Polarization

$$E_x + \frac{(m.E_x\cos\psi - E_y)^2}{m^2.\sin\psi} = E_{ox}^2 \Rightarrow \text{quadratic form}$$

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \longrightarrow$ ellipse $\Delta < 0$

parabola $\Delta = 0$ α } $\rightarrow$ It cannot represent

hyperbola $\Delta > 0$ α

For $\psi = 0, \pi \rightarrow$ linear

* If E_x and E_y are in phase $\psi = 0$ or out phase $\psi = \pi$ one one of them is 0 $\longrightarrow$ linearly polarized

$m=1$ and $\psi = \pm\frac{\pi}{2} \longrightarrow E_x^2 + E_y^2 = E_{ox}^2 \longrightarrow$ circular polarized

According to its direction it can be LH or RH

Ex $E = (E_{ox}\, a_x - j.a_y.E_{oy}).e^{-jkz} \longrightarrow$ RHCP

Plane Waves in Lossy MEDIA

$$\bar\nabla \times H - \bar J + \frac{\partial D}{\partial t} = J + j\omega\varepsilon.E \xrightarrow{\text{phaser domain}} \Rightarrow \bar\nabla \times \bar H = \underbrace{\sigma.\bar E}_{J_c} + \underbrace{j\omega\varepsilon\bar E}_{J_d}$$

J_c conduction current density; $J_d \longrightarrow$ displasement current density

If $\frac{\sigma}{\omega\varepsilon} \gg 1$ good conductor

$\ll$ good Insulator

$= 1$ poor conductor, poor Insulator

$$\Leftarrow \frac{J_c}{J_d} = \frac{\sigma.E}{\omega\varepsilon E} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

$J_c \perp J_d$

$\tan\theta$ ↳ Loss tangent

Simple, source free, lossy medium $\Rightarrow \nabla^2 E - \mu\sigma.\frac{dE}{dt} - \mu\varepsilon.\frac{d^2E}{dt^2} = 0$

$$\nabla^2 E - \underbrace{j\omega\mu(\sigma + j\omega\varepsilon)}_{\gamma^2}.\bar E = 0$$

Let $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$

α: attenuation constant, neper/meter

β: phase constant ↳ radian/meter

$$\frac{dz}{dt} = \frac{\omega}{\beta} = v \qquad \lambda = \frac{2\pi}{\beta}$$

SKIN DEPTH

$$\delta = \frac{1}{\alpha}$$

$$\nabla \times \bar E = -j\omega\mu.H = \frac{1}{-j\omega\mu}(-\gamma).E_0.e^{-\gamma z}.\hat a_y \Rightarrow \frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \eta = |\eta|e^{j\theta}$$

There is a phase difference E and H fields in lossy medium

Complex Permittivity $= \varepsilon_c$

$$\nabla \times \bar{H} = \bar{J} + j\omega\varepsilon.\bar{E} = \sigma\bar{E} + j\omega\varepsilon.\bar{E} = j\omega E\Big(\underbrace{\varepsilon - j\frac{\sigma}{\omega}}_{\varepsilon_c}\Big)$$

$$\nabla \times H = j\omega\varepsilon_c E. \qquad \varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} \qquad \mu = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$e^{-kz} = e^{-j\omega\sqrt{\mu.\varepsilon_c}\,z} \Rightarrow j\omega\sqrt{\mu\varepsilon_c} = \gamma \qquad jk \longleftrightarrow \gamma \quad \varepsilon \longleftrightarrow \varepsilon_c$$

Perfect dielectric $(\sigma = 0)$

$$\gamma = \alpha + j\beta \Rightarrow \begin{matrix}\alpha = 0 \longrightarrow \\ \beta = \omega\sqrt{\mu\varepsilon} = k\end{matrix} \qquad \delta = \frac{1}{\alpha} = 0 \qquad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{k} \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Good Ins $\frac{\sigma}{\omega\varepsilon} \ll 1$

