EE 303 Summer '96

FINAL EXAM

Aug. 16, 1996

Time allowed = 150 minutes

303

Useful Information:

$\epsilon_0 = \frac{10^{-9}}{36\pi}$ F/m , $\mu_0 = 4\pi \times 10^{-7}$ H/m

$$\sin^2\theta_{B\parallel} = \frac{1-(\mu_2\epsilon_1/\mu_1\epsilon_2)}{1-(\epsilon_1/\epsilon_2)^2}$$

$$\sin^2\theta_{B\perp} = \frac{1-(\mu_1\epsilon_2/\mu_2\epsilon_1)}{1-(\mu_1/\mu_2)^2}$$

$$Z(d) = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

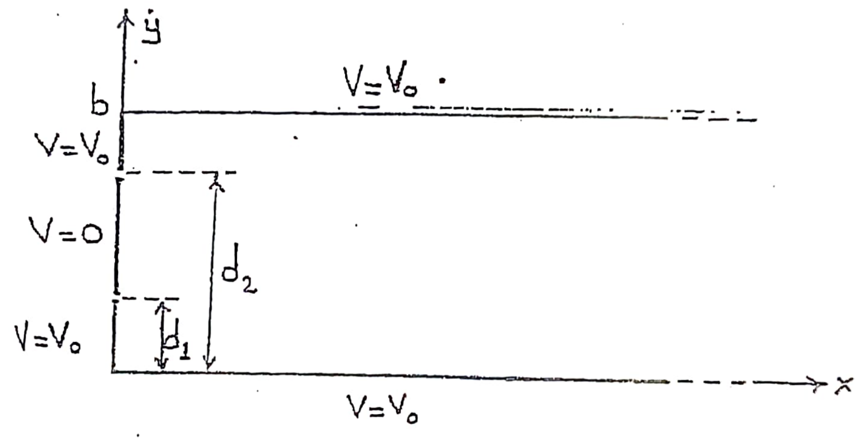
The capacitance per unit length and inductance per unit length of a coaxial transmission line:

$C = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$ F/m $\qquad L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$ H/m.

Good Luck!

Q.1 (20 points)

Consider the two-dimensional boundary value problem shown in the figure. The structure is semi-infinite in x direction. Find the electrostatic potential in $\{0 \leq x, 0 \leq y \leq b\}$.



Q.2 (10 points)

Consider the elliptically polarized wave

$$\bar{E} = (3\hat{a}_x + j5\hat{a}_y)e^{-jkz}.$$

Show that this wave can be decomposed as

$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

where $\bar{E}_1$ is a left-handed circularly polarized wave, and $\bar{E}_2$ is a right-handed circularly polarized wave. (i.e find the phasor expressions for $\bar{E}_1$ and $\bar{E}_2$).

Q.3 (20 points)

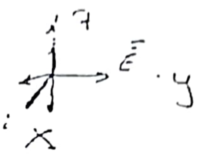
The phasor electric and magnetic fields of a uniform plane wave propagating in a lossless and non-magnetic ($\mu = \mu_0$) medium are given as

$$\bar{E} = 10\, \hat{a}_y\, e^{-j6\pi \hat{n}\cdot\bar{r}} \quad V/m$$

$$\bar{H} = 10^{-2}(4\hat{a}_x + 3\hat{a}_z)\, e^{-j6\pi \hat{n}\cdot\bar{r}} \quad A/m$$

where $\hat{n}$ is the unit vector in the direction of propagation. Find

a) time-average Poynting vector



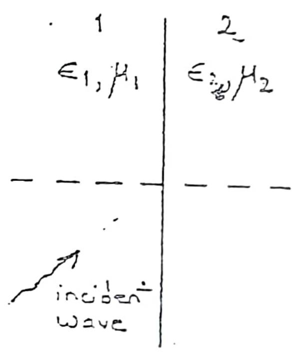
b) $\hat{n}$

c) intrinsic impedance (η)

d) wavelength (λ), angular frequency (ω), phase velocity (v).

Q.4 (15 points)

Consider a uniform plane wave in a lossless medium 1 which is incident upon the planar interface made by a second lossless medium 2.



a) Define the 'critical angle' and 'Brewster angle'.

b) Assume that the electric field of the incident wave is perpendicular to the plane of incidence. Is there a critical angle and a Brewster angle if

i) $\dfrac{\epsilon_1}{\epsilon_2} = 4$, $\dfrac{\mu_1}{\mu_2} = 1$

ii) $\dfrac{\epsilon_2}{\epsilon_1} = 4$, $\dfrac{\mu_2}{\mu_1} = 1$?

If they exist, calculate them in degrees.

continues →

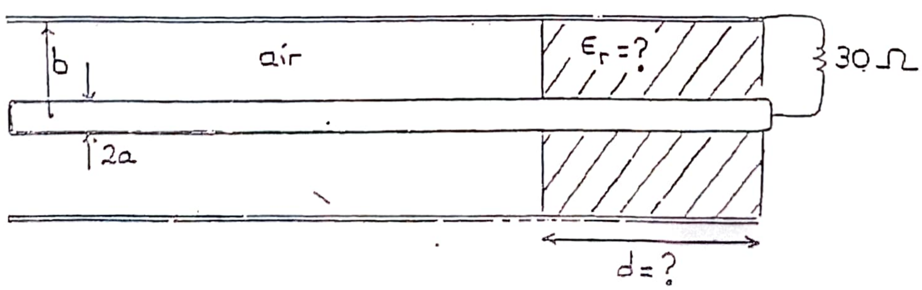
c) Repeat part b) if the electric field of the incident wave is in the plane of incidence.

Summarize your results in the following table:

	i)		ii)	
	θ_c	θ_B	θ_c	θ_B
b)				
c)				

Q.5 (15 points) QUARTER-WAVE TRANSFORMER DESIGN

A certain air-insulated coaxial cable has a characteristic impedance of 75 ohms, and operates at a frequency of 1.2 GHz. It is to be terminated in a resistance of 30 ohms. A quarter-wave "slug" of dielectric material is to be inserted in the line so as to fill the annular space completely. This is to act as a quarter-wave transformer to match the load to the line. What should be the relative permittivity (ϵ_r) of the material (assume $\mu=\mu_0$)? How many centimeters long should the slug be?



