EE 303 Electromagnetic Waves – Fall Semester 2020
Homework 3
Due Date: November 16, 2020 Monday before 13:00.
Please upload your solutions to ODTUClass next Monday no later than 13:00.

Problem 1. (1.4 pts)Phasor electric field intensity of a uniform plane wave propagating in a simple, lossless non-magnetic dielectric medium ($\mu = \mu_0, \epsilon = \epsilon_r \epsilon_0$) is given by

$$\vec{E} = (12\hat{a}_y + A\hat{a}_z)e^{j\pi(By-6z)}, V/m$$

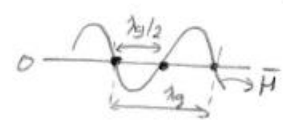
where A and B are real constants.

At $t=0$, the variation of magnetic field intensity is observed with respect to position. The maximum magnitude of the magnetic field intensity vector, $\overline{H}$ is $\frac{2}{3\pi}$ A/m and the distance between the locations of two consecutive zeros of $\overline{H}$ field in space along the direction of propagation is *10 cm*.

a) Find the positive real constant B.
b) Find the unit vector along the direction of propagation.
c) Find the constant A
d) Find the intrinsic impedance of the medium.
e) Find the dielectric constant of the medium (ϵ_r)
f) Find the phase velocity of the wave.
g) Find the the expression for the time-domain $\overline{H}$.

Solution of Question 1;

• The information on H-fields zero locations enables us to find λ_g (wavelength in medium with ϵ_r)



$\frac{\lambda_g}{2} = 10cm$ $\quad \lambda_g = 20cm = 0{,}2\,m$ $\quad k = 2\pi/\lambda_g = 10\pi$

a-) The exponential part of E-field expression must be $e^{-j\bar{k}\cdot\bar{r}}$

where $\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$\bar{k} = k.\hat{n}$ → propagation direction of wave (unit vector)

k → wave number (constant)

$e^{j\pi(By-6z)} = e^{-j\underbrace{k\hat{n}\cdot\bar{r}}_{k_x x + k_y y + k_z z}}$ $\xrightarrow[\text{is satisfied}]{\text{this equation}}$ $k_x x = 0 \rightarrow k_x = 0$

$k_y y = -\pi By \rightarrow k_y = -\pi B$

$k_z z = 6\pi z \rightarrow k_z = 6\pi$

$|k| = 10\pi$ is known so,

$$|k| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{(-\pi B)^2 + (6\pi)^2} = \pi\sqrt{B^2 + 6^2} = 10\pi \quad \text{is satisfied for } \boxed{B = 8}$$

$\cancel{B = -8}$ Question states B is positive

b-) $\bar{k} = |k| . \hat{n}$

$$-8\pi \hat{a_y} + 6\pi \hat{a_z} = 10\pi \left(n_x \hat{a_x} + n_y \hat{a_y} + n_z \hat{a_z} \right) \longrightarrow \left. \begin{array}{l} n_x = 0 \\ n_y = -0{,}8 \\ n_z = 0{,}6 \end{array} \right\} \boxed{\hat{n} = -0{,}8\hat{a_y} + 0{,}6\hat{a_z}}$$

c-) $\bar{E}$ and $\hat{n}$ must be perpendicular for U.P.W. So $\bar{E} . \hat{n} = 0$ must be satisfied.

$$\bar{E} . \hat{n} = (12\hat{a_y} + A\hat{a_z}) . (-0{,}8\hat{a_y} + 0{,}6\hat{a_z}) = 0$$

$$-9{,}6 + 0{,}6 . A = 0 \longrightarrow \boxed{A = 16}$$

d-) $$\eta = \frac{|E_{max}|}{|H_{max}|} = \frac{20}{\frac{2}{3\pi}} = \boxed{30\pi} \ [\Omega]$$

e-) Another η definition can be made as, $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ by equating this to $30\pi [\Omega]$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{120\pi}{\sqrt{\varepsilon_r}} = 30\pi \longrightarrow \sqrt{\varepsilon_r} = 4 \ , \ \boxed{\varepsilon_r = 16}$$

f-) $$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{4} = \boxed{75 \times 10^6} \ [m/s]$$ inside medium g-) $\bar{H}_{ph} = \frac{1}{\eta} \cdot \hat{n} \times \bar{E}_{ph}$ for U.P.W.

$$= \frac{1}{30\pi} \underbrace{(-0{,}8\hat{a_y} + 0{,}6\hat{a_z}) \times (12\hat{a_y} + 16\hat{a_z})}_{-20\hat{a_x}} \cdot e^{-j\pi(6z-8y)} \qquad [A/m]$$

$$\bar{H}_{pn} = \frac{-2}{3\pi} \hat{a_x} \cdot e^{-j\pi(6z-8y)}$$

$$\bar{H}_{(t)} = Re\left\{ \bar{H}_{pn} \cdot e^{jwt} \right\}$$

$$= Re\left\{ \frac{-2}{3\pi} \hat{a_x} \cdot e^{j(wt - \pi(6z-8y))} \right\}$$

$$\boxed{\bar{H}_{(t)} = \frac{-2}{3\pi} \hat{a_x} \cdot \cos(wt - \pi(6z-8y))} \qquad [A/m]$$

Problem 2. (0.8 pts) Time-domain electric field expression for a uniform plane wave (propagating in a simple, lossless and source-free medium) is given as

$$\vec{E}(z,t) = \hat{a}_x A_x cos(\omega t - kz) + \hat{a}_y A_y cos(\omega t - kz + \theta)$$