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ = Attenuation constant $\qquad \delta = \frac{1}{\alpha} \cong \frac{2}{\sigma}.\sqrt{\frac{\varepsilon}{\mu}} \qquad \eta \cong \sqrt{\frac{\mu}{\varepsilon}}\left(1 + j.\frac{\sigma}{2\omega\varepsilon}\right)$

$\beta \cong k = \omega\sqrt{\mu\varepsilon}$ = phase constant $\qquad v = \frac{\omega}{\beta} \cong \frac{\omega}{k} \qquad \lambda = \frac{2\pi}{\beta} \cong \frac{2\pi}{k}$

Good Conductor $\frac{\sigma}{\omega\varepsilon} \gg 1$

$$\gamma \cong \sqrt{j\omega\mu\sigma} = \sqrt{j}.\sqrt{\omega\mu.\sigma} = \sqrt{\frac{\omega\mu\sigma}{2}}(1+j) \qquad \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} \qquad \eta \cong \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$$

Surface Resistance $= \mathrm{Re}\{\eta\} = \frac{1}{\delta\sigma}$

Perfect Conductor

$$\bar{E} = \bar{D} = \bar{B} = \bar{H} = 0$$

$$R_{DC} = \frac{\ell}{\sigma.S} \qquad R_{AC} = \frac{\ell}{\sigma.S} = \frac{1}{\sigma(2\pi a\delta)}$$

$$\bar{\nabla}(\bar{E}\times\bar{H}) = \bar{H}(\underbrace{\bar{\nabla}\times\bar{E}}_{-\frac{\partial B}{\partial t}}) - \bar{E}(\underbrace{\bar{\nabla}\times\bar{H}}_{J+\frac{\partial D}{\partial t}}) = \frac{\mu}{2}\cdot\frac{d}{dt}|H|^2 - \frac{\varepsilon}{2}\cdot\frac{d}{dt}\cdot|E|^2 - \bar{E}.\bar{J}$$

$$-\frac{d}{dt}\Big[\underbrace{\tfrac{1}{2}\mu|H|^2}_{w_M} + \underbrace{\tfrac{1}{2}\cdot\varepsilon|E|^2}_{w_E}\Big] - \bar{E}\bar{J}$$

$$-\bar{\nabla}(\bar{E}\times\bar{H}) = \frac{d}{dt}(w_M + w_E) + \bar{E}.\bar{J}$$

$$-\int_v \nabla(E\times H).dv = \int_v \frac{d}{dt}(w_M + w_E) + \bar{E}.\bar{J}$$

$$\rightarrow \oint_S (E\times H)\, ds = \cdots\cdots = \frac{d}{dt}(W_M + W_E) + \int_v \bar{E}.\bar{J}\, . dv$$

POYNTING'S THEOREM

$\vec{n} \parallel \wp = E\times H = \text{watts}/m^2$ $\quad \bar{\wp}\perp\bar{E} \quad \bar{P}\perp\bar{H}$ $\quad \wp(\bar{r}) \triangleq \frac{1}{T}\int_{t=0}^{t=T=1/f} \wp(\bar{r},t).dt$

$$P_{av}(\vec{r}) \triangleq \frac{1}{2}\text{Re}\{\bar{P}\}$$

$$-\int_v \bar{E}.\bar{J}_i\, dv = \frac{\partial}{\partial t}(W_T) + \oint_S \bar{P}.d\bar{S} + \int_v \sigma E^2 dv$$

$\Rightarrow$ If there is a lossy media $\sigma \neq 0$

$$J = \bar{J}_i + \sigma.\bar{E}$$

$$-\oint_S \bar{P}.d\bar{S} = \frac{\partial}{\partial t}(W_T) + \int_v \sigma E^2 dv$$

$\Rightarrow$ lossy but $J_i = 0$

(*) If medium is lossless $\sigma=0$, last term is zero (ohmic loss)

all power = EM waves

(*) Static Case $\frac{d}{dt} = 0$

Normal Incidence $\longrightarrow$ reflected ve transmitted are in the opposite way

