Question 1

$$\bar{E}_i = E_1 \cdot \hat{a}_y \cdot e^{-jk_1 \cdot (\sin\theta_i x + \cos\theta_i z)}$$

$$\bar{E}_r = E_2 \cdot \hat{a}_y \, e^{-jk_1 \cdot (\sin\theta_r x - \cos\theta_r z)}$$

for $z<0$

$$\bar{E}_a = E_3 \cdot \hat{a}_y \cdot e^{-jk_2 (\sin\theta_a x + \cos\theta_a z)}$$

$$\bar{E}_b = E_4 \cdot \hat{a}_y \, e^{-jk_2 (\sin\theta_b x - \cos\theta_b z)}$$

for $0<z<d$

$$\bar{E}_t = E_5 \cdot \hat{a}_y \, e^{-jk_1 (\sin\theta_t x + \cos\theta_t z)}$$

for $z>d$

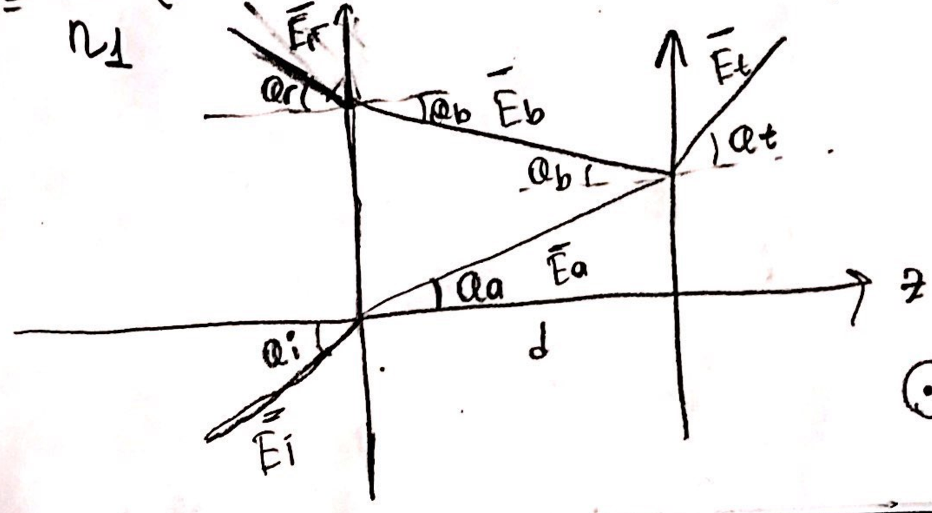
$$\bar{H}_i = \frac{E_1}{\eta_1} (-\cos\theta_i \, \hat{a}_x + \sin\theta_i \, \hat{a}_z) \, e^{-jk_1 (\sin\theta_i x + \cos\theta_i \cdot z)}$$

$$\bar{H}_r = \frac{E_2}{\eta_1} (\cos\theta_r \cdot \hat{a}_x + \sin\theta_r \cdot \hat{a}_z) \, e^{-jk_1 (\sin\theta_r x - \cos\theta_r \cdot z)}$$

$$\bar{H}_a = \frac{E_3}{\eta_2} (-\cos\theta_a \cdot \hat{a}_x + \cos\theta_a \cdot \hat{a}_z) e^{-jk_2 (\sin\theta_a x + \cos\theta_a z)}$$

$$\bar{H}_b = \frac{E_4}{\eta_2} (\cos\theta_b \, \hat{a}_x + \sin\theta_b \cdot \hat{a}_z) e^{-jk_2 (\sin\theta_b x - \cos\theta_b z)}$$

$$\bar{H}_t = \frac{E_5}{\eta_1} (-\cos\theta_t \cdot \hat{a}_x + \sin\theta_t \cdot \hat{a}_z) \, e^{-jk_1 (\sin\theta_t x + \cos\theta_t z)}$$



For $z=0$

$$d' = d.(\tan\theta_a + \tan\theta_b)$$

① $E^{(1)}_{tang} = E^{(2)}_{tang} \Big|_{z=0}$

$$E_1 . e^{-jk_1 \sin\theta_i . x} + E_2 e^{-jk_1 \sin\theta_r (x+d')} = E_a . e^{-jk_2 \sin\theta_a x} + E_b . e^{-jk_2 \sin\theta_b (x+d')}$$