Q.6 (20 points)

A signal source of 80 MHz is connected to a load of unknown impedance Z_L using a lossless transmission line of length 2.1 m. The characteristic impedance of the line is 50 ohms, the voltage standing wave ratio is 3, the first and second voltage minima occur at 0.3 m and 1.5 m both measured from the load terminals.

a) Sketch the voltage standing wave pattern (a rough sketch is sufficient) for $0 \leq d \leq 2.1$ m.

b) Find the wavelength and the wave velocity on this line.

c) Find the reflection coefficient at the load.

d) Find the value of the load impedance.

e) Find the input impedance seen at the source.

1. (20 pts)
Consider a source free medium with constant parameters ϵ, μ, σ which can be assumed to be a good conductor at ω, the angular frequency concerned.

(a) Write down Maxwell's equations in differential form for this medium at angular frequency ω in terms of phasor $\bar{E}$ and $\bar{H}$ by entirely neglecting the displacement current (*4 equations with no* $\bar{B}$, *no* $\bar{D}$).

(b) Using these equations, and eliminating $\bar{H}$ derive the differential equation phasor $\bar{E}$ satisfies.

(c) Assuming $\bar{E} = E_x(z)\ \hat{a}_x$ find the simplified differential equation $E_x(z)$ satisfies.

(d) Find the general solution for $E_x(z)$ from the equation of part c).

2. (20 pts)
Consider a uniform plane wave propagating in a lossless (i.e. $\sigma = 0$) and
nonmagnetic ($\mu = \mu_o = 4\pi \times 10^{-7}\ H/m$) medium. The wavelength is $\lambda = 1/3m$.
The phasor electric and magnetic fields at the origin are

$$\bar{E}(0,0,0) = 4\hat{a}_x + 3\hat{a}_z \quad V/m$$
$$\bar{H}(0,0,0) = 2 \times 10^{-2}\hat{a}_y \quad A/m$$

Find (and give the units of)

(a) The time average Poynting's vector.
(b) The unit vector in the direction of propagation.
(c) Expressions for phasor $\bar{E}(x,y,z)$ and $\bar{H}(x,y,z)$ at an arbitrary point (x,y,z).
(d) Intrinsic impedance η of the medium.
(e) Angular frequency ω.

3. (20 pts)

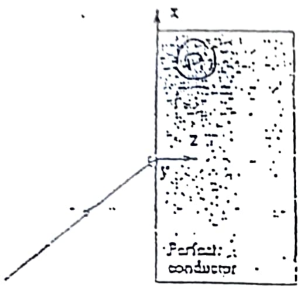
The phasor electric field of a uniform plane wave is given by

$$\bar{E}^i = E_o[\hat{a}_x \frac{\sqrt{3}}{2} + j\hat{a}_y - \hat{a}_z \frac{1}{2}]e^{-jk(\frac{1}{2}x + \frac{\sqrt{3}}{2}z)}$$

(a) Find the polarization (type and sense) of this wave.

(b) Consider now this wave to be incident upon a perfectly conducting boundary coincident with $xy-$ plane as shown. Find the expression for the electric field of the reflected wave.

(c) Find the polarization (type and sense) of the reflected wave.

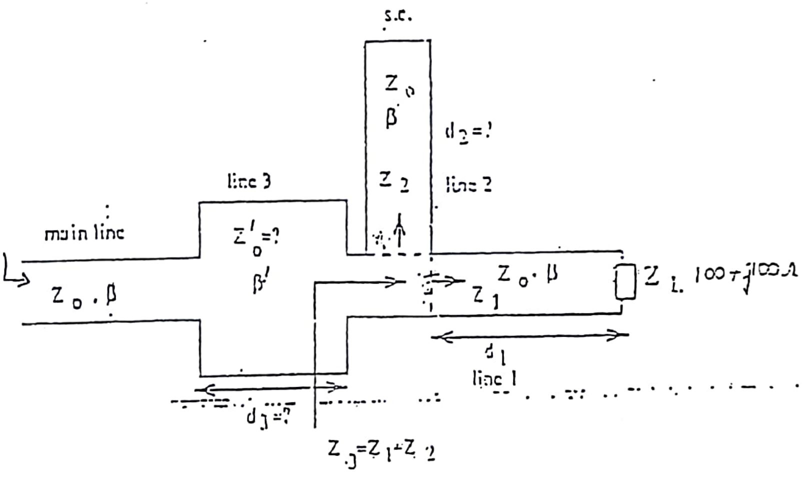


4. (20 pts)
The configuration shown in the figure is to be used to match the load impedance $Z_L = 100 + j100\ \Omega$ to the main transmission line. All transmission lines are lossless. The main line and lines 1 and 2 have characteristic impedance of $Z_o = 50\ \Omega$, and a phase constant of $\beta = 6.25\pi$ rad/m at the frequency of operation. The phase constant of line 3 is $\beta' = 5\pi$ rad/m. The length of line 1 is given as $d_1 = 4$cm. Note that the impedance seen at the end of line 3 is the series connection of Z_1 and Z_2 which are the input impedances of lines 1 and 2, respectively.

(a) Find the length of the short-circuited stub, d_2 (in cm), such that Z_3 is purely real.

(b) For the value of Z_3 found, calculate the characteristic impedance Z_o' (in Ω), and length d_3 (in cm) of line 3 such that the main line sees a matched load.

(c) For the conditions established above, calculate the voltage standing wave ratios in the main line and line 3.



5. A rectangular waveguide with dimensions $a = 4$ cm and $b = 3$ cm is shown in the figure below. Consider a TM mode whose axial electric field component is given by,

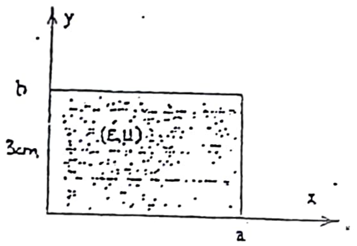
$H_z = 0$

$$E_z = E_0 \sin\frac{2\pi x}{a} \sin\frac{3\pi y}{b} e^{-\gamma z}.$$

(a) Assume the waveguide is filled with air and the frequency of the wave is $f = 18$ GHz. Find the propagation constant γ. Is this a propagating mode or an evanescent mode? Explain.

