EE 303-HW6

$|\bar{E}_i| = 10 \, V/m$

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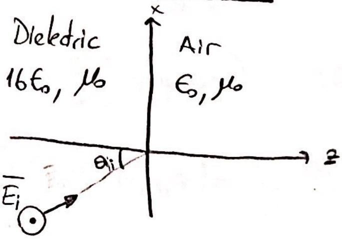

(a) $k_1 \sin\theta_i = k_2 \sin\theta_t$

$k_1 = \omega\sqrt{\mu_1\epsilon_1}$; $k_2 = \omega\sqrt{\mu_2\epsilon_2}$

$k_1 = \omega 4\sqrt{\mu_0\epsilon_0}$; $k_2 = \omega\sqrt{\mu_0\epsilon_0}$

$$\frac{k_1}{k_2} = \frac{\sin\theta_t}{\sin\theta_i} \Rightarrow \sin\theta_t = \frac{k_1}{k_2} \cdot \sin\theta_i$$

$$\Rightarrow \sin\theta_t = 4 \cdot \sin(30^\circ) = 2$$

$$\Rightarrow \cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm j\sqrt{3}$$

• Since $\sin\theta_t > 1$ and $\cos\theta_t$ is purely imaginary, it means that θ_i is higher than θ_c (critical angle). Also, θ_t is a complex angle and, transmitted wave is no longer uniform plane wave, it will be non-uniform plane wave.

(b) From Frensel formulas for perpendicular ($\perp$) polarization,

$$\Gamma = \frac{E_2}{E_1} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}, \text{ where } \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \frac{1}{4} 120\pi = 30\pi \, \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = 120\pi \, \Omega$$

$\cos\theta_i = \frac{\sqrt{3}}{2}$; $\cos\theta_t = -j\sqrt{3}$ due to decaying in medium (2)

• Then, $\Gamma = \dfrac{60\sqrt{3}\pi + j30\pi\sqrt{3}}{60\sqrt{3}\pi - j30\pi\sqrt{3}} = \dfrac{2+j}{2-j} = 1\angle 53^\circ$

$\star \; T_\perp = 1 + \Gamma = 1 + \dfrac{2+j}{2-j} = \dfrac{4}{2-j} = 1.79\angle 26.6^\circ$

(c) $\bar{E}^r = E_2 \, e^{-jk_1(\hat{n}_r \cdot \bar{r})} \hat{a}_y$; $\hat{n}_r = -\cos\theta_i \, \hat{a}_z + \sin\theta_i \, \hat{a}_x$

$\hat{n}_r = -\frac{\sqrt{3}}{2}\hat{a}_z + \frac{1}{2}\hat{a}_x$

$E_2 = E_1 \Gamma = 10\angle 53^\circ$

Then,

$k_1 = \omega\sqrt{16\mu_0\epsilon_0} = \dfrac{4\omega}{c}$

$$\bar{E}^r = (6+j8) \, e^{-j\frac{4\omega}{c}\left(-\frac{\sqrt{3}}{2}z + \frac{1}{2}x\right)} \hat{a}_y \left(\frac{V}{m}\right)$$

(d) $\bar{E}^r(x,y,z,t) = Re\{\bar{E}^r e^{j\omega t}\}$

$$\bar{E}^r(x,y,z,t) = 10\cos\left(\omega t + 53^\circ + \frac{4\omega\sqrt{3}}{2c}z - \frac{4\omega}{2c}x\right)\hat{a}_y \text{ V/m}$$

(e) $\bar{E}^t(x,y,z) = \hat{a}_y E_3 e^{-jk_2(\hat{n}_t\cdot\bar{r})}$; $\hat{n}_t = \cos\theta_t\,\hat{a}_z + \sin\theta_t\,\hat{a}_x$

$E_3 = TE_1 = 17.9\angle 26.6^\circ$ V/m ; $\hat{n}_t = -j\sqrt{3}\,\hat{a}_z + 2\hat{a}_x$

