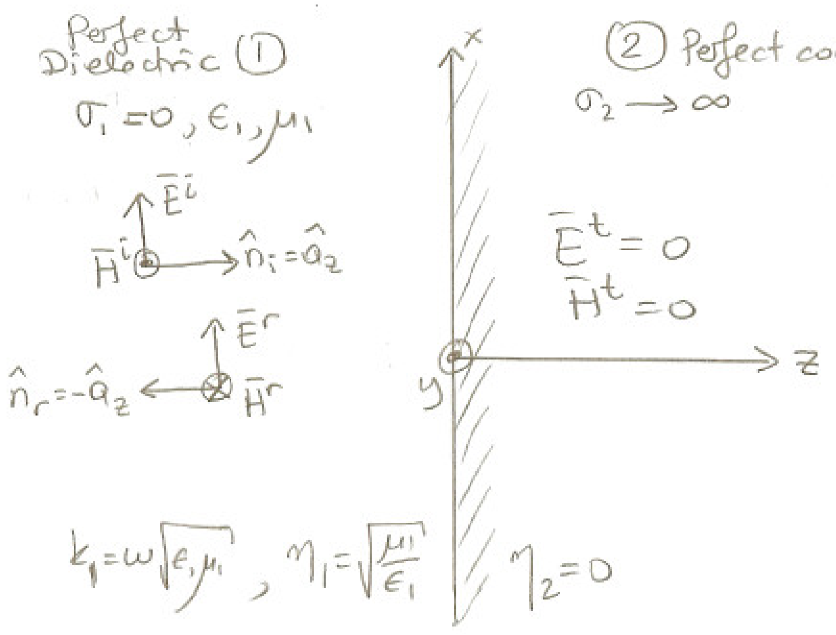
Case 3: Perfect Dielectric / Perfect Conductor Boundary



Remember, we have

$$\eta \cong \sqrt{\frac{j\omega\mu}{\sigma}} \quad \text{in good conductors}$$

$$\lim_{\sigma\to\infty} \eta = \boxed{0 = \eta_2}$$

in this medium ② which is a perfect conductor with $\sigma \to \infty$.

(Also, $\lim_{\sigma\to\infty} \frac{1}{\sqrt{\pi f \mu \sigma}} = 0 \Rightarrow \boxed{\delta_2 = 0}$ zero skin depth in a perfect conductor.

Then, $$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\Bigg|_{\eta_2 = 0} = -1$$

and $$T = \frac{2\eta_2}{\eta_2 + \eta_1}\Bigg|_{\eta_2=0} = 0$$

$\boxed{\Gamma = -1}$ and $\boxed{T = 0}$ as expected

can also be obtained by directly applying the BC's at $z=0$ as shown below:

B.C: $E^{total}_{tang①} = E^{total}_{tang②}$ at $z = 0$

$$\bar{E}^i = \hat{a}_x E_1 e^{-jk_1z}$$
$$\bar{E}^r = \hat{a}_x E_2 e^{+jk_1z}$$

$$\bar{E}^{total}_{①} = \bar{E}^i + \bar{E}^r = \hat{a}_x E_1 e^{-jk_1z} + \hat{a}_x E_2 e^{+jk_1z}$$

(tangential to the boundary)

$$\bar{E}^t = 0$$

$$\Rightarrow \left(E_1 e^{-jk_1z} + E_2 e^{+jk_1z}\right)\Bigg|_{z=0} = 0$$

$$\Rightarrow E_1 + E_2 = 0$$

$$\Rightarrow \boxed{E_2 = -E_1} \Rightarrow \boxed{\Gamma = \frac{E_2}{E_1} = -1}$$

In this case, presence of the perfect conductor causes "Standing waves" to form in the perfect dielectric side.

To see this, let's write the total fields in medium ①:

$$\bar{E}_{①}^{total} = \bar{E}^{i} + \bar{E}^{r} = \hat{a}_x E_1 e^{-jk_1 z} + \hat{a}_x \underbrace{E_2}_{\Gamma E_1 = -E_1} e^{+jk_1 z}$$

$$\bar{E}_{①}^{total} = \hat{a}_x E_1 \big(\underbrace{e^{-jk_1 z} - e^{+jk_1 z}}_{-2j\sin(k_1 z)} \big) \Rightarrow \boxed{\bar{E}_{①}^{total} = -j2E_1 \sin(k_1 z)\, \hat{a}_x}$$

in phasor domain.