(b) If the same waveguide is to be filled with a dielectric material with $\epsilon = \epsilon_o\epsilon_r$ and $\mu = \mu_o$, what should be the range of values of ϵ_r so that the given mode is a propagating mode at $f = 6$ GHz?

note:
$1\ GHz = 10^9\ Hz$
$\frac{1}{\sqrt{\epsilon_o\mu_o}} = 3 \times 10^8\ m/s = 3 \times 10^{10}\ cm/s$



Question 1.

In a source-free, lossless medium ($\sigma = 0$) described by permittivity ε and permeability μ, the phasor electric field expression for an electromagnetic wave of angular frequency ω is given as

$$\overline{E} = \hat{a}_z \sin(\alpha x) \quad \sinh(\beta y) \qquad \text{V/m} \qquad \text{where } \alpha \text{ and } \beta \text{ are real}$$

(a) Show that this field satisfies the phasor Gauss' Law.
(b) Find the associated phasor $\overline{H}$ field. (Hint: Use phasor Faraday's Law)
(c) Using phasor $\overline{H}$ field expression found, re-evaluate the phasor electric field $\overline{E}$. (Hint: Use phasor Ampere's Law)
(d) Find a relationship satisfied by α, β, ω, ε and μ so that the given and re-evaluated $\overline{E}$ field expressions are identical.
(e) Which of the following is a compulsary requirement ? Explain.

(i) $\alpha^2 > \beta^2$ (ii) $\alpha^2 = \beta^2$ (iii) $\alpha^2 < \beta^2$

(f) Is this a plane wave type electromagnetic field or not? Explain your answer.

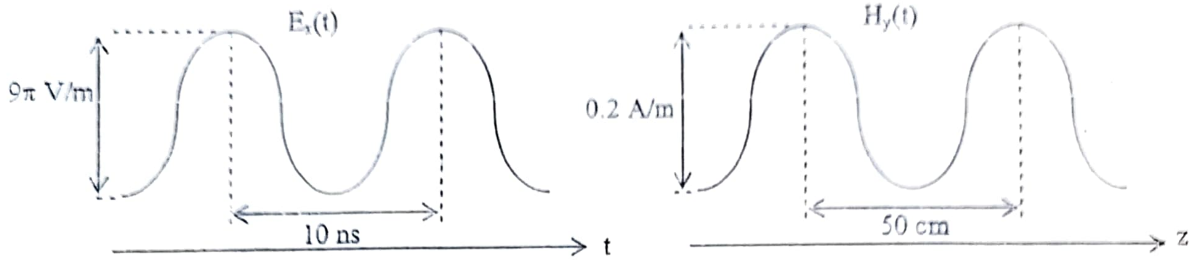
Question 2.

(a) Starting from Maxwell's Equations, derive the wave equation satisfied by the time-domain $\overline{E}$ field in a lossless ($\sigma = 0$) and source-free ($J = 0,\ \rho = 0$) medium with $\mu = \mu_0$ and $\varepsilon = \varepsilon_r \varepsilon_0$.

(b) If the field $\overline{E} = 45 \sin(10^9 t) \cos(5z)\ \hat{a}_y$ V/m is present in the medium described in part (a) find the value of ε_r.

Question 3.

Consider a uniform plane wave having $\vec{E} = E_x(z,t)\hat{a}_x$ and $\vec{H} = H_y(z,t)\hat{a}_y$ and propagating in the +z direction in a lossless dielectric medium. The time variation of E_x in a constant z-plane and the distance variation of Hy for a fixed time are observed to be periodic and shown in the following figures.



a) What is the intrinsic impedance of the medium?
b) What is the propagation constant?
c) What is the phase velocity?
d) Find the relative permittivity and permeability of the medium.
e) Write instantaneous expressions for electric field intensity and magnetic field intensity vectors.

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Question 4.

Determine the type and sense of the polarization for the following plane waves:

(a) $\overline{E}(\overline{r}) = \left[\hat{a}_y\sqrt{2} + \hat{a}_x(1+j)e^{j\frac{\pi}{4}}\right]e^{-jkz}$

(b) $\overline{E}(\overline{r}) = \left(e^{j\frac{\pi}{2}}\,\hat{a}_x - 5\,\hat{a}_y\right)e^{-jkz}$

(c) $\overline{E}(\overline{r}, t) = \hat{a}_x\, \sin\,(\omega t - kz + \frac{\pi}{4}) - \hat{a}_y\, \cos\,(\omega t - kz - \frac{\pi}{4})$

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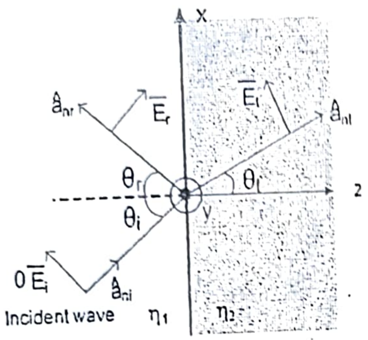
EE 303 Final Examination

Fresnel Formulas :

E // Plane of Incidence Case

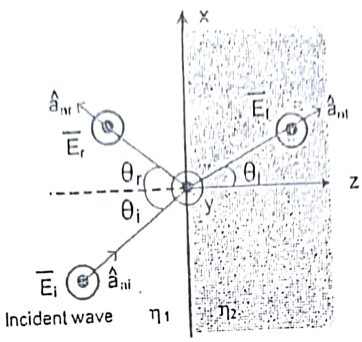


$$\Gamma_{//} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T_{//} = \frac{\cos\theta_i}{\cos\theta_t}(1+\Gamma_{//})$$

$$\sin\theta_B^{//} = \sqrt{\frac{1-(\varepsilon_1\mu_2/\mu_1\varepsilon_2)}{1-(\varepsilon_1/\varepsilon_2)^2}}$$

E ⊥ Plane of Incidence Case



$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T_{\perp} = 1 + \Gamma_{\perp}$$

$$\sin\theta_B^{\perp} = \sqrt{\frac{1-(\mu_1\varepsilon_2/\varepsilon_1\mu_2)}{1-(\mu_1/\mu_2)^2}}$$

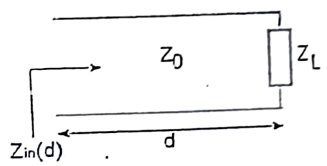
$$\cos(45^o) = \sin(45^o) = \frac{\sqrt{2}}{2}$$

$$\cos(60^o) = \sin(30^o) = \frac{1}{2}$$

$$\cos(30^o) = \sin(60^o) = \frac{\sqrt{3}}{2}$$

$$\cos(36.87^o) = \frac{4}{5}$$

$$\sin(36.87^o) = \frac{3}{5}$$



$$Z_{in}(d) = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

For air and free space : $\mu_0 = 4\pi 10^{-7}$ H/m $\quad \varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m

