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Answer 1

Answer 2

To find the generating function for the sequence

$$1, 4, 7, 10, 13, \ldots$$

we can use the formula for an arithmetic sequence.

The n^{th} term of an arithmetic sequence can be represented as:

$$a_n = a_1 + (n-1) \cdot d$$

where:

$$\begin{aligned} a_n &\text{ is the } n^{th} \text{ term of the sequence,} \\ a_1 &\text{ is the first term of the sequence,} \\ n &\text{ is the term number (position),} \\ d &\text{ is the common difference between consecutive terms.} \end{aligned}$$

From the given sequence, we can observe that:

$$a_1 = 1$$

(first term) and

$$d = 3$$

(since the difference between consecutive terms is 3).

Plugging these values into the formula, we get:

$$a_n = 1 + (n-1) \cdot 3$$

which simplifies to:

$$a_n = 1 + 3n - 3$$

and further simplifies to:

$$a_n = 3n - 2$$

Starting with the basic generating functions:

1. The generating function for the sequence $< 1, 1, 1, 1, \cdots >$ is:

$$\frac{1}{1-x}$$

2. Differentiating the above generating function gives:

$$\frac{d}{dx}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$$

   Which represents the sequence $< 1, 2, 3, 4, \cdots >$.

3. Using the scaling theorem:

   Multiplying $\frac{1}{1-x}$ by 2 gives:
   $\frac{2}{1-x}$
   Which represents the sequence $< 2, 2, 2, 2, \cdots >$.

   Multiplying $\frac{1}{(1-x)^2}$ by 3 givx“es:
   $\frac{3}{(1-x)^2}$
   Which represents the sequence $< 3, 6, 9, 12, \cdots >$.

4. Subtracting the generating function for the sequence $< 2, 2, 2, 2, \cdots >$ from the generating function for the sequence $< 3, 6, 9, 12, \cdots >$ gives:

$$\frac{3}{(1-x)^2} - \frac{2}{1-x}$$

   This resulting generating function represents the sequence $< 1, 4, 7, 10, 13, \cdots >$.

Therefore, the difference of $\frac{3}{(1-x)^2}$ and $\frac{2}{1-x}$ is the generating function for the sequence $< 1, 4, 7, 10, 13, \cdots >$.

Answer 3

Answer 4