Assuming E_1 to be real for computational simplicity,

$$\bar{\mathcal{E}}_{①}^{total}(t) = Re\left\{ \bar{E}_{①}^{total} e^{j\omega t} \right\} = Re\left\{ -j2E_1 \sin(k_1 z)(\cos\omega t + j\sin\omega t)\hat{a}_x \right\}$$

$$\boxed{\bar{\mathcal{E}}_{①}^{total}(t) = 2E_1 \sin(k_1 z)\sin(\omega t)\, \hat{a}_x}$$

in time-domain

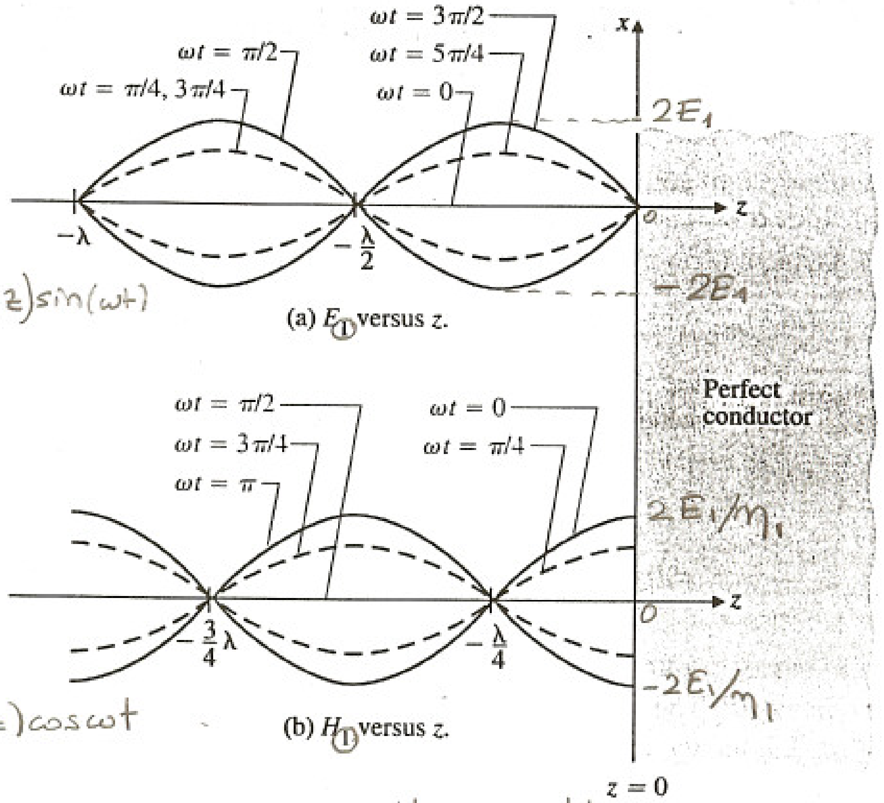
Similarly, the total H-field in medium ① can be obtained as:

$$\bar{H}_{①}^{total} = \bar{H}^{i} + \bar{H}^{r} = \hat{a}_y \frac{E_1}{\eta_1}\big(e^{-jk_1 z} - \underbrace{\Gamma}_{-1} e^{jk_1 z}\big) = \hat{a}_y \frac{E_1}{\eta_1}\big(\underbrace{e^{-jk_1 z} + e^{jk_1 z}}_{2\cos(k_1 z)}\big)$$

$$\boxed{\bar{H}_{①}^{total} = \frac{2E_1}{\eta_1}\cos(k_1 z)\,\hat{a}_y} \longleftrightarrow \boxed{\bar{\mathcal{H}}_{①}^{total} = \frac{2E_1}{\eta_1}\cos(k_1 z)\cos\omega t\, \hat{a}_y}$$

in phasor domain in time-domain

Now, let's plot amplitudes $\mathcal{E}_x^{tot}$ and $\mathcal{H}_y^{total}$ of medium ① as a function of distance z for several time instances (for $\omega t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi$)



$E_{①} = 2E_1 \sin(k_1 z)\sin(\omega t)$

$H_{①} = \dfrac{2E_1}{\eta_1}\cos(k_1 z)\cos\omega t$

FIGURE 7-9 Standing waves of $\mathbf{E}_{①}^{tot} = \mathbf{a}_x E_{①}$ and $\mathbf{H}_{①}^{tot} = \mathbf{a}_y H_{①}$ for several values of ωt.