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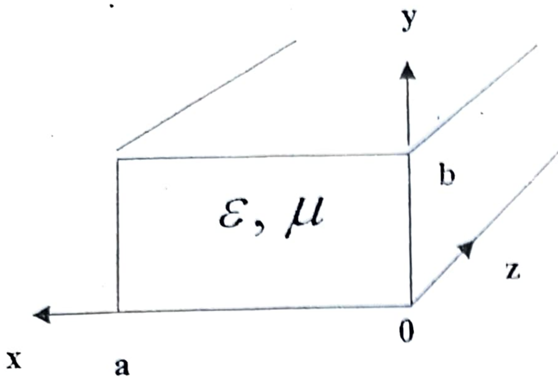


Wave impedance of WG

$$Z = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

Expression of the propagation constant for this rectangular waveguide is given as

$$\gamma = \sqrt{(\frac{m\pi}{a})^2 + (\frac{n\pi}{b})^2 - k^2}$$

Fields for TM modes:

$$E_z = E_o \sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$E_x = -\frac{\gamma}{h^2}(\frac{m\pi}{a})E_o \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$E_y = -\frac{\gamma}{h^2}(\frac{n\pi}{b})E_o \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$H_x = \frac{j\omega\varepsilon}{h^2}(\frac{n\pi}{b})E_o \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$H_y = -\frac{j\omega\varepsilon}{h^2}(\frac{m\pi}{a})E_o \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-\gamma z}$$

Fields for TE modes:

$$H_z = H_o \cos(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$H_x = \frac{\gamma}{h^2}(\frac{m\pi}{a})H_o \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-\gamma z}$$

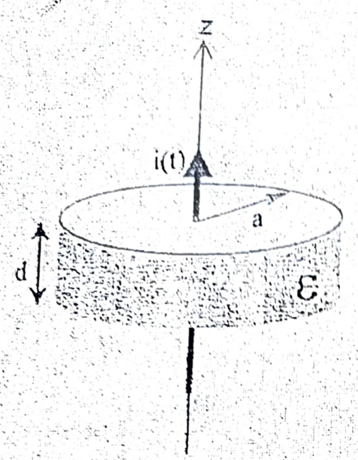
$$H_y = \frac{\gamma}{h^2}(\frac{n\pi}{b})H_o \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$E_x = \frac{j\omega\mu}{h^2}(\frac{n\pi}{b})H_o \cos(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$E_y = -\frac{j\omega\mu}{h^2}(\frac{m\pi}{a})H_o \sin(\frac{m\pi}{a}x)\cos(\frac{n\pi}{b}y)e^{-\gamma z}$$

$$h^2 = k_{c,mn}^2 = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

Question 3 (20 points)



A parallel plate capacitor with circular ($r = a$) perfectly conducting plates is shown in the figure. Neglect fringe fields ($d \ll a$) and find:

a) the displacement current density inside the capacitor

b) the magnetic field intensity $\bar{H}$ inside the capacitor

both in terms of the capacitor current i(t).

$\bar{E} = \frac{V}{d}$ for // plate capacitor

$\bar{D} = \epsilon \bar{E} = \epsilon \frac{V}{d}$

Displacement current density $\frac{\partial \bar{D}}{\partial t} = \frac{\epsilon}{d} \frac{\partial V}{\partial t}$

on the other hand $i_c = C \frac{dV}{dt} \Rightarrow \frac{\partial V}{\partial t} = \frac{i_c}{C}$

$\Rightarrow \boxed{\frac{\partial \bar{D}}{\partial t} = \frac{\epsilon}{d} \frac{i_c \hat{a}_z}{C}}$ since $C = \frac{\pi a^2 \epsilon}{d}$ for the given capacitor

$\frac{\partial \bar{D}}{\partial t} = \frac{\cancel{\epsilon}}{\cancel{d}} \frac{\cancel{d}}{\pi a^2 \cancel{\epsilon}} i_c \hat{a}_z = \frac{i_c(t)}{\pi a^2} \Rightarrow \boxed{\frac{\partial \bar{D}}{\partial t} = \frac{i_c(t)}{\pi a^2} \hat{a}_z}$

$\bar{H} = H_\phi \hat{a}_\phi$

$\oint \bar{H} \cdot d\bar{l} = \int \frac{\partial \bar{D}}{\partial t} \cdot d\bar{s} \Rightarrow H_\phi \cdot 2\pi r = \frac{i_c(t)}{\cancel{\pi} a^2} \cancel{\pi} r^2$

$\Rightarrow \boxed{H_\phi = \frac{i_c(t)}{2\pi a^2} r}$ $\quad r \le a$

Question 4 (20 points)

Consider the following phasor magnetic field intensity that exists in a source free lossless medium characterized by $\mu=\mu_0\mu_r$ and $\varepsilon=\varepsilon_0$.

$\vec{H} = 8j\sin(6x)e^{-j8z}\hat{a}_x + 6\cos(6x)e^{-j8z}\hat{a}_z$ A/m (x and z are in meters)

Note that this is not a uniform plane wave

a) Find the time-domain electric field intensity in terms of the angular frequency ω and ε_0.
b) Find the value of μ_r if $\omega=15\times10^8$ rad/sec

a) $$\vec{E} = \frac{1}{j\omega\varepsilon_o}\vec{\nabla}\times\vec{H}$$

$$= \frac{1}{j\omega\varepsilon_o}\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 8j\sin 6x\, e^{-j8z} & 0 & 6\cos 6x e^{-j8z} \end{vmatrix}$$

$$= \frac{1}{j\omega\varepsilon_o}\left(36\sin 6x + 64\sin 6x\right)e^{-j8z}\,\hat{a}_y$$

$$= -j\frac{100}{\omega\varepsilon_o}\sin 6x\, e^{-j8z}\,\hat{a}_y \quad V/m$$

$$\vec{\mathcal{E}}(x,z,t) = \frac{100}{\omega\varepsilon_o}\sin 6x\,\sin(\omega t - 8z)\,\hat{a}_y \; V/m$$

