EE 303 - HW 8 - 2304814

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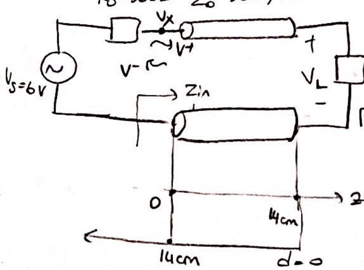
Q1)

$R_S = 50\Omega$ $Z_0 = 50\Omega$, $\lambda = 16cm$



- $\lambda = \frac{2\pi}{\beta} \rightarrow \beta = \frac{2\pi}{\lambda \rightarrow 16cm}$
- $\beta = \frac{\pi}{8} \times 10^3$ rad/m
- $d = 14cm$

a) For lossless cases,

$$Z_{lossless}(d) = Z_0 \cdot \frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)} = Z_0 \cdot \frac{Z_L + jZ_0\tan(\frac{14\pi}{8})}{Z_0 + jZ_L\tan(\frac{14\pi}{8})}$$

$$Z_{Lossless}(d) = Z_0 \cdot \frac{Z_L - jZ_0}{Z_0 - jZ_L} = 50 \cdot \frac{40-j30-j50}{50-j(40-j30)} = 50 \cdot \frac{40-j80}{20-j40} = \boxed{100\Omega}$$

$Z_0 = 50\Omega$
$Z_L = 40-j30\Omega$

b) $\Gamma_L = ?$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-10-j30}{90-j30} = \frac{-1-j3}{9-j3} = \frac{1}{j3} = -\frac{j}{3} = \frac{1}{3}\angle -\frac{\pi}{2}$$

c) $V_s = 6\cos(2\pi f_0)$ V. By using phasor domain,

$$V_x = V^+ + V^- = \frac{V_S \cdot Z_{in}}{R_S + Z_{in}} = 6 \cdot \frac{2}{3} = 4V \Rightarrow V^+ + V^- = 4V.$$ Also we know that

at $z = d = 14cm$ and $\beta = \frac{\pi}{8\times10^{-3}}$ rad/m

From previous part $\Gamma_L = -\frac{j}{3}$

$$\Gamma_L = \frac{V_L^-}{V_L^+} = \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}} = \frac{V^-}{V^+} e^{j2\beta z} = \frac{V^-}{V^+} e^{j\frac{7\pi}{2}} = \frac{V^-}{V^+}(-j) = \frac{1}{3}(-j)$$

$$\frac{V^-}{V^+} = \frac{1}{3} \text{ and } V^- + V^+ = 4V \Rightarrow \boxed{V^- = 1V \text{ and } V^+ = 3V}$$

② $f = 1MHz = 10^6 Hz$

$L = 0.128\mu H = 0.128\times10^{-6}H \Rightarrow Z_{sc} = j Z_0 \tan(\beta l) = jwL = Z_L$

$C = 20pF = 20\times10^{-12}F \Rightarrow Z_{oc} = -j Z_0 \cot(\beta l) = \frac{1}{jwC} = Z_c$

by using this equations we get (*)

(*) $Z_0^2 = \frac{L}{C} \Rightarrow Z_0 = \sqrt{\frac{L}{C}} = 80\Omega$, where $L = 0.128\times10^{-6}H$, $C = 20\times10^{-12}F$

• Then,

$$Z_{oc} = -\cancel{j}\, 80\cot(\beta l) = -\frac{\cancel{j}}{wC} \Rightarrow \cot(\beta l) = \frac{1}{80\times2\times\pi\times10^6\times20\times10^{-12}}$$

$\cot(\beta l) = 99.47$

$\beta l = \text{arccot}(99.47) = 0.573°$

$\beta l = 0.01$ rad

$$\beta = \frac{0.01}{\underset{\uparrow\, l = 31cm}{l}} = \underline{\underline{32.3\times10^{-3}\, rad/m}}$$

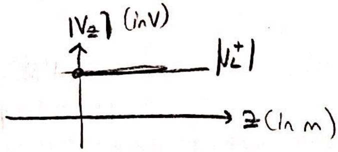
• $\frac{w}{\beta} = \frac{2\pi\times10^6}{32.3\times10^{-3}} = 1.94\times10^8\, m/s$

③ For lossless:

$$V_z = V^+ e^{-j\beta z} + V^- e^{j\beta z} \overset{z = l-d}{=} V_L^+ e^{j\beta d} + V_L^- e^{-j\beta d} = V_L^+ (e^{j\beta d} + \Gamma_L e^{-j\beta d})$$

a) Matched Load: $(\Gamma_L = 0)$

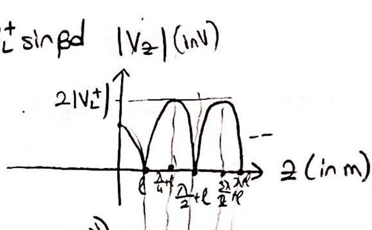
$V_z = V_L^+ e^{j\beta d} \Rightarrow V_z = V_L^+ e^{j\beta(l-z)}$



b) Short-Circuited Line $(\Gamma_L = -1)$

$$V_z(d) = V_L^+ \underbrace{(e^{j\beta d} - e^{-j\beta d})}_{2j\sin\beta d} = 2j\, V_L^+ \sin\beta d$$

$|V_z| = 2\,|V_L^+|\,|\sin(\beta(l-z))| \Rightarrow$



c) open-circuited Line $(\Gamma_L = 1)$

$V_z(d) = V_L^+ (e^{j\beta d} + e^{-j\beta d}) = V_L^+\, 2\cos\beta d$

$V_z = V_L^+\, 2\cos(\beta(l-z)) \Rightarrow$