(*) From the superposition of oppositely traveling waves, STANDING WAVES are formed in the dielectric medium (i.e for $z<0$).

ωt	$\sin \omega t$	$\cos \omega t$	$E_{①}$	$H_{①}$
$0, 2\pi$	0	1	0	$\frac{2E_1}{\eta_1}\cos k_1 z$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	$2E_1 \frac{1}{\sqrt{2}} \sin k_1 z$	$\frac{2E_1}{\eta_1}\frac{1}{\sqrt{2}}\cos k_1 z$
$\pi/2$	1	0	$2E_1 \sin k_1 z$	0
$3\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$2E_1 \frac{1}{\sqrt{2}} \sin k_1 z$	$-\frac{2E_1}{\eta_1}\frac{1}{\sqrt{2}}\cos k_1 z$
π	0	-1	0	$-\frac{2E_1}{\eta_1}\cos k_1 z$
$5\pi/4$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-2E_1 \frac{1}{\sqrt{2}} \sin k_1 z$	$-2\frac{E_1}{\eta_1}\frac{1}{\sqrt{2}}\cos k_1 z$
$3\frac{\pi}{2}$	-1	0	$-2E_1 \sin k_1 z$	0

Basic observations about the STANDING WAVES

① Zeros (nulls) of $\bar{\mathcal{E}}_{①}^{tot}(t)$ and Maxima/minima of $\bar{\mathcal{H}}_{①}^{tot}(t)$ $\Big\}$ occur when $\boxed{k_1 z = -n\pi}$

or using $k_1 = \frac{2\pi}{\lambda}$

when $\boxed{z = -n\frac{\lambda}{2} \quad (n = 0, 1, 2, 3, \cdots)}$

Similarly,

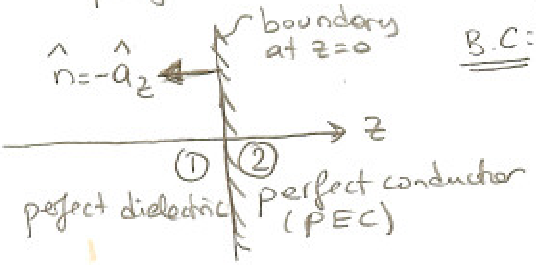
Maxima/minima of $\bar{\mathcal{E}}_{①}^{tot}(t)$ and Zeros (nulls) of $\bar{\mathcal{H}}_{①}^{tot}(t)$ $\Big\}$ occur when $\boxed{k_1 z = -(2n+1)\frac{\pi}{2}}$

or $\boxed{z = -(2n+1)\frac{\lambda}{4} \quad (n = 0, 1, 2, 3, \cdots)}$

② $\bar{\mathcal{E}}_{①}^{total}$ always becomes <u>zero</u> on the perfect conductor boundary, i.e, <u>at $z = 0$</u> due to the $\sin(k_1 z)$ term.

③ $\bar{\mathcal{H}}_{①}^{total}$ takes its <u>either maximum</u> or <u>minimum</u> value (depending on the value of $\cos\omega t$ term at a given time instant) <u>at the boundary $z=0$</u> due to the $\cos(k_1 z)$ term.

④ The surface current density vector $\bar{J}_s$ at $z=0$ boundary can be computed using the boundary condition on tangential H-field (taking into account the fact that $\bar{H} = 0$ in the perfect conductor medium.



