Definition 1: An electromagnetic wave is called a PLANE WAVE if its "constant phase surfaces" are PLANES!

Definition 2: A plane wave is called a UNIFORM PLANE WAVE (u.p.w.) if the "constant magnitude surfaces" are the same as the "constant phase planes".

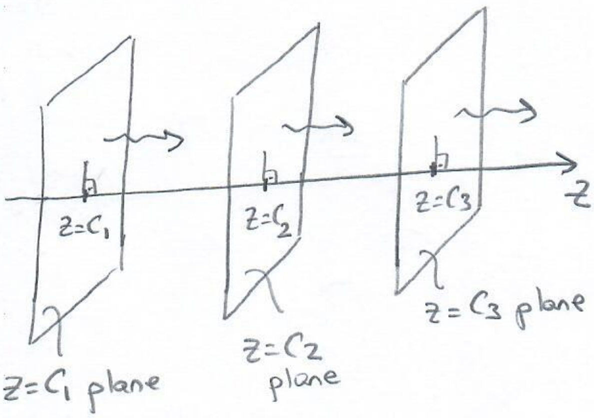
Example: (Let E_0 be a real positive constant)

$$E_x(z,t) = E_0 \cos(\underbrace{\omega t - kz}_{\text{phase}}) \longleftrightarrow E_x(z) = \underbrace{E_0}_{\text{amplitude}}\, e^{\overbrace{j(-kz)}^{\text{phase}}}$$

Phase of solution $= \angle E_x = -kz$ which remains constant if $kz = \text{constant}$ $\left(\begin{matrix} k = \omega\sqrt{\epsilon\mu} \\ \text{a constant}\end{matrix}\right)$

$$\Downarrow$$

$$\boxed{z = \text{constant}}$$ Equation of a plane!



$\therefore$ "Constant phase surfaces" are planes (perpendicular to z-axis).

$\Rightarrow$ Solution is a plane wave (p.w.)

Now, check the "constant magnitude surfaces" of this solution:

$$|\bar{E}(\bar{r})| = |\hat{a}_x E_0 e^{j(-kz)}| = \underbrace{|\hat{a}_x|}_{1}\,|E_0|\,\underbrace{|e^{-jkz}|}_{1} = E_0 = \text{constant}$$

$\therefore$ In this case $|\bar{E}| = E_0 = \text{constant}$ $\Rightarrow$ magnitude is constant everywhere including $z = \text{constant}$ surfaces!

$\Rightarrow$ Each possible "constant phase plane" is also a "constant magnitude plane" $\Rightarrow$ $\boxed{\text{The solution is a UNIFORM PLANE WAVE (u.p.w)}}$

Example: $\bar{E} = \hat{a}_x E_o e^{-(\alpha + j\beta)z}$ (E_o, α, β: real positive constants)

$\bar{E} = \hat{a}_x E_o e^{-\alpha z} e^{-j\beta z} \Longrightarrow \boxed{\text{phase} = \angle \bar{E} = -\beta z}$

$-\beta z = \text{constant} \Rightarrow \boxed{z = \text{constant}}$ (eqn. of a plane)

$\Downarrow$

magnitude $= |\bar{E}|$

$= |\hat{a}_x E_o e^{-\alpha z} e^{-j\beta z}|$

$= \underbrace{|\hat{a}_x|}_{1} \underbrace{|E_o|}_{E_o} \underbrace{|e^{-\alpha z}|}_{e^{-\alpha z}} \underbrace{|e^{-j\beta z}|}_{1}$

$\boxed{|\bar{E}| = E_o e^{-\alpha z}}$

$E_o e^{-\alpha z} = \text{constant} \Rightarrow \boxed{z = \text{constant}}$ (eqn. of a plane)

"Constant phase" and "Constant magnitude" surfaces are both $\boxed{z = \text{constant}}$ family of planes