Perfect dialectric / Perfect dia boundary

in medium ①
$$\bar{E}^i = \hat{a}_x E_1 e^{-jk_1 z} \longleftrightarrow \bar{H}^i = \hat{a}_y \frac{E_1}{\eta_1} e^{-jk_1 z}$$
$$\bar{E}^r = \hat{a}_x E_2 e^{jk_1 z} \longleftrightarrow \bar{H}^r = -\hat{a}_y \frac{E_2}{\eta_1} e^{jk_1 z}$$

in medium ②
$$\bar{E}^t = \hat{a}_x E_3 e^{-jk_2 z} \longleftrightarrow \bar{H}^t = \hat{a}_y \frac{E_3}{\eta_2} e^{-jk_2 z}$$

$$E^{total}_{tan_1} = E^{total}_{tan_2}$$

$$E^i_{tan} + E^r_{tan} = E^t_{tan}$$

$$E_1 + E_3 = E_2$$

Define:
$$\boxed{\Gamma = \frac{E_2}{E_1}}$$: reflection coefficient ($E_2 \to r$, $E_1 \to i$)

$$E_i \cdot \Gamma = E_2$$

$$\boxed{T = \frac{E_3}{E_1}}$$: transmission coefficient ($E_3 \to t$, $E_1 \to i$)

combine them

$J_s = 0$ perfect dialectric media

$$H^{total}_{tan_1} = H^{total}_{tan_2}$$

$$\frac{E_1}{\eta_1} - \frac{E_2}{\eta_1} = \frac{E_3}{\eta_2}$$

$$\frac{1}{\eta_1}(1-\Gamma) = \frac{1}{\eta_2} T$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_2}}$$

$$P_{av_1} = P_{av_2}$$

$$P^r_{av} = \Gamma^2 \cdot P^i_{av} \qquad P^i_{av} + \bar{P}^r_{av} = \bar{P}^t_{av}$$

only normal indices problem

Perfect Dia / Lossy Boundary

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad T = \frac{2\eta_2}{\eta_1 + \eta_2} \quad \text{are still valid}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \qquad \eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma + j\omega\varepsilon}} \qquad \text{in ②} \quad \gamma^2 = \alpha_2 + j\beta_2$$

Perfect Dia / Perfect Conduc

$$\eta_2 \cong \sqrt{\frac{j\omega\mu}{\sigma}} \qquad 0 = \eta_2 \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1 \quad T = 0 \qquad E_1 = -E_2 \qquad \text{no transmission}$$

① zeros of E_1^{total}
Max/Min H_1^{total}
$$\left.\right\} \; k_1 z = -n\pi \quad \text{or} \quad k_1 = \frac{2\pi}{\lambda}$$

$\downarrow$

vice verse $\quad k_1 z = -(2n+1)\frac{\pi}{2}$

$$\hat{n} \times (\bar{H}_1 - \underbrace{\bar{H}_2}_{0}) = \bar{J}_s$$

$$J_s = -\hat{a}_z \times H_1^{total}$$

$$J_s = \frac{2E_1}{\eta_1} \cdot \cos\omega t \cdot a_x$$

H_1^{tota} and $E_\perp^{total}$ are in time quadrate (sin cos

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FINAL

Prependicular polarization — $E \Rightarrow \odot \;\otimes$

Parallel polarization — $H \Rightarrow \odot \;\otimes$

Their normal directions are same way

BREWSTER ANGLE (θ_B)

It is the value of incidence angle for which $\Gamma = \frac{E_2}{E_1} = 0$ (i.e. reflected wave is zero).