$k_2 = \omega\sqrt{\mu_2\epsilon_2} = \frac{\omega}{c}$; $\bar{E}^t(x,y,z) = \hat{a}_y(17.9\angle 26.6^\circ)\,e^{-j\frac{\omega}{c}(2x - j\sqrt{3}z)}$

$$\bar{E}^t = \hat{a}_y(16 + j8)\,e^{-j\frac{2\omega}{c}x}\cdot e^{-\frac{\omega\sqrt{3}}{c}z} \text{ V/m}$$

(f)

$\bar{E}^t(x,y,z,t) = Re\{\bar{E}^r e^{j\omega t}\}$

$$= 17.9 \times e^{-\frac{\omega\sqrt{3}z}{c}}\cos\left(\omega t + 26.6^\circ - \frac{2\omega x}{c}\right)\hat{a}_y \text{ (in V/m)}$$

(g) No, it is not a uniform plane wave. If we look in detail at result, we obtained:

$$\bar{E}^t(x,y,z,t) = 17.9\,e^{-\frac{\omega\sqrt{3}}{c}z}\cos\left(\omega t + 26.6^\circ - \frac{2\omega x}{c}\right)\hat{a}_y$$

Constant Amplitude ⇒ $z = $ cons.
Constant phase ⇒ $x = $ cons

} Therefore, they don't belong same family. So it is non-uniform plane wave.

(h) $v_p = \frac{\omega}{\beta} = \frac{\omega}{k_2\sin\theta_t} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{\mu_0\epsilon_0}\cdot 2} = \frac{c}{2} = 1.5\times 10^8$ m/s

If it were uniform plane wave, v_p would be 3×10^8 m/s (c). However, since it is not traveling with c, it is traveling with $\frac{c}{2}$. That's why it is called "Slow wave".

Question 2)

- Since there are no browster angle for figures 1 and 3, and medias have $\mu = \mu_0$, figures 1 and 3 should be perpendicular polarizations. Then, figures, 2 and 4, are parallel (//) polarizations.

- Also, we know that $\epsilon_{TiO_2} > \epsilon_{quartz}$.

- From figure 1 and 3 $\Rightarrow \theta_{c_1} > \theta_{c_2}$

$\epsilon_{a,b}$ represents permittivity in media ⓐ in figure ⓑ.

$$\Rightarrow \sin^{-1}\left(\sqrt{\frac{\mu_{2,1}\epsilon_{2,1}}{\mu_{1,1}\epsilon_{1,1}}}\right) > \sin^{-1}\left(\sqrt{\frac{\mu_{2,2}\epsilon_{2,2}}{\mu_{1,3}\epsilon_{1,3}}}\right)$$

$$\Rightarrow \sqrt{\frac{\epsilon_{2,1}}{\epsilon_{1,1}}} > \sqrt{\frac{\epsilon_{2,3}}{\epsilon_{1,3}}}$$. Therefore,

$$\rightarrow \frac{\epsilon_{2,1}}{\epsilon_{1,1}} > \frac{\epsilon_{2,3}}{\epsilon_{1,3}}$$, also, ϵ_2 or ϵ_1 are ϵ_{TiO_2} and ϵ_{quartz}.

* It is obvious that for fig:1 $\Rightarrow \boxed{\epsilon_{2,1} = \epsilon_{TiO_2} \text{ and } \epsilon_{1,1} = \epsilon_{quartz}}$. Also, for fig. 3 $\Rightarrow \boxed{\epsilon_{2,3} = \epsilon_{quartz}, \ \epsilon_{1,3} = \epsilon_{TiO_2}.}$ (Fig 1 $\Rightarrow$ quad $\perp$ TiO_2; Fig 3 $\Rightarrow$ TiO $\perp$ q)