$\Downarrow$

$\boxed{\text{Given } \bar{E}\text{-phasor belongs to a u.p.w.}}$

Example: $\bar{E} = \hat{a}_x E_o e^{-\alpha y} e^{-j\beta z}$ (E_o, α, β: real positive constants)

Constant phase surfaces

$\angle \bar{E} = -\beta z = \text{constant}$

$\Rightarrow \boxed{z = \text{constant}}$ family of planes ($\perp$) to z-axis

$\Downarrow$

$\boxed{\text{plane wave (p.w.) solution}}$

Constant magnitude surfaces

$|\bar{E}| = |\hat{a}_x E_o e^{-\alpha y} e^{-j\beta z}|$

$|\bar{E}| = E_o e^{-\alpha y} = \text{constant}$

$\Rightarrow \boxed{y = \text{constant}}$ family of planes ($\perp$) to y-axis

Different families of planes! (Amplitude of the solution does not remain constant on "constant phase surfaces")

$\boxed{\text{NOT a u.p.w!}}$

Example:

Let $\bar{E} \cong \hat{a}_\theta E_0(\theta) \dfrac{e^{-jkR}}{R}$ in spherical coordinates (R, θ, ϕ) (for a small dipole at far field)

phase of $\bar{E} = \angle \bar{E} = -kR$ (k: propagation constant)

const.-phase surfaces: $-kR = \text{const} \Rightarrow \boxed{R = \text{const.}}$ equation for a family of spherical surfaces

$\Downarrow$

Given $\bar{E}$-phasor belongs to a SPHERICAL WAVE!

Example:

Let $\bar{E} \cong \hat{a}_z E_0(\beta) \dfrac{e^{-j\beta r}}{\sqrt{r}}$ in cylindrical coordinates (r, ϕ, z) (far field generated by an ∞'ly long line current)

phase of $\bar{E} = \angle \bar{E} = -\beta r$ (β = propagation constant)

const. phase surfaces: $-\beta r = \text{const.} \Rightarrow \boxed{r = \text{constant}}$ equation for a family of cylindrical surfaces.

$\Downarrow$

Given $\bar{E}$-phasor belongs to a CYLINDRICAL WAVE!

Example:

$$\bar{E} = \underbrace{\hat{a}_x E_0 e^{jkz}}_{\substack{\text{a u.p.w} \\ \text{traveling in} \\ \hat{n}_1 = -\hat{a}_z \text{ direction}}} + \underbrace{\hat{a}_x E_0 e^{-jkz}}_{\substack{\text{a u.p.w} \\ \text{traveling in} \\ \hat{n}_2 = +\hat{a}_z \text{ direction}}}$$

$$\bar{E} = \hat{a}_x E_0 \underbrace{\left(e^{jkz} + e^{-jkz}\right)}_{2\cos(kz)} = \hat{a}_x 2 E_0 \cos kz$$

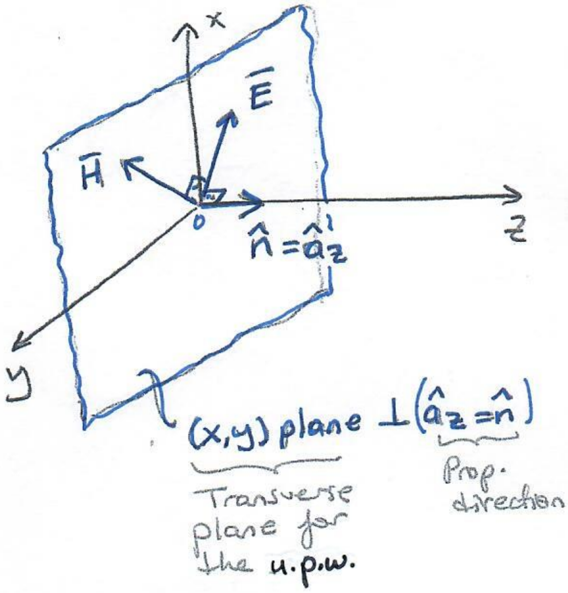
Note that superposition of these two uniform plane waves is not a plane wave anymore! It does not propagate! (It is actually a STANDING WAVE!)