$\theta_a = \theta_b \Rightarrow$ we show this in the next part

and

$\theta_i = \theta_r$ $\quad k_1.\sin\theta_i = k_2.\sin\theta_a$

$$E_1 + E_2 . e^{-jk_1 . \sin\theta_r . d'} = E_3 + E_4 . e^{-jk_2.\sin\theta_b . d'}$$

For $z=d$

② $E^{(1)}_{tang} = E^{(2)}_{tang} \Big|_{z=d}$

$$E_3 . e^{-jk_2 [\sin\theta_a . (x + d . \tan\theta_a) + \cos\theta_a . d]}$$

$$+$$

$$E_4 . e^{-jk_2 [\sin\theta_b (x + d \tan\theta_a) - \cos\theta_b \, d]}$$

$$=$$

$$E_5 . e^{-jk_1 [\sin\theta_t (x + d\tan\theta_a) + \cos\theta_t \, d]}$$

$\sin\theta_a . k_2 = \sin\theta_b . k_2$

$= \sin\theta_t . k_1$

(for all x)

$\Downarrow$

$\theta_a = \theta_b$

$$E_3 . e^{-jk_2 \cos\theta_a . d} + E_4 \, e^{+jk_2 \cos\theta_a . d} = E_5 . e^{-jk_1 \cos\theta_t . d}$$

$\theta_t = \theta_i \Rightarrow$ does not depend en d

(in question $\theta_e = \theta_i$)

Final results

$\theta_a = \theta_b$ $\quad \sin\theta_a . k_2 = \sin\theta_i . k_1$

$\theta_t = \theta_i$

$\theta_r = \theta_i$

For $z=0$

① $H_{tang}^{(1)} = H_{tang}^{(2)}$

$$-\frac{E_1}{\eta_1}\cos\theta_i + \frac{E_2}{\eta_1}\cos\theta_i \cdot e^{-jk_1 \sin\theta_i \cdot d'} = -\frac{E_3}{\eta_2}\cos\theta_a + \frac{E_4}{\eta_2}\cos\theta_a \cdot e^{-jk_2 \sin\theta_b d'}$$

For $z=d$

② $$-\frac{E_3}{\eta_2}\cos\theta_a \cdot e^{-jk_2\cos\theta_a d} + \frac{E_4}{\eta_2}\cos\theta_b \cdot e^{+jk_2\cos\theta_b d}$$

$$= -\frac{E_5}{\eta_1}\cos\theta_t \, e^{-jk_1 \cdot \cos\theta_t d}$$

$$A = \begin{bmatrix} -e^{-jk_1 \cdot \sin\theta_i d'} & 1 & e^{-jk_2 \cdot \sin\theta_b d'} & 0 \\ 0 & e^{-jk_2 \cdot \cos\theta_a d} & e^{jk_2 \cdot \cos\theta_a d} & -e^{jk_1\cos\theta_t d} \\ \frac{1}{\eta_1}\cos\theta_i e^{-jk_1 \sin\theta_i d'} & \frac{1}{\eta_2}\cos\theta_a & -\frac{1}{\eta_2}\cos\theta_a \cdot e^{-jk_2 \sin\theta_b d'} & 0 \\ 0 & -\frac{1}{\eta_2}\cos\theta_a \cdot e^{-jk_2\cos\theta_a d} & \frac{1}{\eta_2}\cos\theta_b \cdot e^{jk_2\cos\theta_b d} & \frac{1}{\eta_1}\cos\theta_t \cdot e^{-jk_1\cos\theta_t d} \end{bmatrix}$$

$$\begin{bmatrix} E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = E_1 \cdot A^{-1} \begin{bmatrix} 1 \\ 0 \\ 1/\eta_1 \cos\theta_i \\ 0 \end{bmatrix}$$

$$\underline{A} \begin{bmatrix} E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = E_1 \begin{bmatrix} 1 \\ 0 \\ 1/\eta_1 \cos\theta_i \\ 0 \end{bmatrix}$$

$\Rightarrow$ Find E_2, E_3, E_4, E_5 in terms of E_1

Then calculate H_2, H_3, H_4, H_5

$$P_{av} = \frac{1}{2} Re\{\bar{E} \times \bar{H}^*\}$$

Question 2

$$\sin\theta_c = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}} = \frac{\sqrt{3}}{2} \qquad \sin^2\theta_B = \frac{1-(\mu_2\varepsilon_1/\mu_1\varepsilon_2)}{1-(\varepsilon_1/\varepsilon_2)^2} = \frac{1}{4}$$

$$\bar{P}_{av}^{r} = \Gamma^2 \bar{P}_{av}^{i} \quad \text{and} \quad \bar{P}_{av}^{t} = \bar{P}_{av}^{i} + \bar{P}_{av}^{r}$$

$$\downarrow$$

$$\left(\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right)^2, \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$4\mu_2\varepsilon_2 = 3\mu_1\varepsilon_1 \quad (1)$$

$$3 - \frac{4\mu_2\varepsilon_1}{\mu_1\varepsilon_2} = \frac{\varepsilon_1^2}{\varepsilon_2^2} \quad (2)$$

