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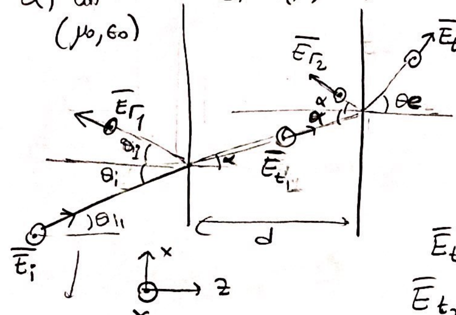
HW7-

Q1)

$k_0 = w\sqrt{\mu_0\epsilon_0}$ $k_g = w\sqrt{2.1\epsilon_0\mu_0}$ $k_0 = w\sqrt{\mu_0\epsilon_0}$

a) air (μ_0, ϵ_0) Glass(μ_0, 2.1ϵ_0)



$\cos\theta_{i_1}\hat{a}_z + \sin\theta_i\hat{a}_x$

Let's investigate one by one:

$$\bar{E}_i = \hat{a}_y\left(E_i e^{-j\bar{k}\cdot\bar{r}}\right) = \hat{a}_y E_i e^{-jk_0(\cos\theta_i z + \sin\theta_i x)}$$

$$\bar{E}_{r_1} = \hat{a}_y\left(E_{r_1} e^{-j\bar{k}\cdot\bar{r}}\right) = \hat{a}_y E_{r_1} e^{-jk_0(-\cos\theta_i z + \sin\theta_i x)}$$

$$\bar{E}_{t_1} = \hat{a}_y\left(E_{t_1} e^{-j(\bar{k}\cdot\bar{r})}\right) = \hat{a}_y E_{t_1} e^{-jk_g(\cos\alpha z + \sin\alpha x)}$$

$$\bar{E}_{t_2} = \hat{a}_y\left(E_{t_2} e^{-j(\bar{k}\cdot\bar{r})}\right) = \hat{a}_y E_{t_2} e^{-jk_0(\cos\theta_e z + \sin\theta_e x)}$$

$$\bar{E}_{r_2} = \hat{a}_y\left(E_{r_2} e^{-j(\bar{k}\cdot\bar{r})}\right) = \hat{a}_y E_{r_2} e^{-jk_g(-\cos\theta_e z + \sin\theta_e x)}$$

- From boundary condition:
  - → (tangential of $\bar{E}$ should be continous)
  - -at left side: $\bar{E}_{i_{tan}} + \bar{E}_{r_{1_{tan}}} = \bar{E}_{t_{1_{tan}}} + \bar{E}_{r_{2_{tan}}}$ (at $z=0$)

    From this condition we know that:

    $\boxed{k_0 \sin\theta_{i_1} = k_g \sin\alpha}$

- at right side: $\bar{E}_{t_{1_{tan}}} + \bar{E}_{r_{2_{tan}}} = \bar{E}_{t_{2_{tan}}}$

  → $\boxed{k_0 \sin\theta_e = k_g \sin\alpha}$

$\cancel{k_0}\sin\theta_i = \cancel{k_0}\sin\theta_e$

$\theta_i = \theta_e + k\pi$ (take zero)

$\theta_i = \theta_e$

Question 2) $① \to \epsilon_1, \mu_1$, $② \to \epsilon_2, \mu_2$ } parallel polarization

$\theta_c = 60°$, critical angle.

$\sin(\theta_c) = \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}}$; we know it. Let $\frac{\mu_2}{\mu_1} = S_\mu$; $\frac{\epsilon_1}{\epsilon_2} = S_\epsilon$.

$$\sin 60° = \frac{\sqrt{3}}{2} = \sqrt{\frac{\mu_2 \epsilon_2}{\mu_1 \epsilon_1}} \Rightarrow \frac{S_\mu}{S_\epsilon} = \frac{3}{4} \Rightarrow \begin{cases} S_\mu = 3k \\ S_\epsilon = 4k \end{cases}$$

Also, we know that brewster angle is 30°.

θ_B for parallel case;

$$\sin^2\theta_B = \frac{1 - (\mu_2\epsilon_1/\mu_1\epsilon_2)}{1 - (\epsilon_1/\epsilon_2)^2} = \frac{1 - S_\mu S_\epsilon}{1 - (S_\epsilon)^2} \qquad \sin^2 30 = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1 - 12k^2}{1 - 16k^2} \Rightarrow 1 - 16k^2 = 4 - 48k^2$$

$$32k^2 = 3$$

$$k^2 = \frac{3}{32} \Rightarrow k = \sqrt{\frac{3}{32}}$$

$$S_\epsilon = 4k = 4 \cdot \frac{\sqrt{3}}{4\sqrt{2}} = \frac{\sqrt{6}}{2} = \frac{\epsilon_1}{\epsilon_2}$$

$$S_\mu = 3k = \frac{3\sqrt{3}}{4\sqrt{2}} = \frac{3\sqrt{6}}{8} = \frac{\mu_2}{\mu_1}$$

• Then, we know that $\Gamma = \frac{n_2 - n_1}{n_2 + n_1}$

$$\Gamma = \frac{n_2 - n_1}{n_2 + n_1} = \frac{\sqrt{\mu_2\epsilon_1} - \sqrt{\epsilon_2\mu_1}}{\sqrt{\mu_2\epsilon_1} + \sqrt{\mu_1\epsilon_2}} = \frac{\sqrt{\frac{\mu_2}{\mu_1}} - \sqrt{\frac{\epsilon_2}{\epsilon_1}}}{\sqrt{\frac{\mu_2}{\mu_1}} + \sqrt{\frac{\epsilon_2}{\epsilon_1}}} = \frac{\sqrt{\frac{3\sqrt{6}}{8}} - \sqrt{\frac{2}{\sqrt{6}}}}{\sqrt{\frac{3\sqrt{6}}{8}} + \sqrt{\frac{2}{\sqrt{6}}}} = 0.03$$

$*$by $\left(\frac{1}{\sqrt{\mu_1\epsilon_1}}\right)$

• As a result

$$\frac{|P_{refl}|}{|P_i|} = \Gamma^2 = \underline{8.7 \times 10^{-4}} \qquad \frac{|P_T|}{|P_i|} = (1 - \Gamma^2) = \underline{0.99}$$

Q3)

$$C \cong \frac{\pi \epsilon}{\ln\left(\frac{d}{a}\right)} = \frac{\pi \epsilon}{\cosh^{-1}\left(\frac{d}{2a}\right)} \text{ F/m} \; ; \; L = \frac{\mu}{\pi} \ln\left(\frac{d}{a}\right) = \frac{\mu}{\pi} \cosh^{-1}\left(\frac{d}{2a}\right)$$

$$R_{AC} = \frac{1}{\pi a} \sqrt{\frac{\pi f \mu_c}{\sigma}} = \frac{1}{a} \sqrt{\frac{f \mu_c}{\pi \sigma}}$$ (for good conductor assumption) (Ω/m)

$$G \cong \frac{\pi \sigma}{\ln\left(\frac{d}{a}\right)} \; (S/m) = \frac{\pi \sigma}{\cosh^{-1}\left(\frac{d}{2a}\right)} \; S/m$$

. The terms with cosh's are taken from our reference book