Note that **uniform plane waves** belong to a larger set of electromagnetic waves called TEM (Transverse ElectroMagnetic) waves. $\bar{E}$ and $\bar{H}$ fields of a TEM wave does NOT have components along the propagation direction ($\hat{n}$) but they lie completely in the TRANSVERSE PLANE which is normal to $\hat{n}$.

Assume $\hat{n} = \hat{a}_z$ for a **uniform plane wave** (u.p.w.)

Then, $\bar{E}$ and $\bar{H}$ fields may have both x and y components on the Transverse plane (which in the (x,y) plane $\perp \hat{a}_z$)

in general
$$\begin{cases} \bar{E} = E_x \hat{a}_x + E_y \hat{a}_y \\ \bar{H} = H_x \hat{a}_x + H_y \hat{a}_y \end{cases}$$

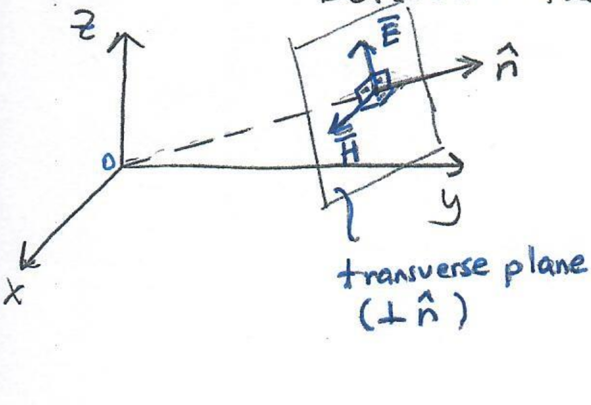


such that

$$\boxed{\bar{H} = \frac{1}{\eta} \hat{a}_z \times \bar{E}}$$

$$\left(\text{or} \quad \hat{a}_z \times \bar{E} = \eta \bar{H}\right)$$

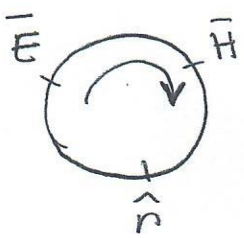
Note that for a **given** u.p.w. propagating in an arbitrary direction $\hat{n}$, there is Right Hand Cyclic relation between the vectors $\bar{E}$, $\bar{H}$ and $\hat{n}$, in general.



$$\boxed{\bar{H} \times \hat{n} = \frac{\bar{E}}{\eta}}$$

or

$$\boxed{\underbrace{\hat{n} \times \bar{E}}_{(V/m)} = \underbrace{\eta}_{(\Omega)} \underbrace{\bar{H}}_{(A/m)}}$$



Example: A u.p.w.-type EM wave propagating in a simple, lossless, source-free medium has the time-domain electric field intensity vector

$$\bar{E}(z,t) = \hat{a}_x 2 \cos(6\pi \times 10^9 t - 40\pi z) \quad (V/m)$$

Find:

(a) Radian frequency (ω), frequency (f), period (T), wave number (k), wavelength (λ) and velocity of propagation (v). Give units!

(Using the general form for comparison)

$$\bar{E}(z,t) = \hat{a}_x E_0 \cos(\omega t - kz)$$

$$\bar{E}(z,t) = \hat{a}_x 2 \cos(6\pi \times 10^9 t - 40\pi z)$$

$\Rightarrow$ $\boxed{E_0 = 2 \ V/m}$ $\boxed{\omega = 6\pi \times 10^9 \ \text{radian/sec}}$

$\omega = 2\pi f$

$\Rightarrow f = \frac{\omega}{2\pi} = 3 \times 10^9 \ Hz.$

(frequency of oscillation of the planewave) $\boxed{f = 3 \ GHz}$

$$T = \frac{2\pi}{\omega} = \frac{1}{f} = \frac{1}{3 \times 10^9} \cong 0.333 \times 10^{-9} \ \text{sec} \Rightarrow \boxed{T \cong 0.333 \ \text{nanosec.}}$$

(Time period of the u.p.w)

Also, $\boxed{k = 40\pi \ \text{radian/meter}}$ $\Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{40\pi} = \boxed{0.05 \ \text{meter} = \lambda}$

($\lambda = 5$ cm.)