b) $\nabla^2\vec{E} + k^2\vec{E} = 0$ Homogeneous Helmholtz Equation

$$-j\frac{100}{\omega\varepsilon_o}\left(-36\sin 6x e^{-j8z} - 64\sin 6x e^{-j8z} + k^2\sin 6x e^{-j8z}\right) = 0$$

$$k^2 = 100 \Rightarrow k = 10 = \omega\sqrt{\mu_r\mu_o\varepsilon_o} = \frac{\omega\sqrt{\mu_r}}{c} = \frac{15\times10^8\sqrt{\mu_r}}{3\times10^8}$$

$$\sqrt{\mu_r} = 2 \quad \boxed{\mu_r = 4}$$

EE 303 Final Examination

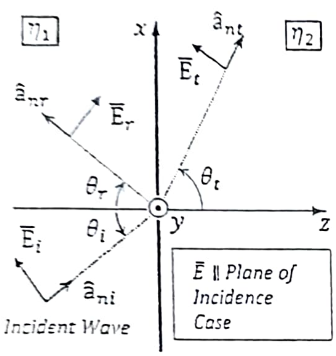
January 7th 2015

COVER SHEET

Only simple calculators are allowed.

Mobile phones must be turned off during the exam.

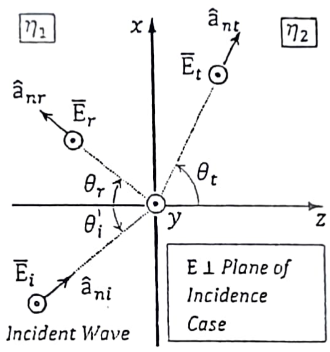
The following (or similar) set of formulas will be available on the cover sheet of the final examination. Your teachers may change the cover sheet for the actual exam booklets if needed.



$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$T_\parallel = \frac{\cos\theta_i}{\cos\theta_t}(1 + \Gamma_\parallel)$$

$$\sin\theta^{\parallel}_{Brew} = \sqrt{\frac{1 - (\epsilon_1\mu_2)/(\epsilon_2\mu_1)}{1 - (\epsilon_1/\epsilon_2)^2}}$$

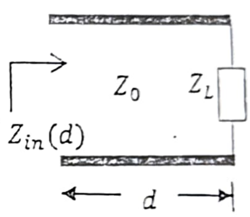


$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

$$T_\perp = 1 + \Gamma_\perp$$

$$\sin\theta^{\perp}_{Brew} = \sqrt{\frac{1 - (\epsilon_2\mu_1)/(\epsilon_1\mu_2)}{1 - (\mu_1/\mu_2)^2}}$$

TL Input Impedance

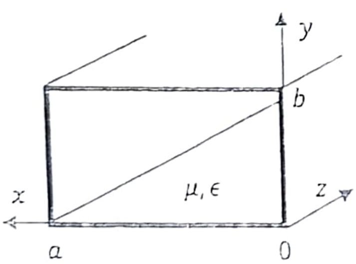


$$Z_{in}(d) = Z_0 \frac{Z_L + jZ_0 \tan\beta d}{Z_0 + jZ_L \tan\beta d}$$

For air and free space

$\mu_0 = 4\pi 10^{-7} H/m$

$\epsilon_0 = \dfrac{10^{-9}}{36\pi} F/m$



$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$Z_{TM} = \frac{\gamma}{j\omega\epsilon}$$

$$Z_{TE} = \frac{j\omega\mu}{\gamma}$$

$$\boxed{\overline{\nabla} \times \overline{\nabla} \times \overline{A} = \overline{\nabla}(\overline{\nabla} \cdot \overline{A}) - \nabla^2 \overline{A}}$$

$$\boxed{\overline{\nabla} \cdot (\psi\overline{A}) = \overline{\nabla}\psi \cdot \overline{A} + \psi\overline{\nabla} \cdot \overline{A}}$$

$$\boxed{\overline{\nabla} \times (\psi\overline{A}) = \overline{\nabla}\psi \times \overline{A} + \psi\overline{\nabla} \times \overline{A}}$$

Q1

a. (2 pts) Write down the Maxwell's Equations in their most general, differential form in phasor domain.

b. Consider a source-free, lossy medium with parameters ϵ, μ, σ where ϵ and μ are assumed to be given constants but the conductivity $\sigma = \sigma(\bar{r})$ is a given function of position.

  i. (4 pts) Write down the Maxwell's Equations for this specific case in phasor domain. Express the equations only in terms of $\bar{E}$ and $\bar{H}$ phasor fields, parameters ε, μ, σ and the radian frequency ω.

  ii. (8 pts) Derive the vector differential equation (wave equation) satisfied by the phasor $\bar{H}$ field. Express your equation in a form $\nabla^2 \bar{H} + \cdots = 0$.
  ($\bar{H}$ should be the only unknown phasor field in your equation.)

c. (1 pts) Simplify the differential equation you obtained in part (b-ii) for $\sigma = \sigma_0 = constant$ case.

Hint:

$$\bar{\nabla} \times \bar{\nabla} \times \bar{A} = \bar{\nabla}(\bar{\nabla} \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\bar{\nabla} \times (\alpha \bar{A}) = \bar{\nabla}\alpha \times \bar{A} + \alpha \bar{\nabla} \times \bar{A}$$

where α is a scalar and $\bar{A}$ is a vector.

Solution (GTS)

a) $\bar\nabla\times\bar E = -j\omega\bar B$
$\bar\nabla\times\bar H = \bar J + j\omega\bar D$
$\bar\nabla\cdot\bar D = \rho$
$\bar\nabla\cdot\bar B = 0$

b-i) $\bar\nabla\times\bar E = -j\omega\mu\bar H$
$\bar\nabla\times\bar H = \sigma\bar E + j\omega\epsilon\bar E$
$\bar\nabla\cdot\bar D = 0 \Rightarrow \bar\nabla\cdot\bar E = 0$
$\bar\nabla\cdot\bar B = 0 \Rightarrow \bar\nabla\cdot\bar H = 0$

b-ii) Compute the curl of both sides of $\bar\nabla\times\bar H = \sigma\bar E + j\omega\epsilon\bar E$
where $\sigma = \sigma(\bar r)$