* For fig 2 and 4 we can say that

$$\theta_{B_2} > \theta_{B_4} \Rightarrow \frac{\epsilon_{2,2}}{\epsilon_{1,2}} > \frac{\epsilon_{2,4}}{\epsilon_{1,4}} \Rightarrow \epsilon_{2,2} = \epsilon_{TiO_2}\ ; \ \epsilon_{1,2} = \epsilon_{quartz}$$

$$\epsilon_{2,4} = \epsilon_{quartz}\ ; \ \epsilon_{1,4} = \epsilon_{TiO_2}.$$

from $\theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$ since $\mu_1 = \mu_2$

(Fig 2 $\rightarrow$ q // TiO_2; Fig 4 $\rightarrow$ TiO_2 // q)

* Therefore

a) Incident wave is in TiO_2 and $E \perp$ POI in figure 3.

b) Incident wave is in TiO_2 and E // POI in figure 4.

c) Incident wave is in Quartz and $E \perp$ POI in figure 1.

d) Angle A is called brewster angle (θ_B, $\Gamma = 0$)

e) Angle B is called critical angle (θ_c, $|\Gamma| = 1$)

* For $\theta_i = 0° \Rightarrow \theta_T = 0°$.

→ From figure 1:

However, we know that $n_t < n_q$ since $\epsilon_q < \epsilon_t$

$$\Gamma_\perp(\theta_i = 0°) = \left|\frac{n_2 - n_1}{n_2 + n_1}\right| = 0.5 = \left|\frac{n_{tio_2} - n_q}{n_{tio_2} + n_q}\right| = \frac{n_q - n_t}{n_q + n_t} = \frac{1}{2}$$

$$\Rightarrow 2n_q - 2n_t = n_t + n_q \Rightarrow n_q = 3n_t$$

→ For figure 2:

$$f) \; M = \Gamma_{\parallel}(\theta_i = 0°) = \left|\frac{n_2 - n_1}{n_2 + n_1}\right| = \frac{n_q - n_t}{n_t + n_q} = \frac{1}{2}$$

⇒ For figure 3:

$$g) \; N = \Gamma_\perp(\theta_i = 0°) = \left|\frac{n_2 - n_1}{n_2 + n_1}\right| = \frac{n_q - n_t}{n_q + n_t} = \frac{1}{2}$$

⇒ For figure 4:

$$h) \; P = \Gamma_{\parallel}(\theta_i = 0) = \left|\frac{n_2 - n_1}{n_2 + n_1}\right| = \frac{n_q - n_t}{n_q + n_t} = \frac{1}{2}$$

i) $\theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$ for $\mu_1 = \mu_2$.

For figure 2: ② → quartz, ① → TiO_2. also we know that $n_q = 3n_t$

$$\frac{n_q}{n_t} = 3 = \sqrt{\frac{\epsilon_t}{\epsilon_q}}$$

$$\boxed{\frac{\epsilon_t}{\epsilon_q} = 9}$$

· Then, for figure 2:

$$(i) \; A = \theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{\frac{\epsilon_t}{\epsilon_q}}\right) = \tan^{-1}(3) = \underline{71.57}$$

· Also, for figure 3:

$$(J) \; \theta_c = \arcsin\left(\frac{\sqrt{\mu_2\epsilon_2}}{\sqrt{\mu_1\epsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{\epsilon_q}{\epsilon_t}}\right) = \sin^{-1}\left(\frac{1}{3}\right) = \underline{19.47° = B}$$

$\mu_1 = \mu_2 = \mu_0$
$\epsilon_2 = \epsilon_q \,;\, \epsilon_1 = \epsilon_{tio_2}$

· Also, for figure 4:

$$(k) \; C = \theta_B = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{\frac{\epsilon_q}{\epsilon_t}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = \underline{18.43° = C}$$

$$(l) \; D = \theta_c = \arcsin\left(\sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{\epsilon_q}{\epsilon_t}}\right) = \sin^{-1}\left(\frac{1}{3}\right) = \underline{19.47° = B}$$