(space period of the u.p.w)

$k = \omega\sqrt{\epsilon\mu} = \frac{\omega}{v} \Rightarrow \boxed{v = \frac{\omega}{k}}$

or

(or, using $\lambda = \frac{v}{f} \Rightarrow \boxed{v = \lambda f}$

$\Big\}$ $\boxed{v = 1.5 \times 10^8 \ \text{meter/sec}}$

(b) If $\mu = \mu_0$ is given, find ϵ_r of the propagation medium. Also find the intrinsic impedance (η) of this medium!

In such a simple and lossless medium, we know that

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}} = \frac{1}{\sqrt{\epsilon_0\mu_0}}\frac{1}{\sqrt{\epsilon_r\mu_r}} = c\frac{1}{\sqrt{\epsilon_r\underbrace{\mu_r}_{\mu_r=1}}}$$

(as $\mu = \mu_0$)

$$\Rightarrow 1.5\times10^8 = \frac{3\times10^8}{\sqrt{\epsilon_r}} \Rightarrow \sqrt{\epsilon_r} = 2 \Rightarrow \boxed{\epsilon_r = 4}$$

Also, we know

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{1}{\epsilon_r}}$$

$$\eta = \eta_0\frac{1}{\sqrt{\epsilon_r}} \quad \left(\text{where, } \eta_0 = 120\pi \cong 377\ \Omega\right)$$

$$\Rightarrow \eta = \frac{120\pi}{2} = \boxed{60\pi\ (\Omega) = \eta}$$

(c) Find phasor $\bar{E}$-field expression. Specify the $\hat{n}$ unit vector.

$$\bar{\mathcal{E}}(z,t) = \hat{a}_x \underbrace{2}_{\text{amplitude}} \cos(6\pi\times10^9 t \underbrace{- 40\pi z}_{\text{phase}})$$

$$\Rightarrow \boxed{\bar{E}(z) = \hat{a}_x\, 2\, e^{j(-40\pi z)} \quad \text{V/m}}$$

$\bar{E}_0 = 2\hat{a}_x$ (V/m), $\boxed{\hat{n} = \hat{a}_z}$ unit vector in the dir. of prop.

$\bar{k} = k\hat{n} = 40\pi\,\hat{a}_z$ (rad/m)

$\bar{k}$: propagation vector!

d) Find $\overline{H}$ (magnetic field intensity phasor) and $\overline{\mathcal{H}}$ (time-domain expression of mag. field intensity vector)

(*) We can find $\overline{H}$ phasor by two different approaches:

(i) Using the Maxwell's Equations in phasor domain (this approach is applicable to all types of EM fields)

$$\overline{\nabla} \times \overline{E} = -j\omega\mu \overline{H} \Rightarrow \overline{H} = \frac{1}{-j\omega\mu} \overline{\nabla} \times \overline{E} \qquad \left(\mu = \mu_0 \text{ here}\right)$$

$$\Rightarrow \overline{H} = j\frac{1}{\omega\mu_0} \overline{\nabla} \times \left[\hat{a}_x 2 e^{-j40\pi z}\right]$$

$$= \frac{j}{\omega\mu_0} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2e^{-j40\pi z} & 0 & 0 \end{vmatrix} = \hat{a}_y \frac{2j}{\omega\mu_0} (-j40\pi z) e^{-j40\pi z}$$

$$\Rightarrow \boxed{\overline{H} = \hat{a}_y \frac{1}{30\pi} e^{-j40\pi z} \quad \text{Amp/meter}}$$

(*) Note that $\overline{E}$ and $\overline{H}$ have the same phase as $\eta = 60\pi\ (\Omega)$ is real in this medium

(ii) Since the given EM wave is a uniform plane wave, we can use $\overline{H} = \frac{1}{\eta} \hat{n} \times \overline{E}$ in phasor domain.