B.C: $\underbrace{\hat{n}}_{-\hat{a}_z} \times (\bar{H}_{①} - \overset{0}{\cancel{\bar{H}_{②}}}) = \bar{J}_s \Rightarrow \boxed{\bar{J}_s = -\hat{a}_z \times \bar{\mathcal{H}}_{①}^{tot}\Big|_{z=0}}$

$$\Rightarrow \boxed{\bar{J}_s = \frac{2E_1}{\eta_1} \cos\omega t \; \hat{a}_x \quad (A/m)}$$

(5) Distance between two successive nodes is $\frac{\lambda}{2}$

Distance " " " maxima is $\frac{\lambda}{2}$

Distance " " " minima is $\frac{\lambda}{2}$

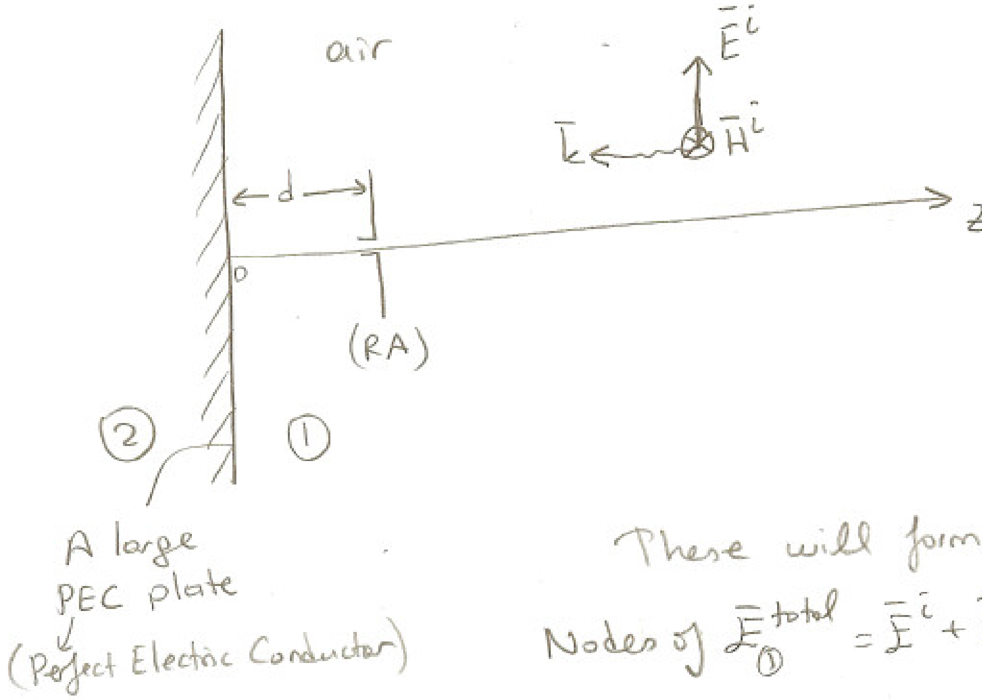
Distance between a node and the closest extremum point (minimum or maximum) is $\frac{\lambda}{4}$.

(6) Standing waves $\bar{E}_{①}^{tot}$ and $\bar{\mathcal{H}}_{①}^{tot}$ are in <u>time quadrature</u> (having $\sin \omega t$ and $\cos \omega t$ terms) and in <u>space quadrature</u> (i.e, shifted by a quarter wavelength $(\frac{\lambda}{4})$ with respect to each other due to the $\sin k_1 z$ and $\cos k_1 z$ terms).

<u>Problem:</u> In the figure below, a receiving antenna (RA) is placed "d" meters in front of a large PEC plate. The antenna responds to the <u>total</u> field $\bar{E}^{total} = \bar{E}^i + \bar{E}^r$.

If the incoming EM radiation is a u.p.w oscillating at $f = 100$ MHz,

a) For which d values, antenna picks up the max. field?
b) For which d values, antenna senses no field at all?



<u>Soln:</u> We know that there will be both incident and reflected field in air due to the presence of PEC plate.

These will form standing waves in air region.

Nodes of $\bar{E}_{①}^{total} = \bar{E}^i + \bar{E}^r$ occur at $z = n\frac{\lambda}{2}$

Extremum points of $\bar{E}_{①}^{total}$ occur at $z = (2n+1)\frac{\lambda}{4}$

where $\lambda = \frac{c}{f} = \frac{3\times10^8}{100\times10^6} = 3$ m. $\boxed{\lambda = 3 \text{ m.}}$ $\rightarrow \lambda/2 = 1.5$m. $\rightarrow \lambda/4 = 0.75$m.

So, (a) for $d = 0, 1.5$m, 3m, 4.5m, 6m., ... RA senses <u>zero</u> intensity
(b) For $d = 0.75$m., 2.25m, 3.75m., 5.25m, ... RA senses <u>maximum</u> intensity.