Example: A good conductor case.

Compute α, β, v, δ, λ and η for copper given that

$$\mu \cong \mu_0 = 4\pi \times 10^{-7}\ H/m \qquad \epsilon \approx \epsilon_0 = \frac{1}{36\pi} 10^{-9}\ F/m.$$

$$\sigma = 5.8 \times 10^{7}\ \mho/m.$$

a) At $f = 1\ MHz = 10^6\ Hz$.

b) At $f = 4\ mHz$

c) Observe that as $f \uparrow \Rightarrow \alpha\uparrow, \beta\uparrow, v\uparrow, \delta\downarrow, \lambda\downarrow$.

Soln: we need to check the ratio $\frac{\sigma}{\omega\epsilon}$ first:

$$\left.\frac{\sigma}{\omega\epsilon}\right|_{f=1MHz} = \frac{\sigma}{2\pi f\epsilon} = \frac{5.8\times10^7}{2\pi\times10^6\times\frac{1}{36\pi}10^{-9}} = 1.044\times10^{12} >> 1$$ good conductor at $f = 1\ MHz$.

$$\left.\frac{\sigma}{\omega\epsilon}\right|_{f=4MHz} = \frac{5.8\times10^7}{2\pi\times4\times10^6\times\frac{1}{36\pi}10^{-9}} = 2.61\times10^{11} >> 1$$ good conductor at $f = 4\ MHz$. too

(a) for $f = 1\ MHz$.

$$\alpha = \beta \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi\times10^6\times4\pi\times10^{-7}\times5.8\times10^7} \cong 1.513\times10^4$$

$\alpha = 1.513\times10^4\ Np/m.$

$\beta = 1.513\times10^4\ Rad/m.$

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi\times10^6}{1.513\times10^4} \cong 415.23\ m/sec.$$

$$\delta = \frac{1}{\alpha} \cong \frac{1}{1.513\times10^4} \cong 6.6\times10^{-5}\ m.$$

$\lambda = \frac{2\pi}{\beta}$ or for this good conductor $\lambda = \frac{2\pi}{\alpha} = 2\pi\delta \cong 4.15\times10^{-4}\ m$

$$\eta = \frac{(1+j)}{\sigma\delta} = \frac{(1+j)}{5.8\times10^7\times6.6\times10^{-5}} \cong 0.00026\,(1+j)\ \Omega$$

(b) for $f = 4$ MHz

$$\alpha = \beta \approx \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 4 \times 10^6 \times 4\pi \times 10^{-7} \times 5.8 \times 10^7} \cong 3.026 \times 10^4 \; (\uparrow)$$

$$v = \frac{\omega}{\beta} = \frac{2\pi \times 4 \times 10^6}{3.026 \times 10^4} \cong 830.46 \text{ m/sec.} \; (\uparrow)$$

$$\delta = \frac{1}{\alpha} = \frac{1}{3.026 \times 10^4} \cong 3.3 \times 10^{-5} \text{ m.} \; (\downarrow)$$

$$\lambda = 2\pi\delta = 2\pi \times 3.3 \times 10^{-5} \cong 2.073 \times 10^{-4} \text{ m.} \; (\downarrow)$$

$$\eta \doteq \frac{1+j}{\delta\sigma} = \frac{1+j}{3.3 \times 10^{-5} \times 5.8 \times 10^7} \cong 0.00052\,(1+j)\;(\Omega) \; (\uparrow)$$

Note: Compare the values computed for the phase velocity (v) to the free-space velocity value of 3×10^8 m/sec.

Note: Compare the values computed for the wavelength (λ) to the free-space value $\lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{10^6} = 300$ m. at $f = 1$ MHz.

and $\lambda_0 = \frac{3 \times 10^8}{4 \times 10^6} = 75$ m. at $f = 4$ MHz.

Note: Compare the values computed for the intrinsic impedance (η) to the free space value $\eta_0 \doteq 377\ \Omega$.

$\therefore$ v, λ and η are very small in a good conductor

Example: Propagation of plane waves in sea water

For the sea-water, the following parameters are given:

$\epsilon = 81\,\epsilon_0$

$\mu = \mu_0$

$\sigma = 4$ ℧/m.

a) Compute the ratio $\left(\frac{\sigma}{\omega\epsilon}\right)$ in terms of frequency f.

b) Comment on the behaviour of sea water for $f \geq 10$ GHz. Is electromagnetic communication is possible for this frequency range within sea water? Why?

c) Comment on the behaviour of sea water for $f \leq 100$ KHz. similarly.

Solution:

(a) $$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi f \times 81 \times \frac{1}{36\pi} \times 10^{-9}} \cong 0.9 \times \frac{10^9}{f}$$

(b) for $f \geq 10$ GHz $= 10^{10}$ Hz. $\Rightarrow \frac{\sigma}{\omega\epsilon} \leq 0.9 \times \frac{10^9}{10^{10}} = 0.09 \ll 1$

$\therefore$ For this range, sea water behaves as a good insulator

$$\Rightarrow \gamma = \alpha + j\beta \cong \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} + jk \quad \text{where } k = \omega\sqrt{\mu\epsilon}$$

$$\alpha \cong \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{4}{2}\sqrt{\frac{4\pi \times 10^{-7}}{81 \times \frac{1}{36\pi} \times 10^{-9}}} \cong 83.77 \text{ Neper/m}$$

using 1 Neper $= 8.686$ dB $\Rightarrow \alpha \cong 728$ dB/m

a very large attenuation!

$$\delta = \frac{1}{\alpha} \cong \frac{1}{84 \text{ Nep/m}} \cong 1.2 \text{ cm.}$$

$\therefore$ Over a propagation distance of only 1.2 cm., the wave amplitude drops to $\frac{1}{e}$ or approximatiely 37 % of its initial value. $\Rightarrow$ Communication in sea water, within such a high frequency range, is NOT possible using electromagnetic waves.

(c) for $f \leq 100$ kHz $= 10^5$ Hz.

$$\frac{\sigma}{\omega\epsilon} \tilde{\geq} 0.9 \frac{10^9}{10^5} = 0.9 \times 10^4 \gg 1 \Rightarrow$$ Sea water behaves as a good conductor at such lower frequencies.

$\Longrightarrow \gamma = \alpha + j\beta$ where $\alpha = \beta \approx \sqrt{\pi f \mu \sigma} = \frac{1}{\delta}$

The attenuation factor α decreases as frequency decreases, but the wavelength λ increases at the same time

For example,

f (kHz)	α (Np/m)	δ (m)	λ (m) ($\lambda = 2\pi\delta$)
100	1.25	0.8	5.02
10	0.4	2.5	15.71
1	0.13	8	50.3

Note that decreasing frequency for lower losses is not practical as the increasing wavelengths make antenna sizes too large to be feasible.

For instance, for a half-dipole antenna, the antenna length is $\frac{\lambda}{2} \approx 25$ meters at $f = 1$ kHz.

Conclusion: Communication using electromagnetic waves is not practical in sea water. Therefore SONAR devices (which use acoustic waves) are much better alternatives to RADARS for submarine communication.