Case 1: Perpendicular Polarization

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

at $\theta_i = \theta_{B_\perp} \Rightarrow \Gamma_\perp = 0$

$$\Rightarrow \boxed{\eta_2 \cos\theta_{B_\perp} - \eta_1 \cos\theta_t = 0}$$

using

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$$

$$\sin\theta_t = \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}} \sin\theta_i \Big|_{\theta_i = \theta_{B_\perp}} \quad \text{(Snell's Law)}$$

$$\cos\theta_t = \sqrt{1 - \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2} \sin^2\theta_{B_\perp}}$$

gives

$$\boxed{\sin^2\theta_{B_\perp} = \frac{1 - (\mu_1\epsilon_2/\mu_2\epsilon_1)}{1 - (\mu_1/\mu_2)^2}}$$

For $\mu_1 = \mu_2$ case, $\sin\theta_{B_\perp} \to \infty$

$\therefore \theta_{B_\perp}$ does not exist

For non-magnetic media with $\mu_1 = \mu_2 = \mu_0$, for example, a Brewster angle $\theta_{B_\perp}$ can not be found to make reflections zero!

Case 2: Parallel Polarization

$$\Gamma_{//} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

at $\theta_i = \theta_{B_{//}} \Rightarrow \Gamma_{//} = 0$

$$\boxed{\eta_2 \cos\theta_t - \eta_1 \cos\theta_{B_{//}} = 0}$$

Using Snell's Law of refraction, it can be shown that

$$\boxed{\sin^2\theta_{B_{//}} = \frac{1 - (\mu_2\epsilon_1/\mu_1\epsilon_2)}{1 - (\epsilon_1/\epsilon_2)^2}}$$

$\therefore \theta_{B_{//}}$ exists unless $\epsilon_1 = \epsilon_2$

For $\mu_1 = \mu_2$ case,

$$\boxed{\theta_{B_{//}} = \sin^{-1}\left(\frac{1}{\sqrt{1 + \frac{\epsilon_1}{\epsilon_2}}}\right) = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

OBLIQUE INCIDENCE

$$E^i = E_1 \cdot e^{-j\bar{k}_i \cdot \bar{r}} \quad \text{with} \quad \bar{k}_i = k_i \cdot \hat{n}_i$$

$$E_r = E_2 \cdot e^{-j\bar{k}_r \cdot \bar{r}} \quad \text{with} \quad \bar{k}_r = k_1 \cdot \hat{n}_r$$

$$E_t = E_3 \cdot e^{-j\bar{k}_t \cdot \bar{r}} \quad \text{with} \quad k_t = k_2 \cdot \hat{n}_t$$

$$k_1 = \omega\sqrt{\varepsilon_1 \cdot \mu_1} \qquad k_2 = \omega\sqrt{\varepsilon_2 \cdot \mu_2}$$

$$z=0 \longrightarrow B.C \longrightarrow (E_i + E_r)_{tan}\Big|_{z=0} = E^t_{tan}\Big|_{z=0} \qquad x, y \in$$

$$k_{ix} \cdot x + \cancel{k_{iy}} y = k_{rx} \cdot x + \cancel{k_{ry}} y = k_{tx} \cdot x + \cancel{k_{ty}} y \longrightarrow$$

$$k_{ix} = k_{rx} = k_{tx} \qquad n_i = \underbrace{k_1 \sin\theta_i}_{k_{ix}} \cdot \hat{a}_x + \underbrace{k_1 \cos\theta_i}_{k_z} \cdot \hat{a}_z = k_i = k_1 \cdot \hat{n}_i$$

$\theta_i = \theta_r \longrightarrow$ snell law of reflection

$$\sqrt{\mu_1 \cdot \varepsilon_1} \cdot \sin\theta_i = \sqrt{\mu_2 \cdot \varepsilon_2} \cdot \sin\theta_t$$

Index of refraction

$$v_1 = \frac{1}{\sqrt{\mu_1 \cdot \varepsilon_1}} \qquad v_2 = \frac{1}{\sqrt{\mu_2 \varepsilon_2}} \qquad \frac{\sin\theta_t}{\sin\theta_i} = \frac{v_2}{v_1} = \sqrt{\frac{\mu_1 \cdot \varepsilon_1}{\mu_2 \cdot \varepsilon_2}} = \frac{n_1}{n_2}$$