$$\underbrace{\bar\nabla\times(\bar\nabla\times\bar H)}_{\bar\nabla(\underbrace{\bar\nabla\cdot\bar H}_{0}) - \nabla^2\bar H} = \underbrace{\bar\nabla\times(\sigma(\bar r)\bar E)}_{(\bar\nabla\sigma)\times\bar E + \sigma\bar\nabla\times\bar E} + \underbrace{\bar\nabla\times(j\omega\epsilon\bar E)}_{j\omega\epsilon\bar\nabla\times\bar E}$$

$$-\nabla^2\bar H = (\bar\nabla\sigma)\times\underbrace{\bar E}_{\frac{\bar\nabla\times\bar H}{\sigma+j\omega\epsilon}} + (\sigma+j\omega\epsilon)\underbrace{\bar\nabla\times\bar E}_{-j\omega\mu\bar H}$$

$$\pm\nabla^2\bar H = \mp(\bar\nabla\sigma)\times\frac{(\bar\nabla\times\bar H)}{\sigma+j\omega\epsilon} \pm j\omega\mu(\sigma+j\omega\epsilon)\bar H$$

$$\boxed{\nabla^2\bar H - j\omega\mu(\sigma+j\omega\epsilon)\bar H + \frac{(\bar\nabla\sigma)\times(\bar\nabla\times\bar H)}{\sigma+j\omega\epsilon} = 0}$$

c) If $\sigma = 0 \Rightarrow \bar\nabla\sigma = 0$

$$\Rightarrow \nabla^2\bar H - \underbrace{j\omega\mu(\sigma+j\omega\epsilon)}_{\gamma^2}\bar H = 0$$

$$\Rightarrow \boxed{\nabla^2\bar H - \gamma^2\bar H = 0}$$ where $\gamma = \sqrt{j\omega\mu(\sigma+j\omega\epsilon)}$
propagation constant

Q2 (15 pts)

Write down (in phasor domain) the expression for the electrical field of a uniform plane wave that:

a) propagates in a homogeneous medium with parameters $\epsilon = 3\epsilon_0, \mu = 12\mu_0, \sigma = 2.5$ S/m,

b) has a frequency $f = 300$ MHz,

c) is right hand circularly polarized,

d) has a time average Poynting vector $\bar{P}_{av} = \frac{6}{\pi}\hat{a}_x$ at ~~z~~ x $= 0$.

You must find the numerical values of all constants.

The form of the electric field is:

$$\bar{E} = \frac{E_0}{\sqrt{2}}(\hat{a}_y - j\hat{a}_z)\, e^{-\alpha x}\, e^{-j\beta x}$$

The medium at this frequency is good conductor

Since we have

$$\frac{\sigma}{\omega\epsilon} = \frac{2.5}{2\pi \times 300\times10^6 \times 8.854\times10^{-12}\times 3} \cong 50 \gg 1$$

Thus we have

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi\times300\times10^6\times12\times4\pi\times10^{-7}\times2.5}{2}} = 60\pi$$

$$P_{av} = \frac{1}{2}\mathrm{Re}\left\{\frac{E_0^2}{\eta^*}\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{\sigma E_0^2}{(1-j)\alpha}\right\} = \frac{\sigma E_0^2}{4\alpha} = \frac{6}{\pi}$$

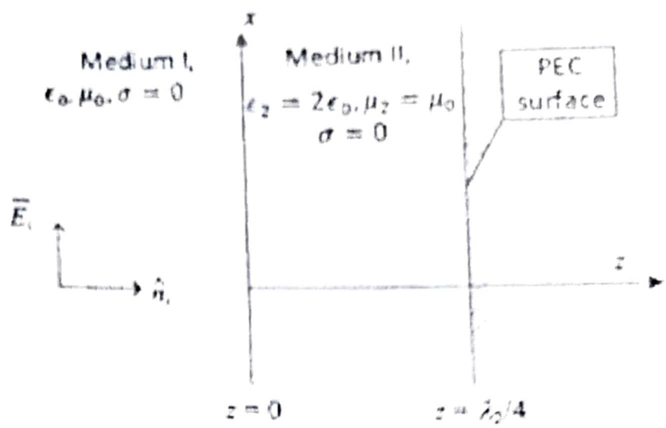
$\Rightarrow E_0 = 24$ Thus

$$\bar{E}(x) = 24\left(\frac{\hat{a}_y - j\hat{a}_z}{\sqrt{2}}\right) e^{-60\pi x}\, e^{-j60\pi x} \quad \text{V/m}$$

Q3. (20 pts)

Suppose that a uniform plane wave polarized linearly in x direction with an amplitude E_0 is propagating in +z direction and is normally incident on a boundary between two lossless dielectric media as shown in the figure below. The second medium is a slab of length $\lambda_2/4$ where λ_2 is the wavelength in this medium. The other side of the slab is perfect electric conductor.

Determine the electric fields everywhere.



Fields in the first medium:

$$\overline{E}_I(z) = \left(E_0 e^{-j\beta_0 z} + E_r e^{+j\beta_0 z}\right)\hat{a}_x$$

$$\overline{H}_1(z) = \left(E_0 e^{-j\beta_0 z} - E_r e^{+j\beta_0 z}\right)\frac{1}{\eta_0}\hat{a}_y$$

Field in third medium:

$$\overline{E}_{III} = 0 \; ; \; \overline{H}_{III} = 0$$

Fields in the second medium:

$$\overline{E}_{II}(z) = \left(E_1 e^{-j\beta_2 z} + E_2 e^{+j\beta_2 z}\right)\hat{a}_x$$

B.C. on tangential $\overline{E}$ @ $z = \lambda_2/4$:

$$E_1 e^{-j\frac{2\pi}{\lambda_2}\frac{\lambda_2}{4}} + E_2 e^{+j\frac{2\pi}{\lambda_2}\frac{\lambda_2}{4}} = -jE_1 + jE_2 = 0 \Rightarrow \boxed{E_1 = E_2}$$

B.C. on tangential $\overline{E}$ @ $z=0$:

$$E_0 + E_r = E_1 + E_2 = 2E_1 \quad (1)$$

B.C. on tangential $\overline{H}$ @ $z=0$: $\left(\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{1}{\sqrt{2}}\eta_0\right)$

$$\frac{E_0 - E_r}{\eta_0} = \frac{E_1 - E_2}{\eta_2} = 0 \Rightarrow \boxed{E_r = E_0}$$