$\mu_2 = \mu_1 = \mu_0$
$\epsilon_2 = \epsilon_q \,;\, \epsilon_1 = \epsilon_{tio_2}$

$$(m)\ \lambda_t = \frac{w_t}{k_t} \; ; \; \lambda_q = \frac{w_q}{k_q} \Rightarrow \frac{\lambda_q}{\lambda_t} = \frac{w_q\, k_t}{w_t\, k_q} = \frac{k_t}{k_q} = \frac{w_t\sqrt{\mu_0 \epsilon_t \epsilon_0}}{w_q\sqrt{\mu_0 \epsilon_q \epsilon_0}}$$

$w_t = w_q = w$

$$\frac{\lambda_q}{\lambda_t} = \sqrt{\frac{\epsilon_t}{\epsilon_q}} = 3 \Rightarrow \underline{\underline{\lambda_q = 3mm}}$$

Question 3)

Def Brewster angle, θ_B, is the value of incidence angle s.t. $\Gamma = \frac{E_2}{E_1} = 0$

<u>For Parallel Polarization</u>

$$\Gamma_{\parallel}(\theta_i, \theta_t) = \frac{n_2 \cos\theta_t - n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_i}$$

at $\theta_i = \theta_B \Rightarrow \Gamma$ shoul be zero.

$$\Gamma_{\parallel}(\theta_i = \theta_B, \theta_t) = \frac{n_2 \cos\theta_t - n_1 \cos\theta_B}{n_2 \cos\theta_t + n_1 \cos\theta_B} = 0 \Rightarrow n_2 \cos\theta_t - n_1 \cos\theta_B = 0$$

$$\cos\theta_t = \frac{n_1}{n_2}\cos\theta_B$$

$$\boxed{\cos^2\theta_t = \frac{n_1^2}{n_2^2}\cos^2\theta_B} \ \star$$

$k_1 \cdot \sin\theta_B = k_2 \sin\theta_t$ From Snell's Law

$$\sin\theta_t = \frac{k_1}{k_2}\sin\theta_B \Rightarrow \sin^2\theta_t = \frac{k_1^2}{k_2^2}\sin^2\theta_B \Rightarrow 1-\sin^2\theta_t = 1-\frac{k_1^2}{k_2}\sin^2\theta_B = \cos^2\theta_t \quad -\star\star$$

- Combining equations $\star$ and $\star\star$

$$1-\frac{k_1^2}{k_2^2}\sin^2\theta_B = \frac{n_1^2}{n_2^2}\cos^2\theta_B = \frac{n_1^2}{n_2^2}(1-\sin^2\theta_B) \quad \text{say } \sin^2\theta_B = A$$

$$1-\frac{k_1^2}{k_2^2}A = \frac{n_1^2}{n_2^2} - \frac{n_1^2}{n_2^2}A$$

$$\sin^2\theta_B = A = \frac{1-\frac{n_1^2}{n_2^2}}{\frac{k_1^2}{k_2^2}-\frac{n_1^2}{n_2^2}} = \frac{1-\frac{\mu_1\epsilon_2}{\epsilon_1\mu_2}}{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}-\frac{\mu_1\epsilon_2}{\epsilon_1\mu_2}} = \frac{(\epsilon_1\mu_2-\mu_1\epsilon_2)\cdot\epsilon_2}{\epsilon_1^2\mu_1-\epsilon_2^2\mu_2} = \frac{1-(\mu_2\epsilon_1/\mu_1\epsilon_2)}{1-(\epsilon_1/\epsilon_2)^2}$$

$\epsilon_1 \quad \epsilon_2$

$$\text{For } \mu_1=\mu_2=\mu_0 \Rightarrow \sin^2\theta_B = \frac{1-\epsilon_1/\epsilon_2}{1-(\epsilon_1/\epsilon_2)^2} = \frac{1}{1+\frac{\epsilon_1}{\epsilon_2}} \Rightarrow \theta_B = \arcsin\left(\sqrt{\frac{1}{1+\frac{\epsilon_1}{\epsilon_2}}}\right)$$