$\mu_2 \cdot \varepsilon_2 > \mu_1 \cdot \varepsilon_1 \longrightarrow$ Second medium is denser $\longrightarrow$ first medium is denser

$\theta_i > \theta_t$ $\qquad$ $\theta_t > \theta_i$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1 \cdot \varepsilon_1}{\mu_2 \varepsilon_2}} = \frac{1}{\sin\theta_c}$$

If the first medium is denser than the second medium and $\theta_i > \theta_c \longrightarrow$

θ_t becomes complex angle. $\longrightarrow$ $\cos\theta_t = \pm j\sqrt{\frac{\mu_1 \cdot \varepsilon_1}{\mu_2 \cdot \varepsilon_2} \cdot \sin^2\theta_i - 1}$

$\sin\theta_t > 1$

$$E^t = E_3 \cdot e^{-j\bar{k}_t \cdot \bar{r}} = E_3 \cdot e^{-j(k_2 \cdot \sin\theta_t \cdot x + k_2 \cdot \cos\theta_t \cdot z)}$$

$$= E_3 \cdot \underbrace{e^{-j \overset{\beta_x}{k_2 \cdot \sin\theta_t} \cdot x}}_{\text{complex exponential}} \cdot \underbrace{e^{j \overset{\alpha_x}{k_2 \cdot \cos\theta_t} \cdot z}}_{\text{Real exponential}}$$

$\longrightarrow$ if it is large enough we can call it "surface wave"

$$E^t = \bar{E}_3 \cdot \underbrace{e^{-\alpha_z \cdot z}}_{\text{same}} \cdot \underbrace{e^{-j\beta_x \cdot x}}_{1} = |E_3| \cdot e^{-\alpha_z \cdot z}$$

$$\boxed{\angle E^t = -\beta_x \cdot x}$$

For $\theta_i \leq \theta_c$ $\quad v = \frac{w}{k_2} \longrightarrow \theta_t$ is real

For $\theta_i > \theta_c$ $\quad$ the phase velocity $= v_{sw} = \frac{w}{B_x} = \frac{w}{k_2 \cdot \sin\theta_t}$

when $\theta_i \geq \theta_c \longrightarrow$ total reflection

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Prependicular Polarization

$$E_I = E_{\perp} \cdot e^{-jk_1 \cdot \hat{n}_i \cdot \hat{r}} \longrightarrow \bar{H}_i = \frac{1}{\eta_i} \cdot \hat{n}_i \times E_i = \frac{E_\perp}{\eta_1}\left(-\cos\theta_i \cdot \hat{a}_x + \sin\theta_i \cdot a_z\right) \cdot e^{-jk_1 n_i \cdot \hat{r}}$$

$r \longrightarrow \cos\theta_i \cdot a_x$

$t \longrightarrow -\cos\theta_i \cdot a_x$

Apply BC's at $z=0$

1) $E^{tot}_{tang}\Big|_{z=0} = E^{tot}_{tang}\Big|_{z=0}$ $\qquad E_1 + E_2 = E_3$

2) $H^{tot}_{tang}\Big|_{z=0} = H^{total}_{tang}\Big|_{z=0}$ $\qquad (H^i_x + H^r_x)\Big|_{z=0} = H^t_x\Big|_{z=0}$ $\qquad \frac{1}{\eta_1}(E_2 - E_1) \cdot \cos\theta_i = \frac{1}{\eta_2}(-E_3 \cdot \cos\theta_t)$

$\Gamma = \frac{E_2}{E_1} \longrightarrow$ reflected $\qquad T = \frac{E_3}{E_1} \longrightarrow$ transpored

$$\Gamma_\perp = \frac{E_2}{E_1} = \frac{\eta_2 \cdot \cos\theta_i - \eta_i \cdot \cos\theta_t}{\eta_2 \cdot \cos\theta_i + \eta_i \cdot \cos\theta_t} \qquad T_\perp = \frac{E_3}{E_1} = \frac{2\eta_2 \cdot \cos\theta_i}{\eta_2 \cdot \cos\theta_i + \eta_i \cos\theta_t}$$