(Be careful! This approach is valid only for uniform plane waves.)

$$\overline{H} = \frac{1}{\eta} \hat{n} \times \overline{E} = \frac{1}{\cancel{60}\pi_{30}} \underbrace{\hat{a}_z \times \left[\hat{a}_x \cancel{2}\right.}_{\hat{a}_y} \left. e^{-j40\pi z}\right] = \boxed{\hat{a}_y \frac{1}{30\pi} e^{-j40\pi z} \ (A/m) = \overline{H}}$$

Finally, $\overline{\mathcal{H}}(z,t) = Re\left\{\overline{H} e^{j\omega t}\right\} = Re\left\{\hat{a}_y \frac{1}{30\pi} e^{-j40\pi z} e^{j6\pi \times 10^9 t}\right\}$

$$\Rightarrow \boxed{\overline{\mathcal{H}}(z,t) = \hat{a}_y \frac{1}{30\pi} \cos\left(6\pi \times 10^9 t - 40\pi z\right) \ (A/m)}$$

Exercise:

Given $\bar{E}(\bar{r}) = \hat{a}_x \underbrace{E_0 e^{-\alpha y} e^{-j\beta z}}_{E_x}$ (V/m) (E₀, α, β are all real positive constants.)

a) Find the corresponding $\bar{H}$ phasor.

b) Is this given electromagnetic field a TEM wave?
(TEM Wave: Transverse Electromagnetic wave)

Solution:

a) Given $\bar{E}$ phasor belongs to a non-uniform plane wave as constant phase surfaces are $-\beta z = \text{const} \Rightarrow z = \text{Constant planes}$

and constant magnitude surfaces are $E_0 e^{-\alpha y} = \text{const.} \Rightarrow y = \text{const. planes}$

Any Electromagnetic wave should satisfy the Maxwell equations $\Rightarrow$ $\bar{\nabla} \times \bar{E} = -j\omega\mu\bar{H}$ should be satisfied (assuming a linear medium)

$$\Rightarrow \bar{H} = \frac{1}{-j\omega\mu} \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \frac{j}{\omega\mu}\left[-\hat{a}_y\left(0 - \frac{\partial E_x}{\partial z}\right) + \hat{a}_z\left(-\frac{\partial E_x}{\partial y}\right)\right]$$

$$\bar{H} = \frac{j}{\omega\mu}\left[\hat{a}_y \underbrace{\frac{\partial}{\partial z}\left(E_0 e^{-\alpha y} e^{-j\beta z}\right)}_{-j\beta E_0 e^{-\alpha y} e^{-j\beta z}} - \hat{a}_z \underbrace{\frac{\partial}{\partial y}\left(E_0 e^{-\alpha y} e^{-j\beta z}\right)}_{-\alpha E_0 e^{-\alpha y} e^{-j\beta z}}\right]$$

$$\boxed{\bar{H} = \frac{\beta}{\omega\mu} E_0 e^{-\alpha y} e^{-j\beta z}\, \hat{a}_y + \alpha E_0 e^{-\alpha y} e^{-j\beta z}\, \hat{a}_z}$$

(Note that using $\bar{H} = \frac{1}{\eta}\hat{n} \times \bar{E}$ gives <u>incorrect</u> results here as the wave is <u>not</u> a u.p.w.)

b) As the $\bar{H}$ field has a component in the propagation direction ($\hat{n} = \hat{a}_z$), this electromagnetic wave does not belong to TEM waves.