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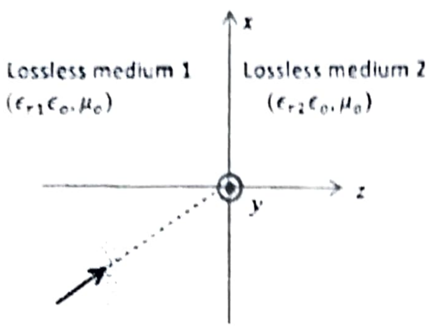
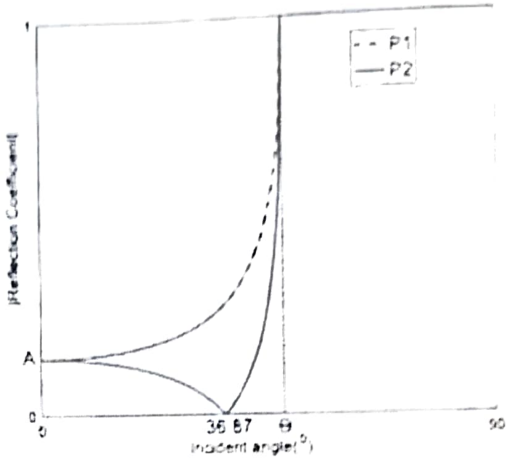
Q4 (25 pts)

Consider an interface between two lossless <u>non-magnetic</u> media as shown in the figure. The magnitudes of the reflection coefficients are plotted for perpendicular and parallel polarization as a function of the incidence angle in the figure below.





a) (3 pts) Which plot (P1/P2) corresponds to which polarization (parallel/perpendicular)? Explain your reasoning.

Since both media are nonmagnetic, there isn't a real Brewster angle ($\Gamma = 0$) for perpendicular polarization. So P1: Perpendicular
P2: Parallel

b) (3 pts) Without making any calculations state whether the reflection coefficient is positive or negative when its magnitude is A. Explain your reasoning.

There is a critical angle (θ). Therefore $\epsilon_1 > \epsilon_2$
$\Rightarrow \eta_2 > \eta_1$. A is the magnitude of ref. coef.
at normal incidence $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} > 0$ Positive

c) (5 pts) Find the relation between the wave number in the second medium, k_2 and the wave number in the first medium, k_1.

$$\sin \theta_{B}^{\parallel} = \sqrt{\frac{1-(\epsilon_1/\epsilon_2)}{1-(\epsilon_1/\epsilon_2)^2}} \qquad \text{Let } \frac{\epsilon_1}{\epsilon_2} = a$$

$$\sin 36.87 = \sqrt{\frac{1-a}{1-a^2}} = \sqrt{\frac{1}{1+a}} = \frac{3}{5} \qquad \Rightarrow a = \frac{16}{9} = \frac{\epsilon_1}{\epsilon_2}$$

$$\frac{k_1}{k_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{4}{3} \qquad \boxed{3k_1 = 4k_2}$$

d) (4 pts) Calculate the numerical values of A and θ.

$$A = \left|\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right| = \left|\frac{\eta_2/\eta_1 - 1}{\eta_2/\eta_1 + 1}\right| = \left|\frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1}\right| = \left|\frac{\frac{4}{3} - 1}{\frac{4}{3} + 1}\right| = \frac{1}{7}$$

θ is the critical angle ⇒ $k_1 \sin\theta = k_2$

$$\sin\theta = \frac{k_2}{k_1} = \frac{3}{4} \Rightarrow \theta = 48.6^\circ$$

e) (5 pts) If the incident electric field intensity is $\bar{E}_i = (4\hat{a}_x - 5j\hat{a}_y - 3\hat{a}_z)e^{-jk_1(0.6x+0.8z)}$, find the reflected electric field intensity.

$$\frac{\sin\theta_i}{\cos\theta_i} = \frac{0.6}{0.8} \Rightarrow \tan\theta_i = \frac{3}{4} \Rightarrow \theta_i = 36.87^\circ = \theta_B^{\parallel}$$

$$\Rightarrow \Gamma^{\parallel} = 0$$

$\cos\theta_t = \sin\theta_i$ since $\theta_i = \theta_B^{\parallel}$

$$\Gamma_\perp = \frac{\eta_2/\eta_1 \; 0.8 - 0.6}{\eta_2/\eta_1 \; 0.8 + 0.6} = 0.28$$

$$\bar{E}_r = -1.4j\,\hat{a}_y\, e^{-jk_1(0.6x - 0.8z)}$$

f) (5 pts) If the incident electric field intensity is $\bar{E}_i = (E_o\hat{a}_y)e^{-jk_1(\sqrt{3}x+z)/2}$ and the transmitted electric field intensity is $\bar{E}_t = (\tau\hat{a}_y)e^{-j\varphi(k_1,x,z)}$, find $\varphi(k_1, x, z)$.

$$\frac{\sin\theta_i}{\cos\theta_i} = \sqrt{3} \Rightarrow \theta_i = 60^\circ > \theta_c$$

$$\sin\theta_t = \frac{4}{3}\sin\theta_i = \frac{4}{3}\frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{3} > 1 \Rightarrow \cos\theta_t$$ is imaginary

$$\cos\theta_t = -j\sqrt{\sin^2\theta_t - 1} = -j\sqrt{\frac{4}{3} - 1} = -j/\sqrt{3}$$

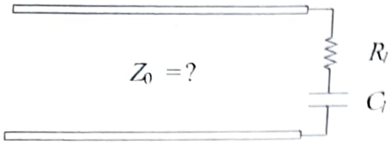
$$\bar{E}_t = \tau\hat{a}_y e^{-jk_2(\sin\theta_t x + \cos\theta_t z)}$$

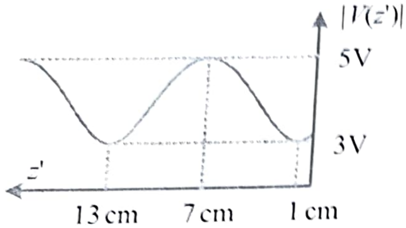
$$= \tau\hat{a}_y e^{-jk_1\frac{3}{4}\left(\frac{2\sqrt{3}}{3}x - \frac{j}{\sqrt{3}}z\right)} = \tau\hat{a}_y \underbrace{e^{-jk_1\frac{\sqrt{3}}{2}x}}\;\underbrace{e^{-k_1\frac{\sqrt{3}}{4}z}}$$

$$= \tau\hat{a}_y e^{-jk_1\frac{3}{4}\left(\frac{2\sqrt{3}}{3}x - \frac{j}{\sqrt{3}}z\right)}$$