(1) and (2)

$$3\varepsilon_2^2 - 4\mu_2\varepsilon_1\varepsilon_2^2 = \mu_1\varepsilon_2\varepsilon_1^2$$

$$3\varepsilon_2^2 - 3\mu_1\varepsilon_1^2\varepsilon_2 = \mu_1\varepsilon_2\varepsilon_1^2$$

$$3\varepsilon_2^2 = 4\mu_1\varepsilon_1^2\varepsilon_2 \Rightarrow 3\varepsilon_2 = 4\mu_1\varepsilon_1^2 \quad (3)$$

Use (1) and (3)

$$4\cdot\mu_2\cdot\frac{4\mu_1\varepsilon_1^2}{3} = 3\mu_1\varepsilon_1 \Rightarrow \boxed{\mu_2\varepsilon_1 = 9/16}$$

$$\frac{\mu_2}{\mu_1} = a \qquad \frac{\varepsilon_2}{\varepsilon_1} = b$$

(1) $$\sqrt{a.b} = \sqrt{\frac{3}{4}} \Rightarrow b = \sqrt{\frac{2}{3}}, \quad a = \boxed{\frac{3\sqrt{3}}{4\sqrt{2}} = \frac{\mu_2}{\mu_1}}$$

$$\frac{\mu_2\varepsilon_1}{\mu_1\varepsilon_2} = \frac{3\sqrt{3}}{4\sqrt{2}}\cdot\frac{\sqrt{3}}{\sqrt{2}} = \frac{9}{8}$$

$$\boxed{\mu_1\varepsilon_2 = 1/2}$$

(2) $$\frac{1-a/b}{1-(1/b)^2} = \frac{1}{4} \qquad \boxed{\frac{\varepsilon_2}{\varepsilon_1} = \sqrt{\frac{2}{3}}}$$

$$\Downarrow \quad ab \to 3/4$$

$$\frac{b^2 - ab}{b^2-1} = \frac{1}{4}$$

$$4b^2 - 3 = b^2 - 1$$

$$3b^2 = 2 \Rightarrow b^2 = \boxed{\frac{2}{3} = \frac{\varepsilon_2^2}{\varepsilon_1^2}}$$

$$\Gamma^2 = \frac{\mu_1/\varepsilon_1 + \mu_2/\varepsilon_2 - 2\sqrt{\dfrac{\mu_1\mu_2}{\varepsilon_1\varepsilon_2}}}{\mu_1/\varepsilon_1 + \mu_2/\varepsilon_2 + 2\sqrt{\dfrac{\mu_1\mu_2}{\varepsilon_1\varepsilon_2}}}$$

$$= \frac{\mu_1\varepsilon_2 + \mu_2\varepsilon_1 - 2\sqrt{\mu_1\varepsilon_1\mu_2\varepsilon_2}}{\mu_1\varepsilon_2 + \mu_2\varepsilon_1 - 2\sqrt{\mu_1\varepsilon_1\mu_2\varepsilon_2}}$$

$$= \frac{1/2 + 9/16 - 2\sqrt{9/32}}{1/2 + 9/16 + 2\sqrt{9/32}} \approx \boxed{8{,}7 \cdot 10^{-4}}$$

$$\boxed{P_{av}^{r} = \Gamma^2 P_{av}^{i}}$$

$$\boxed{P_{av}^{t} = \left(1 - \Gamma^2\right) P_{av}^{i}}$$

Q3. From coursebook page 343.

radius: a distance: D medium parameters: ϵ, μ

$$C = \frac{\pi \cdot \epsilon}{\cosh^{-1}(D/2a)} \left(\frac{F}{m}\right)$$

$$L = \frac{\mu}{\pi} \cosh^{-1}\left(\frac{D}{2a}\right) \left(\frac{H}{m}\right)$$

$$G = \frac{\pi \sigma}{\cosh^{-1}(D/2a)} \left(\frac{S}{m}\right)$$

$$R = \frac{1}{\pi a} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \; (\Omega/m)$$