

Student Information

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Answer 1

a)

| p | q | $\neg q$ | $p \rightarrow q$ | $p \wedge \neg q$ | $p \rightarrow q \oplus p \wedge \neg q$ |
|-----|-----|----------|-------------------|-------------------|--|
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| F | T | F | T | F | T |
| F | F | T | T | F | T |

b)

$$\begin{aligned} p \rightarrow ((q \vee \neg p) \rightarrow r) &\equiv \neg p \vee ((q \vee \neg p) \rightarrow r) && \text{table 7, Equivalence 1} \\ &\equiv \neg p \vee (\neg(q \vee \neg p) \vee r) && \text{table 7, Equivalence 1} \\ &\equiv \neg p \vee ((\neg q \wedge \neg \neg p) \vee r) && \text{table 6, De Morgan's Second Law} \\ &\equiv \neg p \vee ((\neg q \wedge p) \vee r) && \text{table 6, Double Negation Law} \\ &\equiv (\neg p \vee (\neg q \wedge p)) \vee r && \text{table 6, Associative Law} \\ &\equiv ((\neg p \vee \neg q) \wedge (\neg p \vee p)) \vee r && \text{table 6, Distributive Law} \\ &\equiv ((\neg p \vee \neg q) \wedge (T)) \vee r && \text{table 6, Negation First Law} \\ &\equiv (\neg p \vee \neg q) \vee r && \text{table 6, Identity First Law} \\ &\equiv \neg(p \wedge q) \vee r && \text{table 6, De Morgan's Second Law} \\ &\equiv (p \wedge q) \rightarrow r && \text{table 7, Equivalence 3} \end{aligned}$$

c)

1. F

2. F

3. F

4. T

5. T

Answer 2

- a) $\exists x(P(Can, x) \wedge T(x, L))$
- b) $\forall x(T(x, S) \rightarrow \exists y(N(y, Turkish) \wedge P(y, x)))$
- c) $\forall x(T(x, S) \rightarrow \exists y(R(x, y) \wedge T(y, S) \wedge \forall z((R(x, z) \wedge T(z, S)) \rightarrow (y = z))))$
- d) $\forall y(W(M, y) \rightarrow \neg \exists z(P(z, y) \wedge N(z, English)))$
- e) $\exists x \exists y((x \neq y) \wedge N(x, Turkish) \wedge N(y, Turkish) \wedge P(x, G) \wedge P(y, G) \wedge \forall z((N(z, Turkish) \wedge P(z, G)) \rightarrow (z = x \vee z = y)))$
- f) $\exists x \exists y \exists z(T(x, y) \wedge T(x, z) \wedge (y \neq z))$

Answer 3

| $p \rightarrow q, (r \wedge s) \rightarrow p, (r \wedge \neg q) \vdash \neg s$ | | |
|--|------------------------------|-----------------------|
| 1. | $p \rightarrow q$ | <i>premise</i> |
| 2. | $(r \wedge s) \rightarrow p$ | <i>premise</i> |
| 3. | $(r \wedge \neg q)$ | <i>premise</i> |
| 4. | r | $\wedge e, 3$ |
| 5. | $\neg q$ | $\wedge e, 3$ |
| 6. | $r \wedge s$ | <i>assumption</i> |
| 7. | p | $\rightarrow e, 2, 6$ |
| 8. | q | $\rightarrow e, 1, 7$ |
| 9. | \perp | $\neg e, 5, 8$ |
| 10. | $\neg(r \wedge s)$ | $\neg i, 6 - 9$ |
| 11. | s | <i>assumption</i> |
| 12. | $r \wedge s$ | $\wedge i, 4, 11$ |
| 13. | \perp | $\neg e, 10, 12$ |
| 14. | $\neg s$ | $\neg i, 11 - 13$ |

Answer 4

- a)
 - (First premise) $\exists x(P(x) \rightarrow S(x))$
 - (Second premise) $\forall x(P(x))$
 - (Claim) $\exists y S(y)$
- b)

$$\exists x(P(x) \rightarrow S(x)), \forall x(P(x)) \vdash \exists yS(y)$$

| | | |
|----|------------------------------------|----------------------|
| 1. | $\exists x(P(x) \rightarrow S(x))$ | <i>premise</i> |
| 2. | $\forall x(P(x))$ | <i>premise</i> |
| 3. | $P(c) \rightarrow S(c)$ | <i>assumption</i> |
| 4. | $P(c)$ | $\forall e$ 2 |
| 5. | $S(c)$ | $\rightarrow e$ 3, 4 |
| 6. | $\exists yS(y)$ | $\exists i$ 5 |
| 7. | $\exists yS(y)$ | $\exists e$ 3 – 6 |