CENG 222

Statistical Methods for Computer Engineering

Spring 2023-2024

Homework II

Due date: 03/04/2024, Wednesday, 23:59

Introduction

In this homework, there are three classical questions and one classical/programming hybrid question related to the 4^{th} chapter of your text book. While answering the questions, please **show your work** and the steps of your calculations. Give an explanation about what numbers mean in those steps. Otherwise, you may not get any points. Also, include the code for the programming question in your report as appendix.

Questions

Q1. (**25 pts**)
The joint probability density function $f_{X,Y}(x, y)$ of two continuous variables is defined as follows;

$$f_{X,Y}(x,y) = \begin{cases} x + ky^3 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ \\ 0 & \text{otherwise} \end{cases}$$

a) Find the value of k. *(10 pts)*
b) What is the probability that X is equal to $\frac{1}{2}$, i.e. $P(X = \frac{1}{2})$? *(5 pts)*
c) Find $P(0 \leq X \leq \frac{1}{2}, 0 \leq Y \leq \frac{1}{2})$. *(10 pts)*

Q2. (**25 pts**)
The joint probability density function $f_{X,Y}(x, y)$ of two continuous variables is defined as follows;

$$f_{X,Y}(x,y) = \frac{e^{(-y-\frac{x}{y})}}{y} \quad \text{for} \quad 0 < x, y < \infty$$

a) Find the marginal pdf of Y. What distribution family does Y follow? *(15 pts)*
b) What is the expected value of Y? *(10 pts)*

Q3. (**20 pts**)
Turkish Armed Forces consists of approximately 355,000 active personnel. Among them, 75% belong to the armed forces, 15% belong to the air forces and 10% belong to the naval forces. A major has to make a trip by the sea and he wants to make sure that at least some percentage of the soldiers on board are naval soldiers. A retinue of 1000 random soldiers is selected to accompany the major.
a) What is the probability that at least 9% of the soldiers belong to the naval forces? *(10 pts)*
b) What if the retinue is increased to 2000 soldiers and the minimum percentage is same as part **a**? Does the probability increase or decrease? Briefly explain the reasoning. *(10 pts)*

Q4. (**30 pts**)
For parts **b** and **c** of this question use Octave or MATLAB and **provide the screenshots of your resulting plots or outputs**. Also add your code to the end of your report as appendix.

The lifespan of an African bush elephant follows the normal distribution with a mean (μ) of 65 years and a standard deviation (σ) of 6 years.
a) Calculate the probability that a randomly selected elephant will live more than 60 years, but less than 75 years. *(10 pts)*
b) Create a random sample of size 20 and visualize the distribution as a histogram. You can use the histogram with 20 or 25 bins. Do the same for sizes 100 and 1000 respectively. Put the plots for all three of the sizes. *(10 pts)*
c) As a simulation, create a random sample of size 100 and count the number of elephants with a lifespan between 60 and 75 years. Do this simulation over 1000 iterations. In how many of these simulations at least 70% of the elephants had a lifespan in the given range? Answer the same for 85%. Compare your results with part **a** and briefly explain. *(10 pts)*

Suggested Examples (Ungraded)

From the textbook, check the exercises 4.1 and 4.2. Make sure that you practice the integration.
Also check exercises 4.19 and 4.20 for some conceptual insights. Lastly, you may want to read about Buffon's needle problem. Its a historical problem dating back to 18^{th} century.

Specifications

- You are expected to write your answers in LaTeX format. You can use the given template.
- Please do not skip the calculation steps. Show every step of your work.
- You have a total of 2 late days for this homework. For each day you have submitted late, you will lose 25 points. If you submit your homework at least 2 days later than the deadline, you will get zero.
- Cheating is forbidden. The violators will be punished according to the department regulations.
- Follow the course page on ODTUClass for any updates and clarifications. Please ask your questions on ODTUClass instead of e-mailing if they do not contain some part of the solution. If they contain, you can send an email to "mduymus@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUClass. You are expected to submit a **single** PDF file named "hw2.pdf".