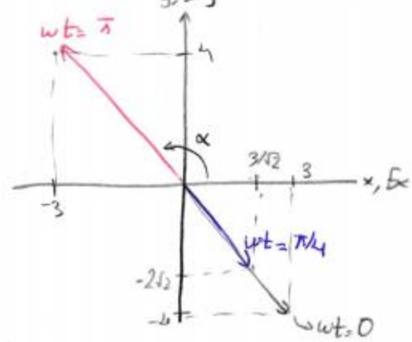
Identify the polarization (both type and sense) and sketch the locus of $\vec{E}(z=0,t)$ for each of the following cases

a) $A_x = 3\ V/m, A_y = 4\ V/m$ and $\theta = 180°$
b) $A_x = 3\ V/m, A_y = 3\ V/m$ and $\theta = 45°$
c) $A_x = 3\ V/m, A_y = 3\ V/m$ and $\theta = -90°$
d) $A_x = 3\ V/m, A_y = 4\ V/m$ and $\theta = -135°$

Solution of Question 2;

a-) $\bar{E}(z=0,t) = \underbrace{3\cos(wt)}_{E_x}\hat{a_x} + \underbrace{4\cos(wt+\pi)}_{E_y}\hat{a_y}$

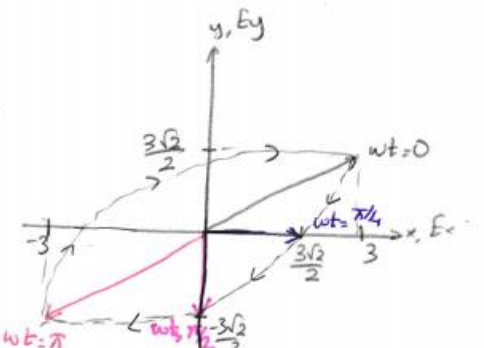
t	wt	Ex	Ey
0	0	3	-4
T/8	$\pi/4$	$3/\sqrt{2}$	$-2\sqrt{2}$
T/4	$\pi/2$	0	0
T/2	π	-3	4



Linear Polarization in a direction making an angle of $\alpha = 127°$ in xy-plane

b-) $\bar{E}(z=0,t) = 3\cos(wt)\hat{a_x} + 3\cos(wt+\pi/4)\hat{a_y}$

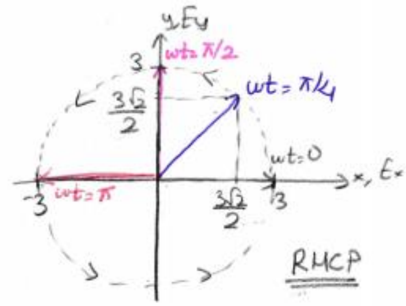
t	wt	Ex	Ey
0	0	3	$\frac{3\sqrt{2}}{2}$
T/8	$\pi/4$	$\frac{3\sqrt{2}}{2}$	0
T/4	$\pi/2$	0	$-\frac{3\sqrt{2}}{2}$
T/2	π	-3	$-\frac{3\sqrt{2}}{2}$



Left Hand Eliptical Polarization (LHEP)

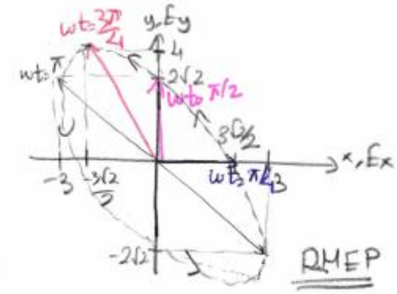
c-) $E(z=0,t) = 3\cos(\omega t)\hat{a_x} + 3\cos(\omega t - \pi/2)\hat{a_y}$

t	ωt	E_x	E_y
0	0	3	0
$T/8$	$\pi/4$	$3\sqrt{2}/2$	$3\sqrt{2}/2$
$T/4$	$\pi/2$	0	3
$T/2$	π	-3	0



d-) $E(z=0,t) = 3\cos(\omega t)\hat{a_x} + 4\cos(\omega t - 3\pi/4)\hat{a_y}$

t	ωt	E_x	E_y
0	0	3	$-2\sqrt{2}$
$T/8$	$\pi/4$	$3\sqrt{2}/2$	0
$T/4$	$\pi/2$	0	$2\sqrt{2}$
$3T/4$	$3\pi/4$	$-3\sqrt{2}/2$	4



Problem 3. (0.3 pts) (Reading Assignment Problem) For a wave travelling in a medium with a skin depth δ, what is the amplitude of electric field intensity vector $\vec{E}$ at a distance of

a) 2δ

b) 4δ

compared with its initial value? Also specify your result in percentage.

Solution of Question 3;

Skin depth is a distance where E-field amplitude of wave reduces to its 1/e value of its initial amplitude.

Assume, $\bar{E} = E_0 . e^{-\alpha z} . e^{-j\beta z} \hat{a_x}$ $\rightarrow$ wave experiences α attenuation when travelling to $\hat{a_z}$ direction.

In its skin depth, ($z = \delta$)

$$|\bar{E}| = |E_0 . e^{-\alpha.\delta} . e^{-j\beta\delta}| = \frac{E_0}{e} \rightarrow e^{-\alpha\delta} = e^{-1} \rightarrow \alpha = 1/\delta$$

a-) at $z = 2\delta$

$$|\bar{E}| = |E_0 . e^{-\alpha.2\delta} . e^{-j\beta 2\delta}| = E_0 . e^{-2} = 0{,}135\, E_0$$

$\hookrightarrow$ E-field amplitude is reduced to its $\underline{\underline{13{,}5\%}}$

b-) at $z = 4\delta$

$$|\bar{E}| = |E_0 . e^{-\alpha.4\delta} . e^{-j\beta 4\delta}| = E_0 . e^{-4} = 0{,}018\, E_0$$

$\hookrightarrow$ E-field amplitude is reduced to its $\underline{\underline{1{,}8\%}}$