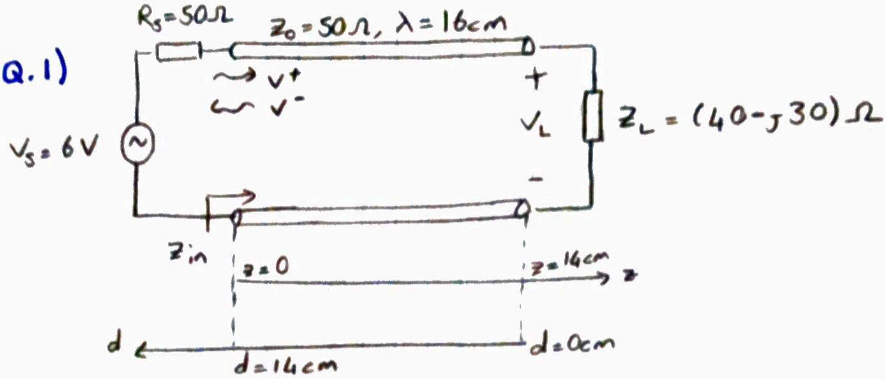
EE303 HW #8 Solutions

Q.1)



a) For a lossless ($\alpha = 0$) TL

$$Z_{lossless}(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} \quad \& \quad \beta = \frac{2\pi}{\lambda}$$

$$Z_{in} = Z_{lossless}(d = 14cm) = 50 \frac{(40 - j30) + j50 \tan\left(2\pi \frac{14cm}{16cm}\right)}{50 + j(40 - j30)\tan\left(2\pi \frac{14cm}{16cm}\right)}$$

$$\Rightarrow Z_{in} = 50 \frac{40 - j30 - j50}{50 - j40 - 30} = 100\Omega$$

$$\boxed{Z_{in} = 100\Omega}$$

b)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40 - j30 - 50}{40 - j30 + 50} = -j\,0.3333$$

$$\boxed{\Gamma_L = -j\,0.33}$$

c)

voltage division

$$V(z=0) = V^+ + V^- = 4V \quad \left[V(z=0) = V_S \frac{Z_{in}}{Z_{in} + R_S} = 4V\right]$$

$$V_L = V(z = 14cm) = \underbrace{V^+ e^{-j\beta z}}_{V_L^+} + \underbrace{V^- e^{j\beta z}}_{V_L^-} \quad \& \quad \Gamma_L = \frac{V_L^-}{V_L^+}$$

$$\Rightarrow V^+ = \frac{V^-}{\Gamma_L} e^{+j2\beta z \to 14cm} = \frac{V^-}{-j\,1/3}(-j) = 3V^-$$

$$\Rightarrow V(z=0) = V^+ + V^- = 4V^- = 4 \Rightarrow \boxed{V^- = 1\ V}$$

$$\Rightarrow \boxed{V^+ = 3\ V}$$

Q.2) $f = 1\,MHz$

31 cm lossless transmission line (characteristic impedance: Z_0)

→ when terminated with a short circuit ($Z_L = 0$)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} = jZ_0 \tan(\beta d) \quad (1)$$

$$\& \; Z_{in} = j\omega \underbrace{L}_{0.128\,\mu H} = j\omega\, 0.128 \times 10^{-6}$$

→ when terminated with an open circuit ($Z_L \to \infty$)

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)} = \frac{Z_0}{j\tan(\beta d)}$$

$$\& \; Z_{in} = \frac{1}{j\omega \underbrace{C}_{20\,pF}} = \frac{1}{j\omega\, 20 \times 10^{-12}} \quad (2)$$

→ Combining (1) & (2)

$$Z_{in}^2 = \frac{1}{j\omega C} j\omega L = L/C = \frac{Z_0}{j\tan(\beta d)} jZ_0 \tan(\beta d) = Z_0^2$$

$$\Rightarrow Z_0 = \sqrt{L/C}$$

characteristic impedance $\boxed{Z_0 = 80\,\Omega}$

From (1) → $Z_{in} = j\underbrace{\omega}_{2\pi \times 10^6} 0.128 \times 10^{-6} = j\,80 \tan(\beta \underbrace{d}_{0.31\,m})$

or

From (2) → $Z_{in} = \dfrac{1}{j\omega\, 20 \times 10^{-12}} = \dfrac{80}{j\tan(\beta d)}$

$\Rightarrow \tan(\beta d) \cong 0.01 \Rightarrow \beta = 0.0324\ rad/m$

phase velocity: $v_p = \dfrac{\omega}{\beta} = \dfrac{2\pi \overbrace{f}^{1\,MHz}}{\beta} \cong 1.938 \times 10^8\ m/s$

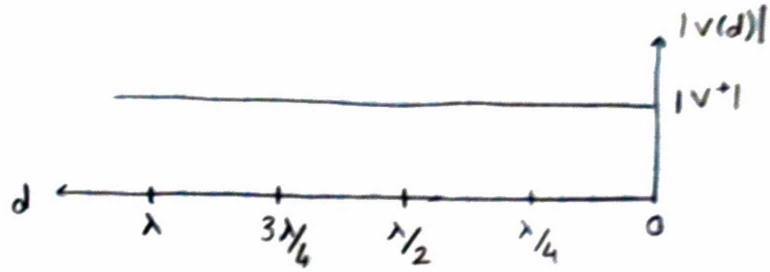
$$\boxed{v_p = 1.938 \times 10^8\ m/s}$$

$\Rightarrow v_p = \dfrac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} = \dfrac{1}{\sqrt{\varepsilon_r}} 3 \times 10^8 \Rightarrow \boxed{\varepsilon_r \cong 2.397}$

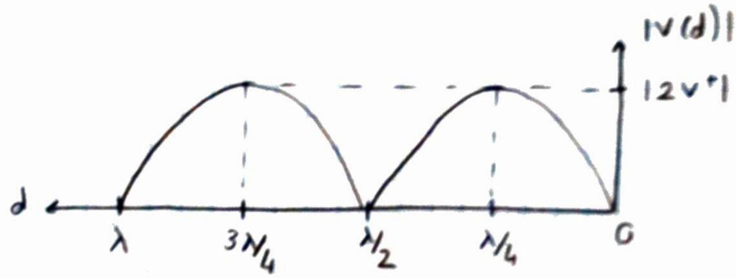
relative permittivity

Q.3) VSWR patterns

a) Matched load:



b) Short circuited line:



c) Open circuited line: