0.37-j0.29 \tag{12}$$

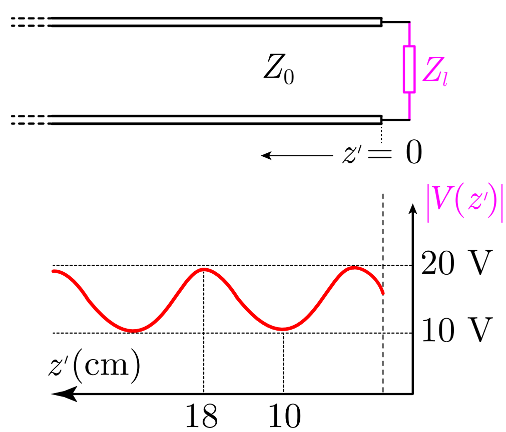
VSWR for the first transmission line:

$$\text{VSWR}_2 = \frac{1+|\Gamma_{l1}|}{1-|\Gamma_{l1}|} \approx 2.77 \tag{13}$$

Input impedance of the overall combination:

$$Z_{in} = \frac{Z_{01}^2}{Z_{in,1}} = \frac{100^2}{(160-j120)} = (40+j30)\ \Omega \tag{14}$$

Question 2
Consider a lossless transmission line terminated by a load Z_l with a given voltage pattern. Assume that the phase velocity is 3×10^8 m/s along the line.



- Find the wavelength and VSWR in the line.
- Find the magnitudes of the incident and reflected voltage waves.
- Find the reflection coefficient at the load (Γ_l).

- Find the load impedance if the characteristic impedance of the transmission line is $Z_0 = 200\ \Omega$.
- If the load involves a single inductor or capacitor (in addition to a resistor), find the value of the inductance or capacitance.
- Find the impedance value measured at $z' = 8$ cm.

Solution

$$\lambda = 0.32 \text{ m} \qquad \text{(from peak-to-peak distance)} \tag{15}$$

$$\text{VSWR} = 20/10 = 2 = \frac{1+|\Gamma_l|}{1-|\Gamma_l|} \longrightarrow |\Gamma_l| = 1/3 \tag{16}$$

$$z'_{\text{max,first}} = \lambda\varphi_l/(4\pi) = 2 \text{ cm} \longrightarrow \varphi_l = 0.08\pi/\lambda = \pi/4 \text{ rad} \tag{17}$$

$$\Gamma_l = |\Gamma_l|\exp(j\varphi_l) = (1/3)\exp(j\pi/4) = \sqrt{2}(1+j)/6 \tag{18}$$

From the maximum voltage:

$$|V_0^+|(1+|\Gamma_l|) = 20 \longrightarrow |V_0^+| = 15 \text{ V} \quad \text{and} \quad |V_0^-| = |\Gamma_l||V_0^+| = 5 \text{ V} \tag{19}$$

Load impedance:

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0} \longrightarrow Z_l = Z_0\frac{1+\Gamma_l}{1-\Gamma_l} \tag{20}$$

$$Z_l = 200\frac{1+\sqrt{2}/6 + j\sqrt{2}/6}{1-\sqrt{2}/6 - j\sqrt{2}/6} \approx (277.9 + j147.4)\ \Omega \tag{21}$$

Note: $f = u_p/\lambda = (3/0.32) \times 10^8$ Hz and $\omega \approx 5.9 \times 10^9$ rad/s

Load: A resistor with 277.9 Ω resistance and an inductor with an inductance of

$$L \approx 147.4/\omega \approx 25 \text{ nH} \tag{22}$$

Impedance at $z' = 8$ cm (capacitive, even though the load is inductive):

$$Z(z' = 8 \text{ cm}) = Z_0^2/Z_l \approx (112.3 - 59.6)\ \Omega \tag{23}$$

Question 3

A series combination of a 30 Ω resistor and a $1/(80\pi)$ nF capacitor is connected to a lossless transmission line as a load at 1 GHz. The characteristic impedance of the transmission line is 50 Ω, while the phase velocity is 3×10^8 m/s. Find the impedance values measured at $z' = 7.5$ cm and $z' = 15$ cm. Also find how the values change if the frequency is increased to 2 GHz.

Solution

At 1 GHz, the load impedance can be found as

$$Z_l(1\text{ GHz}) = 30 + \frac{1}{j2\pi \times 10^9 \times 10^{-9}/(80\pi)} = (30 - j40)\ \Omega \tag{24}$$

$$z_1 = 0.075\text{ m} = \lambda/4 \text{ [inverter] at 1 GHz} \tag{25}$$

$$z_2 = 0.15\text{ m} = \lambda/2 \text{ [repeater] at 1 GHz} \tag{26}$$

$$Z(z_1, 1\text{ GHz}) = \frac{50^2}{(30 - j40)} = (30 + j40)\ \Omega \tag{27}$$

$$Z(z_2, 1\text{ GHz}) = (30 - j40)\ \Omega \tag{28}$$

$$Z_l(2\text{ GHz}) = 30 + \frac{1}{j2\pi \times 2 \times 10^9 \times 10^{-9}/(80\pi)} = (30 - j20)\ \Omega \tag{29}$$

$$z_1 = 0.075\text{ m} = \lambda/2 \text{ [repeater] at 2 GHz} \tag{30}$$

$$z_2 = 0.15\text{ m} = \lambda \text{ [repeater] at 2 GHz} \tag{31}$$

$$Z(z_1, 2\text{ GHz}) = Z(z_2, 2\text{ GHz}) = (30 - j20)\ \Omega \tag{32}$$