Case 1 $\quad$ Perfect Die / Perfect Conductor

$\hookrightarrow \eta_2 = \lim_{\sigma_2 \to \infty} \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = 0$

$\eta_2 = 0$

$\Gamma_\perp = -1 \qquad T_1 = 0$ $\qquad$ for prependicular

$E_1 = E_2$ and E_3

case 2 $\quad \mu_1 \cdot \epsilon_1 > \mu_2 \cdot \epsilon_2$

i) Let $\theta_i = \theta_c \longrightarrow \theta_t = \pi/2$ $\qquad E_1 = E_2 \qquad 2E_1 = E_3$

$\Gamma_\perp = 1$

$T_\perp = 2$

$$P_{av} = \frac{1}{2} \cdot Re\{\vec{E}^t \times \vec{H}^{t*}\}$$

$$\frac{1}{2} Re\left\{\frac{|E_3|}{\eta_2} \cdot a_y \times a_z\right\} = \frac{1}{2} \cdot \frac{|E_3|^2}{\eta_2} \cdot \hat{a}_x \left(\frac{W}{m^2}\right)$$

Case 3

$\theta_i = 0 \Rightarrow \theta_t = 0 \Rightarrow \theta_r = 0 \qquad T_\perp = 1 + \Gamma_\perp$

Parallel Polarization Case

BC 1 $\quad H^{total}_{tang} = H_1 - H_2 = H_3 \Longrightarrow \frac{1}{\eta_1}(E_1 - E_2) = \frac{1}{\eta_2} E_3$

BC 2 $\quad$ cont of $E \qquad (E_1 + E_2).\cos\theta_i = E_3 . \cos\theta_t$

$$\Gamma_{\parallel} = \frac{E_2}{E_1} = \frac{\eta_2 . \cos\theta_t - \eta_1 . \cos\theta_i}{\eta_2 . \cos\theta_t + \eta_1 . \cos\theta_i} \qquad T_{\parallel} = \frac{E_3}{E_1} = \frac{2\eta_2 . \cos\theta_i}{\eta_2 . \cos\theta_t + \eta_1 . \cos\theta_i}$$

$$1 + \Gamma_{\parallel} = T_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

Case 1 $\longrightarrow$ Perfect cond

$\Gamma_{\parallel} = -1 \qquad T_{\parallel} = 0$

Case 2 $\longrightarrow \epsilon_1 . \mu_1 > \epsilon_2 . \mu_2 \qquad \theta_i > \theta_c$

$\Gamma_{\parallel}$ and $T_{\parallel}$ will be complex

$\Gamma_{\parallel} = e^{j\phi_{\parallel}} \qquad |\Gamma_{\parallel}| = 1$ and $\phi_{\parallel} \neq \phi_t$

Case 3

$$\theta_i = \theta_r = \theta_t = 0 \qquad \Gamma_{\parallel} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad T_{\parallel} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Brewster Angle

It is the angle of incidence angle for which $\Gamma = \frac{E_2}{E_1} = 0$

For $\mu_1 = \mu_2$ case θ_B does not exist.