Phase matching same as E_i at $z=0$

negative root is chosen for $\cos\theta_t$ to obtain decaying function





Q5. Consider a lossless transmission line with unknown characteristic impedance. An engineer connects a load consisting of a $R_l = 75\ \Omega$ resistor and $1/(10\pi)$ nF capacitor (connected in series), as shown in the figure (source side is not shown). Then, she obtains the voltage pattern (absolute value of the voltage in phasor domain) with respect to position at $f = 250$ MHz. Maximum/minimum values of the voltage, as well as maximum/minimum locations are labeled in the figure.

d. Find the reflection coefficient, Γ_l.

e. Find the characteristic impedance Z_0.

f. Find the capacitance per unit length and inductance per unit length in the equivalent model of the transmission line.

Please show your steps clearly.

Solution:

$$\lambda/4 = (7-1) = 6 \text{ cm} = 0.06 \text{ m} \longrightarrow \lambda = 0.24 \text{ cm} \tag{1}$$

$$\frac{\lambda\varphi_l}{4\pi} - \frac{\lambda}{4} = 0.01 \longrightarrow \frac{\lambda\varphi_l}{4\pi} = 0.07 \longrightarrow \varphi_l = \frac{7\pi}{6} \text{ rad} \tag{2}$$

$$\text{VSWR} = \frac{5}{3} = \frac{1+|\Gamma_l|}{1-|\Gamma_l|} \longrightarrow |\Gamma_l| = \frac{1}{4} \tag{3}$$

$$\Gamma_l = \frac{1}{4}\exp(j7\pi/6) = \frac{1}{4}\left(-\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) \tag{4}$$

$$Z_l = 75 - \frac{j}{\omega 10^{-9}/(10\pi)} = 75 - \frac{j}{2\pi \times 250 \times 10^{6} \times 10^{-9}/(10\pi)} = (75-20j)\ \Omega \tag{5}$$

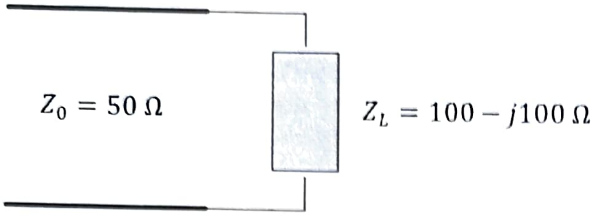
$$Z_0 = Z_l\frac{1-\Gamma_l}{1+\Gamma_l} \approx Z_l(1.489+0.397j) \approx \boxed{120\ \Omega} \tag{6}$$

$$\sqrt{\frac{L_0}{C_0}} = 120 \longrightarrow L_0 = 14400C_0 \tag{7}$$

$$u_p = f\lambda = 250 \times 10^{6} \times 0.24 = 6 \times 10^{7} = \frac{1}{\sqrt{L_0C_0}} \longrightarrow L_0C_0 = \frac{1}{36} \times 10^{-14} \tag{8}$$

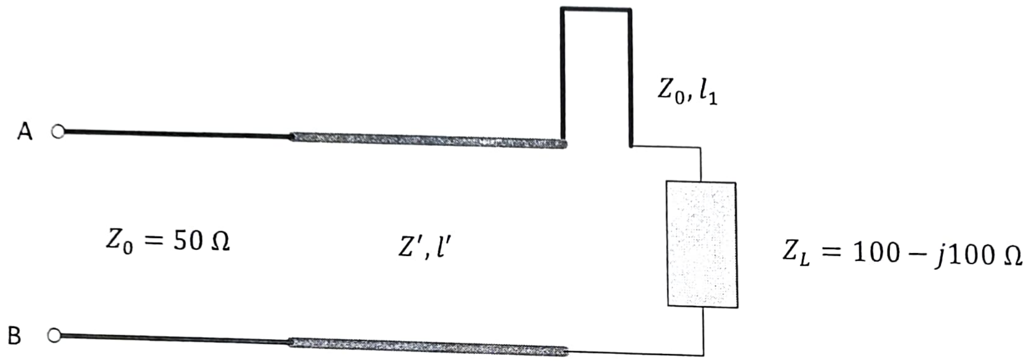
$$\longrightarrow C_0^2 = \frac{1}{36 \times 144} \times 10^{-16} \longrightarrow C_0 = \frac{1}{72} \times 10^{-8} = \boxed{\frac{5}{36}\text{ nF}} \longrightarrow L_0 = \boxed{2\ \mu\text{H}} \tag{9}$$

Q6



a. A transmission line circuit consists of a 50 Ω transmission line connected to a $100 - j100\ \Omega$ load as shown in the figure. Find the VSWR (Voltage Standing Wave Ratio) measured on the line.

b. Transmission line circuit of part (a) is now connected to a matching circuit consisting of a short circuited stub of length l_1, characteristic impedance of $Z_0 = 50\ \Omega$ and another piece of transmission line of characteristic impedance Z' and length l' as shown in the figure below. Find l_1, and l' in terms of the wavelengths of the corresponding transmission lines and Z' so that VSWR at the port A-B becomes unity.



a) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{\cancel{100}^{2} - j\cancel{100}^{2} - \cancel{50}^{1}}{\cancel{100}_{2} - j\cancel{100}_{2} + \cancel{50}_{1}} = \dfrac{1 - j2}{3 - j2}$

$= \dfrac{\sqrt{5} \angle \tan^{-1}(-2)}{\sqrt{13} \angle \tan^{-1}(-\frac{2}{3})}$ $|\Gamma| = \sqrt{\dfrac{5}{13}}$

$S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = 4.27$

180 π

63.43

b) $Z_0 \tan\beta l = \cancel{100}^{2}$ (with $\cancel{50}$ above Z_0)

$\tan\beta l = 2$

$\beta l = 0.35\pi \Rightarrow l \dfrac{2\pi}{\lambda} = 0.35\cancel{\pi}$

$l_1 = \dfrac{0.35}{2} \cancel{\lambda} = 0.175\lambda$ //

$l' = \dfrac{\lambda}{4}$ $50 = \dfrac{Z'^2}{100} \Rightarrow Z' = 10\sqrt{50}\ \Omega$