Note: $\quad (\parallel)$ case $\longrightarrow \mu_1 = \mu_2 = \mu$ and $\theta_i = \theta_B$

Snell's Law : $k_1 . \sin\theta_{B_\parallel} = k_2 . \sin\theta_t \longrightarrow \sqrt{\epsilon_1} . \sin\theta_{B_\parallel} = \sqrt{\epsilon_2} . \cos\theta_t$

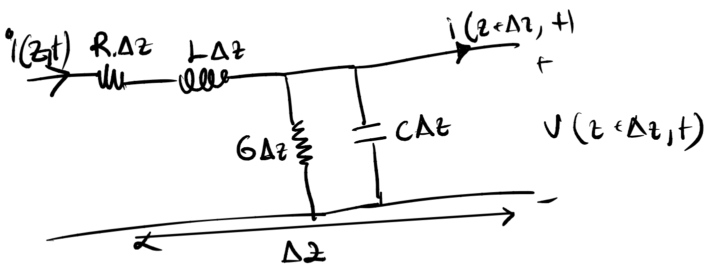
Brewster Condition $\eta_1 . \cos\theta_{B_\parallel} = \eta_2 . \cos\theta_t \longrightarrow \frac{1}{\sqrt{\epsilon_1}} . \cos\theta_{B_\parallel} = \frac{1}{\sqrt{\epsilon_2}} . \cos\theta_t$

x

$$\sin 2\theta_{B_\parallel} = \sin 2\theta_t$$

$$\theta_{B_\parallel} = \frac{\pi}{2} - \theta_t \longrightarrow \theta_i = \theta_B \quad \mu_1 = \mu_2$$

TRANSMISSION LINES



C: Capacitive effect
L: Inductive effective
R: Conductor losses $\rightarrow \Omega$
G: Dialectric losses $\rightarrow 1/\Omega$

KVL

$$\frac{d^2 v}{dz^2} = -R \cdot \frac{di}{dz} - L \cdot \frac{d^2 i}{dz \cdot dt}$$

KCL

$$-\frac{d^2 i}{dz \cdot dt} = G \cdot \frac{dv}{dt} + C \cdot \frac{d^2 v}{dt^2}$$

$\rightarrow$ We can create functions with v and i

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

$$\frac{\partial v^2(z)}{\partial z^2} - \gamma^2 V(z) = 0 \qquad \frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0 \Rightarrow \text{Travelling } I(z)$$

$\rightarrow$ Travelling $V(z)$

$$v(z) = V^+ \cdot e^{-\gamma z} + V^- \cdot e^{\gamma z}$$

$$\gamma = \alpha + j \cdot \beta$$

α: attenuation, β: phase

$$V(z) = V^+ \cdot e^{-\alpha z} \cdot e^{-j\beta z} + V^- \cdot e^{\alpha z} \cdot e^{j\beta z}$$

$\longrightarrow a_z$ $\qquad$ $\longrightarrow -a_z$

$$u = \frac{\omega}{Im(\gamma)} = \frac{\omega}{\beta} \qquad \lambda = \frac{2\pi}{Im(\gamma)} = \frac{2\pi}{\beta}$$

$$\gamma_{TL} = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma_{upw} = \sqrt{j\omega\mu\,(\sigma + j\omega\varepsilon)}$$

$$I(z) = \frac{-1}{R + j\omega L} \cdot \frac{dV(z)}{dz} = -\frac{1}{R + j\omega L} \cdot \frac{d}{dz}\left[V^+ e^{-\gamma z} + V^- \cdot e^{\gamma z}\right]$$

$$Z_0 = \frac{1}{Y_0} \triangleq \sqrt{\frac{R + j\omega L}{G + j\omega C}} \Longleftrightarrow \eta_{upw} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

$$\frac{V(z)}{I(z)} \neq Z_0 = \frac{V_+}{I_+} = \frac{V_-}{I_-}$$

If T_L is lossless $R=0$ and $G=0$

$\gamma = j\omega\sqrt{LC}$ $\quad \gamma = \alpha + j\beta$ $\quad \alpha = 0$ $\quad \beta = \omega\sqrt{LC}$

$z_0 = \sqrt{\frac{L}{C}} \Longrightarrow \sqrt{\frac{\mu}{\varepsilon}}$ $\quad v = \frac{1}{\sqrt{LC}}$

Low Loss

Reflection Coef

$\Gamma_L \triangleq \frac{V_L^-}{V_L^+}$ $\quad z_L = \frac{V_L}{I_L} = \frac{V_+ + V_-}{\frac{1}{z_0}(V_L^+ - V_L^-)} = z_0 \frac{1 + \frac{V_L^-}{V_L^+}}{1 - \frac{V_L^-}{V_L^+}}$

$z_L = z_0 \cdot \frac{1+\Gamma_L}{1-\Gamma_L}$ $\quad = |\Gamma_L| \cdot e^{j\theta_L} = \frac{z_L - z_0}{z_L + z_0}$

$V(d) = V^+ \cdot e^{-\gamma(\ell - d)} + V^- \cdot e^{\gamma(\ell - d)}$ $\quad \Gamma(d) = \frac{V_L^-(d)}{V_L^+(d)} = \Gamma_L \cdot e^{-2\gamma d}$

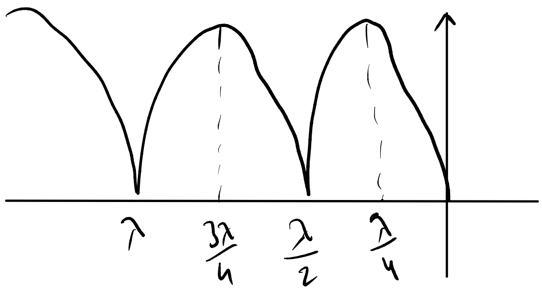
Standing

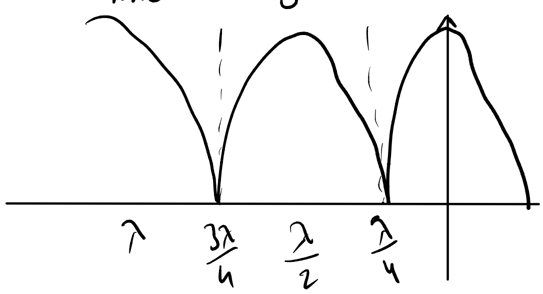
$V(d) = V_L^+ \cdot e^{j\beta d} + V_L^- \cdot e^{-j\beta \cdot d}$ $\quad V(d) = V_L^+ \left(e^{j\beta d} + \frac{V_L^-}{V_L^+} \cdot e^{-j\beta d} \right)$

$V(d) = V_L^+ \left(e^{j\beta d} + \Gamma_L \cdot e^{-j\beta \cdot d} \right)$ $\quad \Gamma_L \triangleq \frac{V_L^-}{V_L^+}$

VSWR = Voltage Standing Wave Ratio $\triangleq s$ $\quad \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$

$S = \frac{V_{max}}{V_{min}}$ $\Longrightarrow$ short cct $\Longrightarrow$ $\frac{V_{max}}{V_{min}} = \frac{2|V_L^+|}{0} = \infty$





⇒ open

Arbitrary Load Impedance

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma_L| \cdot e^{j\theta_L} \qquad |\Gamma_L| < 1$$

$$V(d) = V_L^+ \cdot e^{j\beta d} + V_L^- \cdot e^{-j\beta d}$$

$$|V_L^+| = |V^+ \cdot e^{-\gamma \ell}| = |V^+ \cdot e^{-\alpha \ell} \cdot e^{-\beta \ell}| = |V^+| \cdot |e^{-\alpha \ell}| = |V^+| \cdot e^{-\alpha \ell}$$

$$\left.\begin{aligned} |V_L^+| &= |V^+| \\ |V_L^-| &= |V^-| \end{aligned}\right\} \Rightarrow \text{lossles} \quad \alpha = 0$$

$$\frac{\lambda}{2} = 50cm \qquad \frac{V_{max}}{V_{min}} = \frac{3V_{min}}{V_{min}} = S = 3 \qquad Z_0 = 300\Omega$$

$$d_{min} = 30cm$$

$$\lambda = 100cm \qquad \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{100} = \frac{\pi}{50} \qquad \theta_L = 2\beta \cdot d_{min} - \pi$$